A Consensus Model in Group Decision Making Based on Interpolative Boolean Algebra

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Abstract

The aim of this paper is to propose a soft consensus model based on interpolative Boolean algebra for group decision making problems. Consensus degrees are calculated on three levels (on pairs of alternatives, alternatives, collective level) by means of pseudo-logical aggregation. The relation of equivalence is employed as a similarity measure among experts' opinions. In fact real-valued realization of equivalence is used as a generalized Boolean polynomial.

In the illustrative example of sustainable development problem, we have shown that the proposed model is appropriate to determine the level of agreement among experts.

Keywords: Consensus, Interpolative Boolean algebra, Group decision making, Similarity measure, Equivalence relation

1. Introduction

In today's knowledge-based world, various intelligent tools and techniques are used to support decision making. Decision making is an integral part of every aspect of life and involves choosing an alternative which best fulfills the entire set of goals.

Complex problem analysis requires the knowledge and experience of a group of experts usually from different fields of expertise and different backgrounds. A group decision making (GDM) problem may be defined as a decision situation in which there are two or more experts (i) each of them characterized by his/her own perceptions, attitudes, motivations, and personalities, (ii) who recognize the existence of a common problem, and (iii) who attempt to reach a collective decision [1]. In other words, a GDM problem is a decision problem with several alternatives and a panel of decision makers or experts that try to achieve a common solution taking into account their opinions or preferences [2].

Experts come from different background/specialty fields and their opinions are unlikely to be identical. Since they are usually either close or conflicting, it is valuable to measure the level of agreement i.e. consensus degree. In GDM before applying the selection process, it is desirable that experts reach a high degree of consensus.

In this paper, the focus is not on the methods and/or reasoning that experts use to evaluate a set of alternatives, but on the appropriate way to measure consensus of experts' opinions on the solution set of alternatives. The existing soft consensus models can be perceived from the perspective of similarity/distance measures and aggregation functions employed.

Common distance functions for modeling soft consensus in GDM problems are: Manhattan, Euclidean, Cosine, Dice, and Jaccard distance functions [3]. In general, the weighted average is the simplest and widely used aggregation tool.

In this paper we propose a soft consensus model based on interpolative Boolean algebra (IBA) [4, 5]. In fact, we employ the IBA equivalence [6] as a logic-based similarity measure. To obtain the collective consensus degrees we propose (pseudo) logical aggregation (LA) - universal aggregation method [7]. Depending on the nature of a problem, LA can be realized throughout various aggregation functions [8, 9, 10]. In the proposed model, pseudo-logical functions are employed on three different levels introduced in [1]: pair of alternatives, alternatives and collective level.

The paper is structured as follows. In Section 2 an overview of consensus approaches in group decision making is given. Section 3 and 4 provide the basic concepts of Interpolative Realization of Boolean Algebra and similarity based on IBA respectively. In Section 5 the proposed consensus model based on IBA is introduced and an illustrative example is provided in Section 6. Finally, Section 7 concludes the paper.

2. Consensus approaches in group decision making

Many complex decisions are usually made in groups because a single decision maker is not able to consider a problem from different perspectives.
Before experts are able to make a decision, they need to evaluate the alternatives by either numerical or linguistic assessments. Typically it is not easy to reach the absolute consensus among expert' opinions. For that reason it is important to find the appropriate way to measure consensus. Consensus refers to a general agreement within the group of people, or agents, both human or software, in a more general setting [11]. In the broader sense, consensus is a dynamic and iterative process and is repeated until experts' opinions become sufficiently similar or a target level of consensus is reached [2, 12].

In analyzing consensus, different approaches have been proposed [13]:

- hard consensus measures that vary between 0 (no consensus or partial consensus) and 1 (full consensus) (e.g. [14, 15]);

- soft consensus measures as more realistic approach.

Kacprzyk and Fedrizzi [16] introduced fuzzy majority using linguistic quantifiers to define soft consensus measures. Furthermore, the issue of measuring consensus based on fuzzy preferences and majorities is discussed in [17].

Herrera et. al [12] introduced a soft consensus model with fuzzy linguistic preferences in GDM problems, and defined two types of soft consensus measures: consensus degrees and proximity measures. Further, several soft consensus models were developed; models able to process different representation formats of experts' preferences [18], consensus models under fuzzy multigranular linguistic preferences [19], and consensus models with incomplete fuzzy preference relations [20]. An extensive review of soft consensus models can be found in [11, 21]. Advantages and drawbacks of consensus approaches in fuzzy GDM, and their future trends are analyzed in [13].

