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**Random walks and non-linear paths in macroeconomic time series: Some evidence and implications**

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Abstract

This paper investigates whether the inherent non-stationarity of macroeconomic time series is entirely due to a random walk or also to non-linear components. Applying the numerical tools of the analysis of dynamical systems to long time series for the United States, we reject the hypothesis that these series are generated solely by a linear stochastic process. Contrary to Real Business Cycle theory, that attributes the irregular behavior of the system to exogenous random factors, we maintain that the fluctuations in the time series cannot be explained only by means of external shocks plugged into linear autoregressive models. A dynamic and non-linear explanation may be useful for the double aim of describing and forecasting more accurately the evolution of the system. Linear macroeconomic models that find empirical verification on linear econometric analysis are therefore seriously called in question. On the contrary non-linear dynamical models may enable us to educe more complete information about economic phenomena from the same data sets used in the empirical analysis from Real Business Cycle Theory. We conclude that Real Business Cycle theory, and the unit root autoregressive models in general, are an inadequate device for a satisfactory understanding of economic time series. A theoretical approach grounded on non-linear metric methods, may however allow to identify non-linear structures that endogenously generate fluctuations in macroeconomic time series.

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# 1 Introduction

The aim of this paper is to identify the nature of the dynamics of macroeconomic time series. When time series are characterized by zero autocorrelation for all possible leads and lags, the issue of distinguishing between deterministic and stochastic components becomes an impossible task when linear metric methods are used (Hommes 1998).

This impasse arises from the fact that linear methods are only appropriate to detect regularities in time series like autocorrelations and dominant frequencies (Graybill 1961, Conover 1971, Oppenheim and Schafer 1989), while fluctuations in real economic time series are generally characterized by zero autocorrelation and zero dominant frequency. Economic fluctuations look really similar to background noise, which does not possess dominant frequencies (except zero frequency) and each noisy impulse is serially uncorrelated. The spectral analysis of economic fluctuations, seemingly as complex as noise, has lead many economists to consider these fluctuations like random variables with identically independently distributed (*i.i.d.*) events.

As a matter of fact the *i.i.d.* hypothesis is an obvious necessity for all linear models (not only the economic ones) to describe, at least approximately, the irregularities in the observed data. In the past two kind of linear economic models based on the *i.i.d.* hypothesis in the residuals have been presented. In the first model, known as the *deterministic trend* model, variables evolve as a function of time along a linear trend while shocks temporarily shift the value of the variable from the value of the linear trend. In the second model (the *stochastic trend* model) variables evolve as a function of their foregoing values and a shock shifts the value of the variable from the lagged value (Rappoport and Reichlin 1989). In this second case any shock does evidently affect the value of the variable at all leads and, therefore, it has a persistent effect. Moreover the time series is entirely determined by the occurrence of all past shocks (Fuller 1998, Maddala and Kim 1998).

Following the seminal article the Nelson and Plosser (1982), the empirical evidence in the last twenty years has contradicted the linear deterministic and the stationary models. The stochastic trend model put forward by Nelson and Plosser seemed, instead, not to be contradicted by empirical results.

In this paper the Nelson and Plosser model will be called in question because it is based on the hypothesis that fluctuations are *i.i.d.* while they are not. This hypothesis, in our opinion, obscures existent non-linearities that may be endogenized in non-linear models.

This article is organized as follows. In section 2 the main stylized facts offered by the recent linear econometric analysis are presented. In section 3 we put forward the hypothesis that non-linearities of the system may be a deterministic cause of the irregularities in economic time series and we introduce a procedure, based on recent signal processing techniques, that allows to identify the existence of non-linearities in the system and, hopefully, to disentangle non-linearities (signals) from stochastic components (noise). In section 4 we present results obtained using artificial non-linear and autoregressive models. In particular we use the arsenal of tools from non-linear dynamics to identify the hidden deterministic structure that is underlying the time series. In section 5 we present results obtained using non-linear metric techniques applied to monthly seasonally adjusted time series of some US macroeconomic time series (industrial production, employment, consumer price index, hourly wages, etc.) and some sector specific like the production in the Hi-Tech sector. The common result that stands out from this analysis is that all the time series we analyzed are also characterized by non-random structures in the residuals and therefore the *i.i.d.* hypothesis is simply inconsistent with facts. The choice of assuming the residual components as random neglects the existence of a complex phenomenon. Instead, it is even theoretically possible to reduce any stochastic component that perturbs unpredictably the system and thus peak the non-linear deterministic component[1] . In section 6 we illustrate some theoretical implications that we can infer from our empirical results about the Real Business Cycle theory grounded on stochastic components with persistent effects.

## 2 Empirical evidence

In the last twenty years we have witnessed a huge progress in the statistical and econometric analysis of time series which as given economists a more profound knowledge about the relations between economic variables. The discovery and the realization that time series do not show any tendency to evolve along a deterministic log-linear growth path, while the cyclical reversible component, assumed by classical econometricians, does not exist at all, has deeply marked the direction of the empirical research in the last two decades.

Recent econometrics works have provided a solid empirical basis that is in contrast to the theoretical results of the early neoclassical growth models a la Solow (1956) and the Business Cycles models a la Lucas (1972, 1977 and 1980). Nelson and Plosser (1982) have provided empirical verification to the theoretical alternative of Real Business Cycle, despite the conventional wisdom of classical econometrics that assumed ex-ante stationarity for all the economic variables. Nelson and Plosser have shown that many macroeconomic time series[2] are not stationary at all, and the stationary stochastic models developed in the '70s do not actually find any empirical foundation[3].

[1] See Kantz and Schreiber 1997.

[2] Nelson and Plosser have analyzed fourteen macroeconomic time series for the US (with starting date between 1860 and 1909 and with final date 1970). Among these there are real GNP, nominal GNP, industrial production, employment, the unemployment rate, the consumer index rate, nominal wages and real wages.

[3] In the classical econometric works, time series were considered stationary along a deterministic trend, that is variables are a linear function of time:

$x_t = \beta t + \alpha + \varepsilon_t$

with $\varepsilon_t \sim N\left(0, \sigma^2\right)$ *i.i.d.* residuals, $\alpha$ and $\beta$ are parameters, $t$ is time, and $x_t$ is a single

On the contrary, Nelson and Plosser have shown that the irregularity present in macroeconomic time series could simply be explained by the introduction of random shocks with persistent effects as happens in unit root processes[4].

These results were in sharp contrast with the classic econometric works, which affirmed that the irregularity in economic time series were due to transitory shocks, and have been crucial in bringing the research direction towards the theory of Real Business Cycle.

The acknowledged contribution of the Nelson and Plosser work has been to have discovered the non-stationarity in the time series and the absence of any deterministic trend. The introduction of random external shocks as the unique generator of the irregularity in the behavior of economic systems, does not contradict the results put forward by Nelson and Plosser.

Without the injection of external shocks, time series would move exactly in the direction that the neoclassical theory predicts[5]. However in the presence of external shocks, economic systems move irregularly in the way that is described by the Real Business Cycle models (Prescott 1998).

In this article we try to move a step forward starting from this empirical evidence.

Our aim is to identify the process that generates the non-stationarity in time series without stating ex ante, contrary to Nelson and Plosser, that the non-stationarity is the direct consequence of a stochastic process. Actually there may be many possible non-linear deterministic alternatives to the stochastic explanation to the non-stationarity in time series.

Treating economic fluctuations as endogenous non-linear process, and therefore object of analysis, may contribute to a better understanding about the temporal evolution of time series. Our aim is to understand the dynamics of fluctuations as the evolution of the system may depend entirely on them. We believe that the practice of assuming fluctuations as random *i.i.d.* variables with a probability distribution equivalent to noise is basically wrong since, as we shall see in section 5, residuals are characterized by a structure that is very different from noise and even from any other kind of random variable. This result leads us to conclude that it is feasible to discover deterministic laws that

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observation of the variable $x$ at time $t$.

In this case the time series of the variable $x$ is stationary along a time trend and residuals have only temporary effects. The short run component may be insulated regressing $x_t$ against time and assuming the regression line as the abscissa This procedure was approximately the one that was used in the '70s to analyze short run cycles.

[4] In the *unit root* processes, time series are not stationary and follow a *random walk* like: $x_t = \rho x_{t-1} + \varepsilon_t$ with $\varepsilon_t = N(0, \sigma^2)$ *i.i.d.* residuals and $\rho = 1$. This process is called *unit root* because $x_{t-1}$ is multiplied by a parameter that is equal to one (or close to one). Residuals have persistent effects since, as we can see, each fluctuation will not be reabsorbed in the future: $x_t = x_{t-1} + \varepsilon_t = x_{t-2} + \varepsilon_{t-1} + \varepsilon_t = ... = \varepsilon_0 + \varepsilon_1 + ... + \varepsilon_{t-1} + \varepsilon_t$. The *signal* $x_t$ is therefore generated by the past and present noise $\varepsilon$. Since noise is a *i.i.d.* random and exogenous variable, we conclude that the variable $x_t$ depends entirely on a variable which we don't know anything about.

[5] See about this argument King, Rebelo and Plosser (1988 a, b).

shape the underlying non-linear structures[6].

Before we introduce in section 5 a procedure for the analysis of economic time series inspired by chaos theory, we present some recent developments in the literature, which we call *unit root* literature, that confirms the results put forward by Nelson and Plosser. Thus, we compare the results of the unit root literature with those from the alternative literature stream that we call *broken trend* literature. We will show that these two streams obtain only seemingly opposite empirical results, and that both fail to resolve the issues they claim to explain (the nature of time series).

Nelson and Plosser showed that many macroeconomic time series were indeed not stationary and that the stationary models developed in the '70s did not have any empirical ground. Nelson and Plosser showed that the irregularity in time series could be explained by random shocks that have persistent effects over time. This view sharply contrasts with the classical econometric works that maintained that the irregularities in time series were due to transitory and reversible shocks.

The empirical findings by Nelson and Plosser were theoretically fundamental since, until the '80s, economists presented models which solution was a linear trend (in a logarithmic scale) for many of macroeconomic variables, and cycles were essentially seen as short term fluctuations without effects in the long run. Just as an example of these models, consider the growth models of the '60s a la Solow (1956) and the business cycle models a la Lucas (1972, 1977 and 1980) based on monetary disturbances with transitory effects; at those times it was believed that the development path of macroeconomic variables was a linear trend (e.g. long-run growth for GDP, consumption etc.) and all the irregularities, mainly due to monetary factors, averaged out in the long run. The unjustified and unsupported belief of the existence of a long run stable solution has been hard to eradicate and still many economists seem not to have realized the basic message of the paper by Nelson and Plosser. For example, many growth models have presented steady state long run solutions and have still to respond to the multitude of econometric works on unit roots in the '80s and '90s, which prove that random productivity shocks are the driving force of growth in an inherently linear economic system. The linear growth models mathematically need to incorporate some exogenous random shocks to generate artificial time series that can resemble the real ones. If we wish not to introduce random shocks and we wanted that the model were able to generate similar non-stationary time series like the real one, the only truly way would be to make our model non-linear.

In section 5 we show that the structure of the exogenous shocks that have been observed does not look random at all, and therefore we need to look at these shocks not as exogenous factors but as a result of endogenous interactions. Our aim is to understand these interactions that has been hitherto considered

[6] In order to identify successfully the deterministic laws in the time series from the observed fluctuations, the removal of truly stochastic component is a critical factor. If we were able to remove all the stochastic components we could better focus on the dynamics of the determinant components.

as exogenous shocks. Consequently the usual procedure of plugging into models normally *i.i.d.* random shocks is very approximative. The probability of a normal distribution is the one typical of *white noise* while, as we will show in more detail in section 5, the residual components have indeed a structure far different from white noise. This result leads us to think that it would be worthwhile to unravel the hidden structures of the residuals.

While the residual component may also contain a white noise structure, we will show that this structure is not the prevailing one.

Before we present in more detail a procedure to analyze economic time series, we briefly review the recent unit root literature. We compare the results suggested by the unit root literature with those empirical works of the so called *broken trend* literature. We show that the broken trend literature turns out to generate seemingly opposite results to the unit root literature but, as the unit root literature, it does not provide an ultimate response to the issues it claims to explain.

## 2.1 Recent results from the Unit Root literature

Many recent related works have been published after the Nelson and Plosser paper and their results differ mainly for the test function that has been used in the verification of the non-stationarity hypothesis.

Some papers simply confirm that the non-stationarity of economic time series is a recurrent characteristic in many countries. Similarly to Nelson and Plosser, Lee and Siklos (1991) found that macroeconomic time series for Canada are not stationary. Mills (1992) obtained basically the same results for the UK, McDougall (1995) for New Zealand, Rahman and Mustafa (1997) for the Asian countries, Sosa for Argentina (1997), Gallegati (1996), de Haan and Zelhorst (1994) for Italy.

The macroeconomic variables that are more frequently analyzed are GDP, GNP, GDP and GNP per capita, industrial production, employment, unemployment rate and the consumer price index. Occasionally other variables like savings (Coakley, Kulasi and Smith 1995), investments (Coorey 1991, Coakley, Kulasi and Smith 1995), wages (Coorey 1991), exchange rates (Durlauf 1993, Parikh 1994, Wu and Crato 1995, Serletis and Zimonopoulos 1997, Welivita 1998), money and velocity of money (Al Bazai 1998, Serletis 1994) have been analyzed.

The remarkable result from these studies is to have pointed out that almost every time series in any country is characterized by the presence of a unit root, or equivalently by a stochastic process like a random walk.

This result also seems not to depend on the frequency of observation: Wells (1997), Osborn, Heravi and Birchenhall (1999) have found similar results using both quarterly and monthly data[7].

[7] Since the power of the Dickey-Fuller test increases with the frequency of observations, the

The one exception to the existence of unit root in macroeconomic time series is the unemployment rate. This non-conformity was first noticed by Nelson and Plosser and has been confirmed by the majority of unit roots researchers afterwards[8].

In table 1 we list the main works that ascertained the existence of a unit root in macroeconomic time series. For each author we mark with the "+" sign the variable that was found to follow a random walk, and with the "=" sign the variable for which the results were mixed.

## 2.2 The broken trend hypothesis

Rappoport and Reichlin (1988) put forward the hypothesis that there could exist a broken deterministic trend that cannot be identified by the Dickey-Fuller test. Rappoport and Reichlin showed that in the case of a broken and deterministic trend, the Dickey-Fuller test produces spurious results, since it is incapable to reject a false null hypothesis (the unit root hypothesis). Rappoport and Reichlin have moreover revealed empirical evidence concerning the existence of a broken trend in many macroeconomic time series. They indeed rejected the hypothesis of a random walk for many real variables (like industrial production, real GNP, real per capita GNP and money supply) though not for all of them[9].

Since the results obtained by the broken trend literature are open to discussion in the sense that the studies hitherto published do not lead to a general rejection of the random walk hypothesis, we question whether the broken trend hypothesis provides the ultimate answer to the nature of economic time series.

As we shall see in section 4, the Dickey-Fuller test cannot be used to distinguish a random walk from a non-linear dynamics. In the presence of a broken trend or non-linear dynamics the Dickey-Fuller test does not allow us to reject correctly the random walk hypothesis, even when we know a priori that the time series is generated by a deterministic process. If we want to discriminate stochastic processes from the deterministic ones we should use other tests. The BDS test by Brock, Dechert and Scheinkman may allow us to detect non-linear structures in a time series. We will use the BDS test for its good power properties and flexibility in sections 4 and 5 together with other qualitative and quantitative metric tools that allow us to uncover non-linear structures among data[10].

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use of quarterly or monthly data are to be preferred to yearly data (see Maddala and Kim 1998).

[8] Except Banerjee et al. (1992), Bresson and Celimene (1995), Dolado and Lopez (1996), Leybourne et al. (1999).

[9] The consumer price index and nominal wages for instance were found to follow a random walk.

[10] Other tests for nonlinearity have been proposed in the past. The Tsay (1986) and Engle (1982) tests are discussed in Brock et al. (1991) and compared to the BDS tests. The BDS test has been shown to have higher power against a number of non linear alternatives respect to the Tsay test and similar to the Engle test. Contrary to the Engle test, the BDS test has been

Now we briefly review the results from the literature on broken trends claim.

Probably one of the works that most influenced the direction of research of time series analysis has been the paper by Perron in 1989. Perron as well as Rappoport and Reichlin showed that, when fluctuations are stationary along a broken trend, the Dickey-Fuller test is not able to reject the unit root hypothesis. Perron developed a test that allows to correctly reject the *i.i.d.* null hypothesis if the series is characterized by a broken trend. Perron applied his test to the same time series of the US that were used by Nelson and Plosser, after arbitrarily assigning the date in which the structural break occurs. Perron concludes that the null *i.i.d.* hypothesis can be rejected also at a high confidence level for almost all the time series.

Similar results were obtained by Raj (1992) studying macroeconomic time series for Canada, France and Denmark , Rudebusch (1992) for England, Linden (1992) for Finland, Wu and Chen (1995) for Taiwan, Soejima (1995) for Japan.

Other authors checked for a broken trend in specific time series. Diebold and Rudebush (1989), Duck (1992), Zelhorst and de Haan (1993), Ben, David and Papell (1994), Alba and Papell (1995), McCoskey and Selden (1998) have found a broken trend for the GDP in many countries. Alba and Papell (1995) for GDP per capita and Li (1995), Gil and Robinson (1997) found similar dynamics in industrial production, Simkins (1994) in the wages in 8 OECD countries and McCoskey and Selden (1998) in the G7, Raj and Scottje (1994) in the US income distribution, Culver and Papell 1995, Leislie, Pu and Wharton (1995), and MacDonald (1996) in the exchange rates. Given these results we could check whether the broken trend hypothesis explains the dynamics of unemployment rate better than the unit root hypothesis. However Nelson and Plosser already found that the US unemployment rate tended to be stationary, and the works by Hansen (1991), Li (1995), Leslie, Pu and Warton (1995), Song and Wu (1997, 1998), Gil and Robinson (1997), Hylleberg and Engle (1996) simply confirm the empirical evidence presented by Nelson and Plosser.

In table 2 we present the main works that support the hypothesis of a broken trend in macroeconomic time series. For each author we mark with the "-" sign the variable that was found stationary along a broken trend.

Criticisms to both the broken trend and the unit root hypothesis have been put forward by many authors. Zivot and Andrews (1990, 1992) estimate the position in time of the structural break and find that the existence of the broken trend is not that clear in many of the time series that were analyzed by Perron.

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shown to have power also against generic non linear alternatives with zero autocovariances where the Engle test fail.

Other tests for determinism have been developed in the past. However a statistical work that compares the power of all these tests for a wide number of nonlinear alternatives has not been developed. Moreover some tests may have better power then others for particular non linear alternatives and for a certain number of data points. For instance the Engle test performed better then the BDS test in detecting GARCH structures and the BDS test performs also with data that have zero autocovariances. The choice for a certain statistical test should take into account the type of data we are dealing with. we opted to use the BDS test for the main reason that it has good power properties against an unspecified nonlinear alternative with finite but large samples like we had at our disposal.

Cushing and McGarvey (1996) found that the fluctuations in the macroeconomic time series are more persistent compared to what stationary models indicate, but they are also less persistent than unit root models suggest. Mixed results were also obtained by Leybourne, McCabe and Tremayne (1996) for many US macroeconomic time series, Krol (1992) for the production of many US sectors, and Crosby (1998) for the Australian GDP.

It seems therefore that not every time series is characterized by a unit root. What does this suggest? Are time series generated by a deterministic process or by chance? This issue has not been well formulated neither in the unit root nor in the broken trend literature. It has been indeed associated with the idea that a non-stationary process is a random walk process. As we will see in section 4, not all the non-stationary processes follow a random walk. Indeed, there may exist many deterministic non-linear processes that are not stationary and become stationary after differentiating with respect to time.

## 3 The non-linear hypothesis

After twenty years from the publication of the Nelson and Plosser article, we now have two literature streams that debate around the nature of the time series: the one that underlines the existence of a random walk and the one that asserts the complete linear (though with a break) determinism in the economic time series.

We will show in section 5 that the empirical evidence around the nature of economic time series can be clearer than the one provided by both the unit root literature and the broken trend literature.

Our analysis proceeds with the following steps:

1) Selection of the time series with a minimal number of observations. Brock et al. (1991) have proved that a number of at least 400 observations would be a good starting point, if not a necessary condition, to obtain trustful results from the non-linear dynamics tools, like the BDS test, maximal Liapunov exponents, entropy level etc.[11]. It is therefore necessary to rely on seasonally adjusted monthly data for a sufficiently long period[12]. The time series we used are those of the US and data were provided by the Bureau of Labor and Statistics and the Bureau of Economic Analysis.

2) We take the natural logs of the original time series if the time series tend to diverge exponentially.

3) We differentiate the time series once with respect to time and eventually remove linear autocorrelation in the residuals and we check for stationarity via the augmented Dickey-Fuller test.

[11] See the statistical appendix.

[12] We exclude the possibility to analyze any time series of GDP and GNP because of the dearth of data, since these time series are at most quarterly.

4) We calculate the level of spatio-temporal entropy[13], that is a statistical index of the degree of disorder of the system. If the time series of the residual were generated by a random process the level of entropy should be close to the maximal value. However also non-linear processes may present a high degree of disorder and reach values of entropy close to that of white noise[14]. On the other hand we should expect a low level of entropy for processes that are deterministic and autocorrelated[15]. However we should not overestimate the importance of the calculus of entropy; in fact it does not allow us to distinguish a random process from a complex deterministic one and even between periodic cycles and linear trend. Nevertheless entropy measures may help us to comprehend the level of complexity of a time series.

5) We calculate the values of the maximal Liapunov exponents that characterize the time series, to see how fast nearby trajectories diverge over time. If the maximal Liapunov exponents turns out to be negative, it means that trajectories tend to converge to a stable fixed point. If it were zero we would have found a limit cycle. If it were positive the time series is either characterized by chaos or a random walk. We anticipate that the residuals of the linear models that explain the economic time series generally present a positive value for the maximal Liapunov exponent and a high level of entropy and this indicates how is difficult to forecast them in the long run[16].

6) We proceed with our investigation regarding the nature of the process. We use *Ruelle plots*[17] to uncover, from the qualitative point of view hidden structures in the time series. In addition we use the BDS test to verify quantitatively and in reliable way the existence of non-linearity in data.

