The Time Theory Analysis and MATLAB Simulation of Control System

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Abstract. The time-domain analysis in typical second order control system is given. The performance indicators of the unit step response system are calculated. Based on the MATLAB simulation, the unit step response performance indicators through the M function and Simulink dynamic structure diagram are obtained. The theory analysis and the simulation results are compared.

Introduction

The time-domain analysis method is used to solve the dynamic response of the system according to the Laplasse transform of differential equation system. The typical dynamic responses are included of the unit step response unit, the unit ramp response, the acceleration response and the unit impulse response etc.

One purpose of time domain analysis is to solve the performance indicators of the response system. The general performance indicators of step response include Peak Time (TP), Overshoot, Adjust Time（ts）, and the Steady-state Error (ess).

Theoretical Analysis

The system which can be used to describe the two order differential equation is called second-order system. Its typical structure is shown in Figure 1.
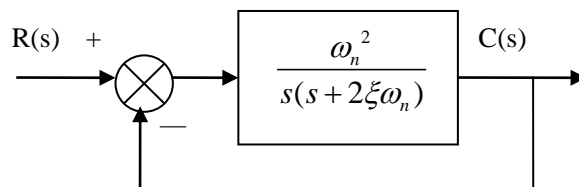

R(s) $+$ $\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ C(s)

Figure 1: Typical structure of second-order system

Now the structure of second-order system is shown in Figure 2.

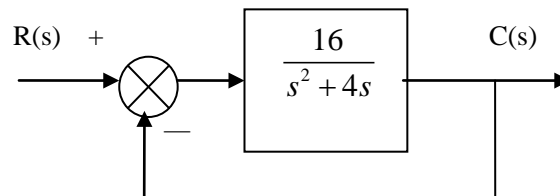R(s) $+$ $\dfrac{16}{s^2+4s}$ C(s)

Figure 2: Structure of second-order system

The above system is the unit negative feedback system. The open-loop transfer function is

$$G(s) = \frac{16}{s^2+4s}.$$

The close - loop transfer function will be got.

$$\phi(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{16}{s^2+4s}}{1+\frac{16}{s^2+4s}} = \frac{16}{s^2+4s+16}$$

The performance indicators and dynamic parameters of the system will be calculated under the unit Step function signal.

So the corresponding relation between the parameters and coefficient is shown as below.

$$\begin{cases} \omega_n^2 = 16 \\ 2\xi\omega_n = 4 \end{cases}$$

The absence of damping angle frequency and damping ratio is got as below.

$$\omega_n = 4, \xi = 0.5$$

So we will get the damping angle frequency.

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4*\sqrt{1-0.5^2} = 3.46$$

$$\beta = \arccos\xi = \arccos 0.5 = 60° = \frac{\pi}{3} = \frac{3.14}{3} = 1.05 \quad \text{(rad)}$$

The dynamic performance indicators of the system will be got as follows.

（1）Delay Time:

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7*0.5}{4} = 0.34 \quad \text{(S)}$$

（2）Raise Time:

$$t_r = \frac{\pi-\beta}{\omega_d} = \frac{3.14-1.05}{3.46} = 0.6 \quad \text{(S)}$$

（3）Peak Time:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{3.46} = 0.91 \quad \text{(S)}$$

（4）Overshoot:

$$\sigma\% = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}*100\% = e^{-\frac{3.14*0.5}{\sqrt{1-0.5^2}}}*100\% = e^{-1.81}*100\% = \frac{1}{e^{1.81}}*100\% = 16.3\%$$

（5）Setting Time

$$t_s = \frac{3.5}{\xi\omega_n} = \frac{3.5}{0.5*4} = 1.75 \quad \text{(S)}$$

（The error band　corresponding to 5%）

$$t_s = \frac{4.5}{\xi\omega_n} = \frac{4.5}{0.5*4} = 2.25 \quad \text{(S)}$$

（The error band　corresponding to 2%）

（6）Peak Magnitude

MP=（1+σ）*1=（1+0.163）*1=1.163

If input signal is unit step function, the output of second order under-damped system （ξ<1）is shown in Figure 3.
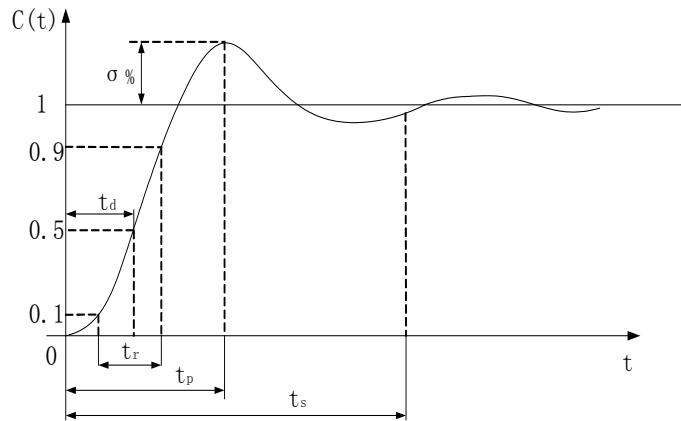
Figure 3: Unit step response of second order under-damped system