The idea of calculating distances among preferences in the context of linear orders has revealed in [22]. Garcia-Lapresta and Perez-Roman [23] extended this notion to weak orders paying special attention to seven well-known distances: discrete, Manhattan, Euclidean, cosine, Chebyshev, Kemeny and Hellinger. According to [3] following distance functions are common for measuring soft consensus in GDM problems: Manhattan, Euclidean, cosine, dice, and Jaccard.

In this paper it is appropriate to consider consensus approaches from the following aspects: representation formats of input data, similarity measures and aggregation functions. We present model with [0,1] inputs, with a novel logic-based similarity measure and LA as a universal method for aggregation.

3. Interpolative Realization of Boolean Algebra

This section brings briefly overview of Interpolative Boolean Algebra, a consistent multi-valued ([0,1]-valued) realization of finite (atomic) Boolean algebra.

3.1. Interpolative Boolean Algebra

IBA is a theoretical framework introduced by Radojevic [4], where all elements of Boolean algebra (BA) have their real-valued realization consistent to Boolean theories and axioms. The real-valued realization of finite BA is adequate for many real problems since gradation offers superior expressiveness in comparison to the black-white outlook [5].

By introducing the principle of structural functionality opposed to truth functional principle, Radojevic [5] has separated two logical levels: symbolic and valued. Structural functionality is an algebraic (value irrelevant) principle whereas truth functionality depends on value realization and it is only valid in classical two-valued case. The structure of any attribute – element of Boolean algebra determines which atomic Boolean elements are included and/or not included in it.

Structural function defines inclusion of the corresponding atom in Boolean function.

A generalized Boolean polynomial uniquely corresponds to any element of the analyzed Boolean algebra. The procedure of transformation of Boolean functions into corresponding GBPs is defined in [24] and is explained on example further in this section.

A GBP has the ability to process values of primary variables from real unit interval [0,1] so as to preserve all algebraic characteristics on value level.

In GBPs there are two standard arithmetic operators + and $-$, and generalized product $\otimes$ as the third. Generalized product is a subclass of T-norms. Operator function for generalized product is any function which maps $\otimes\colon [0,1]\times[0,1]\to[0,1]$ and satisfies all four axioms of T-norms (comutativity, asociativity, monotonicity and boundary condition) and additional axiom - non-negativity condition [5].

In the case of $\Omega=\{a,b\}$ generalized product is from the following interval [7]:

$$\max(a+b-1,0)\le a\otimes b\le \min(a,b) \quad (1)$$

Here we want to note that operator chosen for a generalized product does not have any influence on algebra since algebra is always Boolean.

Depending on the nature of the attributes $\Omega=\{a,b\}$ that are to be aggregated, we can discuss three marginal cases for operator selection. The first case refers to attributes of the same/similar nature and implies the use of min function i.e. $a\otimes b=\min(a,b)$. The second involves attributes of the same/similar nature but inversely (negatively) correlated. In this case Lukasiewicz operator is proposed i.e. $a\otimes b=\max(a+b-1,0)$. In the case of independent attributes (different by nature) ordinary product that belongs to interval is used i.e. $a\otimes b=a\cdot b$.

Only once the transformations have been conducted and the final structure established, will the values be introduced and computed. This is the main difference between the conventional and Boolean consistent approaches which can, in certain cases, lead to different results.

3.2. Transformation of Boolean functions into GBPs – relation of equivalence

In this section we aim to illustrate the transformation of logical Boolean functions into GBPs on the case of the relation of equivalence i.e. logical expression for equivalence.

For a set of primary Boolean variables $\Omega=\{a_1,\dots a_n\}$, that generates Boolean algebra $BA(\Omega)$, the procedure of transformation of Boolean functions into corresponding GBPs is defined in [24]:

- For combined elements $F(a_1,\ldots,a_n), G(a_1,\ldots,a_n) \in BA(\mathbf{\Omega})$:

$$\begin{aligned}
&(F \wedge G)^{\otimes} = F^{\otimes} \otimes G^{\otimes}, \\
&(F \vee G)^{\otimes} = F^{\otimes} + G^{\otimes} - (F \wedge G)^{\otimes}, \quad (2) \\
&(\neg F)^{\otimes} = 1 - F^{\otimes}
\end{aligned}$$

- For primary variables $\{a_1,\ldots a_n\}$:

$$\begin{aligned}
&(a_i \wedge a_j)^{\otimes} = \begin{cases} a_i \otimes a_j, & i \neq j \\ a_i, & i = j \end{cases}, \\
&(a_i \vee a_j)^{\otimes} = a_i + a_j - (a_i \wedge a_j)^{\otimes}, \quad (3) \\
&(\neg a_i)^{\otimes} = 1 - a_i
\end{aligned}$$

The relation of equivalence is defined as following logical expression:

$$a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a) \qquad (4)$$

To transform a Boolean function into a GBP the first step is to assess its structure. In this particular case the following transformation steps have been taken in eq. 5.

