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VERSITA

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On the Lense-Thirring test with the Mars Global Surveyor in the gravitational field of Mars

Comment

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Abstract: I discuss some aspects of a recent frame-dragging test performed by exploiting the Root-Mean-Square (RMS) orbit-overlap differences of the out-of-plane component (N) of the Mars Global Surveyor (MGS) spacecraft's orbit in the gravitational field of Mars. A linear fit to the complete time series for the entire MGS data set (4 February 1999–14 January 2005) yields a normalized slope 1.03 ± 0.41 (with 95% confidence bounds). Other linear fits to different data sets confirm agreement with general relativity. Huge systematic effects induced by mismodeling the martian gravitational field which have been claimed by some authors are absent in the MGS out-of-plane record. The same level of effect is seen for both the classical non-gravitational and relativistic gravitomagnetic forces on the in-plane MGS orbital components; this is not the case for the out-of-plane components. Moreover, the non-conservative forces experience high-frequency variations which are not important in the present case where secular effects are relevant.

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1. Introduction

Orbital data for the Mars Global Surveyor (MGS) spacecraft were collected over 5 years. In Refs. [1, 2] I proposed an interpretation of the time series of RMS orbit-overlap differences [3] for the out-of-plane orbital component, in terms of the general relativitic gravitomagnetic Lense-Thirring effect[1] [6]. The average of such a time series over ΔP (in this case, 5 years: 14 November 1999-14 January 2005 [1]), normalized to the predicted Lense-Thirring out-of-plane mean shift over the same time span, is $\mu = 1.0018 \pm 0.0053$.

My interpretation has recently been questioned by Krogh

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[1] *The Lense-Thirring effect consists of a small precession in the orbit of a test particle which is moving freely around a central rotating mass; the general relativistic component of the gravitational field due to the mass currents generated by the rotation of the body is named "gravitomagnetic" by analogy with the magnetic field induced by electric currents in Maxwellian electromagnetism [4]. For details of some recent, controversial tests performed in the Earth's gravitational field with the LAGEOS satellites, see Ref. [5] and references therein.*

in Ref. [7]. Remarks concerning the analysis presented in Refs. [1, 2] mainly deal with

I) The observable used: could the data in Refs. [1, 2] have been misinterpreted?

II) The discrepancy between the predicted gravitomagnetic Lense-Thirring shift and the data over the chosen time span ΔP: Krogh suggests that I incorrectly compared the 1.6 m value of the out-of-plane average orbit error (released by Konopliv *et al.* in Ref. [3] for the entire MGS data set) to a Lense-Thirring shift calculated for a shorter time interval ΔP;

III) The data set used: in Refs. [1, 2] some of the initial months of the MGS data set were discarded;

IV) The bias –neglected by me in Refs. [1, 2]– due to the multipolar expansion of the Newtonian part of the Martian gravity field, as pointed out in unpublished Refs. [8, 9] quoted by Krogh [7];

V) The impact of atmospheric drag: this was neglected in Refs. [1, 2]

Below I present my reply. Following further, independent tests, I present various linear fits to *different data sets*. These include the *complete* time series of the *entire MGS data* (4 February 1999–14 January 2005); the predictions of general relativity are *always confirmed*. Analytical calculation of competing aliasing effects due to both the gravitational and non-gravitational perturbations (which affect the *in-plane* orbital components of MGS), do *not* show up in the real data. Moreover, non-conservative forces, whose steadily refined modeling mainly improved the *in-plane* orbital components of MGS, *not* the *normal* one, exhibit high-frequency, non-secular time variations.

2. My arguments

2.1. General considerations

Before going into the details of my reply, I will evaluate the possibility that general relativity should be taken into account in describing the motion of test bodies in a given gravitational field. This can be done using a standard argument [10] applied to the MGS-Mars system.
It is necessary to consider the relativistic characteristic length of the problem at hand, (similar to the Schwarzschild radius of the body acting as the source of the gravitational field), and then comparing this length to the accuracy with which the orbit of a test particle can be determined.