7) We check our results randomly shuffling the time series and we verify whether the results that we obtain from the BDS test applied on a randomly shuffled time series are indeed different from the results that we obtained performing the BDS test on the original time series[18] . This verification is extremely important since, if the two results turn out to be different, it means that the time order of the original time series is significant and there exists causality in data.

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[13] As calculated by E. Kononov (1999), VRA 4.2 program.

[14] See section 4.3, the case of the *tent map*.

[15] Like for instance the one depicted in section 4.3 for the Rossler residuals.

[16] See statistical appendix.

[17] They are also called *recurrence plots*.

[18] This step is also sometimes called "shuffle diagnostic" (see Lorentz 1989) via "surrogate time series" (Kantz and Schreiber 1997). A "surrogate" time series is essentially the shuffle of the original time series preserving all the linear properties of the time series like frequencies, amplitudes and eventual linear autocorrelations. We have derived the surrogate time series for all the economic time series we have analyzed, but we called them with the more general and less specialistic term of "shuffled time series".

# 4 Results from artificial time series

Before applying the described procedure to real time series, we present some results obtained from artificial time series, whose deterministic process that generates the data is known. The study of artificial time series serves to test the reliability and effectiveness of the results obtained with the technics at our disposal.

We present some cases of deterministic systems that produce a dynamics very similar to a random walk and we check whether the non-linear dynamics tools allow us to gain more information about the nature and the evolution of the time series under consideration. We will see that the information gain from using the numerical tools of non-linear time series analysis may be relevant and may lead us to consider the issues of dynamics from a very different perspective.

## 4.1 Trends

We consider first the most simple limit case, that is the case of growth along a linear trend. In particular we check the results from the Dickey-Fuller test when a linear time series grows deterministically with time. Thereafter we apply non-linear metric tools to this series to see which kind of information we may extract from a time series. The application of non-linear techniques to a linear system may not seem to be necessary, but this step will allow us to compare the information that we can obtain from a linear system with linear statistics and non-linear dynamics tools.

In the trend stationary case, residuals have no persistent effects and the time series is stationary along a linear trend. If we consider the variable $x_t$ as a linear function of time $t$: $x_t = x_0 + \phi t + \varepsilon_t$ where $x_0$ is the initial value (in our case it is equal to zero), $\phi$ is a parameter and $\varepsilon_t$ is a random variable normally *i.i.d.*. Running the Dickey-Fuller test we should reject correctly the null hypothesis of a unit root and the Durbin-Watson statistics, *DW*, should be around 2 (when $DW \simeq 2$, residuals have no serial correlation)[19].

Suppose that we are interested to study the dynamics of a variable that could be the GDP, $y_t$, in a model of ours. In our model we assume that GDP grows at the yearly rate $g = 2\%$:

$$y_t = y_0(1+g)^t \rightarrow \ln y_t = \ln y_0(1+g)^t \rightarrow \ln y_t = \ln y_0 + t\ln(1+g)$$

Suppose that $\ln y_t$ is perturbed by a normally *i.i.d.* exogenous shock $\varepsilon$:

$$\ln y_t = \ln y_0 + t\ln(1+g) + \varepsilon_t.$$

Set $\ln y_t = x_t$ and $\ln(1+g) = \phi$ we obtain:

[19] See the statistical appendix to for the exact critical values of the DW test.

$$x_t = x_0 + \phi t + \varepsilon_t \text{ where } g = 0.02 \text{ and } \phi = 0.02.$$

The time evolution of $x_t$ is represented in fig. 1.

Applying the Dickey Fuller test we decidedly reject the null hypothesis of unit root (tab. 3). The Dickey-Fuller test turned out to be -21.28 while the critical value at 5% significance level is -3.41. For values less than 3.41, the null hypothesis is rejected, as it is in this case. The Durbin-Watson statistics turned out to be close to 2 and this confirms that the residuals are not serially correlated. In this case, the Dickey Fuller test was able to correctly reject the null hypothesis of a stochastic trend and to accept correctly the alternative hypothesis of a linear trend.)

Let us now turn our attention to some qualitative and quantitative measurements obtained with non-linear dynamics tools. The value of *entropy* that characterizes the levels is 0%, and this indicates that the time series is characterized by an almost null degree of disorder. In fact residuals are all concentrated around a linear trend, which represents a long term equilibrium path. If we analyze the residuals, which were assumed to be normally *i.i.d.*, the level of entropy turns out to be 90%, a value relatively close to the ideal limit of 100% of a purely casual process (a value that is very difficult to reach in series generated by the simple algorithms of a random number generators)[20]. This indicates that the degree of disorder of a system characterized only by a random variable normally *i.i.d.* is very high.

We have calculated the value of the maximal Liapunov exponent for the residuals, in order to measure the rate the sensitive dependance on initial conditions, that is the rate of divergence of nearby initial states. It turned out to be positive (table18, row normal *i.i.d.* process) and so high that residuals follow a unpredictable dynamics[21]. As we will see in section 4.3, high values of the maximal Liapunov exponent and entropy are also typical of many non-linear systems (and not only of stochastic systems).

The calculus of the Liapunov exponents and of entropy gives a scalar output. There are also qualitative visual devices that consent to uncover complex structures in data and even to single out exceptional historical events. They are the *phase portraits* and the recurrence plots. The phase portrait is simply a graphical representation that plots the value $x(t)$ against $x(t-h)$ where $0 < h < t$. In fig. 3 the residuals $\varepsilon(t)$ are plotted against $\varepsilon(t-1)$[22].

The recurrence plots by Eckmann, Kamphorst and Ruelle (1987) are a graphical tool for the qualitative analysis of time series based on phase portraits and allow us to uncover deterministic structures that could not be revealed by phase plots. In the most simple recurrence plots, the distances between each observations are measured and marked by a grey tone. On the axis each point corresponds to a dated observation. The diagonal is the locus where $\|x(t) - x(t-h)\| = 0$ where $h = 0$ and the corresponding tone is white.

[20] See the statistical appendix.

[21] See the statistical appendix also for a numerical example.

[22] We could obviously plot residuals $\varepsilon(t)$ against the residuals of any preceding period like e.g. $\varepsilon(t-4)$. Knowing ex ante that $\varepsilon$ is the result of a random number generator, the $\varepsilon(t) - \varepsilon(t-4)$ plot is qualitatively equivalent to the $\varepsilon(t) - \varepsilon(t-1)$ plot.

In the case of a deterministic trend the distance grows with the temporal distance of observations. The most distant observations are $x(0)$ and $x(T)$, hence the points $[x(0) - x(T)]$ and $[x(T) - x(0)]$ are marked by a black tone (fig. 4). The points along the parallels to the 45 degree line are characterized by the same grey tone and this indicates that the couples of observations that keep the same temporal distance are also characterized by the same spatial distance (represented by the same grey tone).

On the contrary, recurrence plots of normally *i.i.d.* residuals, should neither present any continuous line between points nor particular areas characterized by the same grey tone. The fact that some nearly continuous lines may be noticed (fig. 5), is due to the random number generator , which is a mathematical algorithm and therefore does not produce purely unstructured time series. However fig. 5 shows much less structure than the Ruelle plot in fig. 4 and is close to the one of a purely normal *i.i.d* process.

Actually, Ruelle plots may allow to single out much more hidden structures, when they compare embedded vectors[23] instead of single observations. Ruelle plots mark the distances between points[24] with a tone of gray. If we choose $m = 1$ we obtain the figures 4 and 5. If we chose different values of $m$, we would have also graphs similar to fig. 4 and 5. However, in other cases especially in the cases of chaotic systems, the choice of appropriate values for $m$ allows to uncover structures, otherwise neglected.

To discriminate a stochastic process from a process that contains a deterministic structure we apply the BDS test. The null hypothesis is that the time series is characterized by an *i.i.d.* process, while the alternative hypothesis is that the time series follows a non-linear law. Applying the BDS test to the residuals randomly generated at computer, we have found a value for the BDS function equal to -1.28 and a critical value of 1.96 at 5% significance level. As we expected, we accept the null *i.i.d.* hypothesis[25].

From this simple exercise we have obtained the following results:

[23] The embedded vectors are simply defined as:
$\mathbf{x}_i = \{x_{i-(m-1)}, x_{i-(m-2)}, ..., x_i\}$ where $x_i$ is the observed value at a certain point at time and $m$ is called embedding dimension.
For example suppose to have a series of 10 observed values of a certain variable $x$:
$x = \{8, 5, 6, 9, 4, 4, 1, 7, 3, 2, 7\}$
we obtain the following embedded vectors:
$\mathbf{x}_2 = \left\{x_{2-(2-1)}, x_{2-(2-2)}\right\} = \{x_1, x_2\} = \{8, 5\}$
$\mathbf{x}_3 = \left\{x_{3-(2-1)}, x_{3-(2-2)}\right\} = \{x_2, x_3\} = \{5, 6\}$
$\mathbf{x}_4 = \left\{x_{4-(2-1)}, x_{4-(2-2)}\right\} = \{x_3, x_4\} = \{6, 9\}$
...
$\mathbf{x}_{10} = \left\{x_{10-(2-1)}, x_{10-(2-2)}\right\} = \{x_9, x_{10}\} = \{3, 7\}$
for $m = 2$.
and $\mathbf{x} = \{x_2, x_{3,} x_4, ..., x_{10,}\}$ is the embedded time series for $m = 2$.
The embedded time series are of great importance in nonlinear dynamics because thanks to them, as it has been shown by Takens (1981), we may uncover some properties like the correlation dimension of an unknown underlying motion law that generated the time series itself from the observed values of the process.

[24] i.e. between vectors $\mathbf{x}_i$

[25] See the statistical appendix for a discussion of the BDS test, size and power properties.

- using the Dickey Fuller test we have correctly concluded that the time series on levels is stationary and follows a deterministic trend.
- the entropy indicates that the time series of levels is stable and the time series of residuals is extremely unstable. The maximal Liapunov exponents of residuals is sharply positive, and this indicates that nearby trajectories diverge over time. Both the values of entropy and the maximal Liapunov exponent do not provide a definitive answer to the question as regards the nature of time series.
- recurrence plots and phase portraits allow us to identify the existence of structures that are different from those of a normally *i.i.d.* process.
- the BDS test allows us to better appreciate the importance of the time order in time series, that is to detect the existence of deterministic structures in time series. In this case we were not able to detect any deterministic structure in the residuals since there weren't any (except for the ones of the random number generator algorithm).

## 4.2 Random walks

In a similar way as we have done in the case of deterministic trends, we now analyze an other limiting case, the random walk. The random walk hypothesis is not generally rejected by the unit root literature and it is at the core of Real Business Cycle theory.

In order to verify the results that we obtain with the Dickey-Fuller test and the non-linear dynamics tools, we perform a controlled experiment with a known random walk process.

In the random walk case the residuals, contrary to what happens in the case of deterministic trends, have persistent effects and cumulate over time, without being reabsorbed even partially in the future. The time series is not stationary, does not follow a linear trend, but can still grow in a quite similar way to the case of the deterministic trend. From a visual comparison between a series that grows like a random walk and a series that grows along a deterministic linear path, it is often not possible to distinguish the nature of the two time series. The Dickey-Fuller test serves to single out which of the two time series is the one that follows a random walk.

In a random walk process, the value of the variable $x_t$ depends on its lagged value $x_{t-1}$ and a normally *i.i.d.* shock $\varepsilon_t$:

$$x_t = x_{t-1} + \varepsilon_t$$

Suppose now that we are interested in the dynamics of a variable $y$ that grows yearly at the average rate of 2%, as an effect of the cumulation of shocks:

$\ln y_t = \ln y_{t-1} + \varepsilon_t \rightarrow \ln y_t - \ln y_{t-1} = \varepsilon_t \rightarrow \ln \frac{y_t}{y_{t-1}} = \varepsilon_t \rightarrow y_t = e^{\varepsilon_t} y_{t-1}$[26] with $\varepsilon_t \simeq N(0.02 \ln y_{t-1}, \sigma^2)$ *i.i.d.*

[26] Assuming positive shock, at worst we have null growth.

Plotting the log series against time we can see a dynamics (fig. 6) similar to the case of deterministic trend (fig. 1). It is not possible to determine which of the two time series is the random walk through a direct visual inspection alone. A growth trend exists, but it is a stochastic one.

To distinguish between a stochastic trend and a deterministic trend we apply the Dickey Fuller test and, as we expected, we are not able to reject the unit root hypothesis. The value of the test function turned out to be -1.98 while the critical value is -3.41 at 5% significance level (tab. 4). Residuals turned out not to be serially correlated (Durbin-Watson statistic is 1.99).

The entropy level, the maximal Liapunov exponent, the BDS test and Ruelle plots of the residuals are exactly the same of those obtained for the deterministic trend case[27]. Inasmuch as the aim of non-linear dynamics is to detect complex structures in residuals, both in the case of stochastic growth and deterministic growth, residuals are stochastic and the tools of non-linear dynamics cannot be used to detect linear determinism. The proper instrument to detect linear determinism is indeed the Dickey-Fuller test.

### 4.3 Non-linear walks

*Autoregressive tent map growth*

We check what happens when we apply the Dickey-Fuller test to an artificial time series in which the value of the variable depends on its lagged value and a deterministic non-linear residual. We will apply the BDS test and other tools of non-linear dynamics to identify the deterministic structures that the Dickey-Fuller test is not able to detect.

Suppose that a time series is generated by the following deterministic law:

$$x_t = x_{t-1} + 0.04 x_{t-1}\varepsilon_t \text{ with } \left\{ \begin{array}{c} \varepsilon_t = 2\varepsilon_{t-1} \text{ for } \varepsilon_{t-1} < 0.5 \\ \varepsilon_t = 2(1-\varepsilon_{t-1}) \text{ for } \varepsilon_{t-1} < 0.5 \end{array} \right.$$

this system is known as the *tent map* and was published in the Economic Journal by Scheinkman (1990) and by Vastano and Wolf (1986) in a working paper of the University of Texas. The particularity of this system is that it is a chaotic system but it has the same statistical properties of a uniform distribution.

Similarly to the random walk, $0.04\varepsilon_t$ has an average value equal to 0.02[28].

A visual inspection of the generated time series $x_t$ (fig. 7) may lead us to confusion because the time series $x_t$ looks very similar to the time series with a deterministic or stochastic trend. In order to see whether the system follows a stochastic or a deterministic trend we apply the Dickey-Fuller test and we

[27] The residuals in both cases were obtained from the same random number generator.

[28] Because of the finite approximation of the program we used, we could not obtain more then 50 observations. Consequently we have added a very small ratio of white noise to each $\varepsilon_t$ so that the system does not repeat itself even in the long run. We have added $\mathbf{0.000001} * N(0.5, 1)$ noise.

can't reject the unit root hypothesis. In fact the value of the test function turned out to be -1.8 (table 5) while the null hypothesis is rejected for values less than -3.4 at 5% confidence level. The time series appears to be similar to the stochastic trend or to the deterministic linear trend. But we know that it is neither. The Durbin-Watson statistic turned out to be exactly equal to 2.00, and this indicates that residuals are not serially correlated. At this stage we would apply again the Dickey-Fuller test to the residuals to see whether they are stationary, and we would conclude that the process is autoregressive of order one with *i.i.d.* residuals.

This conclusion would only be partially true. The process is autoregressive of order one and therefore there exist a unit root, but the residuals (represented in fig. 8) are deterministic and therefore, knowing the law that generates the residuals, the process would be perfectly predictable. In this case we must be very careful to interpret the results obtained with the Dickey-Fuller test; it suggests that it is not possible to refuse the null hypothesis of the existence of a unit root, and therefore the hypothesis of autoregressive process of order one where residuals have persistent effects. However the residuals, as in this case, can be non-stochastic. Consequently the Dickey-Fuller test is a tool that is not suitable to uncover whether the series follows a deterministic law, except for the special case that the series follows a deterministic linear trend. The acceptance of a unit root hypothesis and the presence of not serially correlated residuals does not authorize us to take for granted that the time series has a stochastic origin.

From the values of entropy (78%) and the positive maximal Liapunov exponent we may infer that the system is nearly unpredictable. However these characteristics are typical of both stochastic and chaotic processes. In order to infer the existence of non-linear structures we have performed the BDS test. The value of the BDS statistic turned out 99.2 and this allows us to reject the null *i.i.d.* hypothesis with a minimal probability to be in error[29].

*Autoregressive Rossler growth*

Consider the following system:

$x_t = x_{t-1} + 0.02x_{t-1}(\frac{\varepsilon_t}{10} + 1)$ (fig. 10)

where $0.02(\frac{\varepsilon_t}{10} + 1)$ has an average equal to 0.02, and $\varepsilon_t$ is the result of a deterministic chaotic time series that generates aperiodic cycles [30] (see fig. 11).

Applying the Dickey Fuller test we would reject the null hypothesis of autoregressive process of order one and accept the alternative hypothesis of a deterministic trend. The Dickey-Fuller statistic turned out to be -57.52, a value enormously greater than the respective critical value (-3.97 is the corresponding 5% critical value) (table 6). The Durbin-Watson statistic turned out to be 0.09 and residuals are indeed serially correlated. Given these results we would think that the time series follows a deterministic trend and fluctuations are cyclical with reversible effects. However our model is autoregressive of order one, it

[29] See in appendix the high power of the BDS test against the tent map.

[30] For a detailed description of the Rossler process see Lorentz (1989) or Gandolfo (1997).

does not follow a deterministic trend and the time series is entirely generated by fluctuations $\varepsilon_t$ that have persistent effects.

The value of entropy of the residuals is 15% and this low value implies that the system tend to preserve a certain stability. The maximal Liapunov exponent is positive and therefore the evolution of the system is sensitive with respect to its initial conditions, but since its value is close to zero, it suggests that the system is also cyclical. In fact it has aperiodic cycles, thus, the system is also chaotic. The recurrence plots of the residuals (fig. 12), just like a simple graph against time (fig. 11), shows a cyclical and aperiodical dynamical structure.

The confirmation of the existence of non-linear structures in the time series follows from the high value of the BDS statistic (table 6). The null *i.i.d.* hypothesis is rejected. Though the BDS test also in this case is able to detect correctly the existence of non-linear structures in the data, we may better appreciate its use with serially uncorrelated residuals, as in the cases of the tent map and the seasonally adjusted real time series.

# 5 Empirical evidence: the US time series

In the past 15 years the detection of non-linearities in real economic time series has turned out a very difficult task. The main problem is to apply the non-linear dynamics tools to time series that contain a sufficient number of observations. In order to reliably calculate the BDS test a quite high number of observations is needed. Around 400 observations are necessary to detect low dimensional non-linearities. If we wish to discover more complex structures, we need even a higher number of observations. This is due to the fact that the BDS test has a very low power for small finite samples[31]. In tab. 7, we show that using a small sample from a random walk growth process, the BDS test rejects spuriously the null *i.i.d.* hypothesis (see the high value of the BDS statistic[32]). The application of the BDS test, as well as all the tools of non-linear dynamics based as the BDS test on the correlation dimension, on small samples may produce spurious results[33]. In section 4.2 the problem of spurious results did not arise since the sample was sufficiently large and the test power high.

When we have at our disposal time series with a very limited number of observations, as in the case we have tested in tab. 7 where the observations were 160, it is necessary to use linear metrics while the use of non-linear dynamics tools would too often produce wrong results. For instance, the frequency of observations for GDP is only quarterly and data are available starting from 1959. Though the Bureau of Economic Analysis is going to release these data this year from 1929, we could only have a maximum of 280 observations and this

[31] See the statistical appendix.

[32] See column $W_{m,N}$.

[33] In the statistical appendix the finite sample property of the methods of nonlinear dynamics based on the correlation integral are discussed.

limitation would not allow us to prove the existence of a non-linear dynamics[34]. Moreover a series like GDP is somewhat peculiar: it is the result of the sum of the production of all the sectors and the production of each sector may have a dynamics of its own. Time series of different sectors are characterized by specific values of amplitude, frequency and phase. If we simply sum the values of production (that is the amplitude of the signal) of each sector, we generate a time series of GDP, whose signal characteristics like amplitude, frequency and phase are not shared with other time series. The GDP time series may not be appropriate to display structural changes in the sectorial time series.

Chavas and Holt (1991) have chosen to analyze a very specific time series of which it was already known to have a cyclical nature: the *Pork Cycle*. Chavas and Holt have shown the existence of aperiodic cycles in the quarterly time series of the US quantities and prices of pork meat from 1910 till 1984. Chavas and Holt have the great merit to have proved that fluctuations in time series may have a non-linear origin.

In the analysis that follows, we focus on some main macroeconomic and sector time series. We check whether it is possible to extract signals from the residuals that economic literature has assumed to be stochastic. What we want to ascertain whether the residuals also contain present a non-linear component together with a truly stochastic component. What we are trying to find is whether important temporal linkages are present between residuals. We will attempt to falsify the results of rejection of the null *i.i.d.* hypothesis. We will proceed to a random shuffle of the time series in order to break any temporal link among data and we will apply non-linear dynamics tools on the shuffled time series. If the results of non-linear test on both the original and the shuffled time series are similar, it means that time linkages are not important and the time series is generated by a stochastic process, otherwise there is evidence that time cannot be ruled out and there exists a non-linear component.

## 5.1 Industrial production

Time series for industrial production is certainly one of the most complete available. Data go back to 1919 and the frequency of observation is monthly.

Applying the Dickey-Fuller test[35] to the log of the observed values, we cannot reject the null hypothesis of a unit root (table 8).

Afterwards we have estimated the following linear model that best fits the data:

[34] A generally accepted result is that the GDP time series, as pointed out by the vast literature on unit roots and cointegration, is characterized by a stochastic trend, but it cannot reliably tested with the nonlinear numerical tools because of a paucity of observations. Hence we cannot ascertain whether the GDP is really characterized by a nonlinear dynamics.

[35] Since some time series were autocorrelated in the residuals, we have used for all the real time series the "augmented" form of the Dickey-Fuller test including more lags, trend and intercept. The number of lags we have considered is the minimal that consents to obtain uncorrelated residuals. See statistical appendix or Harris (1995) for more details.

$$Y(t) = 0.02 + 0.99Y(t-1) + 0.51[Y(t-1) - Y(t-2)] + (2.89E - 0.5)t + \varepsilon$$

where Y(t) are the observed values of the industrial production in terms of value[36]. The Durbin Watson statistic is 1.95 well within the acceptance range 1.89-2.10 and this indicates that residuals are not serially correlated.