After the transformation has been accomplished focus is transferred to the value level. A GBP that uniquely corresponds to the relation of equivalence satisfies the properties of reflexivity, transitivity and symmetry only when min is used as generalized product ($\otimes := \min$). Therefore a consistent comparison of different objects is only possible by the same criteria [6].

$$\begin{aligned}
(a \Leftrightarrow b)^{\otimes} &= \left((a \Rightarrow b) \wedge (b \Rightarrow a)\right)^{\otimes} = \\
&= (a \Rightarrow b)^{\otimes} \otimes (b \Rightarrow a)^{\otimes} = \\
&= (1 - a + a \otimes b) \otimes (1 - b + a \otimes b) = \qquad (5) \\
&= 1 - b + a \otimes b - a + a \otimes b - a \otimes a \otimes b + a \otimes b - a \otimes b \otimes b + a \otimes b \otimes a \otimes b = \\
&= 1 - b + a \otimes b - a + a \otimes b - a \otimes b + a \otimes b - a \otimes b + a \otimes b = \\
&= 1 - b - a + 2 \cdot a \otimes b
\end{aligned}$$

By applying min function in the previous GBP we obtain the following expression:

$$(a \Leftrightarrow b)^{\otimes} = 1 - a - b + 2 \cdot \min(a,b) \quad (6)$$

The graphical interpretation of multi-valued equivalence by Radojevic is given in Fig 1.

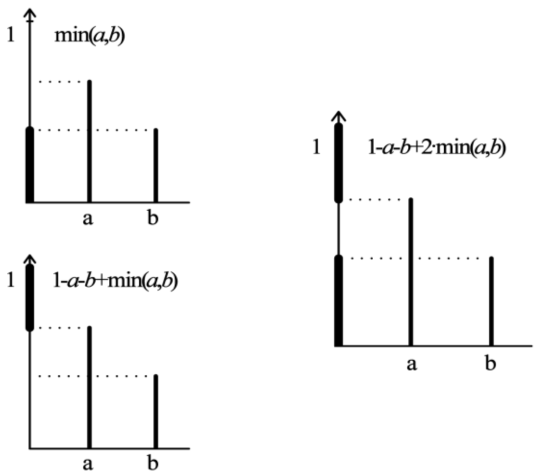

Fig. 1: IBA equivalence.

Two objects *a* and *b* can only be compared by the same property. They are equivalent by the union of two parts: the intensity of both having the same property and the intensity of both not having that property.

3.3. Pseudo-logical aggregation based on IBA

In this paper, we use pseudo-logical aggregation based on IBA to obtain consensus degrees on different levels or one global estimate of consensus for group decisions.

Logical aggregation is a consistent and transparent procedure based on IBA for aggregating factors [7]. The task of LA is the fusion of primary attributes' values into one resulting globally representative value. It has two steps:

- Normalization of attributes' values:

$$\|\cdot\| : \Omega \to [0,1] \qquad (7)$$

- Aggregation of normalized attributes' values into one resulting value by logical or pseudo-logical function used as a LA operator:

$$Aggr\,[0,1]^n \to [0,1] \qquad (8)$$

Pseudo-logical function, called pseudo GBP, is a linear convex combination of generalized Boolean polynomials. A pseudo-logical aggregation depends on the aggregation measure and generalized product. As a special case, LA operator may be realized as weighted sum, arithmetic mean, min or max function, etc. [7]. Logical dependence between aggregating factors may also be taken into account [9]. Thus, LA is able to model various aggregation functions depending on the nature of a problem.

4. Similarity based on IBA

First part of this section is devoted to the theoretical basis for notions of distance, similarity and metrics [25]. Further we define distance and similarity based on IBA.

Definition 1. Let X be a set. A function $d : X \times X \to R$ is called a **distance** (or dissimilarity) on X if, for all $x, y \in X$, there holds:

- $d(x, y) \geq 0$ (non-negativity);
- $d(x, y) = d(y, x)$ (symmetry);
- $d(x, x) = 0$ (reflexivity).

