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Magic and Modulo Magicness of Paley Digraph

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Abstract— ***A special digraph arises in round robin tournaments. More exactly, a tournament T_q with q players 1, 2, ... , q in which there are no draws. This gives rise to a digraph in which either (u, v) or (v, u) is an arc for each pair u, v. Graham and Spencer defined the tournament as, The nodes of digraph D_p are {0, 1, ... , p -1} and D_p contains the arc (u, v) if and only if u - v is a quadratic residue modulo p where $p \equiv 3 (\mathrm{mod}\ 4)$ be a prime. This digraph is referred as the Paley tournament. Raymond Paley was a person raised Hadamard matrices by using this quadratic residues. So to honor him this tournament was named as Paley tournament. These results were extended by Bollobas for prime powers. Modular super edge trimagic labeling and modular super vertex magic total labeling has been investigated in this paper.***

AMS Subject Classification: 05C78

Key words: Super edge tri magic labeling, Paley Digraph and Modular Super vertex magic total labeling.

I. INTRODUCTION

This article has discussion about a strong regular digraph called Paley digraph. Let us consider a graph G with set of nodes collectively with set of arcs which is relating some subset of those nodes. One among different way to label the graph elements is labeling with integers. Labeling is defined as a mapping of a sequence of graph elements send to a sequence of real values (negative integers not allowed). Graph labeling has wide range of application in real life which is categorized as qualitative labeling and quantitative labeling. More than 2000 papers have been published in graph labeling and for more details one can refer [1]. Out of many types of labeling most fascinating labeling is magic labeling. It was originated from number theory especially from magic squares and was pioneered by Sedlacek in 1963 [2] [3]. Stewart [4] have been studied about this labeling and named as super magic provided the labels are continuous integers, starting from 1. Few researchers evidently termed "magic" as an alternative of "supermagic". Magic labeling is defined as a total labeling by Kotzig and Rosa [5], in which the tagging is from 1 to |V|+| E|. Super vertex-magic total labeling was introduced by MacDougall, et.al.[6](some implement the term "super vertex-magic" for the same notion). Magic labeling and super vertex (a,d) antimagic labeling for digraphs was introduced by K.Thirusangu et.al. [7].

C. Jayasekaran et.al.[8] introduced the labeling called edge trimagic total labeling. It is defined as a one to one and onto function f: V ∪ E → {1, 2, …, |V | + |E|} ∋,∀ edge xy ∈ E, the sum of the arc and the node will be equal to any integers k_1 or k_2 or k_3 where k_i are constants. If the nodes of G are labeled with least positive integers then it is called as super edge trimagic total labeling. In 2014, the same labeling was extended for the digraphs by the same author. Using super vertex magic total labeling Modular super vertex magic labeling was introduced by D. Antony Xavier [9]. An injective function f : V ∪ E → {1,2,3... |V | + |E|} is called a modular vertex magic labeling if ∃ a constant k ∋, $f(x) + \sum f(xy) = k (\mathrm{mod}\ p)$ $\forall x \in V$, $0 \le k < p-1$. Paley digraph confess some members of family of magic labeling, also some deficiency of labeling and cordiality in various papers [10] [11] [12]. In the following section, modular edge trimagic and super vertex magic labeling has been discussed.

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RESULT AND DISCUSSION

II. MODULAR SUPER EDGE TRIMAGIC LABELING OF PALEY DIGRAPH

A. Theorem

Suppose P(q) is a Paley digraph with Galois field elements as nodes $v_1, v_2, ..., v_q$, $q \equiv 3 (\mathrm{mod} 4)$. If each nodes of G has p incoming and outgoing arcs with |E| = pq, where p $=\left(\frac{q-1}{2}\right)$ then P(q) is super edge trimagic digraph.

Proof

Let us consider a Galois field element of order q, a prime number which is congruent to 3 mod 4 as the nodes and construct a Paley digraph. We denote the node set V(P) as{ $v_1, v_2, \ldots, v_q$ } and the arc set as E ={ $E_1 \cup E_2 \cup E_3$ }, where $E_1 = \{e_i \quad 1 \le i \le p-1$, $E_2 = \{e_j \quad p \le j \le q-3$, and $E_3 = \{e_K \quad q-2 \le k \le q$. To prove that P(q) is super edge trimagic digraph let us define the function as

$$f(v_i) = i \quad 1 \le i \le q$$

When k is odd

$$f\left(E_{r_k}(e_i)\right) = \begin{cases} kq + p + 2 + 1 - i & 1 \le i \le p-1 \\ kq + 3p + 1 - i & p \le i \le q-3 \\ (k+1)q + 1 - i & q-2 \le i \le q \end{cases}$$

When k is even

$$f\left(E_{r_k}(e_i)\right) = \{kq + i \quad 1 \le i \le q$$

Summing up the labels of respective arc set to each node and the computation of each node gives either k_1 or k_2 or k_3 respectively.