From the original series $Y$ we focused on the residuals $\varepsilon$ (fig. 13). The residuals appears also to be characterized by a very complicate dynamics if we look at its entropy level (80%) (table 8).

The calculus of the maximal Liapunov exponent depends on the parameter of the embedding dimension $m$. There exists a maximal Liapunov exponent for each value of $m$. The maximal Liapunov exponents are all positive for different values of $m$ and this stands to indicate a high sensitivity of the time series with respect to its initial conditions (table 18).

The existence of a structured dynamics seems also corroborated by the Ruelle plot[37] (fig. 14) where the presence of continuous lines is clear. In fig. 14 we can easily detect, without any a priori historical knowledge, the periods in which significant historical events have perturbed the industrial production. From this recurrence plot we can see that the first years of the '20s, the years around 1933 and 1944, have been characterized by anomalous dynamics. The embedded vectors represented by the single points around those dates show a big distance, marked with a dark color, compared to nearly all the other vectors. Moreover, we can see that after the 400th embedded vector, the dynamics is more settled and seems also to repeat (see the bright area on the upper right). What is evident in fig. 14 is the existence of a structure that differs from a random walk (fig. 5).

To ascertain whether the time series is generated by a non-linear deterministic process we have applied the BDS test. The null *i.i.d.* hypothesis is strongly rejected (tab. 9, column $W_{m,N}$). A similar test based on the same statistic of the BDS test is the dimension test (tab. 9, column $d_m$). The correlation dimension $d_m$ grows very slowly with $m$ and tends to converge to a fixed value. This is typical of a process that is not guided by chance (Hommes 1998)[38].

If we randomize the order of the events of the original time series, we find that the values of the BDS test and the correlation dimension turn out to be very different from the values obtained using the original time series and we correctly accept the null *i.i.d.* hypothesis for the shuffled time series. This is evidence that the time order of the residuals of the original time series is not random, and a temporal causality in the fluctuations exists.

We conclude that residuals in industrial production show a structure that cannot come from a mere linear stochastic process and therefore a non-linear explanation is necessary to understand the temporal causality of events. This

[36] All the sector time series we have considered are in terms of value.

[37] obtained setting $m = 5$.

[38] Similar results were also obtained adding a small percentage of noise (5% of the variance). We added noise to the time series simply because, when the nonlinear structure is well defined, adding a small stochastic component should not change significantly the result of the test. Even if there were small *i.i.d.* measure errors these should not call in question the obtained results.

result show that in the residuals there exist clear non-linear structures and consequently residuals should be studied as they were signals rather than noise.

### 5.2 Empirical analysis of other macroeconomic time series: industrial production in the main US sectors, employment, hourly wages and consumer price index

A thorough analysis of each sector would require too much room and would be beyond the aim of this article that wants to obtain general results around the existence of deterministic structures in macroeconomic time series. Shortly we synthesize the results obtained by analyzing some of the main US macroeconomic time series. We have limited our analysis to the main sectors of the American economy[39], employment, hourly wages and the consumer price index. For the economic variables characterized by seasonal cycles we analyzed the seasonally adjusted ones. The frequency of observations is monthly. Data go back to 1947 for the transportation sector, industrial machinery and electrical machinery, 1967 for the hybrid Hi-tech sector (computers, semiconductors and communications), 1939 for employment, 1932 for hourly wages and 1913 for the consumer price index.

All the time series (log transformed), except employment, seem characterized by a unit root, since for most of them we are not able to reject the null *i.i.d.* hypothesis of the Dickey-Fuller test (tables 11, 12, 13, 14, 15, 16 and 17) with high confidence levels (higher than 5%)[40]. These results are qualitative similar to those obtained by Nelson and Plosser. For all the time series, the estimated residuals of the linear model[41] that fits best the data turn out to be serially uncorrelated (the null hypothesis of the Durbin-Watson test is never rejected even at high confidence level for all the time series, tab. 11, 12, 13, 14, 15, 16 and 17).

All the time series we analyzed (tab. 11, 12, 15 and 17) are characterized by high entropy values (generally higher than 70%) that are typical of both chaotic and stochastic processes. For all the time series we found positive values of the corresponding maximal Liapunov exponents (tab. 18) and this result suggests that nearby trajectories diverge over time at a positive exponential rate. The interesting result is that all the time series are characterized by a liapunov exponent decidedly lower than the one of a normal *i.i.d.* process, and lower than the one of the tent map. This means that even if time series have to be considered unpredictable in the long run, in the short run they are more predictable than a *i.i.d.* process and a deterministic process like the tent map[42].

[39] That is those that are the most important with respect to the value added.

[40] However for transportation equipment production and industrial machinery production we are not able to reject the null hypothesis only at 1% significance level.

[41] See the estimated equations directly inside tables 11, 12, 13, 14, 15, 16 and 17.

[42] It is worthwhile to mention that in section 5.1 we found a maximal Liapunov exponent for the industrial production close to zero, indicating the presence of cycles.

The presence of structures different from those typical of a normal stochastic process, has been pointed out by the recurrence plots of all the time series. If we compare fig. 15, 16, 17, 18, 19, 20, 21 with fig. 5 (fig. 5 is typical of an unstructured random process), it is clear the existence of structures (repetitive continuous lines over time) in the distances (represented by the intensity of grey) between the embedded vectors (represented by each single point in the coordinates) [43].

The application of the BDS test provides us further information about the existence of determinism in time series. Applying the BDS test to all the time series at our disposal, we are not able to accept the null *i.i.d.* hypothesis. All the series are characterized by high values of the BDS statistic beyond their respective critical values (column $W_{m,N}$ tables 19, 20, 21, 22, 23, 24 and 25[44]). The dimension test[45], based as the BDS test on the calculus of the correlation dimension, allows us in some cases to measure the dimension of the chaotic attractor that characterizes the time series. Without going into the details, the dimension test is based on the fact that a truly stochastic process is characterized by the growth of the correlation dimension with the increase of the embedding dimension, while a truly chaotic process is characterized by the correlation dimension tending to settle to a constant value when the embedding dimension increases (Hommes 1998). This constant value represents the dimension of the chaotic attractor. In all the series we have analyzed the correlation dimension (column $d_m$ in tables 19, 20, 21, 22, 23, 24 and 25) grows less than proportionally with respect to "$m$", but in many cases we cannot detect a clear tendency of the correlation dimension to settle clearly to a constant value (column $d_m$ in tables 19, 20, 21, 22, 23, 24, 25 and fig. 22). For all the time series we have analyzed, the BDS test suggests that the time series contain a deterministic structure, but it is not possible to quantify, via the dimension test, the dimension of the underlying attractor of the time series[46].

To check furthermore our results we have randomly ordered the real time series and applied BDS and calculated the dimension correlation of the shuffled time series to see whether temporal linkages were relevant. In all the cases

[43] The presence of continuous lines in the recurrence plots indicates that the embedded vectors represented by each point keep approximately the same distance with respect to all the vectors that belong to the continuous line. In a normal *i.i.d.* process, each vector is randomly distant from any other vector and the probability that nearby vectors have similar distances is very low. Thus in a normal *i.i.d.* process we should not notice any continuous line in the recurrence plots).

[44] see also the statistical appendix for critical values and the finite sample characteristics of the test.

[45] Note that the "dimension test", contrary to the BDS test, is not really a statistical test since critical values are not specified. It's a numerical tool that suggests the existence a deterministic dynamics when the calculated correlation dimension tend to a fixed value when the embedding dimension grows.

[46] This phenomenon may be due to the presence of a stochastic component in the time series. It should be therefore important to filter our data in order to separately analyze the only deterministic component and to quantify the dimension of the chaotic attractor. The future application of filters that allow us to reduce and hopefully remove the stochastic component may allow us to detect the dimension of chaos for all the real time series for which we have already uncovered the presence of chaos.

the values of the BDS and the dimension tests of the shuffled time series were notably different. We could not reject the null hypothesis of the BDS test for all the shuffled time series and the correlation dimension also was also higher (tables 26, 27, 28, 29, 30, 31, 32 and fig. 23) with respect to the original time series (tables 19, 20, 21, 22, 23, 24, 25 and fig. 22). This is a confirmation that temporal linkages between residuals are really important and therefore that just a probabilistic hypothesis on the residuals of macroeconomic time series does not have an empirical foundation.

# 6 Concluding remarks

We have first shown the theoretical possibility (section 3 and 4) and latter the empirical evidence (section 5) that in the serially uncorrelated residuals there are present non-linear signals that, in the models with a deterministic (linear or broken) or stochastic trend, are hypothesized to be normally *i.i.d.*, like white noise. Doing in this way, both the purely stochastic and linearly deterministic models substitute non-linear signals with noise. The approach that we put forward is to separate the stochastic component (that is indeed present in the residuals) from the deterministic component and study these two components separately. To be successful in this task we need a data filter based on the concepts of non-linear dynamics. In this paper we have limited our analysis to the detection of the existence of clear non-linearities in the residuals of macroeconomic time series. We have detected non-linearities in all the time series we analyzed. All the time series we have considered are thus characterized by determinism, notwithstanding all the series (except employment) are non-stationary and residuals are serially uncorrelated. If all this is true, in the short run, we may make better predictors than simple autoregressive models.

The problem of distinguishing between the two alternative hypothesis, deterministic trend or stochastic trend, was at the core of unit root and broken trend literature (section 2), but for us it was not the first issue. Our aim was indeed to detect non-linear structures in those components that linear stochastic models have assumed as exogenous factors. As far as in linear stochastic models noise plays the relevant role to make "non-stationary" basically stationary processes, it was for us of primary importance, from the theoretical point of view, to check whether a component of what has been considered noise may have an endogenous explanation. If this is the case as confirmed in section 5, economic variables may not follow a stationary path even in absence of external shocks and the observed non-stationarity may be the consequence of the relations between the economic variables.

## Statistical appendix

Our basic statistical issue is to understand whether the dynamics behind the residuals is the result of non-linearities or just of a random process. In sections 4 and 5 we have analyzed some cases of both artificial and real time series, and we applied to these time series some statistical tools like the Dickey-Fuller unit root test to check for stationarity and the Durbin-Watson statistic to check whether the residuals were serially uncorrelated. This appendix gives some basic information about the statistical tools used in this paper. More technical information about testing for unit roots may be found in Harris (1995) Boswijk (1996), Maddala and Kim (1998). We also provide some introduction for testing non-linear dynamics with the BDS test and measures about the stability of the systems with Liapunov exponents, entropies and the visual tool of recurrence plots. A comprehensive and technical description of the BDS test is found in Brock et Al. (1991), while advanced material about Liapunov exponents, entropies and recurrence plots may be found in Tong (1990) and in Kantz and Schreiber (1997). In this appendix we summarize the logics and some results of both the linear and non-linear time series methods that are strictly necessary for the understanding of the paper.

### a ) The Durbin-Watson test

The Durbin-Watson test is a parametric hypothesis test. The Durbin-Watson statistic measures the relation between adjacent residuals. Serial correlation in the residuals that are adjacent in time constitutes a problem that should be removed. In fact, serial correlation leads ordinary least squares to biased estimates of the parameter coefficients, and is symptomatic of bad model specification (Johnston 1984), that is the functional form of the model ($x_t = x_{t-1}+\varepsilon_t$) is inappropriate because some variables (e.g. lagged errors $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$ with $-1 < \rho < 1$ and $v \sim N\left(0, \sigma^2\right)$) are omitted (Harvey 1990).

The Durbin-Watson statistic $DW$ is defined as $DW = \frac{\sum_{t=2}^{T}\left(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1}\right)^2}{\sum_{t=1}^{T}\hat{\varepsilon}_t^2} \approx 2(1-\rho)$ where $\hat{\varepsilon}_t$ are the $OLS$ estimated residuals. If there is no correlation between adjacent residuals, $DW$ will be around 2. Given the equation $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$ with $v_t \backsim N(0, \sigma)$, the null hypothesis of zero autocorrelation is $H_0 : \rho = 0$,while the alternative is $\rho \neq 0$. Since $DW \approx 2(1-\rho)$, the $DW$ will be close to 2 under the null hypothesis $\rho = 0$. In the case of strong positive serial correlation, it will be near zero. In the case of negative serial correlation, the Durbin-Watson statistic has a value between 2 and 4. Critical values depend on the sample size. In presence of large samples (i.e. more than 200 observations) $DW$ is approximately normally distributed with mean *2* and variance *4*/$N$ with $N$ the number of observations (Harvey 1990). Based on this result, we can easily derive the critical values for any size of the test for one tailed test against either positive or negative autocorrelation. The null hypothesis of zero autocorrelation is rejected if the $DW$ statistic is less than its critical value for the case of positive autocorrelation alternative hypothesis and greater than its critical value for the case of negative autocorrelation alternative hypothesis.

In the case of normal distribution $N\left(2, \frac{4}{N}\right)$ the repartition function $F(z)$ is equal to $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{u^2}{2}}du$ with $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}}$ and $\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du \sim N(0,1)$.

The null hypothesis of zero autocorrelation is rejected if the $DW$ statistic is less than its critical value for the case of positive autocorrelation alternative hypothesis. The probability that $N\left(2, \frac{4}{N}\right)$ assumes values less than its critical value $x$ is $F(z)$. If the size of the test is 5% we have: $\alpha = P(Z < z) = F(z) = 5\% = 1 - 95\% = 1 - F(1.6449) = F(-1.6449)$. $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}} = -1.6449 \rightarrow x = 2 - 1.6449 * \sqrt{\frac{4}{N}}$.

If the size of the test is 3%, $x = 2 - 1.96 * \sqrt{\frac{4}{N}}$ while for $\alpha = 1\%$, the critical value is $x = 2 - 2.326 * \sqrt{\frac{4}{N}}$

e.g. if N=1021 the critical value corresponding to a 5%, 3% and 1% size of the test are respectively 1.897, 1.877 and 1.854.

As the sample size grows the critical values tend to 2! The null hypothesis of no serial correlation is rejected in favor of *positive* serial correlation if $DW$ is less than its critical value at a fixed level of significance. Similarly the null hypothesis of no serial correlation is rejected in favor of *negative* serial correlation if $DW$ is greater than its critical value at a fixed level of significance.

For a 5% size of the test we have: $\alpha = P(Z > z) = 1 - F(z) = 5\% = 1 - 95\%$, $\rightarrow F(z) = 95\% = F(1.6449)$ and $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}} = 1.6449 \rightarrow x = 2 + 1.6449 * \sqrt{\frac{4}{N}}$.

If the size of the test is 3%, $x = 2 + 1.96 * \sqrt{\frac{4}{N}}$ while for $\alpha = 1\%$, the critical value is $x = 2 + 2.326 * \frac{4}{N}$.

e.g. if N=1021 the critical value corresponding to a 5%, 3% and 1% size of the test are respectively 2.103, 2.123, 2.146.

The rule of thumb suggested by some econometric software of considering serial correlation in serious consideration only for values less 1.5 or greater than 2.5 is therefore wrong for large sample, while it could be accepted for small samples (like 20 or 30 observations). As large sample increases for the same size of the test we should calculate its critical values in the way as it has been shown.

### b) The ARCH Test

We have used the Lagrange Multiplier $ARCH$ Test (ML ARCH test) for autoregressive conditional heteroskedasticity in the residuals (Engle 1982).

To test the null hypothesis that there is no $ARCH$ up to a certain order in the residuals, the following regression for the squared residuals is fitted:

$\varepsilon_t^2 = \theta_0 + \theta_1 \varepsilon_{t-1}^2 + ... + u_t$

where $\varepsilon_t$ is the residual and $u_t$ an exogenous input. We have used EViews 3.1 software which reports two test statistics from this test regression. The Obs*R-squared statistic is the Engle LM test statistic. The F-statistic is an omitted variable test for the joint significance of all lagged squared residuals. We reject the null hypothesis of zero heteroskedasticity and no omitted variables if the respective p-values are lower than the significance level (generally set at 1, 3, or 5 %).

**c) The Dickey-Fuller test**

The Dickey Fuller test is a parametric hypothesis test. With the Dickey-Fuller test we are concerned with testing whether the parameter $\phi$ of the regression equation $x_t = \phi x_{t-1} + \varepsilon_t$ is equal to 1 with $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. $x_i$ with $i = 1...T$ are the natural logarithms of the real quantities. Since real time series do not show an ever increasing growth rate we are only concerned whether $\phi = 1$ (i.e. the series is non-stationary) or alternatively $\phi < 1$ (i.e. the series is stationary).

Applying the difference operator $\Delta$: $\Delta x_t = x_t - x_{t-1}$, $x_t = \phi x_{t-1} + \varepsilon_t \rightarrow \Delta x_t = (\phi - 1) x_{t-1} + \varepsilon_t$. The null hypothesis $\phi = 1$ is equivalent to $(\phi - 1) = 0$ and the alternative to $(\phi - 1) < 0$.

Dickey and Fuller (1976)[47], via Monte Carlo techniques, derived a $t$-test from the data generated by the random walk process $\Delta x_t = x_{t-1} + \varepsilon_t$. The critical values of the Dickey-Fuller test for prefixed levels (10, 5, 2.5 and 1%) of significance are:

| $T$ | 10% | 5% | 2.5% | 1% |
|---|---|---|---|---|
| 25 | -1.60 | -1.95 | -2.26 | -2.66 |
| 50 | -1.61 | -1.95 | -2.25 | -2.62 |
| 100 | -1.61 | -1.95 | -2.24 | -2.60 |
| 250 | -1.62 | -1.95 | -2.23 | -2.58 |
| 500 | -1.62 | -1.95 | -2.23 | -2.58 |
| $\infty$ | -1.62 | -1.95 | -2.23 | -2.58 |

The null hypothesis is rejected when the $t$-ratio is smaller than its critical value. Testing for a unit root using the regression equation $\Delta x_t = (\phi - 1) x_{t-1} + \varepsilon_t$ implies that the process has zero mean (i.e. no stochastic trend) and no deterministic trend.

A more general regression equation is: $x_t = \alpha + \beta t + \phi x_{t-1} + \varepsilon_t \rightarrow \Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ where $\alpha$ and $\beta$ are parameters. $\alpha$ indicates that there is a stochastic trend (drift) while $\beta t$ indicates that there is a deterministic trend. Given the regression equation $\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ the critical values of the Dickey-Fuller test are:

[47] See Harris 1995.

| $T$ | 10% | 5% | 2.5% | 1% |
|---|---|---|---|---|
| 25 | -3.24 | -3.60 | -3.95 | -4.38 |
| 50 | -3.18 | -3.50 | -3.80 | -4.15 |
| 100 | -3.15 | -3.45 | -3.73 | -4.04 |
| 250 | -3.13 | -3.43 | -3.69 | -3.99 |
| 500 | -3.13 | -3.42 | -3.68 | -3.98 |
| $\infty$ | -3.12 | -3.41 | -3.66 | -3.96 |

and the null hypothesis is rejected when the $t$-ratio is smaller than its critical value. In the case where the data generating process is unknown, the use of the regression equation $\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ is to be preferred to $\Delta x_t = (\phi - 1) x_{t-1} + \varepsilon$ since the latter is only valid when the mean of the time series is zero while we do not know the true mean of the time series. The more general specification of the regression equation prevents us to get spurious results when there is not any a priori information about the existence of a deterministic or stochastic trend in the time series. However $\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ is an autoregressive model of order 1. If the true data generating process were of order>1, that is $\Delta x_t$ would depend on other lagged terms than $x_{t-1}$, $\varepsilon_t$ would turn out to be autocorrelated as an effect of the mispecification. Autocorrelated errors invalidate the use of the Dickey-Fuller distribution, which are based on the assumption of white noise (Harris 1995). Changing the estimating equation to the *augmented Dickey-Fuller* regression, we have:

$\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \sum_{i=1}^{p-1} (\phi_i - 1) \Delta x_{t-1} + \varepsilon_t$ where $(\phi - 1) = \sum_{i=1}^{p} (\phi_i - 1) - 1$

If $(\phi - 1) = 0$, against the alternative $(\phi - 1) < 0, x_t$ contains a unit root. The same critical values of the case $\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ may be used, although they are valid as an asymptotic approximation (Boswijk 1996). A large negative t-statistic rejects the hypothesis of a unit root and suggests that the series is stationary. In this paper we have used the augmented form of the Dickey-Fuller test and the critical values are those from Mac Kinnon (1991) for various sample size.

**d) Grassberber-Procaccia correlation sum (integral)**

The Grassberber-Procaccia correlation sum is defined as the fraction of all possible pairs of points in a m-dimensional (i.e. vectors of m-elements) lying within a distance $\epsilon$ (Dechert 1994, Hommes 1998). Intuitively the correlation sum is a measure of *concentration* of scattered points.

Its formula is:

$$C_{m,N}(\epsilon) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \chi (\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|)$$

where $N$ is the number of observation of a $m$-dimensional vector time series $\mathbf{x}_i = [x_i, x_{i+1}, ..., x_{i+m}]$,

$x_i$ the observations and $i = 1, 2, ..., N$.

$\|\mathbf{x}_i - \mathbf{x}_j\|$ is the euclidean distance between vectors, i.e.
$\sqrt{(x_i - x_j)^2 + (x_{i+1} - x_{j+1})^2 + ... + (x_{i+m} - x_{j+m})^2}$
$\chi$ is a function that:
$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 1$ if $\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon$,
$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 0$ if $\|\mathbf{x}_i - \mathbf{x}_j\| \geq \epsilon$.

Two important theorems (we refer the reader to the original references for the proves) are related to the correlation sum, the correlation dimension and the BDS statistic that will be discussed below:

**Theorem 1** *as* $N \longrightarrow \infty$, $C_{m,N}(\epsilon) \longrightarrow C_m(\epsilon) = \Pr(\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon)$ *with probability one (Brock et al. 1991).*

Therefore for a sufficiently large number of observations the correlation sum measures the probability that two randomly chosen vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are $\epsilon$-close to each other:

$$C_{m,N}(\epsilon) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) \sim C_m(\epsilon) = \Pr(\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon)$$

**Theorem 2** *if* $\mathbf{x}_i$ *is generated by a stochastic i.i.d. process* $\lim_{n \to \infty} \left(C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m\right) \rightarrow 0$ *with probability 1 (Brock, Dechert1988).*

Therefore for a sufficiently large number of observations $C_{m,N}(\epsilon) \sim C_{1,N}(\epsilon)^m$ if the underlying process is *i.i.d.*.