Simulation Based on MATLAB Command

In order to analyze time domain performance of control system, the following commands are entered in the MATLAB7.0 command window.

```
>> num=[16];
>> den=[1 4 0];
>> sys=tf(num,den)
Transfer function:
      16
---------
  s^2 + 4 s
>> closys=feedback(sys,1)
Transfer function:
        16
  --------------
  s^2 + 4 s + 16
>> step(closys)
```
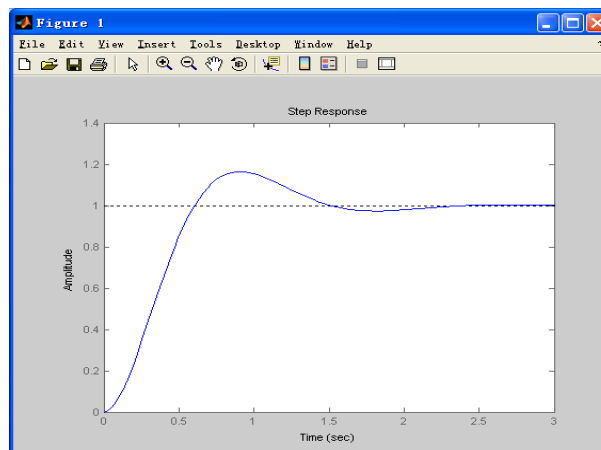

Figure 4: Unit step response of second order system based on MATLAB

```
>> [y,t]=step(closys);
>> [mp,tp,ess,b1,b2,sigma,n,pusi,T,f]=targ(y,t)
mp =1.1630
tp =0.9110
ess = -0.0028
b1 = 0.1603
b2 = 0.0016
sigma = 15.9813
```

n =102.0344
pusi = 0.9902
T = 1.8221
f =0.5488
The targ(y,t) function program is shown as follow.
function[mp,tp,ess,b1,b2,sigma,n,pusi,T,f]=targ(y,t)
[mp,tf]=max(y);
ct=length(t);
mp;
tp=t(tf);
tm=max(tf);
yss=(ct);
q=1;
m=q-1;
  while m<3,
        for a=(tm/100):0.01:tm
            j=[0:a:tm];
            for i=1:length(j);
                if (y(i+1)-y(i))<0 & (y(i)-y(i-1))>0
                    m=m+1;
                    pm(m)=y(i);
                    tp(m)=t(i);
                end
            end
        end
  end
  yss=y(ct);
  ess=1-yss;
  b1=pm(1)-yss;
  b2=pm(2)-yss;
  sigma=100*b1/yss;
  n=b1/b2;
  pusi=(b1-b2)/b1;
  T=(tp(2)-tp(1));
  f=1/T;
  tp=tp(1);

The symbols meanings of above program are given as follows. MP is Peak Magnitude; TP is Peak Time; ESS is Steady-state Error; B1 is Overshoot of first peak; B2 is Overshoot of second peak; Sigma is overshoot of step response; n is attenuation of step response; Pusi is decrement of step response; T is period; F is frequency.

Simulation Based on Simulink of MATLAB

The Simulink dynamic structure diagram of second order system based on MATLAB is given as figure 5. And it is named as "erjie".
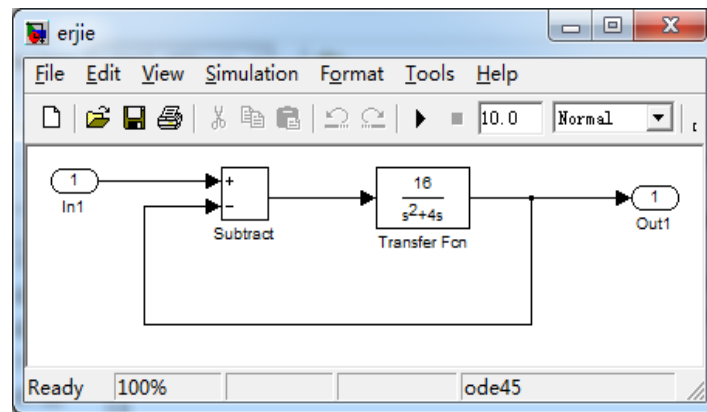
Figure 5: Simulink dynamic structure diagram of second order system based on MATLAB

Parameters of Transfer Fcn are set as figure 6.