For $1 \le i \le p-1$

$v_i = i + q + p + 2 + 1 - i + 2q + i + \cdots + (p-1)q + i + pq + p + 2 + 1 - i$

$$= i + q(1 + 2 + \cdots + p) + p\left(\frac{p+1}{2}\right) + \left(\frac{p+1}{2}\right)(-i) + \left(\frac{p-1}{2}\right)i + \left(\frac{p+1}{2}\right)3$$

$$= p\left(\frac{p+1}{2}\right)(q+1) + \left(\frac{p+1}{2}\right)3$$

$= k_1$.

For $p \le j \le q-3$

$v_j = j + q + 3p + 1 - j + 2q + j + \cdots + (p-1)q + j + pq + 1 - j$

$= j + q(1 + 2 + \cdots + p) + \left(\frac{p+1}{2}\right)3p + \left(\frac{p+1}{2}\right)1 + \left(\frac{p+1}{2}\right)(-j) + \left(\frac{p-1}{2}\right)j$

$$= \left(\frac{p-1}{2}\right)[pq + 3p + 1]$$

$= k_2$.

For $q-2 \le k \le q$

$v_k = k + 2q + 1 - k + 2q + k + \cdots + (p-1)q + k + (p+1)q + 1 - k$

$= q[2 + 4 + \cdots + (p+1)] + \left(\frac{p+1}{2}\right)1 + q[2 + 4 + \cdots + (p-1)]$

$$= 2q\left[1 + 2 + \cdots + \frac{p+1}{2}\right] + \left(\frac{p+1}{2}\right) + 2q\left[1 + 2 + \cdots + \frac{p-1}{2}\right]$$

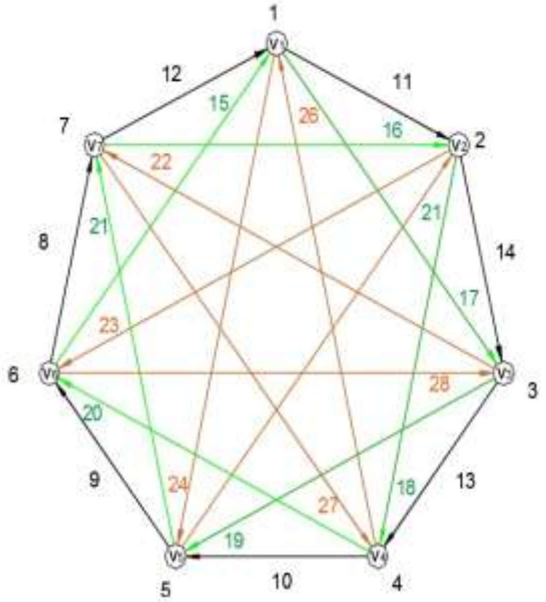
$$= \left(\frac{p+1}{2}\right)[(p+1)q + 1]$$

$= k_3$.

In this the node set is labeled as smaller values and this proved that P(q) is super edge trimagic graph.

A. Illustration

Paley digraph P(7) as super edge trimagic total graph .



B. Corollary

If the directed graph P(q) where q is a prime number is super edge trimagic graph then by the definition of modular super vertex magic one can easily identify that this graph as well as a modular super edge trimagic digraph. But the contrary must not be true.

III. MODULAR SUPER VERTEX MAGIC TOTAL LABELING

A. Proposition

Let G be a digraph with finite field elements as vertices v_1, v_2, ..., v_q, q is prime and $q \equiv 3$ (mod4). If each vertex of G has p incoming and p outgoing arcs with $|E| = pq$, where $p = \frac{q-1}{2}$ then G admits vertex magic total labeling.

B. Algorithm

Input: Galois field elements as nodes.

Step 1: Put together the nodes and arcs as directed graph P(q)

Step 2: The mapping $f : V \cup E \rightarrow \{1,2,\ldots,|V|+|E|\}$ is defined as

$f(v_i) = i \quad 1 \le i \le q$

$$f(E_{p_k}(e_i)) = \begin{cases} (k+1)q + 1 - i & 1 \le i \le q \quad k \text{ odd} \\ kq + i & 1 \le i \le q \quad k \text{ even} \end{cases}$$

Output: Super vertex magic total labeling.

C. Theorem

Let G be a digraph with finite field elements as nodes v_1, v_2, ..., v_q, q a prime and congruent to 3 (mod4). If each node of G has p incoming and p outgoing arcs with $|E| = pq$, where $p = \frac{q-1}{2}$ then G acknowledges super vertex magic total labeling.