**e) Correlation Dimension**

The correlation dimension $d_m$ is defined as: $d_m = \lim_{\epsilon \to 0} \lim_{N \to \infty} \frac{\ln C_{m,N}(\epsilon)}{\ln \epsilon}$ and can be readily obtained once we have computed the correlation sum $C_{m,N}(\epsilon)$.

Let us analyze some limit cases:

$C_{m,N}(\epsilon) = 1$ is defined as the fraction of all possible pairs of points (or vectors) are lying within a small distance $\epsilon$, so it may assume any value between 0 and 1.

Suppose that $C_{m,N}(\epsilon) = 0$, that is there are no pairs of points (or vectors) lying within a small distance $\epsilon$. $d_m = \frac{\ln 0}{0} = \infty$. For a random process the $d_m \rightarrow \infty$.

Suppose that $C_{m,N}(\epsilon)$ increases towards 1, that is the fraction of all possible pairs of points (or vectors) are getting inside within a small distance $\epsilon$. $\ln C_{m,N}(\epsilon)$ decreases towards $\ln 1 = 0$ and with it $d_m$ that tends towards 0. This is the case in which all the observations are all lying close each other and all inside a distance $\epsilon$. The phenomenon is completely stable and determined.

Now non-linear mathematical systems are able to generate time series where $0 < d_m < \infty$. For pseudo random generator $d_m \longrightarrow \infty$, while for any other system $d_m$ tend to a finite value. For example in the case of the tent map it is easy to calculate that $d_m \longrightarrow 1$; for this case and others see Hsieh (1991). With the calculus of the correlation dimension seems therefore to be possible to detect determinism. However this experimental procedure is not a statistical test. Brock, Dechert and Scheinkman (1987) have therefore provided a statistical hypothesis test with a null hypothesis of *i.i.d.* against any departure from *i.i.d.*

**f) The BDS statistic, size and power**

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The BDS test is a non-parametric hypothesis test. Contrary to parametric tests, like the Durbin-Watson and the Dickey Fuller tests, it does not test whether a particular parameter assumes a given value. Indeed it tests whether data are independent and identically distributed.

We have seen that theorem 2) implies that $C_{m,N}(\epsilon) \sim C_{1,N}(\epsilon)^m$ if the underlying process is *i.i.d.*.
Brock et al (1987) have also proved that $\sqrt{N}C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m$ converges to a normal distribution (one can also compute $C_{m,N}(\epsilon)$ and $C_{1,N}(\epsilon)$ and show the same results):

**Theorem 3** *as* $N \longrightarrow \infty$, *if* $\mathbf{x}_i$ *is generated by a stochastic i.i.d. process then,* $\sqrt{N}C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m \longrightarrow N(0,\sigma)$ *and* $W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ *where* $\sigma_{m,N}(\epsilon)$ *is a consistent estimator of the asymptotic standard error of* $[C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m]$.

$W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ is the BDS statistic and converges in distribution to a standard normal $N(0,1)$.

**Size**

As $N \longrightarrow \infty$ the critical value corresponding to a 10%, 5% and 2% size of the two side test are respectively $|1.64|$, $|1.96|$ and $|2.33|$. The null hypothesis of *i.i.d.* is rejected if the $W_{m,N}(\epsilon)$ is greater than its critical value at a fixed level of significance.

However as any other test that relies on its asymptotic distribution, we need the critical values for the finite sample distribution. Brock et al. (1991) and Hsieh (1991). These values were found via Monte Carlo simulations. They have generated random number samples of different sizes (100, 500 and 1000) and 6 distributions (standard normal, student-t with 3 degrees of freedom, double exponential, chi square with 4 degrees of freedom, uniform and bimodal). They applied the BDS test and repeated this experiment 2000 times (5000 for samples of 100 and 500 data points) for different values of $m$ ($m = 2, m = 5$ and $m = 10$) and $\epsilon$ ($\frac{\epsilon}{\sigma} = \frac{1}{4}, \frac{1}{2}, 1, \frac{3}{2}, 2$). If we use a 5%, 2.5% or 1% significance level, we should reject 5%, 2.5% or 1% of the replications. Brock et al. and Hsieh found the size of the test for different critical values ($\pm 1.64, \pm 1.96, \pm 2.33$ which correspond to 5%, 2.5% or 1% size of the standard normal in case of one side test) of the parameters $m$ and $\epsilon$ for different finite sample sizes.

These were the main results from Monte Carlo simulations (see Brock et al. 1991 for all the tables of the BDS test):

1) *The finite sample property is quite poor for samples of 100 points.* We report the results from the normal distribution. We can easily see that the BDS test rejects the null hypothesis *i.i.d.* at least 3 times more than it should.

Size of BDS Statistic, Standard Normal

| $m = 2, N = 100$ | $\epsilon/\sigma$ 0.25 | 0.50 | 1.00 | 1.50 | 2.00 | $N(0,1)$ |
|---|---|---|---|---|---|---|
| $\% < -2.33$ | 28.3 | 13.1 | 3.90 | 3.94 | 5.62 | 1.00 |
| $\% < -1.96$ | 32.1 | 17.2 | 8.02 | 7.1 | 9.18 | 2.50 |
| $\% < -1.64$ | 35.6 | 21.9 | 12.3 | 12.04 | 13.8 | 5.00 |
| $\% > 1.64$ | 27.9 | 16.5 | 10.0 | 9.02 | 10.1 | 5.00 |
| $\% > 1.96$ | 25.2 | 13.4 | 6.44 | 5.66 | 7.12 | 2.50 |
| $\% > 2.33$ | 23.0 | 10.4 | 3.78 | 2.96 | 4.5 | 1.00 |

Using the *i.i.d.* time series generated by the other distribution (especially the uniform and the bimodal) did not change the picture very much. The null hypothesis is spuriously rejected too often when the sample size is small.

If we increase the sample size to 500 and 1000 data points, the BDS distribution becomes more normal and its asymptotic distribution (the standard normal) gives a much better approximation of the finite sample BDS distribution (especially when N=1000). Similar results (see Brock et al.1991) were obtained when the samples were obtained from the other *i.i.d.* processes (student-t etc.)

Size of BDS Statistic, Standard Normal

| $m = 2, N = 500$ | $\epsilon/\sigma$ 0.25 | 0.50 | 1.00 | 1.50 | 2.00 | $N(0,1)$ |
|---|---|---|---|---|---|---|
| $\% < -2.33$ | 8.62 | 1.96 | 1.10 | 1.28 | 1.34 | 1.00 |
| $\% < -1.96$ | 13.0 | 4.44 | 3.04 | 3.26 | 3.52 | 2.50 |
| $\% < -1.64$ | 17.1 | 8.24 | 5.98 | 6.20 | 6.78 | 5.00 |
| $\% > 1.64$ | 16.9 | 9.32 | 6.92 | 6.04 | 6.58 | 5.00 |
| $\% > 1.96$ | 12.6 | 5.76 | 3.76 | 3.36 | 3.86 | 2.50 |
| $\% > 2.33$ | 8.98 | 3.42 | 1.80 | 1.68 | 1.88 | 1.00 |

| $m=2, N=1000$ | $\epsilon/\sigma$ 0.25 | 0.50 | 1.00 | 1.50 | 2.00 | $N(0,1)$ |
|---|---|---|---|---|---|---|
| $\% < -2.33$ | 4.65 | 1.40 | 1.05 | 0.90 | 0.80 | 1.00 |
| $\% < -1.96$ | 8.95 | 3.25 | 2.90 | 2.45 | 2.65 | 2.50 |
| $\% < -1.64$ | 13.3 | 6.55 | 5.60 | 6.30 | 6.15 | 5.00 |
| $\% > 1.64$ | 9.50 | 6.20 | 4.70 | 4.20 | 5.50 | 5.00 |
| $\% > 1.96$ | 6.30 | 3.70 | 2.25 | 2.40 | 2.50 | 2.50 |
| $\% > 2.33$ | 3.60 | 1.55 | 0.90 | 0.70 | 0.90 | 1.00 |

2) increasing the embedding dimension $m$ the asymptotic distribution may provide a better approximation of the finite sample BDS distribution.

Size of BDS Statistic, Standard Normal

| $m=5, N=500$ | $\epsilon/\sigma$ 0.25 | 0.50 | 1.00 | 1.50 | 2.00 | $N(0,1)$ |
|---|---|---|---|---|---|---|
| $\% < -2.33$ | 12.8 | 0.84 | 0.94 | 1.16 | 1.12 | 1.00 |
| $\% < -1.96$ | 17.1 | 2.48 | 2.24 | 2.88 | 2.92 | 2.50 |
| $\% < -1.64$ | 21.8 | 5.58 | 5.52 | 5.62 | 5.86 | 5.00 |
| $\% > 1.64$ | 19.7 | 7.24 | 5.12 | 5.20 | 5.68 | 5.00 |
| $\% > 1.96$ | 16.1 | 4.56 | 3.10 | 2.96 | 3.16 | 2.50 |
| $\% > 2.33$ | 12.9 | 2.84 | 1.56 | 1.28 | 1.6 | 1.00 |

| $m=5, N=1000$ | $\epsilon/\sigma$ 0.25 | 0.50 | 1.00 | 1.50 | 2.00 | $N(0,1)$ |
|---|---|---|---|---|---|---|
| $\% < -2.33$ | 6.05 | 0.70 | 0.70 | 0.85 | 0.60 | 1.00 |
| $\% < -1.96$ | 9.55 | 2.25 | 2.30 | 2.55 | 2.50 | 2.50 |
| $\% < -1.64$ | 13.7 | 4.60 | 5.35 | 5.50 | 5.40 | 5.00 |
| $\% > 1.64$ | 14.4 | 6.80 | 5.35 | 5.75 | 5.90 | 5.00 |
| $\% > 1.96$ | 11.0 | 4.20 | 3.10 | 3.50 | 3.55 | 2.50 |
| $\% > 2.33$ | 7.55 | 2.25 | 1.95 | 1.70 | 1.60 | 1.00 |

However for large values of $m$ the finite sample property gets poor again. The reason is that there may be too few observations. In fact $W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon)-C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ and if we calculate for example $C_{1,N}(0.25)$ we find $C_{1,N}(0.25) = 0.14$. If $m=10$, $C_{1,N}(0.25)^{10} = 2.89255E-09$. If we compute $C_{1,N}(0.25)^{10}$ when we have for 1000 observations $C_{1,N}(0.25)^{10} = 0$, that is the probability to find pairs of 10-dimensional vectors within $\epsilon = 0.25$ is zero. The computed $W_{m,N}(\epsilon)$ becomes large and we spuriously reject the null *i.i.d.* hypothesis. Brock et al. (1991) suggest to keep the maximal value of $m$ around $\frac{N}{200}$. For $\frac{\epsilon}{\sigma} = 0.25$, $N=1000$ and $m=10$ we reject 95% the right null hypothesis instead of 1%. If we increase the ratio $\frac{\epsilon}{\sigma}$ to 1 we have very good results and the

$N(0,1)$ is a good approximation of the finite sample BDS distribution. This is why by increasing $\epsilon$ we also increase the probability to find vectors closer than $\epsilon$. However from the preceding tables increasing $\frac{\epsilon}{\sigma}$ beyond 2.0 is not generally recommended since the size of the BDS tends to be too small compared to the normal distribution (it would spuriously accept too often the null hypothesis). Mostly the choice of $\frac{\epsilon}{\sigma}=1$ or 1.5 gives a good size of the test. We have computed the BDS test for many different values of $\frac{\epsilon}{\sigma}$ between 2 and 0.25.

Size of BDS Statistic, Standard Normal

| $m=10, N=1000$ | $\epsilon/\sigma$ 0.25 | $\epsilon/\sigma$ 1.00 | $N(0,1)$ |
|---|---|---|---|
| $\% < -2.33$ | 95.0 | 0.40 | 1.00 |
| $\% < -1.96$ | 95.15 | 1.35 | 2.50 |
| $\% < -1.64$ | 95.4 | 3.85 | 5.00 |
| $\% > 1.64$ | 3.75 | 6.40 | 5.00 |
| $\% > 1.96$ | 3.70 | 3.90 | 2.50 |
| $\% > 2.33$ | 3.60 | 2.00 | 1.00 |

**Power**

The BDS test has asymptotic power against the following specific alternatives:

- first order autoregression $AR(1)$: $x_t=\rho x_{t-1}+\varepsilon_t$ , $|\rho|\leq 1$ and $\varepsilon_t \sim N(0,1)$

- first order moving average $MA(1)$: $x_t=\rho\varepsilon_{t-1}+\varepsilon_t$, $|\rho|\leq 1$ and $\varepsilon_t \sim N(0,1)$

- tent map: $x_t=2x_{t-1}$ if $x_t<0.5$ and $x_t=2-2x_{t-1}$ if $x_t>0.5$

- threshold autoregression $TAR(1)$ (Lim 1980[48]) : $x_t=\rho x_{t-1}+\varepsilon_t$ , $|\rho|\leq 1$ if $x_t \leq \overline{x}$ and $x_t=\varrho x_{t-1}+\varepsilon_t$ , $|\varrho|\leq 1$ if $x_t \geq \overline{x}$ and $\varepsilon_t \sim N(0,1)$

- non-linear moving average $NMA$ (Robinson 1977[49]): $x_t=\varepsilon_t+\varepsilon_{t-1}\varepsilon_{t-2}$ and $\varepsilon_t \sim N(0,1)$

- autoregressive conditional heteroskedasticity $ARCH$ (Engle 1982): $x_t = z_t\varepsilon_t$ and $z_t^2=z_0^2+\rho x_{t-1}^2$ and $\varepsilon_t \sim N(0,\sigma)$, $0<\rho<1$, $x_t$ has variance $\frac{z_0^2}{1-\rho}$

- generalized autoregressive conditional heteroskedasticity $GARCH$ (Bollerslev 1986): $x_t=z_t\varepsilon_t$ and $z_t^2=z_0^2+\rho x_{t-1}^2+\varrho z_{t-1}^2$ and $\varepsilon_t \sim N(0,1)$, $0<\rho+\varrho<1$, $x_t$ has variance $\frac{z_0^2}{1-\rho-\varrho}$

It rejects the null hypothesis of $i.i.d.$ with probability one for $0\leq\frac{\epsilon}{\sigma}\leq 2$.

For finite samples Monte Carlo simulations showed that (see Brock et al. 1991 for all the tables of the BDS test):

1) the BDS test has different power against different alternatives. As an extreme case see that for a sample size of 100 data points and a significance level of 1%, the power against a GARCH model is only 14.4%, that is the

[48] See Tong 1990.
[49] See Tong 1990.

probability to accept the alternative when this is the true one is only 14.4%. On the contrary in the case the time series is generated by a the tent map the power is maximal.

Power of BDS Statistic

| $m = 2, N = 100$ | $\epsilon/\sigma$ 1.00 | |
|---|---|---|
| | tent | GARCH |
| $\% > 1.64$ | 100% | 25.6% |
| $\% > 1.96$ | 100% | 20.2% |
| $\% > 2.33$ | 100% | 14.4% |

2) the BDS test increases its power with the sample size. As an example, notice that for N=1000, in the case of GARCH model, the power of the test increases to beyond 80%

Power of BDS Statistic

| $m = 2$ | | $\epsilon/\sigma$ 1.00 | |
|---|---|---|---|
| | GARCH $N = 100$ | GARCH $N = 500$ | GARCH $N = 1000$ |
| $\% > 1.64$ | 25.6% | 67.8% | 90.4% |
| $\% > 1.96$ | 20.2% | 58.9% | 85.9% |
| $\% > 2.33$ | 14.4% | 48.3% | 80.3% |

3) the BDS test increases its power with the embedding dimension but for a large embedding dimension the power of the test falls:

Power of BDS Statistic

| $N = 1000$ | | $\epsilon/\sigma$ 1.00 | |
|---|---|---|---|
| | GARCH $m = 2$ | GARCH $m = 5$ | GARCH $m = 10$ |
| $\% > 1.64$ | 90.4% | 98.9 | 0% |
| $\% > 1.96$ | 85.9% | 98.4% | 0% |
| $\% > 2.33$ | 80.3% | 97.2% | 0% |

4) The BDS test has good power properties against all the alternative considered when the number of data points is 1000. When the data points available are around 500, the BDS shows good power except for the GARCH alternative:

| | Power of BDS Statistic | | | | | | |
|---|---|---|---|---|---|---|---|
| $N = 1000$ | | | | $\epsilon/\sigma$ | | | |
| $m = 5$ | | | | 1.00 | | | |
| | $tent$ | $AR(1)$ | $MA(1)$ | $TAR$ | $NMA$ | $ARCH$ | $GARCH$ |
| $\% > 1.64$ | 100% | 100% | 100% | 98.8% | 100% | 100% | 98.9% |
| $\% > 1.96$ | 100% | 100% | 100% | 97.2% | 100% | 100% | 98.4% |
| $\% > 2.33$ | 100% | 100% | 100% | 94.5% | 100% | 100% | 97.2% |

| | Power of BDS Statistic | | | | | | |
|---|---|---|---|---|---|---|---|
| $N = 500$ | | | | $\epsilon/\sigma$ | | | |
| $m = 5$ | | | | 1.00 | | | |
| | $tent$ | $AR(1)$ | $MA(1)$ | $TAR$ | $NMA$ | $ARCH$ | $GARCH$ |
| $\% > 1.64$ | 100% | 100% | 99.6% | 98.8% | 100% | 100% | 87.4% |
| $\% > 1.96$ | 100% | 100% | 99.3% | 97.2% | 100% | 100% | 83.0% |
| $\% > 2.33$ | 100% | 100% | 98.5% | 94.5% | 99.6% | 99.9% | 76.6% |

5) Comparing the power results of the BDS test over a 500 data points time series to that of other non-linear tests, specifically the Tsay and Engle tests, the BDS test performs better or similar to these tests. The Engle test performs slightly better than the BDS tests in the case of GARCH structures. However the BDS contrary to the Engle test (which look for non-zero autocovariances) is able to detect non-linearities independently from the value of autocovariances.

### g) Liapunov exponents

The Liapunov exponent quantifies the sensitive dependence on initial conditions (states). Take for example a one dimensional dynamic system $x_t = f(x_{t-1})$ like the tent map:

$$x_t = 2x_{t-1} \text{ for } x_{t-1} < 0.5$$
$$x_t = 2(1 - x_{t-1}) \text{ for } x_{t-1} < 0.5$$

We know that for the tent map, given an initial state $x_0$, there will correspond one and only one $x_1$between 0 and 1. If the process were uniformly distributed between 0 and 1, $x_1$ could assume any value between 0 and 1 with the same probability. In the case of the tent map, $x_1$ has only one specific correspondent $x_{t+1}$ between 0 and 1. This means that the system is dependent on initial conditions.

If we take another possible initial state $x_0 + \epsilon_0$ close to $x_0$, $f(x_0 + \epsilon_0)$ will be still close to $f(x_0)$, but it will be more distant than $x_0 + \epsilon_0$ from $x_0$. After some periods the two orbits will appear to be totally uncorrelated. This is because the two orbits are divergent. The system is characterized by *sensitive* dependence because two nearby initial states lead to two different orbits which are divergent.

The Liapunov exponent measures the average rate of divergence of nearby initial states.

After $N$ periods, the distance between the two orbits is (Hommes 1998, Kantz and Schreiber 1997):

$$\left|f^N(x_0+\epsilon_0)-f^N(x_0)\right| \approx \left|\left(f^N\right)'(x_0)\,\epsilon_0\right|$$

If we denote with $\epsilon_N$ the distance at time $N$ between the two orbits we may define the exponential divergence of nearby orbits as:

$$\epsilon_N = \epsilon_0 e^{\lambda N}$$

$\epsilon_N = \epsilon_0$ when $\lambda = 0$, that is the case of a cyclical series or a steady state

$\epsilon_N < \epsilon_0$ when $\lambda < 0$, that is the case of convergent series towards a a steady state

$\epsilon_N > \epsilon_0$ when $\lambda > 0$, that is the case of divergent series

$$\left|f^N(x_0+\epsilon_0)-f^N(x_0)\right| \approx \left|\left(f^N\right)'(x_0)\,\epsilon_0\right| = \epsilon_0 e^{\lambda N}$$
$$\left|\left(f^N\right)'(x_0)\right| = e^{\lambda N} \longrightarrow \lambda = \tfrac{1}{N}\ln\left|\left(f^N\right)'(x_0)\right|$$

Using the chain rule for $\left(f^N\right)'(x_0)$ and taking the limit $t \to \infty$

$$\lambda = \lim_{N\to\infty} \frac{1}{N}\sum_{i=0}^{N-1} \ln\left|f'(f^i(x_0))\right|$$

$\lambda$ is the Liapunov exponent. Notice that if we have a positive $\lambda$, say $\lambda = \ln 2$, after 10 periods

$$e^{10\ln 2} = 2^{10} = 1024 \longrightarrow \left|f^{10}(x_0+\epsilon_0)-f^{10}(x_0)\right| = 1024\epsilon_0$$

that is after 10 periods the distance between $f^{10}(x_0+\epsilon_0)$ and $f^{10}(x_0)$ is on average 1024 times greater than $f^0(x_0+\epsilon_0)$ and $f^0(x_0)$. If the initial true value were $x_0+\epsilon_0$ and we had measured $x_0$, after 10 periods we would have an amplified error on average 1024 times greater than the initial error. The larger the Liapunov exponent the more difficult the prediction is, and so the Liapunov exponent is a measure of predictability.

When we have a time series, we do not know the true function $f$, but we have its realizations, that is a time series. We have used the following algorithm by Kantz (1994)[50] to compute numerically the Liapunov exponent, directly from the time series without knowing the true or an estimated function of $f$.

Suppose to observe a point (or vector) $x_j$ that is very close to $x_i$. $x_j$ and $x_i$ are the observed values of the underlying function. Take the distance between these two observations $\epsilon_i = x_j - x_i$. $\epsilon_i$ grows exponentially with time.