Definition 2. Let X be a set. A function $s : X \times X \to R$ is called a **similarity** on X if, for all $x, y \in X$, there holds:

- $s(x, y) \geq 0$ (non-negativity);
- $s(x, y) = s(y, x)$ (symmetry);
- $s(x, y) \leq s(x, x)$ with equality if and only if $x = y$.

The main transforms used to obtain a distance (dissimilarity) d from a similarity s bounded by 1 from above are: $d = 1 - s$, $d = (1-s)/s$, $d = \sqrt{1-s}$, $d = \sqrt{2(1-s^2)}$, $d = \arccos s$, $d = -\ln s$, etc. From the point of view of logic, distance is the negation of similarity.

Definition 3. Let X be a set. A function $d : X \times X \to R$ is called a **metric** on X if function d is distance on X and if, for all $x, y, z \in X$, there holds:

- $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality).

The main idea of this paper is to use the relation of equivalence in the sense of IBA as similarity. Real-valued realization of implication and equivalence as generalized Boolean polynomials were presented by Radojevic in [6, 26]. Their application in the form of exclusive disjunction was considered for measuring distance in [27].

The exclusive disjunction in the sense of IBA, $dIBA : [0,1] \times [0,1] \to [0,1]$, can be used as a distance (and metric) because it satisfies all necessary conditions:

- $dIBA(x, y) = x + y - 2 \cdot x \otimes y \geq 0$, due to the boundary and monotonicity of generalized Boolean product – $x \geq x \otimes y, y \geq x \otimes y$;
- $dIBA(x, y) = x + y - 2 \cdot x \otimes y = = x + y - 2 \cdot y \otimes x = dIBA(y, x)$, due to the commutativity of generalized Boolean product;
- $dIBA(x, z) + dIBA(z, y) - dIBA(x, y) =$
  $x + z - 2 \cdot x \otimes z + z + y - 2 \cdot z \otimes y - x - y + 2 \cdot x \otimes y =$
  $2 \cdot z - 2 \cdot x \otimes z - 2 \cdot z \otimes y + 2 \cdot x \otimes y =$
  $= 2 \cdot (z - x \otimes z - z \otimes y + x \otimes y) \geq 0$
  1. $y \geq z \Rightarrow x \otimes y - z \otimes x \geq 0, z - z \otimes y \geq 0 \Rightarrow$
  $\Rightarrow z - x \otimes z - z \otimes y + x \otimes y \geq 0;$
  2. $z > y \Rightarrow (x \otimes y = x \otimes z) \vee (x \otimes y = z \otimes y) \Rightarrow$
  $\Rightarrow x \otimes z + z \otimes y - x \otimes y =$
  $= \max(x \otimes z, z \otimes y) \leq z \Rightarrow$
  $\Rightarrow z - x \otimes z - z \otimes y + x \otimes y > 0$
  $\Rightarrow z - x \otimes z - z \otimes y + x \otimes y > 0$, due to the boundary and monotonicity of generalized Boolean product - $x \geq x \otimes y, y \geq x \otimes y$ and the rules of transformation of Boolean functions into corresponding GBPs defined in eq. 3.

The relation of equivalence in the sense of IBA, $sIBA : [0,1] \times [0,1] \to [0,1]$, can be used as a similarity measure because it satisfies all necessary conditions:

- $sIBA(x, y) = 1 - x - y + 2 \cdot x \otimes y =$ $= 1 - (x + y - 2 \cdot x \otimes y) \geq 0$, due to the boundary and monotonicity of generalized Boolean product $x \geq x \otimes y$, $y \geq x \otimes y$, $x + y - 2 \cdot x \otimes y \leq 1$;
- $sIBA(x, y) = 1 - x - y + 2 \cdot x \otimes y =$ $= 1 - x - y + 2 \cdot y \otimes x = sIBA(y, x)$, due to the commutativity of generalized Boolean product;
- $sIBA(x, x) = 1 - x - x + 2 \cdot x \otimes x =$ $= 1 - 2 \cdot x + 2 \cdot x = 1 \geq sIBA(x, y)$, due to the rules of transformation of Boolean functions into corresponding GBPs defined in eq. 3.

IBA similarity is the logical negation of IBA distance – $sIBA(x, y) = 1 - x - y + 2 \cdot x \otimes y = 1 - dIBA(x, y) = \neg dIBA(x, y)$.

5. Consensus model based on IBA in GDM

In this section we present a consensus model in group decision making based on IBA.