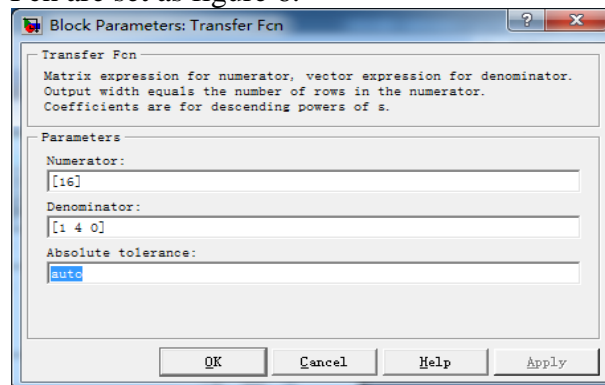Figure 6: Parameters setting of Transfer Fcn

The following commands are entered in the MATLAB7.0 command window.

```
>>   [a,b,c,d]=linmod('erjie')
a =
     -4     -16
      1       0
b =
      1
      0
c =
      0      16
d =
      0
>>   sys=ss(a,b,c,d);
>> figure(1)
>> step(sys)
```
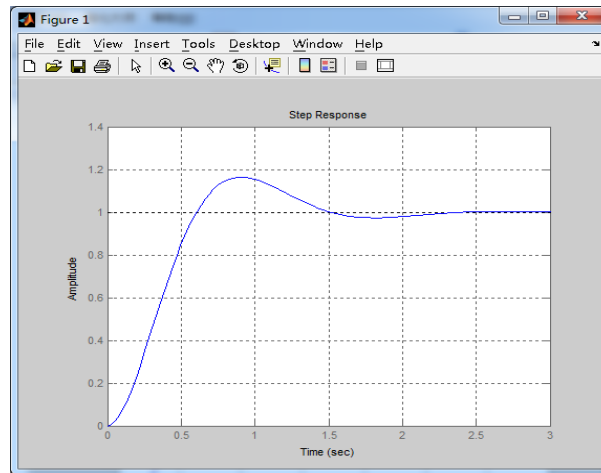
Figure 7: Unit step response of second order system based on Simulink
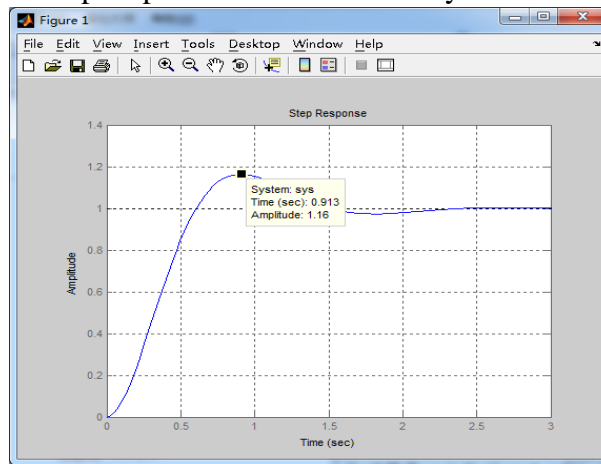

Figure 8: Unit step response of second order system performance based on Simulink
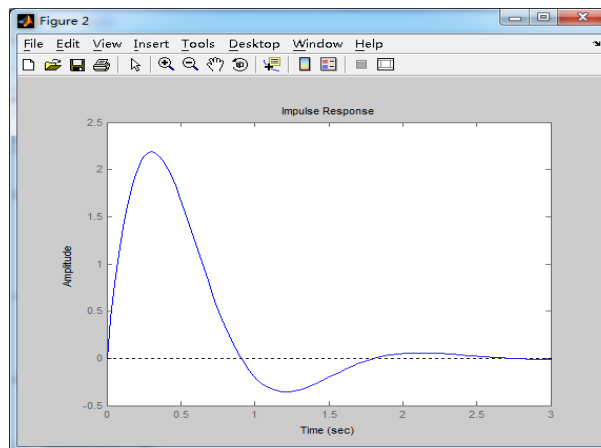
>> figure(2)
>> impulse(sys)

Figure 9: Unit impulse response of second order system performance based on Simulink

From figure 8, it can be seen that the unit pulse response peak time TP=0.913 second and peak Magnitude MP=1.16.

Conclusions

According to the simulation result, we can see that MP= 1.1630, TP=0.9110, B1=0.1603. According to the theory analysis above, we know that MP=（1+σ）*1=（1+0.163）*1=1.163, $t_p = 0.91$ S, $\sigma\%$ =16.3%=0.16. MATLAB simulation Results are consistent with theoretical analysis. MATLAB simulation verifies the theoretical analysis.

The time-domain analysis in typical second order control system is given. The performance indicators of the unit step response system are calculated. Based on the MATLAB simulation, the unit step response performance indicators through the M function and Simulink dynamic structure diagram are obtained. The theory analysis and the simulation results are compared. This paper has good auxiliary function to use and study the typical second order control system.

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