After $N$ periods, the distance between the two points is:

[50] See Kantz, Schreiber (1997).

$$|x_{N+j} - x_{N+i}| = |\epsilon_{N+i}| = \epsilon_i e^{\lambda N}.$$
$$\left|\left(f^N\right)'(x_i)\right| = e^{\lambda N}$$
$$\lambda = \frac{1}{N}\ln\left|\left(f^N\right)'(x_i)\right|$$

$\lambda$ is the value of the Liapunov exponent. Since from one single time series can define as many different Lyapunov exponents as the number of embedding dimension $m$, we can restrict ourself to the maximal Liapunov exponent that is the most relevant for our analysis. Numerically one can derive a robust consistent and unbiased estimator for the maximal Liapunov exponent (Kantz and Schreiber 1997). One computes:

$$\Phi = \frac{1}{N}\sum_{i=0}^{N-1}\ln\left(\frac{1}{|\xi(x_i)|}\sum_{x\in\xi(x_i)}|x_{j+n} - x_{i+n}|\right)$$

$\xi(x_i)$ is the neighborhood of $x_i$ with radius $\epsilon_i$. $|\xi(x_i)|$ denotes the number of observed values within the neighborhood of $x_i$. $n$ is the number of iteration. $\Phi$ varies with $n$ and its slope gives an estimate of the Liapunov exponent.

### h) recurrence plots

A recurrence plot $(\mathbf{x}_i, \mathbf{x}_j)$ is a graphical representation of the euclidean distance $\|\mathbf{x}_i - \mathbf{x}_j\|$ in the correlation integral in two dimensions.

It is easy to produce a recurrence plot via an ordinary excel program or alike for the $\chi$ function in the correlation integral:

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 1$ if $\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon$ and we give to the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ in the recurrence plot the color white

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 0$ if $\|\mathbf{x}_i - \mathbf{x}_j\| \geq \epsilon$ and we give to the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ in the recurrence plot the color black.

When $\|\mathbf{x}_i - \mathbf{x}_j\| = 0$ the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ is white. Along the $45^0$ line $\mathbf{x}_i \equiv \mathbf{x}_j$ so that the $45^0$ line $\mathbf{x}_i\mathbf{x}_j$ is white.

When $\|\mathbf{x}_i - \mathbf{x}_j\|$ is maximal, $(\mathbf{x}_i, \mathbf{x}_j)$ is black.

When $0 < \|\mathbf{x}_i - \mathbf{x}_j\| < 1$, the $(\mathbf{x}_i, \mathbf{x}_j)$ assumes a grey tone proportional to the euclidean distance.

For random signals, the uniform distribution of grey tones over the entire plot is expected. For non-linear systems a more structured recurrence plot may be dominant. Any continuous line and zones characterized by the same grey tone in the plot indicates the existence of correlation between pair of the $m$-dimensional points $(\mathbf{x}_i, \mathbf{x}_j)$ since they maintain a similar euclidean distance.

### h) Kononov entropy

The Kononov Entropy compares the distribution of distances between all pairs of m embedded vectors over the entire recurrence plot with the distribution of grays over each diagonal line of the recurrence plot. The result is normalized and presented as a percentage of "maximum" entropy (randomness). That is, 100% entropy means the absence of any structure (uniform distribution of greys), while other values of entropy implies the stochastic or deterministic presence of structures.

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| Tab.1 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Authors | Y | Y/N | Y ind. | Y agr. | GNP | GNP/N | Exp. | S | I | E | U | r | exch. r. | c.p.i. | M | dM/dt | country |
| Nelson, Plosser 82 | | | | | + | + | | | | + | = | | | + | | | Usa |
| Lee, Siklos 91 | | | | | + | + | | | | + | = | | | + | | | C |
| Coorey 91 | | | | | | | | | + | | | | | | | | Usa |
| Mills 92 | + | | | | | | | | | | | | | | | | UK |
| Banerjee et al. 92 | | | | | + | + | | | | + | + | | | + | | | 7 OECD, J |
| Fung, Lo 92 | | | | | | | | | | | | | + | | | | Usa |
| Durlauf 93 | + | | | | | | | | | | | | | | | | Usa |
| Parikh 94 | | | | | | | | | | | | | + | | | | J, UK, G |
| Moca 94 | | | | | + | | | | | | | | | | | | Usa |
| Gamber, Sorensen 94 | | | | | | | | | | | | + | | | | | Usa |
| Haslag et al. 94 | | | | | | | | | | | | + | | | | | Usa |
| de Haan, Zelhorst 94 | + | | | | | | | | | | | | | | | | I |
| Mc Dougall 95 | | | | | + | + | | | | + | = | | | + | | | NZ |
| Serletis 94 | | | | | | | | | | | | | | | | + | Usa |
| Bresson, Celimene 95 | | | | | | | | | | | + | | | | | | Caraibi |
| Wu, Crato 95 | | | | | | | | | | | | | + | | | | NZ |
| Franses, Kleibergen 96 | | | | | + | + | | | | + | = | | | + | | | Usa |
| Gallegati 96 | + | | + | + | | | | | | | | | | | | | I |
| Serletis, Zimonopoulos 97 | | | | | | | | | | | | | + | | | | 17 OECD |
| Wells 97 | | | | | + | + | | | | + | = | | | + | | | Usa |
| Sosa 97 | + | | | | | | | | | | | | | | | | Arg. |
| Nunes, Newbold, Kuan 97 | | | | | + | + | | | | + | = | | | + | | | Usa |
| Rahaman, Mustafa' 97 | + | | | | | | + | | | | | | | | | | Asia |
| Bohl 98 | | + | | | | | | | | | | | | | | | G7 |
| Weliwita 98 | | | | | | | | | | | | | + | | | | Asia |
| Al Bazai 98 | | | | | | | | | | | | | | | + | | Arabia |
| Choi, Yu 97 | | | | | | | | | | | | | | + | | | OECD |
| Dolado, Lopez 96 | | | | | | | | | | | + | | | | | | Spagna |
| Coakley, Kulasi, Smith 95 | | | | | | | | + | + | | | | | | | | OECD |
| Osborn, Heravi, Birchenhall 99 | | | + | | | | | | | | | | | | | | G, F, UK |
| Leybourne, Mc Cabe 99 | | | | | | | | | | | + | | | | | | Usa |

Y= GDP, Y/N GDP per capita, Y ind.= industrial production, Y agr.= agricolture,
Exp.= exportations, S= savings, I= investments, E= employment, U= unemployment rate, r= interest rate, exch.
r. = exchange rate, c.p.i.=consumer price index, M= money, dM/dt= velocity of money; C= Canada,
J= Japan, I=Italy, NZ= New Zealand, Arg.=Argentina, G=Germany, F=France

| Tab. 2 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Authors | Y | Y/N | Y ind. | Y distr. | GNP | GNP/N | E | U | w | r | exch. r. | c.p.i. | country |
| Rudebush 90 | | | | | - | - | - | - | | | | - | Australia |
| Diebold, Rudebush 89 | - | | | | | | | | | | | | Usa |
| Perron 89 | | | | | - | - | - | - | | | | - | Usa |
| Gokey 90 | | | | | | | | | | - | | | 8 OECD |
| Hansen 91 | | | | | | | | - | | | | | UK |
| Duck 92 | - | | | | | | | | | | | | 9 paesi |
| Capitelli, Scjlegel 91 | | | | | | | | | | - | | | 6 paesi |
| Raj 92 | | | | | - | - | - | - | | | | - | Usa, C, F, DK |
| Rudebush 92 | | | | | - | - | - | - | | | | - | G |
| Kwaiatkowski et al. 92 | | | | | - | - | - | - | | | | - | Usa |
| Linden 92 | | | | | - | - | - | - | | | | - | Finlandia |
| Simkins 94 | | | | | | | - | | - | | | | Usa |
| Raj, Scottje 94 | | | | - | | | | | | | | | Usa |
| Caselli, Marinelli 94 | | | | | - | | | | | | | | I |
| Wu, Chen 95 | | | | | - | - | - | - | | | | - | Taiwan |
| Ben, David, Papell 94 | - | | | | | | | | | | | | 16 paesi |
| Culver Papell 95 | | | | | | | | | | | - | | OECD |
| Soejima 95 | | | | | - | - | - | - | | | | - | J |
| Li 95 | | | - | | | | | | | | | | Usa |
| Alba, Papell 95 | - | - | | | | | | | | | | | Nics |
| Leslie, Pu, Wharton 95 | | | | | | | | | | - | | | UK |
| Wu 96 | | | | | | | | | | | - | | Usa |
| Donald 96 | | | | | | | | | | | - | | OECD |
| Lee 96 | | | | | - | - | - | - | | | | - | Usa |
| Wu, Zhang 96 | | | | | | | | | | - | | | OECD |
| Moosa, Bhatti 96 | | | | | | | | | | - | | | Asia |
| Song, Wu 97 | | | | | | | | - | | | | | 48 stati Usa |
| Lumsdaine, Papell 97 | | | | | - | - | - | - | | | | - | Usa |
| Gil, Robinson 97 | | | - | | | | | - | | | | | Usa |
| Fleissing, Strauss 97 | | | | | | | | | - | | | | G7 |
| Mc Coskey, Selden 98 | - | | | | | | | | | | | | OECD |
| Song, Wu 98 | | | | | | | | - | | | | | OECD |
| Hylleberg, Engle 96 | | | | | | | | - | | | | | OECD |
| Cheung, Chinn 96 | | | | | - | | | | | | | | Usa |
| Dolmas, Raj, Scottje 99 | | - | | | | | | | | | | | Usa |

Y= GDP, Y/N GDP per capita, Y ind.= industrial production, Y distr=income distribution, w=wages
Exp.= exportations, E= employment, U= unemployment rate, r= interest rate, w= wages
exch. r. = exchange rates, c.p.i.=consumer price index, C= Canada,
J= Japan, I=Italy, NZ= New Zealand, Arg.=Argentina, G=Germany, F=France, DK= Denmark

| | | | | |
|---|---|---|---|---|
| Tavola 3: deterministic trend | | | | |
| ADF Test Statistic | -21.2812 | 1% Critical Value* | | -3.972 |
| | | 5% Critical Value | | -3.4166 |
| | | 10% Critical Value | | -3.1303 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Augmented Dickey-Fuller Test Equation | | | | |
| LS // Dependent Variable is D(SER04) | | | | |
| Date: 01/04/00 Time: 16:30 | | | | |
| Sample(adjusted): 4 1024 | | | | |
| Included observations: 1021 after adjusting endpoints | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| SER04(-1) | -0.934349 | 0.043905 | -21.2812 | 0 |
| D(SER04(-1)) | -0.049395 | 0.031317 | -1.577254 | 0.115 |
| C | 4.314082 | 0.201308 | 21.43022 | 0 |
| TREND(1) | 0.018503 | 0.000869 | 21.2812 | 0 |
| R-squared | 0.492919 | Mean dependent var | | 0.019791 |
| Adjusted R-squared | 0.491423 | S.D. dependent var | | 0.007961 |
| S.E. of regression | 0.005678 | Akaike info criterion | | -10.33855 |
| Sum squared resid | 0.032783 | Schwarz criterion | | -10.31924 |
| Durbin-Watson stat | 2.000879 | | | |
| Entropy of levels | 0% | Entropy of residuals | | 90% |
| BDS statistic | -1.28 | 5% Critical Value | | 1.96 |

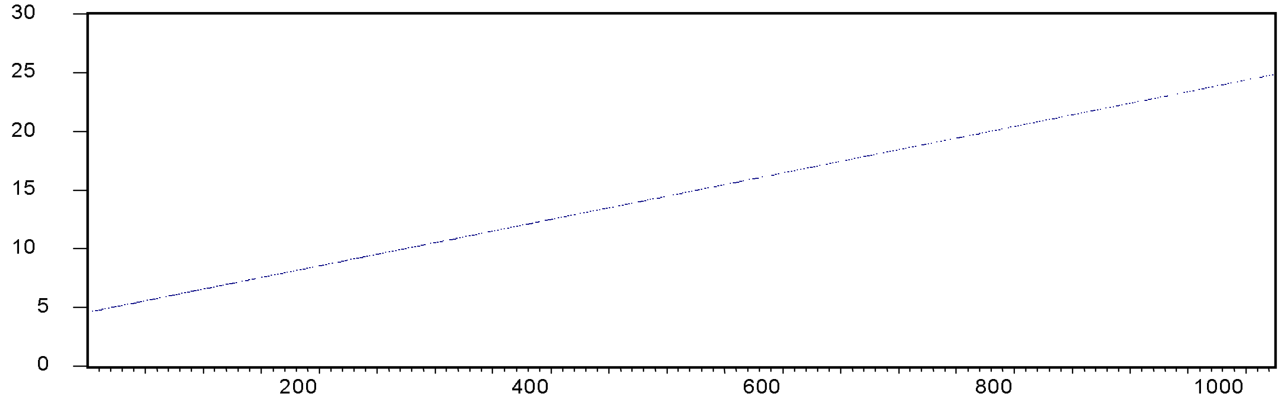
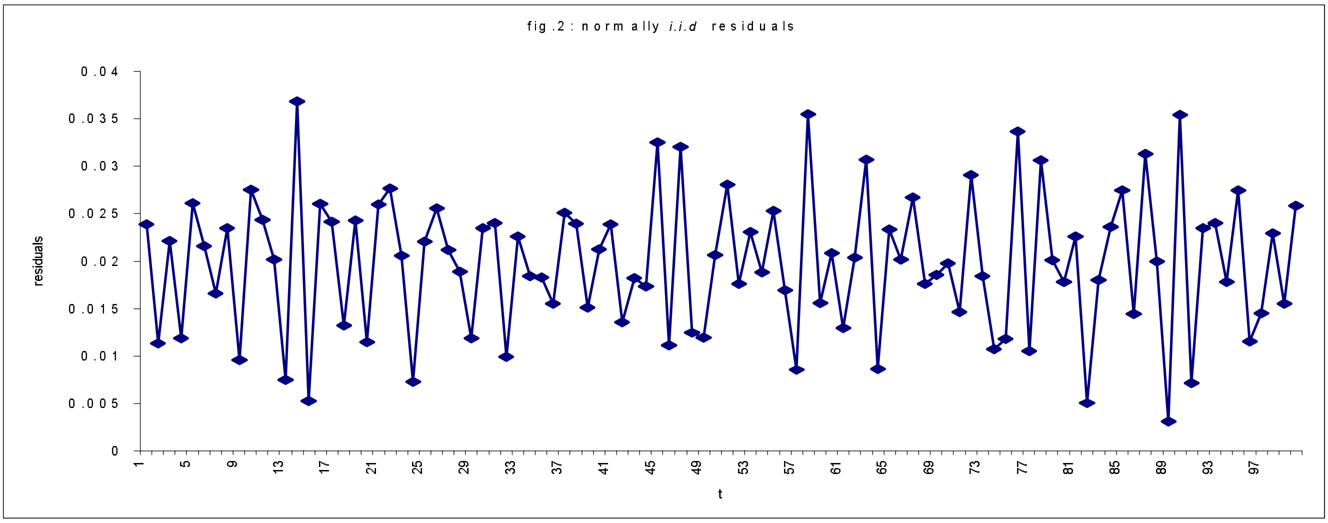
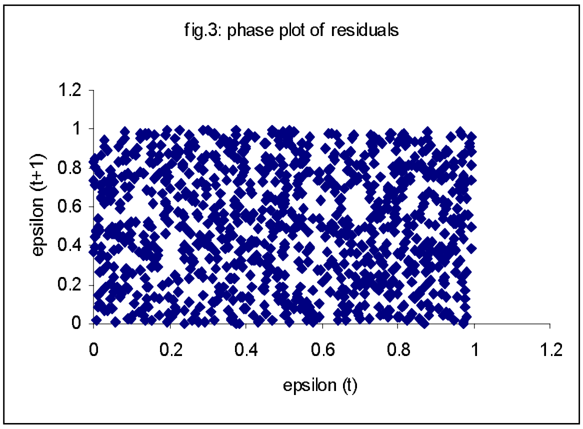



fig.1: trend stationary growth



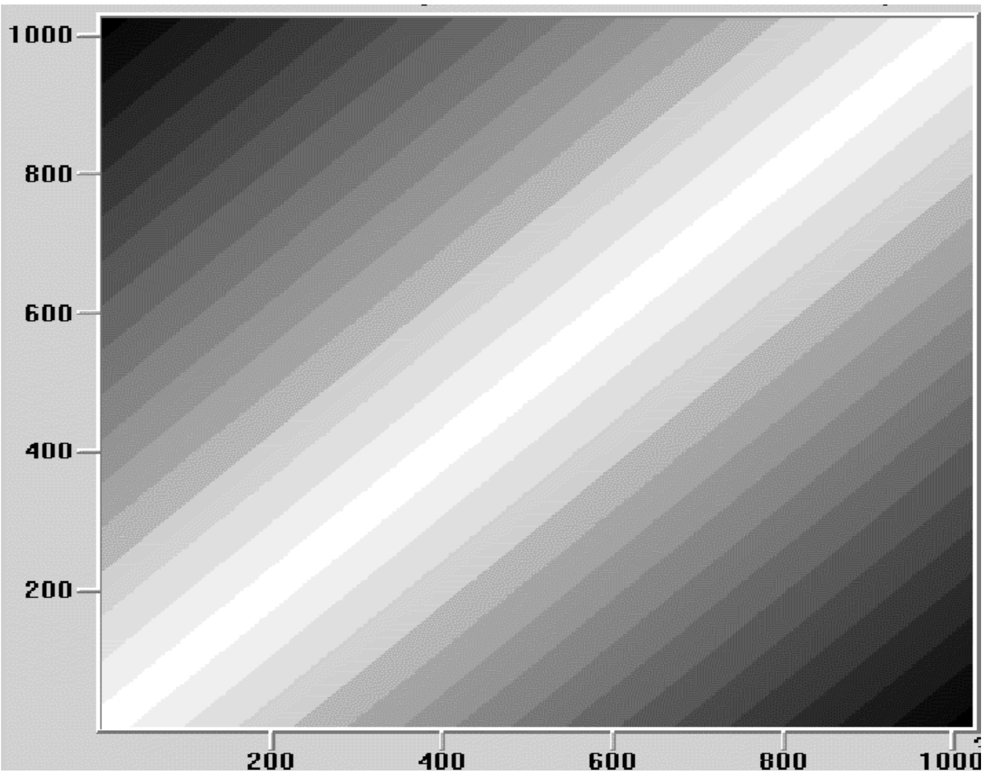



Fig.4: recurrence plot of a linear trend

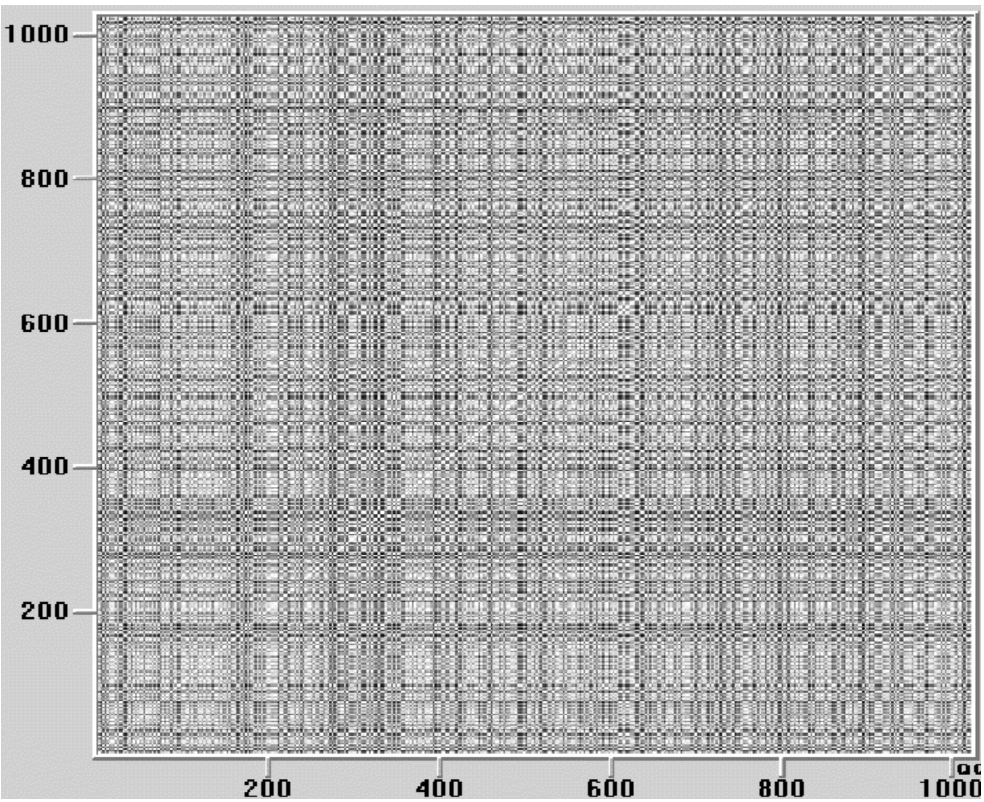



Fig.5: recurrence plot of *i.i.d.* residuals

| Tab 4: random walk | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -1.981912 | 1% Critical Value* | | -3.972 |
| | | 5% Critical Value | | -3.4166 |
| | | 10% Critical Value | | -3.1303 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Augmented Dickey-Fuller Test Equation | | | | |
| LS // Dependent Variable is Y(t)-Y(t-1) | | | | |
| Date: 01/06/00 Time: 03:53 | | | | |
| Sample(adjusted): 3 1024 | | | | |
| Included observations: 1022 after adjusting endpoints | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | -0.007941 | 0.004007 | -1.981912 | 0.0478 |
| Y(t-1)-Y(t-2) | -0.043856 | 0.031348 | -1.399032 | 0.1621 |
| Intercept | 0.056553 | 0.018177 | 3.111216 | 0.0019 |
| TREND | 0.000157 | 7.93E-05 | 1.984763 | 0.0474 |
| R-squared | 0.006147 | Mean dependent var | | 0.019756 |
| Adjusted R-squared | 0.003218 | S.D. dependent var | | 0.011278 |
| S.E. of regression | 0.01126 | Akaike info criterion | | -8.969137 |
| Sum squared resid | 0.129064 | Schwarz criterion | | -8.949843 |
| Durbin-Watson stat | 1.992604 | | | |
| BDS stat | -1.55 | 5% Critical Value | | -1.96 |