In a classical GDM situation there is a problem to solve, a solution set of possible alternatives, $A = \{A_1, A_2, ..., A_n\}$ and a group of two or more experts, $E = \{E_1, E_2, ..., E_m\}$ characterized by their own ideas, attitudes, motivations and knowledge, who express their opinions about this set of alternatives to achieve a common solution [13, 28]. Apart from the selection process, in this paper we consider only consensus process. In literature, there are different ways to perceive consensus: consensus degrees and proximity measures [13]. As proposed in [12] we observe consensus degrees on three different levels. The first level refers to consensus degrees on each pair of alternatives for all experts. The second level measures consensus degrees for each alternative, while the third level aims to evaluate global consensus for all experts on all alternatives.

Our focus is on GDM problems where experts express their preferences on alternatives by numerical values. It is assumed that expert's opinion on each alternative is described by a_{ij} - assessment of i-th alternative by j-th expert. All assessments are normalized on [0,1] interval. As a result consensus degrees within [0,1] interval is obtained..

The starting point for a problem is given in Table 1.

Alternative/ Expert	E_1	E_2	...	E_m
A_1	a_{11}	a_{12}	...	a_{1m}
A_2	a_{21}	a_{22}	...	a_{2m}
…	...	...	...	...
A_n	a_{n1}	a_{n2}	...	a_{nm}

Table 1: Normalized alternative assessments.

The computation of the consensus degrees is carried out in two steps:

1. Calculation of similarity matrices for experts, as well as alternatives;
2. Calculation of consensus degrees on all three levels.

Calculation of similarity matrices for each expert E_j where $j=1,\dots,m$ is given at Table 2.

E_j	A_1	A_2	...	A_n
A_1	$(a_{1j}\Leftrightarrow a_{1j})^{\otimes}$	$(a_{1j}\Leftrightarrow a_{2j})^{\otimes}$	...	$(a_{1j}\Leftrightarrow a_{nj})^{\otimes}$
A_2	$(a_{2j}\Leftrightarrow a_{1j})^{\otimes}$	$(a_{2j}\Leftrightarrow a_{2j})^{\otimes}$	...	$(a_{2j}\Leftrightarrow a_{nj})^{\otimes}$
...	...	...	...	...
A_n	$(a_{nj}\Leftrightarrow a_{1j})^{\otimes}$	$(a_{nj}\Leftrightarrow a_{2j})^{\otimes}$	...	$(a_{nj}\Leftrightarrow a_{nj})^{\otimes}$

Table 2: Similarity matrix for expert E_j.

Clearly, similarity matrix is symmetric and the main diagonal elements are always equal to 1.

Further similarity matrices from the aspect of each alternative A_i, $i=1,\dots,n$ are also calculated (Table 3).

A_i	E_1	E_2	...	E_m
E_1	$(a_{i1}\Leftrightarrow a_{i1})^{\otimes}$	$(a_{i1}\Leftrightarrow a_{i2})^{\otimes}$	...	$(a_{i1}\Leftrightarrow a_{im})^{\otimes}$
E_2	$(a_{i2}\Leftrightarrow a_{i1})^{\otimes}$	$(a_{i2}\Leftrightarrow a_{i2})^{\otimes}$	...	$(a_{i2}\Leftrightarrow a_{im})^{\otimes}$
...	...	...	...	...
E_m	$(a_{im}\Leftrightarrow a_{i1})^{\otimes}$	$(a_{im}\Leftrightarrow a_{i2})^{\otimes}$	...	$(a_{im}\Leftrightarrow a_{im})^{\otimes}$

Table 3: Similarity matrix for alternative A_i.

As beforehand, similarity matrices have the same properties.

In the second step, we proceed with the calculation of consensus degrees on all three levels.

Level 1: On the basis of similarity matrices for each expert E_j, $j=1,\dots,m$ consensus degrees on pair of alternatives $CD^1_{i,k}$ can be defined as a pseudo-logical function e.g. weighted sum:

$$CD^1_{i,k} = \sum_{j=1, i\neq k}^{m} w_j \cdot sm(j)_{i,k}, \quad \sum_{j=1}^{m} w_j = 1 \quad (9)$$

where $sm(j)_{i,k} = \left(a_{i,j} \Leftrightarrow a_{k,j}\right)^{\otimes}$ defines similarity measures of alternatives i and k for expert j.

Depending on experts' field of expertise and experience various weights can be assigned.