In the present case, the characteristic gravitomagnetic length of a rotating body of mass M and angular momentum S is [11]

$$l_g = \frac{S}{cM}, \tag{1}$$

where c is the speed of light in vacuum; l_g is one of the two parameters with dimension of length entering the Kerr metric, [12] which describes the exterior field of a rotating black hole. Since the angular momentum of Mars can be evaluated as

$$S_{\rm M} = (1.92 \pm 0.01) \times 10^{32}\ {\rm kg\ m^2\ s^{-1}} \tag{2}$$

(from the latest spacecraft-based determinations of the areophysical parameters [3]), it turns out that

$$l_g^{\rm M} = \frac{S_{\rm M}}{cM_{\rm M}} = 1.0\ {\rm m}. \tag{3}$$

This value must be compared with the present-day levels of accuracy in the determination of the MGS orbit; it can be evaluated as about 0.15 m [3] in the radial direction. Since it is not affected by the gravitomagnetic force itself, as I will show later, such a figure is truly representative of the impact of all the sources of errors affecting the orbit of MGS. Thus, it makes sense to investigate the possibility of measuring the Lense-Thirring effect in the gravitational field of Mars with MGS in such a way that a possible positive outcome should not be regarded as unlikely.

2.2. Reply to Krogh's points

Point I The entire MGS data set was subdivided by Konopliv *et al.* [3] into 388 arcs (smaller time intervals of data), not 442 arcs, as claimed by Krogh [7]. For MGS, the lengths of the arcs vary from 4 days to 6 days, in order to cover many orbital revolutions (each $\approx$ 2 h). For each arc, the spacecraft position and velocity (amongst other things) were estimated and used as the starting point for a numerical propagation of the satellite's motion by means of the dynamical models. In the case of MGS, these models did not include the general relativistic gravitomagnetic force. Contiguous arcs had an overlap just 2 h (one orbital revolution). The differences in predicted RMS spacecraft positions, relative to those in the previous and subsequent arcs were determined. Since the arc overlaps cover just one orbit, such RMS differences may account for any of: measurement errors, random errors, or systematic bias due to mismodeling/unmodeling dynamical forces yielding secular (*i.e.* averaged over one orbital revolution) effects, whatever their physical origin may be.
Indeed, RMS orbit solution overlaps are *commonly used in satellite geodesy as useful and significant indicators*

of the overall orbit accuracy [13, 14]. Conversely, they are also used to gain information about *systematic errors coming from inaccurate modeling of the forces acting on the spacecraft*. For details see Refs. [13, 14]. Of course, such a technique is insensitive to short-period effects, *i.e.* those having frequencies higher than the orbital frequency: only dynamical features of motion with timescales equal to, or larger than one orbital period can be sensed by such orbit-overlap differences. Moreover, the average orbit error $\langle \Delta N_{\rm diff} \rangle$ of about 1.6 m does not refer to any particular arc overlap; instead, it comes from the *mean of the entire set* of RMS orbit-overlap differences for the chosen time span ΔP and is well representative of those un-modelled/mis-modelled forces yielding effects which do not average out over ΔP (such as the Lense-Thirring signal). Time-varying patterns exhibiting well-defined periodicities –including measurement errors such as those related to Earth-Mars geometry –are mainly averaged out, yielding little or no contribution to the average orbit error. Incidentally, it should be apparent that there is no sense in looking for the error of the error, as seemingly required by Krogh [7] when he blames me [1] for not having included the uncertainty in $\langle \Delta N_{\rm res} \rangle$. Another criticism by Krogh [7] is that the RMS overlap differences would be unable to specify any orbital precession.

Points II and III In order to reply to all such criticisms I performed another, independent test of my hypothesis. First, by linearly fitting[2] the *complete time series* of Ref. [3], after rescaling the data to centre the zero point of the time-series, I obtained a slope of -0.64 ± 0.26 m yr^{-1}, (with 95% confidence bounds), while the predicted Lense-Thirring MGS out-of-plane rate (customarily defined as *positive along* the direction of the spacecraft's orbital angular momentum) amounts to 0.62 m yr^{-1}. The negative sign is due to the fact that Konopliv *et al.* [3] defined the *normal* direction to be positive in the *opposite* direction to the MGS orbital angular momentum (Konopliv 2007, private communication). Should such a linear fit be used as indicator of the existence of the Lense-Thirring effect, its relativistic prediction would be fully confirmed within the experimental error; the null hypothesis would be rejected at 2.4 sigma level. Then, I also repeated the procedure by fitting a straight line to the data set *without full January 2001*, (which was mainly affected by measurement errors which, according to Krogh [7], would mimic the Lense-Thirring effect) obtaining a value of -0.61 ± 0.26 m yr^{-1}. *Removal of data from the entire year 2001* (which was mainly affected by angular momentum wheel desaturation operations), yields a value of -0.57 ± 0.28 m yr^{-1}. A different linear fit to the time series after removing data from the final month (December 2004-January 2005) yields -0.62 ± 0.27 m yr^{-1}.