fig.6: random walk growth

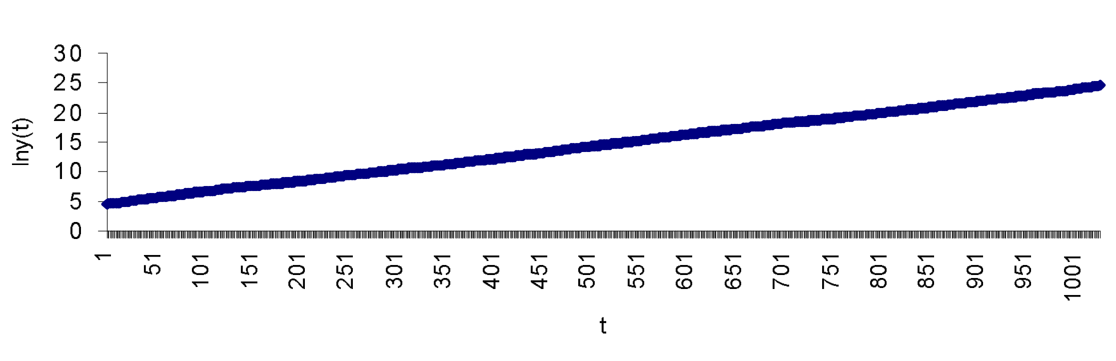



fig. 7: tent map growth

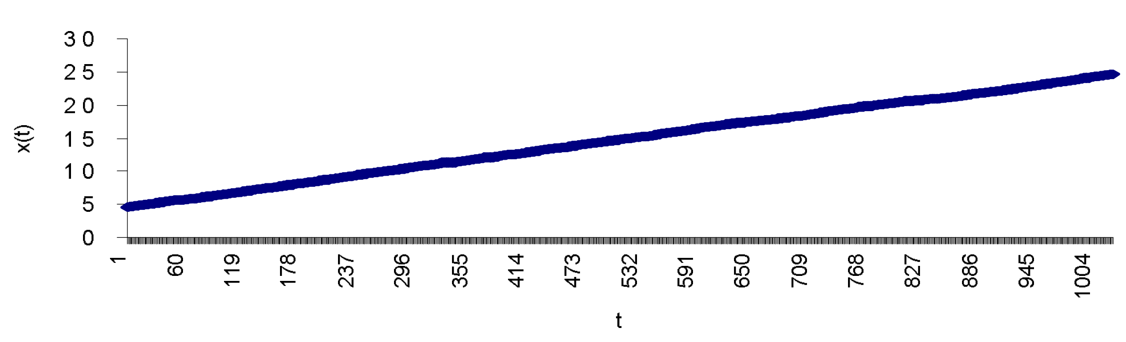
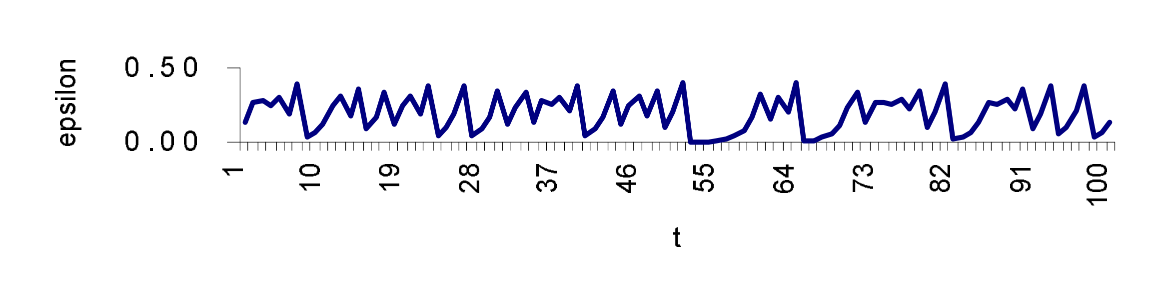



fig. 8: tent growth residuals



| Tab 5: Tent map growth | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -1.793037 | 1% Critical Value* | | -3.9721 |
| | | 5% Critical Value | | -3.4166 |
| | | 10% Critical Value | | -3.1303 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Augmented Dickey-Fuller Test Equation | | | | |
| LS // Dependent Variable is Y(t)-Y(t-1) | | | | |
| Date: 01/07/00 Time: 21:51 | | | | |
| Sample(adjusted): 3 1019 | | | | |
| Included observations: 1017 after adjusting endpoints | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | -0.005239 | 0.002922 | -1.793037 | 0.0733 |
| Y(t-1)-Y(t-2) | 0.040709 | 0.031371 | 1.297662 | 0.1947 |
| Intercept | 0.044087 | 0.013835 | 3.18672 | 0.0015 |
| TREND | 0.000101 | 5.69E-05 | 1.775155 | 0.0762 |
| R-squared | 0.005345 | Mean dependent var | | 0.019587 |
| Adjusted R-squared | 0.002399 | S.D. dependent var | | 0.011415 |
| S.E. of regression | 0.011401 | Akaike info criterion | | -8.94416 |
| Sum squared resid | 0.131676 | Schwarz criterion | | -8.924791 |
| Durbin-Watson stat | 2.000374 | Prob(F-statistic) | | 0.14281 |
| Entropy on residuals | 78% | | | |
| BDS stat | 99.2 | 5% Critical Value | | -1.96 |

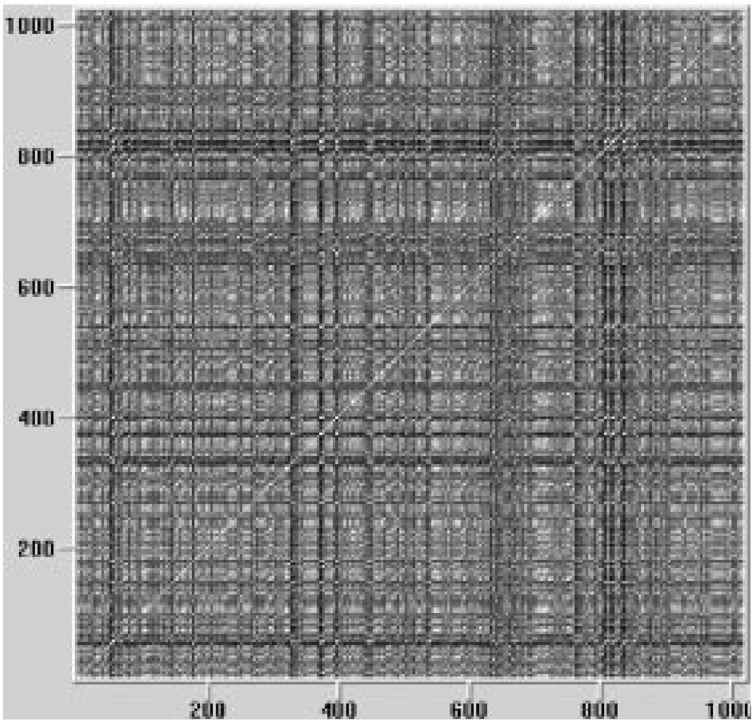



Fig.9: recurrence plot of "tent map" residuals

Fig 10: Rossler growth

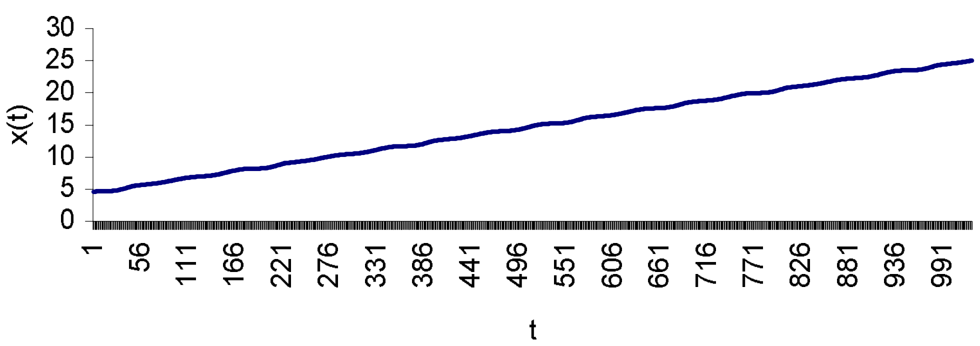
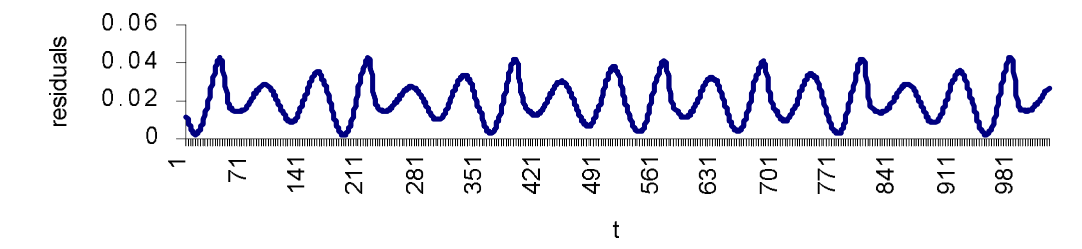


Fig. 11: rossler residuals




| | | | | |
|---|---|---|---|---|
| Tab 6: Rossler growth | | | | |
| ADF Test Statistic | -57.52551 | 1% Critical Value* | | -3.972 |
| | | 5% Critical Value | | -3.4166 |
| | | 10% Critical Value | | -3.1303 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Augmented Dickey-Fuller Test Equation | | | | |
| LS // Dependent Variable is Y(t)-Y(t-1) | | | | |
| Date: 01/08/00 Time: 00:42 | | | | |
| Sample(adjusted): 3 1024 | | | | |
| Included observations: 1022 after adjusting endpoints | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | -0.010692 | 0.000186 | -57.52551 | 0 |
| Y(t-1)-Y(t-2) | 0.997213 | 0.001786 | 558.3107 | 0 |
| Intercept | 0.047578 | 0.000826 | 57.61396 | 0 |
| TREND | 0.000215 | 3.74E-06 | 57.51543 | 0 |
| R-squared | 0.996759 | Mean dependent var | | 0.019975 |
| Adjusted R-squared | 0.996749 | S.D. dependent var | | 0.010042 |
| S.E. of regression | 0.000573 | Akaike info criterion | | -14.92687 |
| Sum squared resid | 0.000334 | Schwarz criterion | | -14.90758 |
| Durbin-Watson stat | 0.093964 | | | |
| Entropy on residuals | 15% | | | |
| BDS stat | 355 | 5% Critical Value | | -1.96 |

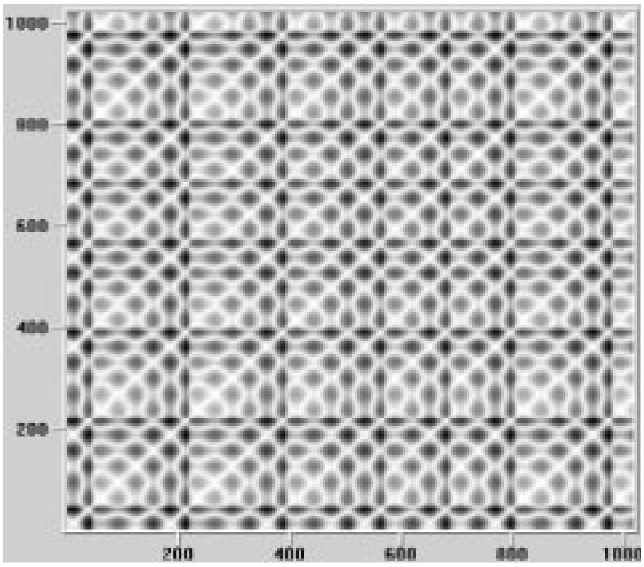



Fig 12: recurrence plot of Rossler growth residuals

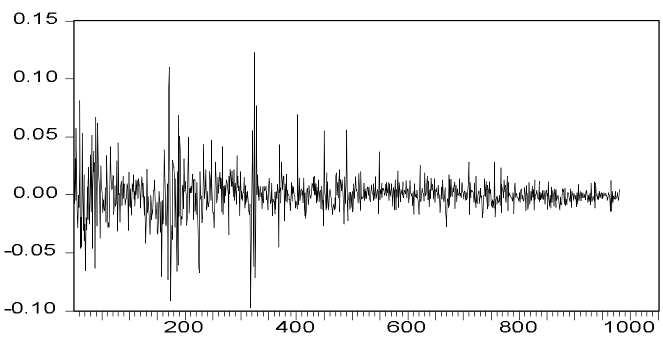



Fig. 13 industrial production residuals

| Tab 7 | | Random residuals | | | | |
|---|---|---|---|---|---|---|
| | | N=160 | SD/Spread=0.23988 | | | |
| $\varepsilon$ | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ |
| 0.3 | 2 | 7548 | 6686 | 2.27E+00 | 3.51E-02 | 6.48E+01 |
| 0.3 | 4 | 7548 | 5466 | 4.04E+00 | 4.71E-02 | 8.58E+01 |
| 0.3 | 6 | 7548 | 4502 | 4.15E+00 | 3.36E-02 | 1.24E+02 |
| 0.3 | 8 | 7548 | 3745 | 3.79E+00 | 1.92E-02 | 1.97E+02 |
| 0.3 | 10 | 7548 | 3138 | 3.34E+00 | 9.88E-03 | 3.38E+02 |
| 0.235801 | 2 | 6184 | 5268 | 2.31E+00 | 2.44E-02 | 9.47E+01 |
| 0.235801 | 4 | 6184 | 3981 | 3.37E+00 | 2.21E-02 | 1.53E+02 |
| 0.235801 | 6 | 6184 | 3031 | 3.00E+00 | 1.06E-02 | 2.83E+02 |
| 0.235801 | 8 | 6184 | 2310 | 2.42E+00 | 4.10E-03 | 5.89E+02 |
| 0.235801 | 10 | 6184 | 1792 | 1.94E+00 | 1.42E-03 | 1.36E+03 |
| 0.18534 | 2 | 4982 | 3993 | 2.07E+00 | 1.60E-02 | 1.30E+02 |
| 0.18534 | 4 | 4982 | 2651 | 2.42E+00 | 9.46E-03 | 2.56E+02 |
| 0.18534 | 6 | 4982 | 1735 | 1.76E+00 | 2.97E-03 | 5.95E+02 |
| 0.18534 | 8 | 4982 | 1162 | 1.23E+00 | 7.50E-04 | 1.64E+03 |
| 0.18534 | 10 | 4982 | 798 | 8.65E-01 | 1.70E-04 | 5.10E+03 |

| Tab 8: industrial production | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -3.054169 | 1% Critical Value* | | -3.9724 |
| | | 5% Critical Value | | -3.4167 |
| | | 10% Critical Value | | -3.1304 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Sample(adjusted): 3 982 | | | | |
| Included observations: 980 after adjusting endpoints.<br>White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: Y(t) = 0.02+0.99*Y(t-1) + 0.51*(Y(t-1)-Y(t-2)+2.98E-05*t+$\varepsilon_t$ | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 0.990541 | 0.005035 | 196.7389 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.515856 | 0.058228 | 8.859306 | 0.0000 |
| Intercept | 0.020007 | 0.010582 | 1.890640 | 0.0590 |
| TREND | 2.98E-05 | 1.52E-05 | 1.959742 | 0.0503 |
| R-squared | 0.999633 | Mean dependent var | | 3.512698 |
| Adjusted R-squared | 0.999632 | S.D. dependent var | | 0.926381 |
| S.E. of regression | 0.017778 | Akaike info criterion | | -5.217661 |
| Sum squared resid | 0.308465 | Schwarz criterion | | -5.197712 |
| Durbin-Watson stat | 1.95 | 5% Critical Value<br>3% Critical Value<br>1% Critical Value | 2.10<br>2.13<br>2.15 | 1.89<br>1.87<br>1.85 |
| Entropy on residuals | 80% | | | |

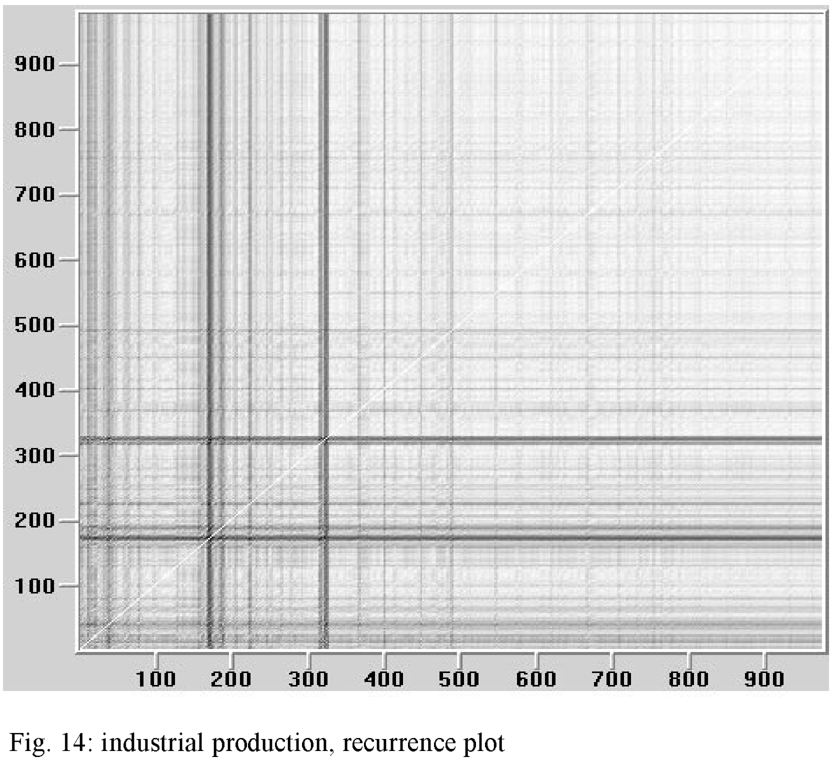



Fig. 14: industrial production, recurrence plot

| Tab 9: industrial production | | | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =981 | SD/Spread | 0.0778 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 417276 | 374424 | 0.84 | 0.07 | 11.27 | 0.13 |
| 0.15 | 4 | 417276 | 320652 | 3.23 | 0.21 | 15.14 | 0.21 |
| 0.15 | 6 | 417276 | 283148 | 5.22 | 0.32 | 16.18 | 0.28 |
| 0.15 | 8 | 417276 | 253273 | 6.62 | 0.39 | 16.83 | 0.34 |
| 0.15 | 10 | 417276 | 229020 | 7.55 | 0.43 | 17.64 | 0.39 |
| 0.10 | 2 | 366655 | 301100 | 1.44 | 0.10 | 13.87 | 0.21 |
| 0.10 | 4 | 366655 | 229816 | 4.46 | 0.23 | 19.40 | 0.32 |
| 0.10 | 6 | 366655 | 186404 | 6.09 | 0.27 | 22.46 | 0.42 |
| 0.10 | 8 | 366655 | 154549 | 6.60 | 0.26 | 25.64 | 0.50 |
| 0.10 | 10 | 366655 | 129222 | 6.46 | 0.22 | 29.49 | 0.58 |
| 0.07 | 2 | 299137 | 211787 | 1.70 | 0.11 | 15.15 | 0.31 |
| 0.07 | 4 | 299137 | 126771 | 3.61 | 0.17 | 21.52 | 0.50 |
| 0.07 | 6 | 299137 | 84243 | 3.72 | 0.13 | 27.83 | 0.66 |
| 0.07 | 8 | 299137 | 58473 | 3.15 | 0.09 | 36.70 | 0.79 |
| 0.07 | 10 | 299137 | 41336 | 2.46 | 0.05 | 49.79 | 0.92 |

| Tab 10: indudrial production *after randomization* | | | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | Num | Obs : N =972 | SD/Spread | 0.078183 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 407865 | 352843 | 0.07 | 0.08 | 0.92 | 0.15 |
| 0.15 | 4 | 407865 | 266237 | 0.30 | 0.21 | 1.41 | 0.30 |
| 0.15 | 6 | 407865 | 198895 | 0.28 | 0.32 | 0.88 | 0.46 |
| 0.15 | 8 | 407865 | 150347 | 0.37 | 0.39 | 0.95 | 0.60 |
| 0.15 | 10 | 407865 | 114661 | 0.46 | 0.42 | 1.11 | 0.75 |
| 0.10 | 2 | 356288 | 269396 | 0.06 | 0.10 | 0.61 | 0.25 |
| 0.10 | 4 | 356288 | 154845 | 0.16 | 0.23 | 0.72 | 0.49 |
| 0.10 | 6 | 356288 | 87165 | 0.04 | 0.26 | 0.17 | 0.74 |
| 0.10 | 8 | 356288 | 50171 | 0.07 | 0.24 | 0.29 | 0.98 |
| 0.10 | 10 | 356288 | 30078 | 0.15 | 0.20 | 0.73 | 1.21 |
| 0.07 | 2 | 288555 | 176733 | 0.04 | 0.11 | 0.39 | 0.37 |
| 0.07 | 4 | 288555 | 66483 | 0.06 | 0.16 | 0.38 | 0.74 |
| 0.07 | 6 | 288555 | 24021 | -0.03 | 0.12 | -0.21 | 1.12 |
| 0.07 | 8 | 288555 | 9039 | 0.00 | 0.08 | -0.04 | 1.49 |
| 0.07 | 10 | 288555 | 3582 | 0.01 | 0.04 | 0.31 | 1.84 |

| Tab 11: transportation equipment production | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -3.90 | 1% Critical Value* | | -3.97 |
| | | 5% Critical Value | | -3.42 |
| | | 10% Critical Value | | -3.13 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:08 | | | | |
| Sample(adjusted): 3 633 | | | | |
| Included observations: 631 after adjusting endpoints<br>White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: $Y(t) = 0.13+0.96*Y(t-1) + 9.07E-05*t+\varepsilon$ | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 0.961671 | 0.011496 | 83.65159 | 0.0000 |
| Intercept | 0.130879 | 0.039174 | 3.340986 | 0.0009 |
| TREND | 9.07E-05 | 2.82E-05 | 3.221789 | 0.0013 |
| R-squared | 0.994651 | Mean dependent var | | 4.088223 |
| Adjusted R-squared | 0.994634 | S.D. dependent var | | 0.486078 |
| S.E. of regression | 0.035606 | Akaike info criterion | | -3.827893 |
| Sum squared resid | 0.798694 | Schwarz criterion | | -3.806800 |
| Durbin-Watson stat | 1.86 | 5% Critical Value | 2.13 | 1.86 |
| | | 3% Critical Value | 2.16 | 1.84 |
| | | 1% Critical Value | 2.19 | 1.81 |
| Entropy | 73% | | | |