Level 2: On the basis of similarity matrices for each alternative A_i, $i=1,\dots,n$ consensus degrees on alternatives CD^2_i can be defined as s pseudo-logical function e.g. weighted sum:

$$CD^2_i = \frac{\sum_{j=1}^{m} \sum_{k=1, j\neq k}^{m} sm(j,k)_i}{m\cdot(m-1)/2} \quad (10)$$

Where $sm(j,k)_i = \left(a_{i,j} \Leftrightarrow a_{i,k}\right)^{\otimes}$ defines similarity measures of experts j and k on alternative i.

In this case, we use the simple average of similarities.

Level 3: On the basis of consensus degrees on alternatives on second level, collective consensus degree as a single measure CD^3 can be defined as:

$$CD^3 = \frac{\sum_{i=1}^{n} CD^2_i}{n} \quad (11)$$

It should be noted that in this consensus model pseudo LA operator can be realized in various forms. For instance, LA operator realized as min function can be employed to detect minimal (guaranteed) level of agreement among experts on an alternative (Level 2).

On the other hand, a LA realization as max function may be appropriate in case it is satisfactory for high level of consensus that at least two experts have a high level of agreement.

As previously indicated this model is able to support different situations that may occur in the consensus process. Thus it is a flexible and universal tool.

6. Illustrative example

In this section we illustrate the proposed consensus model in GDM in the perspective of sustainable development. The four projects P_1, P_2, P_3 and P_4 are to be evaluated by various experts (Table 4). To take into account all components of sustainable environment, these experts should be appointed from economic, social and environmental fields.

Project / Expert	E_1	E_2	E_3
P_1	0.90	0.82	0.77
P_2	0.37	0.30	0.23
P_3	0.89	0.85	0.25
P_4	0.94	0.55	0.20

Table 4: Projects assessments by experts – normalized values.

Consensus degrees on Level 1 are calculated on the bases of similarity matrices for each expert, as simple average (all the experts have same importance) and presented at Table 5.

CD^1	P_1	P_2	P_3	P_4
P_1	-	0.470	0.813	0.707
P_2		-	0.637	0.717
P_3			-	0.867
P_4				-

Table 5: Consensus degrees on pair of alternatives - simple average.

Consensus degrees on a pair of alternatives can be calculated with different weights, not as simple average. For example, weights for each expert can be 0.4, 0.35, and 0.25, respectively that better describes the importance of economic and social aspects of sustainable environment. In this case, consensus degrees on Level 1 are given in Table 6.

As expected, economic and social experts have greater influence on consensus degrees on this level. As a result, values $CD^1_{1,3}$ and $CD^1_{1,4}$ are lower and $CD^1_{2,3}$ and $CD^1_{2,4}$ are greater compared to the previous case.

CD^1	P_1	P_2	P_3	P_4
P_1	-	0.471	0.856	0.747
P_2		-	0.595	0.677
P_3			-	0.863
P_4				-

Table 6: Consensus degrees on pair of alternatives – weighted sum.

Consensus degrees on alternatives, computed using eq. 10, are $CD^2_1 = 0.913$, $CD^2_2 = 0.907$, $CD^2_3 = 0.537$, and $CD^2_4 = 0.507$. In general, experts agree on an alternative if their opinions are either both good or both bad regarding the alternative. Bearing that in mind, we can see from the Table 4. that the greatest consensus is achieved for projects P1 and P2.

Values obtained on Level 2. are constituent parts of collective consensus degree. The consensus degree on the relation is $CD^3 = 0.725$ and it provides information about level of agreement among all experts on all projects considered.

7. Conclusion

In this paper we propose a soft consensus model based on IBA for GDM problems. The main benefits of this model are:

- It includes logic in perceiving/measuring consensus;
- It is a general and flexible approach.

In general, experts agree on an alternative if their opinions are either both good or both bad regarding the alternative. Equivalence relation is proposed as a natural and intuitive way to describe the notion of agreement. In order to determine a level of agreement among experts, IBA equivalence is employed as a logic-based similarity measure.

Consensus degrees on three levels (on pairs of alternatives, alternatives, collective level) are calculated by means of pseudo-logical aggregation. Depending on the applied aggregation functions it is possible to model various decision situations. Thus, a common model is provided.

In the illustrative example we have shown that the proposed model is appropriate to determine the level of agreement among experts. Further these values can be used to direct the process of reaching consensus.

For future research we aim to develop new proximity measures based on IBA. Another potential direction of our research is to compare different logic-based similarity measures employed within our model.

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