Proof

Construct a strong regular directed graph by using the Galois field elements as its nodes and by Paley digraph definition. We symbolize the node set V(P) as$\{ v_1, v_2, \ldots, v_q \}$ and the arc set as $E = \{ E_1 \cup E_2 \cup \ldots \cup E_p \}$, $p = \frac{q-1}{2}$. To establish every directed Paley graph is super vertex magic total graph, exemplify that for every vertex v_i, the totting up of the labels of v_i with its leaving arcs raise a constant. Using the function from the above algorithm, for each vertex the totting up of the labels is given by

$$= q\left(\frac{p+1}{2}\right)\left[\frac{p+3}{2} + \frac{p-1}{2} + \frac{1}{q}\right]$$

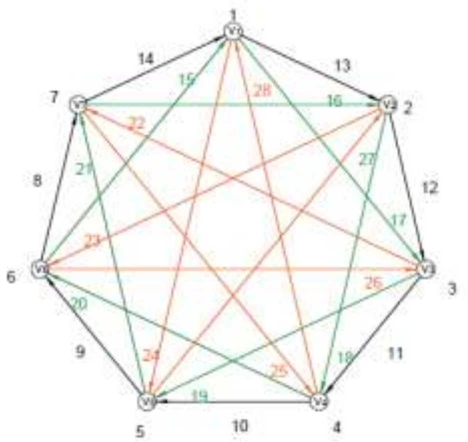
$$= \frac{(q+pq)(q+pq+1)}{2q}$$

$= k$ a constant.

Here the nodes are labeled with smaller numbers. From this one can conclude that the digraph is super vertex magic total graph.

D. Illustration

Paley digraph P(7) with super vertex magic total labeling.



E. Corollary

If the directed graph P(q) where q is a prime number is super vertex magic total graph then it is also a modular super vertex magic total digraph. But the converse need not be true.

IV. BOUND ON MAGIC CONSTANT

A. Theorem

Let G be a quadratic residue digraph with q vertices and pq edges. Suppose the sum of the edge labeling of the Hamiltonian circuit is αq where 'α' is mq + p + 1, m = 1, 2, …, $\frac{q-1}{2}$, a constant. If G is admits super vertex magic total labeling with a magic constant k, then $\frac{q(q+p+1)+1}{2} \leq k \leq q\left(\frac{q+2pq+1}{2}\right)$.

Proof

Every strong regular digraph has Hamiltonian circuit. Since P(q) is also a strong regular directed graph it has a Hamiltonian circuit and its measurement lengthwise is q. The computation of the arc labeling of the tour is αq and α = mq+p+1 where m = 1, 2, …, $\frac{q-1}{2}$ .The node labeling is excluded in the route. The accumulation of labeled arc of the circuit with minimal residue lessen the value and the accumulation of labeled arc with maximal residue maximize the value as$\frac{q(q+p+1)+1}{2} \leq k \leq q\left(\frac{q+2pq+1}{2}\right)$.

V. CHARACTERIZATION OF EDGE GRACEFUL LABELING ON PALEY DIGRAPH

Based on the structural properties of Paley digraph and the definition of edge graceful labeling and antimagic labeling on digraphs, a characterization of labeling is derived on Paley digraph.

A.Theorem

Let G be a quadratic residue digraph with finite field elements as vertices $v_1, v_2, \ldots, v_q$, q is prime and q ≡ 3 (mod 4). If each vertex of G has p incoming and p outgoing arcs with |E| = pq, where p = $\frac{q-1}{2}$ then G is edge graceful if and only if it is antimagic.

Proof

Consider the Paley digraph P(q) with q vertices and pq edges where p = $\frac{q-1}{2}$. Under edge graceful labeling, we define a bijective mapping f : E → {1, 2, ..., pq} such that the induced mapping $f^* : V \to \{0, 1, ..., q-1\}$ given by $f^*(v_j) = \sum_{i=1}^{p} f(e_{ij})$ (mod q), 1 ≤ j ≤ q is distinct. In the case of antimagic labeling we define a bijective mapping f : E → {1, 2, ..., pq} such that the induced mapping $f^* : V \to \{1, 2, ..., N\}$ given by $f^*(v_j) = \sum_{i=1}^{p} f(e_{ij})$, 1 ≤ j ≤ q is distinct. If P(q) admits edge graceful labeling then $f^*(v_j) = \sum_{i=1}^{p} f(e_{ij})$ (mod q), 1 ≤ j ≤ q gives q distinct values. Since $f^*(v_j)$ is independent of vertex labels, $f^*(v_j) = \sum_{i=1}^{p} f(e_{ij})$, 1 ≤ j ≤ q also gives distinct q values. Hence P(q) admits antimagic labeling.

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