| Tab 18: maximal liapunov exponents | M=1 | M=2 | M=3 | M=4 | M=5 |
|---|---|---|---|---|---|
| Normal i.i.d. process | 3.40 | 1.41 | 1.24 | 0.77 | 0.85 |
| Tent map | 2.93 | 0.91 | 0.54 | 0.36 | 0.33 |
| Rossler map | 0.67 | 0.06 | 0.1 | 0.09 | 0.11 |
| Industrial production | 2.68 | 0.75 | 0.39 | 0.33 | 0.26 |
| transportation eq. production | 1.71 | 0.60 | 0.44 | 0.36 | 0.36 |
| industrial machinery and eq. | 1.75 | 0.64 | 0.33 | 0.28 | 0.30 |
| electrical machinery | 1.81 | 0.49 | 0.3 | 0.26 | 0.26 |
| Hi-Tech | 1.59 | 0.46 | 0.27 | 0.21 | 0.25 |
| employment | 1.55 | 0.67 | 036 | 0.30 | 0.27 |
| hourly earnings | 1.81 | 0.7 | 0.51 | 0.39 | 0.36 |
| consumer price index | 1.88 | 0.93 | 0.58 | 0.45 | 0.36 |

| Tab 12: industrial machinery production | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -3.80 | 1% Critical Value* | | -3.98 |
| | | 5% Critical Value | | -3.42 |
| | | 10% Critical Value | | -3.13 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:16 | | | | |
| Sample(adjusted): 3 632 | | | | |
| Included observations: 630 after adjusting endpoints. Newey-West HAC Standard Errors & Covariance (lag truncation=6)<br>Estimated equation:<br>Y(t)=0.05+Y(t-1)+(0.98-1)Y(t-1)+0.09(Y(t-1)-Y(t-2))+ 0.29(Y(t-2)-Y(t-3))+ 0.26(Y(t-3)-Y(t-4))+7.58E-05*t | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 0.981976 | 0.004596 | 213.6797 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.092080 | 0.048112 | 1.913847 | 0.0561 |
| Y(t-2)-Y(t-3) | 0.297701 | 0.038453 | 7.741935 | 0.0000 |
| Y(t-3)-Y(t-4) | 0.266935 | 0.038098 | 7.006599 | 0.0000 |
| Intercept | 0.046728 | 0.012004 | 3.892819 | 0.0001 |
| TREND | 7.58E-05 | 1.81E-05 | 4.183524 | 0.0000 |
| R-squared | 0.999672 | Mean dependent var | | 3.854838 |
| Adjusted R-squared | 0.999669 | S.D. dependent var | | 0.751639 |
| S.E. of regression | 0.013667 | Akaike info criterion | | -5.738187 |
| Sum squared resid | 0.116555 | Schwarz criterion | | -5.695847 |
| Durbin-Watson stat | 2.04 | 5% Critical Value | 2.13 | 1.86 |
| | | 3% Critical Value | 2.16 | 1.84 |
| | | 1% Critical Value | 2.19 | 1.81 |
| Entropy | 77% | | | |

| Tab 13: electric machinery production | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -2.77 | 1% Critical Value* | | -3.98 |
| | | 5% Critical Value | | -3.42 |
| | | 10% Critical Value | | -3.13 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:19 | | | | |
| Sample(adjusted): 3 468 | | | | |
| Included observations: 466 after adjusting endpoints.<br>White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: Y(t) = 0.05+0.97*Y(t-1)+0.17*(Y(t-1)-Y(t-2)) +1.46E-04*t+ε | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 0.972734 | 0.010734 | 90.61905 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.168208 | 0.065447 | 2.570158 | 0.0105 |
| Intercept | 0.051494 | 0.018378 | 2.801982 | 0.0053 |
| TREND | 0.000146 | 5.94E-05 | 2.463291 | 0.0141 |
| R-squared | 0.999237 | Mean dependent var | | 2.984028 |
| Adjusted R-squared | 0.999232 | S.D. dependent var | | 0.738549 |
| S.E. of regression | 0.020463 | Akaike info criterion | | -4.931857 |
| Sum squared resid | 0.193454 | Schwarz criterion | | -4.896285 |
| Durbin-Watson stat | 2.06 | 5% Critical Value | 2.15 | 1.85 |
| | | 3% Critical Value | 2.18 | 1.81 |
| | | 1% Critical Value | 2.21 | 1.78 |

| Tab 14: Hi-Tech | | | | |
|---|---|---|---|---|
| ADF Test Statistic | 0.578766 | 1% Critical Value* | | -3.9854 |
| | | 5% Critical Value | | -3.4230 |
| | | 10% Critical Value | | -3.1341 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:23 | | | | |
| Sample(adjusted): 3 394 | | | | |
| Included observations: 392 after adjusting endpoints | | | | |
| White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: Y(t) = 1.00*Y(t-1)+0.12*(Y(t-1)-Y(t-2))+ε | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 1.003030 | 0.000331 | 3031.013 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.117184 | 0.072128 | 1.624684 | 0.1050 |
| R-squared | 0.999882 | Mean dependent var | | 3.616656 |
| Adjusted R-squared | 0.999881 | S.D. dependent var | | 1.363614 |
| S.E. of regression | 0.014846 | Akaike info criterion | | -5.577049 |
| Sum squared resid | 0.085960 | Schwarz criterion | | -5.556787 |
| Durbin-Watson stat | 2.05 | 5% Critical Value | 2.17 | 1.83 |
| | | 3% Critical Value | 2.20 | 1.80 |
| | | 1% Critical Value | 2.23 | 1.74 |

| tab 15: employment | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -4.205271 | 1% Critical Value* | | -3.9754 |
| | | 5% Critical Value | | -3.4182 |
| | | 10% Critical Value | | -3.1313 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:26 | | | | |
| Sample(adjusted): 3 726 | | | | |
| Included observations: 723 after adjusting endpoints | | | | |
| White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: Y(t) = 0.16+0.98*Y(t-1)+0.27*(Y(t-1)-Y(t-2))+0.27(Y(t-2)-Y(t-3))+2.65E-05*t+ε | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 0.984867 | 0.004407 | 223.4963 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.273272 | 0.101724 | 2.686414 | 0.0074 |
| Y(t-2)-Y(t-3) | 0.273409 | 0.107786 | 2.536593 | 0.0114 |
| Intercept | 0.159671 | 0.046410 | 3.440470 | 0.0006 |
| TREND | 2.65E-05 | 7.56E-06 | 3.503781 | 0.0005 |
| R-squared | 0.999894 | Mean dependent var | | 11.13037 |
| Adjusted R-squared | 0.999894 | S.D. dependent var | | 0.379272 |
| S.E. of regression | 0.003908 | Akaike info criterion | | -8.244650 |
| Sum squared resid | 0.010966 | Schwarz criterion | | -8.212953 |
| Durbin-Watson stat | 2.07 | 5% Critical Value | 2.12 | 1.88 |
| | | 3% Critical Value | 2.15 | 1.85 |
| | | 1% Critical Value | 2.17 | 1.83 |
| Entropy | 68% | | | |

| tab 16: hourly earnings of production workers | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -1.066504 | 1% Critical Value* | | -3.9742 |
| | | 5% Critical Value | | -3.4176 |
| | | 10% Critical Value | | -3.1309 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/12/00 Time: 05:31 | | | | |
| Sample(adjusted): 3 812 | | | | |
| Included observations: 810 after adjusting endpoints | | | | |
| White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model: Y(t) = 1.00*Y(t-1)+0.20*(Y(t-1)-Y(t-2))+0.24(Y(t-2)-Y(t-3))+ε | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 1.000936 | 0.000226 | 4435.214 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.208131 | 0.122095 | 1.704661 | 0.0886 |
| Y(t-2)-Y(t-3) | 0.244610 | 0.104343 | 2.344289 | 0.0193 |
| R-squared | 0.999928 | Mean dependent var | | 1.073049 |
| Adjusted R-squared | 0.999928 | S.D. dependent var | | 1.040165 |
| S.E. of regression | 0.008846 | Akaike info criterion | | -6.614069 |
| Sum squared resid | 0.063067 | Schwarz criterion | | -6.596656 |
| Durbin-Watson stat | 2.04 | 5% Critical Value | 2.12 | 1.88 |
| | | 3% Critical Value | 2.14 | 1.86 |
| | | 1% Critical Value | 2.16 | 1.84 |
| Entropy | 71% | | | |

| Tab 17: Consumer Price Index | | | | |
|---|---|---|---|---|
| ADF Test Statistic | -0.846908 | 1% Critical Value* | | -3.9719 |
| | | 5% Critical Value | | -3.4165 |
| | | 10% Critical Value | | -3.1302 |
| *MacKinnon critical values for rejection of hypothesis of a unit root. | | | | |
| Date: 01/09/00 Time: 04:31 | | | | |
| Sample(adjusted): 3 1042 | | | | |
| Included observations: 1040 after adjusting endpoints | | | | |
| White Heteroskedasticity-Consistent Standard Errors & Covariance | | | | |
| Model:<br>Y(t) = 1.00*Y(t-1)+0.33*(Y(t-1)-Y(t-2))+0.16*(Y(t-2)-Y(t-3))+ 0.13*(Y(t-3)-Y(t-4))+ε | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| Y(t-1) | 1.000281 | 5.11E-05 | 19561.14 | 0.0000 |
| Y(t-1)-Y(t-2) | 0.330581 | 0.044570 | 7.417036 | 0.0000 |
| Y(t-2)-Y(t-3) | 0.166066 | 0.049745 | 3.338342 | 0.0009 |
| Y(t-3)-Y(t-4) | 0.139494 | 0.051642 | 2.701158 | 0.0070 |
| R-squared | 0.999950 | Mean dependent var | | 3.509444 |
| Adjusted R-squared | 0.999950 | S.D. dependent var | | 0.836538 |
| S.E. of regression | 0.005928 | Akaike info criterion | | -7.414489 |
| Sum squared resid | 0.036333 | Schwarz criterion | | -7.395433 |
| Durbin-Watson stat | 2.05 | 5% Critical Value | 2.10 | 1.90 |
| | | 3% Critical Value | 2.12 | 1.88 |
| | | 1% Critical Value | 2.14 | 1.86 |
| Entropy | 71% | | | |

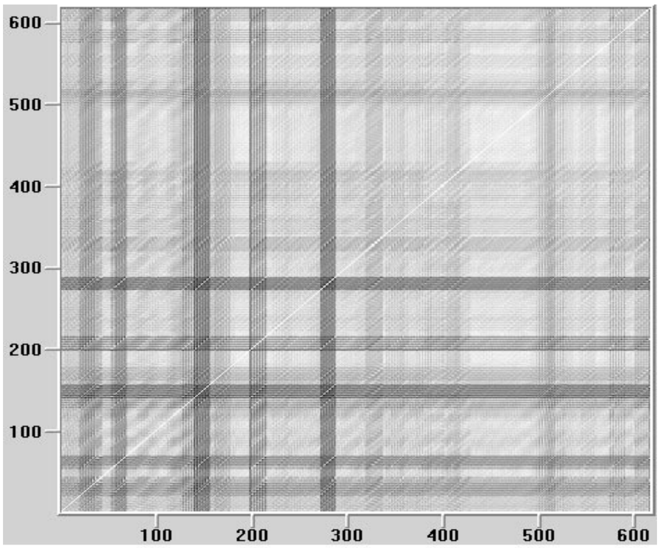



Fig. 15: Transportation equipment production, recurrence plot

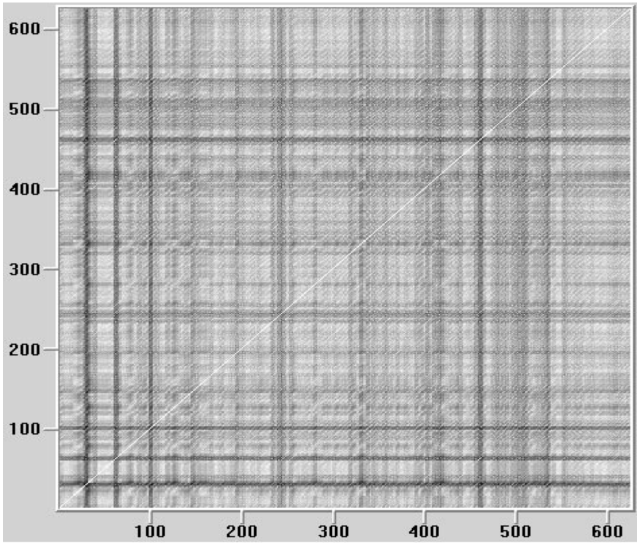



Fig. 16: industrial machinery production, recurrence plot

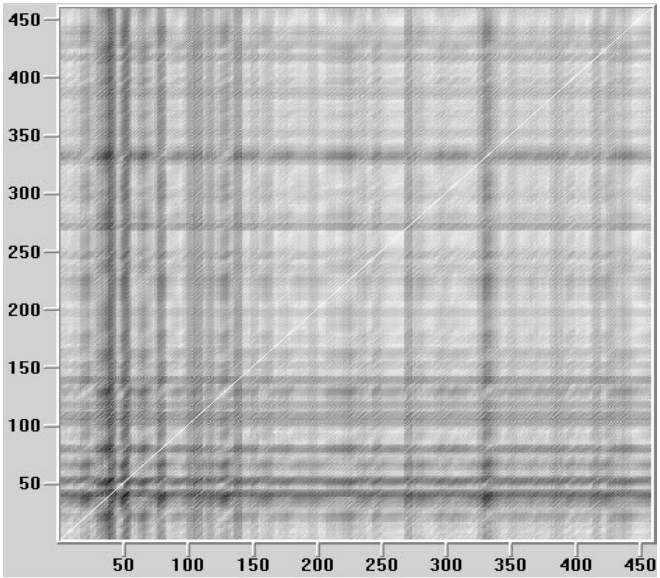



Fig. 17: electric machinery production, recurrence plot

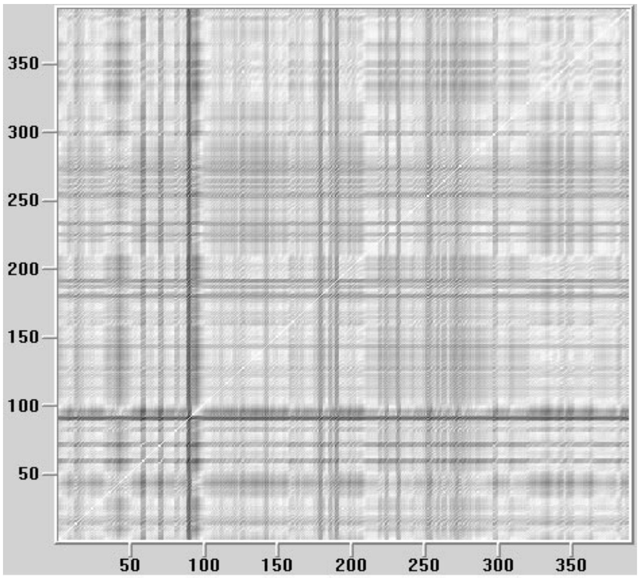



Fig. 18: HITEK production, recurrence plot

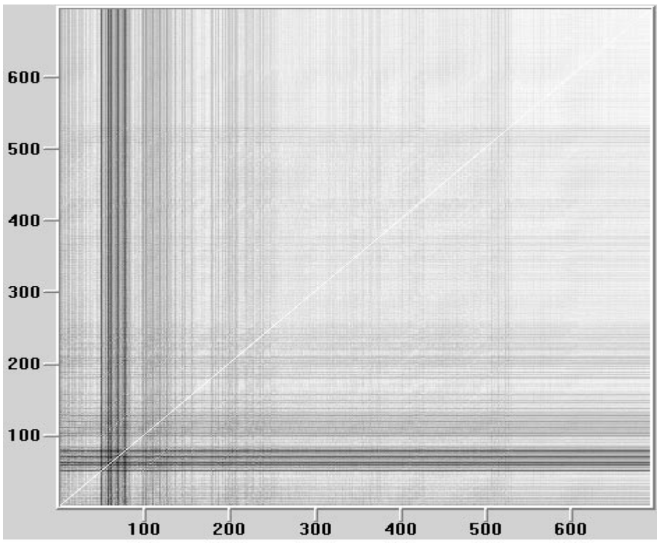



Fig. 19: employment, recurrence plot

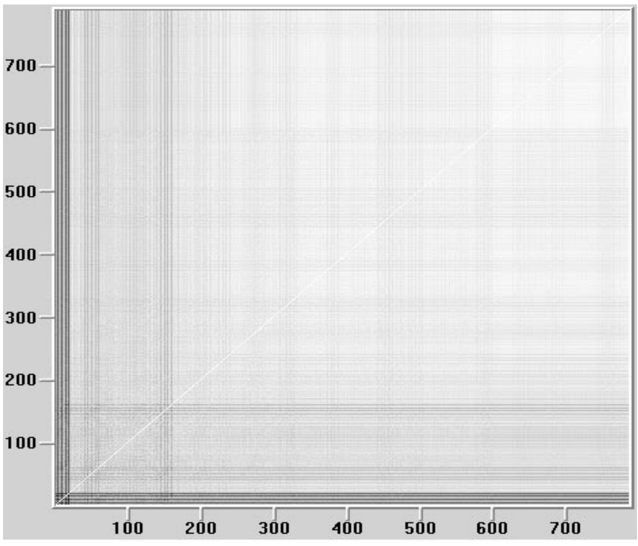



Fig. 20: hourly earnings, recurrence plot

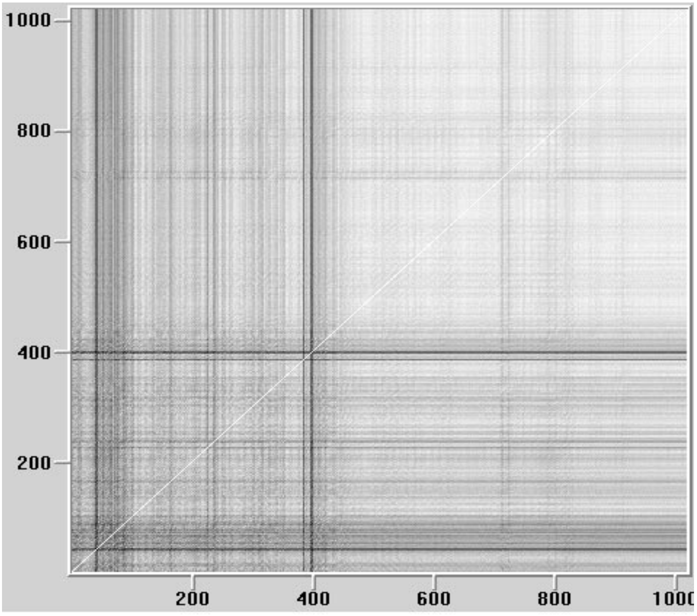



Fig. 21: consumer price index, recurrence plot

| Tab 19: | | transportation eq. production | | | | | |
|---|---|---|---|---|---|---|---|
| Initial O | | Num | Obs : N =633 | SD/Spread | 0.084826 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 173909 | 156653 | 0.75 | 0.07 | 10.44 | 0.13 |
| 0.15 | 4 | 173909 | 130459 | 2.19 | 0.20 | 10.67 | 0.23 |
| 0.15 | 6 | 173909 | 108159 | 2.94 | 0.31 | 9.48 | 0.32 |
| 0.15 | 8 | 173909 | 89195 | 3.24 | 0.38 | 8.55 | 0.43 |
| 0.15 | 10 | 173909 | 73545 | 3.29 | 0.41 | 7.94 | 0.53 |
| 0.08 | 2 | 137452 | 104128 | 1.26 | 0.11 | 11.78 | 0.26 |
| 0.08 | 4 | 137452 | 64000 | 2.51 | 0.19 | 12.98 | 0.46 |
| 0.08 | 6 | 137452 | 39852 | 2.43 | 0.19 | 13.05 | 0.65 |
| 0.08 | 8 | 137452 | 25326 | 2.00 | 0.14 | 13.77 | 0.84 |
| 0.08 | 10 | 137452 | 16302 | 1.51 | 0.10 | 14.97 | 1.02 |
| 0.05 | 2 | 92404 | 50532 | 1.01 | 0.09 | 11.73 | 0.45 |
| 0.05 | 4 | 92404 | 16889 | 1.00 | 0.07 | 13.89 | 0.81 |
| 0.05 | 6 | 92404 | 5762 | 0.49 | 0.03 | 15.30 | 1.17 |
| 0.05 | 8 | 92404 | 2210 | 0.23 | 0.01 | 19.99 | 1.48 |
| 0.05 | 10 | 92404 | 884 | 0.10 | 0.00 | 27.45 | 1.79 |

| Tab 20: | | industrial machinery production | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs : 1 | | Obs : N =633 | SD/Spread | 0.15206 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.24 | 2 | 163689 | 136658 | 0.24 | 0.06 | 4.07 | 0.26 |
| 0.24 | 4 | 163689 | 95830 | 0.56 | 0.15 | 3.77 | 0.51 |
| 0.24 | 6 | 163689 | 68468 | 0.83 | 0.20 | 4.06 | 0.74 |
| 0.24 | 8 | 163689 | 49788 | 1.00 | 0.22 | 4.44 | 0.97 |
| 0.24 | 10 | 163689 | 36668 | 1.06 | 0.22 | 4.77 | 1.18 |
| 0.12 | 2 | 101842 | 54157 | 0.25 | 0.07 | 3.49 | 0.61 |
| 0.12 | 4 | 101842 | 15594 | 0.24 | 0.07 | 3.30 | 1.19 |
| 0.12 | 6 | 101842 | 4765 | 0.15 | 0.04 | 3.76 | 1.74 |
| 0.12 | 8 | 101842 | 1496 | 0.07 | 0.02 | 4.08 | 2.28 |
| 0.12 | 10 | 101842 | 481 | 0.03 | 0.01 | 4.41 | 2.81 |
| 0.06 | 2 | 53464 | 15021 | 0.08 | 0.03 | 2.78 | 0.90 |
| 0.06 | 4 | 53464 | 1231 | 0.02 | 0.01 | 2.90 | 1.78 |
| 0.06 | 6 | 53464 | 111 | 0.00 | 0.00 | 3.57 | 2.62 |
| 0.06 | 8 | 53464 | 11 | 0.00 | 0.00 | 4.52 | 3.43 |
| 0.06 | 10 | 53464 | 2 | 0.00 | 0.00 | 11.83 | 4.03 |

| Tab 21: | | electrical machinery production | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =633 | SD/Spread | 0.118 | | |
| $\varepsilon$ | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.20 | 2 | 82091 | 64531 | 0.52 | 0.08 | 6.53 | 0.32 |
| 0.20 | 4 | 82091 | 42307 | 1.42 | 0.17 | 8.17 | 0.58 |
| 0.20 | 6 | 82091 | 28393 | 1.69 | 0.20 | 8.38 | 0.83 |
| 0.20 | 8 | 82091 | 19154 | 1.58 | 0.19 | 8.39 | 1.08 |
| 0.20 | 10 | 82091 | 13191 | 1.37 | 0.16 | 8.75 | 1.31 |
| 0.12 | 2 | 59256 | 35263 | 0.60 | 0.08 | 7.25 | 0.54 |
| 0.12 | 4 | 59256 | 13961 | 0.88 | 0.10 | 9.26 | 0.98 |
| 0.12 | 6 | 59256 | 5682 | 0.58 | 0.06 | 9.80 | 1.41 |
| 0.12 | 8 | 59256 | 2403 | 0.32 | 0.03 | 10.91 | 1.82 |
| 0.12 | 10 | 59256 | 1013 | 0.16 | 0.01 | 12.08 | 2.23 |
| 0.08 | 2 | 39337 | 16090 | 0.37 | 0.05 | 7.27 | 0.74 |
| 0.08 | 4 | 39337 | 3056 | 0.24 | 0.03 | 9.05 | 1.39 |
| 0.08 | 6 | 39337 | 631 | 0.08 | 0.01 | 10.69 | 2.00 |
| 0.08 | 8 | 39337 | 132 | 0.02 | 0.00 | 12.46 | 2.61 |
| 0.08 | 10 | 39337 | 23 | 0.00 | 0.00 | 11.78 | 3.28 |

| Tab 22: | | HI-TECH | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =393 | SD/Spread | 0.163 | | |
| $\varepsilon$ | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.30 | 2 | 70894 | 65691 | 0.11 | 0.03 | 3.66 | 0.13 |
| 0.30 | 4 | 70894 | 56821 | 0.41 | 0.10 | 4.03 | 0.25 |
| 0.30 | 6 | 70894 | 49710 | 0.76 | 0.17 | 4.44 | 0.36 |
| 0.30 | 8 | 70894 | 43569 | 1.04 | 0.24 | 4.41 | 0.47 |
| 0.30 | 10 | 70894 | 38278 | 1.26 | 0.29 | 4.34 | 0.58 |
| 0.15 | 2 | 50696 | 34942 | 0.41 | 0.09 | 4.69 | 0.41 |
| 0.15 | 4 | 50696 | 17959 | 0.92 | 0.15 | 6.27 | 0.75 |
| 0.15 | 6 | 50696 | 10173 | 1.03 | 0.13 | 7.91 | 1.05 |
| 0.15 | 8 | 50696 | 5714 | 0.80 | 0.09 | 8.52 | 1.35 |
| 0.15 | 10 | 50696 | 3150 | 0.53 | 0.06 | 8.79 | 1.66 |
| 0.10 | 2 | 38684 | 20840 | 0.37 | 0.07 | 4.91 | 0.57 |
| 0.10 | 4 | 38684 | 6942 | 0.53 | 0.07 | 7.24 | 1.05 |
| 0.10 | 6 | 38684 | 2638 | 0.37 | 0.04 | 9.52 | 1.47 |
| 0.10 | 8 | 38684 | 1010 | 0.19 | 0.02 | 11.25 | 1.89 |
| 0.10 | 10 | 38684 | 354 | 0.07 | 0.01 | 11.76 | 2.35 |

| Tab 23 | | | Employment | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =729 | SD/Spread | 0.065 | | |
| $\varepsilon$ | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.12 | 2 | 238186 | 222742 | 0.62 | 0.06 | 10.83 | 0.07 |
| 0.12 | 4 | 238186 | 198517 | 1.97 | 0.18 | 11.00 | 0.13 |
| 0.12 | 6 | 238186 | 179940 | 3.25 | 0.30 | 10.92 | 0.18 |
| 0.12 | 8 | 238186 | 164760 | 4.35 | 0.40 | 10.90 | 0.22 |
| 0.12 | 10 | 238186 | 152373 | 5.29 | 0.48 | 11.05 | 0.25 |
| 0.06 | 2 | 200714 | 164181 | 1.06 | 0.11 | 9.99 | 0.17 |
| 0.06 | 4 | 200714 | 118494 | 2.90 | 0.24 | 12.12 | 0.29 |
| 0.06 | 6 | 200714 | 93491 | 4.19 | 0.29 | 14.60 | 0.37 |
| 0.06 | 8 | 200714 | 75910 | 4.66 | 0.28 | 16.82 | 0.45 |
| 0.06 | 10 | 200714 | 62865 | 4.67 | 0.24 | 19.47 | 0.52 |
| 0.03 | 2 | 139066 | 83116 | 0.95 | 0.10 | 9.14 | 0.34 |
| 0.03 | 4 | 139066 | 37180 | 1.69 | 0.12 | 14.67 | 0.57 |
| 0.03 | 6 | 139066 | 20379 | 1.51 | 0.07 | 22.15 | 0.75 |
| 0.03 | 8 | 139066 | 12786 | 1.16 | 0.03 | 35.82 | 0.89 |
| 0.03 | 10 | 139066 | 8455 | 0.84 | 0.01 | 60.39 | 1.01 |

| Tab 24 | | Hourly earnings of production workers | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =811 | SD/Spread | 0.045 | | |
| $\varepsilon$ | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.10 | 2 | 300298 | 281709 | 0.56 | 0.05 | 12.17 | 0.06 |
| 0.10 | 4 | 300298 | 251488 | 1.78 | 0.15 | 12.04 | 0.11 |
| 0.10 | 6 | 300298 | 229524 | 3.10 | 0.25 | 12.48 | 0.15 |
| 0.10 | 8 | 300298 | 212731 | 4.37 | 0.34 | 12.97 | 0.19 |
| 0.10 | 10 | 300298 | 199799 | 5.55 | 0.41 | 13.58 | 0.21 |
| 0.05 | 2 | 242722 | 195545 | 1.37 | 0.11 | 12.28 | 0.17 |
| 0.05 | 4 | 242722 | 140183 | 3.63 | 0.24 | 15.40 | 0.28 |
| 0.05 | 6 | 242722 | 109290 | 4.84 | 0.26 | 18.31 | 0.37 |
| 0.05 | 8 | 242722 | 87283 | 5.08 | 0.24 | 21.23 | 0.44 |
| 0.05 | 10 | 242722 | 72064 | 4.93 | 0.19 | 25.51 | 0.51 |
| 0.03 | 2 | 163092 | 96483 | 1.33 | 0.10 | 12.89 | 0.33 |
| 0.03 | 4 | 163092 | 42048 | 1.92 | 0.10 | 19.00 | 0.56 |
| 0.03 | 6 | 163092 | 21130 | 1.42 | 0.05 | 26.94 | 0.74 |
| 0.03 | 8 | 163092 | 11565 | 0.91 | 0.02 | 41.09 | 0.91 |
| 0.03 | 10 | 163092 | 6958 | 0.59 | 0.01 | 70.26 | 1.04 |

| Tab 25 | c.p.i | | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N = 1041 | SD/Spread | 0.076 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 476963 | 429828 | 0.48 | 0.06 | 8.22 | 0.12 |
| 0.15 | 4 | 476963 | 363475 | 2.01 | 0.17 | 11.69 | 0.21 |
| 0.15 | 6 | 476963 | 320728 | 3.77 | 0.27 | 13.96 | 0.27 |
| 0.15 | 8 | 476963 | 291773 | 5.43 | 0.34 | 15.90 | 0.32 |
| 0.15 | 10 | 476963 | 268953 | 6.73 | 0.39 | 17.41 | 0.37 |
| 0.08 | 2 | 375583 | 282995 | 1.28 | 0.10 | 12.33 | 0.26 |
| 0.08 | 4 | 375583 | 186959 | 3.61 | 0.19 | 18.60 | 0.43 |
| 0.08 | 6 | 375583 | 142259 | 4.87 | 0.19 | 25.28 | 0.54 |
| 0.08 | 8 | 375583 | 117981 | 5.33 | 0.15 | 34.63 | 0.62 |
| 0.08 | 10 | 375583 | 101467 | 5.28 | 0.11 | 47.91 | 0.68 |
| 0.05 | 2 | 256438 | 145002 | 1.38 | 0.09 | 15.28 | 0.43 |
| 0.05 | 4 | 256438 | 60809 | 2.00 | 0.08 | 24.87 | 0.72 |
| 0.05 | 6 | 256438 | 32519 | 1.58 | 0.04 | 41.64 | 0.92 |
| 0.05 | 8 | 256438 | 19554 | 1.10 | 0.01 | 75.29 | 1.09 |
| 0.05 | 10 | 256438 | 12362 | 0.73 | 0.01 | 145.90 | 1.24 |

| Tab 26 | | Shuffled transportation eq. production | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =625 | SD/Spread | 0.085 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 167844 | 143825 | -0.02 | 0.07 | -0.32 | 0.17 |
| 0.15 | 4 | 167844 | 106236 | 0.03 | 0.21 | 0.14 | 0.33 |
| 0.15 | 6 | 167844 | 78347 | 0.04 | 0.31 | 0.13 | 0.49 |
| 0.15 | 8 | 167844 | 58772 | 0.17 | 0.37 | 0.47 | 0.65 |
| 0.15 | 10 | 167844 | 44223 | 0.25 | 0.39 | 0.65 | 0.80 |
| 0.08 | 2 | 130256 | 86133 | -0.08 | 0.11 | -0.73 | 0.34 |
| 0.08 | 4 | 130256 | 38509 | 0.01 | 0.18 | 0.04 | 0.67 |
| 0.08 | 6 | 130256 | 17089 | 0.01 | 0.16 | 0.03 | 1.00 |
| 0.08 | 8 | 130256 | 7769 | 0.03 | 0.12 | 0.23 | 1.32 |
| 0.08 | 10 | 130256 | 3571 | 0.03 | 0.08 | 0.36 | 1.63 |
| 0.05 | 2 | 86275 | 37323 | -0.09 | 0.08 | -1.17 | 0.55 |
| 0.05 | 4 | 86275 | 7233 | -0.02 | 0.06 | -0.35 | 1.09 |
| 0.05 | 6 | 86275 | 1435 | 0.00 | 0.03 | -0.02 | 1.63 |
| 0.05 | 8 | 86275 | 303 | 0.00 | 0.01 | 0.36 | 2.14 |
| 0.05 | 10 | 86275 | 63 | 0.00 | 0.00 | 0.45 | 2.66 |

| Tab 27 | | Shuffled machinery eq. production | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =625 | SD/Spread | 0.145 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.24 | 2 | 160906 | 132293 | -0.01 | 0.06 | -0.12 | 0.27 |
| 0.24 | 4 | 160906 | 88987 | -0.07 | 0.15 | -0.48 | 0.55 |
| 0.24 | 6 | 160906 | 59473 | -0.14 | 0.20 | -0.70 | 0.83 |
| 0.24 | 8 | 160906 | 39268 | -0.22 | 0.22 | -1.00 | 1.12 |
| 0.24 | 10 | 160906 | 25647 | -0.27 | 0.22 | -1.23 | 1.42 |
| 0.16 | 2 | 124144 | 78379 | -0.05 | 0.08 | -0.64 | 0.49 |
| 0.16 | 4 | 124144 | 31025 | -0.09 | 0.12 | -0.73 | 0.99 |
| 0.16 | 6 | 124144 | 12110 | -0.09 | 0.10 | -0.84 | 1.50 |
| 0.16 | 8 | 124144 | 4497 | -0.08 | 0.07 | -1.24 | 2.03 |
| 0.16 | 10 | 124144 | 1614 | -0.06 | 0.04 | -1.48 | 2.58 |
| 0.10 | 2 | 88619 | 39787 | -0.05 | 0.06 | -0.73 | 0.70 |
| 0.10 | 4 | 88619 | 7965 | -0.04 | 0.05 | -0.69 | 1.40 |
| 0.10 | 6 | 88619 | 1605 | -0.01 | 0.02 | -0.51 | 2.10 |
| 0.10 | 8 | 88619 | 313 | 0.00 | 0.01 | -0.59 | 2.82 |
| 0.10 | 10 | 88619 | 57 | 0.00 | 0.00 | -0.81 | 3.56 |

| Tab 28 | | Shuffled electrical machinery | | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =623 | SD/Spread | 0.119 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.20 | 2 | 75400 | 55812 | -0.04 | 0.08 | -0.56 | 0.37 |
| 0.20 | 4 | 75400 | 31027 | 0.02 | 0.17 | 0.12 | 0.73 |
| 0.20 | 6 | 75400 | 17044 | -0.01 | 0.19 | -0.03 | 1.11 |
| 0.20 | 8 | 75400 | 9108 | -0.07 | 0.17 | -0.39 | 1.50 |
| 0.20 | 10 | 75400 | 4733 | -0.10 | 0.14 | -0.72 | 1.90 |
| 0.12 | 2 | 54030 | 28568 | -0.04 | 0.08 | -0.53 | 0.60 |
| 0.12 | 4 | 54030 | 8256 | 0.02 | 0.09 | 0.24 | 1.20 |
| 0.12 | 6 | 54030 | 2331 | 0.00 | 0.05 | 0.08 | 1.80 |
| 0.12 | 8 | 54030 | 632 | -0.01 | 0.02 | -0.20 | 2.43 |
| 0.12 | 10 | 54030 | 152 | -0.01 | 0.01 | -0.70 | 3.11 |
| 0.08 | 2 | 35635 | 12299 | -0.05 | 0.05 | -0.95 | 0.82 |
| 0.08 | 4 | 35635 | 1539 | 0.00 | 0.02 | -0.04 | 1.62 |
| 0.08 | 6 | 35635 | 170 | 0.00 | 0.01 | -0.71 | 2.48 |
| 0.08 | 8 | 35635 | 20 | 0.00 | 0.00 | -0.58 | 3.31 |
| 0.08 | 10 | 35635 | 2 | 0.00 | 0.00 | -0.80 | 4.21 |

| Tab 29 | | | Shuffled Hi-Tech | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =384 | SD/Spread | 0.127 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.30 | 2 | 67823 | 62231 | 0.00 | 0.03 | 0.02 | 0.17 |
| 0.30 | 4 | 67823 | 53248 | 0.23 | 0.10 | 2.26 | 0.30 |
| 0.30 | 6 | 67823 | 45897 | 0.49 | 0.18 | 2.80 | 0.43 |
| 0.30 | 8 | 67823 | 39428 | 0.63 | 0.24 | 2.65 | 0.55 |
| 0.30 | 10 | 67823 | 33623 | 0.66 | 0.29 | 2.26 | 0.68 |
| 0.15 | 2 | 48171 | 31344 | -0.01 | 0.09 | -0.14 | 0.46 |
| 0.15 | 4 | 48171 | 14464 | 0.31 | 0.14 | 2.14 | 0.87 |
| 0.15 | 6 | 48171 | 6946 | 0.35 | 0.12 | 2.80 | 1.25 |
| 0.15 | 8 | 48171 | 3236 | 0.23 | 0.09 | 2.60 | 1.64 |
| 0.15 | 10 | 48171 | 1462 | 0.12 | 0.06 | 2.20 | 2.06 |
| 0.10 | 2 | 36686 | 18189 | 0.00 | 0.07 | -0.06 | 0.63 |
| 0.10 | 4 | 36686 | 4974 | 0.13 | 0.07 | 1.87 | 1.20 |
| 0.10 | 6 | 36686 | 1430 | 0.09 | 0.04 | 2.43 | 1.74 |
| 0.10 | 8 | 36686 | 398 | 0.03 | 0.02 | 2.29 | 2.30 |
| 0.10 | 10 | 36686 | 118 | 0.01 | 0.01 | 2.51 | 2.83 |

| Tab 30 | | | Shuffled employment | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =729 | SD/Spread | 0.065 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.12 | 2 | 235370 | 212902 | -0.06 | 0.06 | -0.97 | 0.09 |
| 0.12 | 4 | 235370 | 174017 | -0.15 | 0.18 | -0.87 | 0.19 |
| 0.12 | 6 | 235370 | 141667 | -0.28 | 0.29 | -0.94 | 0.28 |
| 0.12 | 8 | 235370 | 115229 | -0.36 | 0.39 | -0.92 | 0.38 |
| 0.12 | 10 | 235370 | 94385 | -0.34 | 0.47 | -0.72 | 0.48 |
| 0.06 | 2 | 195086 | 145688 | -0.10 | 0.11 | -0.92 | 0.21 |
| 0.06 | 4 | 195086 | 82607 | -0.02 | 0.23 | -0.10 | 0.41 |
| 0.06 | 6 | 195086 | 47320 | 0.06 | 0.26 | 0.21 | 0.62 |
| 0.06 | 8 | 195086 | 26960 | 0.06 | 0.25 | 0.23 | 0.82 |
| 0.06 | 10 | 195086 | 15476 | 0.06 | 0.20 | 0.28 | 1.02 |
| 0.03 | 2 | 132187 | 67031 | -0.03 | 0.10 | -0.30 | 0.40 |
| 0.03 | 4 | 132187 | 17853 | 0.04 | 0.10 | 0.41 | 0.79 |
| 0.03 | 6 | 132187 | 4644 | 0.01 | 0.05 | 0.22 | 1.19 |
| 0.03 | 8 | 132187 | 1167 | 0.00 | 0.02 | -0.03 | 1.59 |
| 0.03 | 10 | 132187 | 294 | 0.00 | 0.01 | -0.12 | 2.00 |

| Tab 31 | | | Shuffled hourly earnings of prduction workers | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N =811 | SD/Spread | 0.045 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.10 | 2 | 292001 | 265248 | -0.08 | 0.05 | -1.61 | 0.08 |
| 0.10 | 4 | 292001 | 219041 | -0.18 | 0.15 | -1.18 | 0.16 |
| 0.10 | 6 | 292001 | 180107 | -0.31 | 0.25 | -1.24 | 0.25 |
| 0.10 | 8 | 292001 | 148842 | -0.33 | 0.34 | -0.97 | 0.33 |
| 0.10 | 10 | 292001 | 123654 | -0.27 | 0.40 | -0.67 | 0.41 |
| 0.05 | 2 | 232257 | 167814 | -0.05 | 0.11 | -0.44 | 0.22 |
| 0.05 | 4 | 232257 | 87685 | -0.07 | 0.22 | -0.31 | 0.43 |
| 0.05 | 6 | 232257 | 45437 | -0.09 | 0.24 | -0.39 | 0.65 |
| 0.05 | 8 | 232257 | 23140 | -0.12 | 0.21 | -0.55 | 0.88 |
| 0.05 | 10 | 232257 | 11830 | -0.09 | 0.16 | -0.56 | 1.10 |
| 0.03 | 2 | 151578 | 71891 | 0.02 | 0.10 | 0.17 | 0.40 |
| 0.03 | 4 | 151578 | 15812 | -0.02 | 0.08 | -0.25 | 0.82 |
| 0.03 | 6 | 151578 | 3355 | -0.02 | 0.04 | -0.53 | 1.24 |
| 0.03 | 8 | 151578 | 723 | -0.01 | 0.02 | -0.48 | 1.65 |
| 0.03 | 10 | 151578 | 157 | 0.00 | 0.01 | -0.40 | 2.07 |

| Tab 32 | | | Shuffled c.p.i. | | | | |
|---|---|---|---|---|---|---|---|
| Initial | Obs :1 | | Obs : N = 1024 | SD/Spread | 0.077 | | |
| ε | m | $C_{1,N}$ *N*(N-1)/2 | $C_{m,N}$ *N*(N-1)/2 | BDS | SD | $W_{m,N}$ | $d_m$ |
| 0.15 | 2 | 462429 | 406123 | -0.08 | 0.06 | -1.40 | 0.13 |
| 0.15 | 4 | 462429 | 313711 | -0.16 | 0.17 | -0.94 | 0.27 |
| 0.15 | 6 | 462429 | 239980 | -0.35 | 0.27 | -1.29 | 0.41 |
| 0.15 | 8 | 462429 | 183392 | -0.45 | 0.34 | -1.34 | 0.55 |
| 0.15 | 10 | 462429 | 141013 | -0.44 | 0.38 | -1.15 | 0.69 |
| 0.08 | 2 | 361883 | 247197 | -0.14 | 0.10 | -1.38 | 0.30 |
| 0.08 | 4 | 361883 | 117856 | -0.05 | 0.19 | -0.26 | 0.60 |
| 0.08 | 6 | 361883 | 57152 | 0.05 | 0.18 | 0.24 | 0.90 |
| 0.08 | 8 | 361883 | 28241 | 0.09 | 0.14 | 0.60 | 1.18 |
| 0.08 | 10 | 361883 | 14054 | 0.08 | 0.10 | 0.79 | 1.47 |
| 0.05 | 2 | 245503 | 113306 | -0.09 | 0.09 | -1.07 | 0.50 |
| 0.05 | 4 | 245503 | 24815 | -0.02 | 0.08 | -0.26 | 1.00 |
| 0.05 | 6 | 245503 | 5793 | 0.02 | 0.03 | 0.52 | 1.48 |
| 0.05 | 8 | 245503 | 1417 | 0.01 | 0.01 | 1.02 | 1.95 |
| 0.05 | 10 | 245503 | 302 | 0.00 | 0.00 | 0.56 | 2.46 |

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