Point IV Krogh [7] quotes Sindoni *et al.* [8]. They present analytical calculations about the corrupting impact of various physical parameters of Mars through the classical node precessions. Such rates are induced by the even zonal harmonic coefficients J_ℓ of the multipolar expansion of the Newtonian part of the martian gravitational potential. In particular, Sindoni *et al.* [8] use the first five *even* zonals $J_2 \ldots J_{10}$ (together with errors from former global solutions for Mars's gravity field, the uncertainty in Mars's GM, the uncertainty in the MGC semi-major axis and errors in the inclination) in their analytical formulae for classical secular node precessions. They conclude that, since the resulting effect is tens of thousands of times larger than the Lense-Thirring effect on the MGS, this would be disasterous for any attempt at detecting the gravitomagnetic frame-dragging with such a spacecraft.

The point is that *such figures* (as with others which can be obtained from more accurate calculation) *must ultimately be compared with the reality of the data*, *i.e.* the RMS orbit-overlap differences of MGS.

I repeated such a calculation by considering the other even zonals up to J_{20}, along with the latest errors of the MGS95J global solution and the uncertainties in the radius of Mars. A root-sum-square summation of these terms resulted in a mean bias of 78.9 m d^{-1} in the out-of-plane MGS orbital component. Using a linear summmation an upper bound of 111.6 m d^{-1} was obtained. Such figures clearly show that they are *not* representative of the real MGS orbit. Indeed, over a time span of 5 years they would result in a *mean shift as large as 144 km* (root-sum-square calculation) or *203 km* (linear sum). Interestingly, even if the set of MGS RMS-overlap differences was to be considered as representative of a single orbital arc 6 d long only, the conclusion would be the same: indeed, in *this* case, the total cross-track mean shift due to the martian gravitational potential *would amount to 473.1 m* (root-sum-square) or *669.6 m* (linear sum).

Regarding to Ref. [9], (also quoted in Ref. [7]), recall that RMS orbit-overlap differences are used mainly to give an indication of the overall orbit accuracy, taking into account all the measurement and systematic errors [13, 14]. The important point to note is that *such differences cancel out, by construction,* errors, systematic or not, *common to consecutive arcs*—it would just be the case of a bias like that described by Felici in Ref. [9]—, while effects like the Lense-Thirring one, accumulating in time, are, instead, singled out [14].

[2] *Note that, since the plots in Fig. 3 of Ref. [3] are semi-logarithmic, one should not visually look for a straight line in them.*

Point V In regard to the impact of non-gravitational perturbations, Sindoni *et al.* [8] yield a total accelleration (un-modelled and non-gravitational) of $\approx 10^{-11}$ m s^{-2}, which is the same order of magnitude of the Lense-Thirring acceleration induced by Mars on MGS. The authors neither present any detailed calculation of the effect of such an acceleration on the normal portion of the MGS orbit, nor specify if such a magnitude refers to the out-of-plane component. However, some simple considerations can be easily traced: a hypothetical, generic out-of-plane perturbing force 6.7 times larger than the Lense-Thirring one, and having the same time signature (*i.e.* linear in time), should induce a 10.8 m cross-track shift, on average, over the considered time span ΔP. Again, *such a bias is absent from the data*.

Ref. [3] clearly states that it is the *along-track* portion of the MGS orbit –left unaffected by the Lense-Thirring force– *which is mainly perturbed by the non-gravitational forces*: indeed, the along-track empirical accelerations fitted by Konopliv *et al.* [3] amount just to $\approx 10^{-11}$ m s^{-2}, which shows that the guess made by Sindoni *et al.* [8] is somewhat correct, but it refers only to the *along-track* component.

Time-dependent, periodic signatures would, instead, be averaged out, provided that their characteristic time scales are relatively short, as is the case. Indeed, non-conservative accelerations, which are especially active in the MGS *in-plane* orbital components [3, 15], exhibit time-varying patterns over 12 hr [15] which are averaged out over multi-year time spans (and, incidentally, over 6 d as well) when hypothetically mapped to the out-of-plane direction. To be more definite regarding the issue of the impact of atmospheric drag on the cross-track portion of the MGS orbit, (a point raised by Krogh [7]), let us note that it requires not only consideration of the node Ω, as apparently claimed by Krogh [7], but also the inclination i, according to Ref. [16]

$$\Delta N = a\sqrt{\left(1+\frac{e^2}{2}\right)\left[\frac{(\Delta i)^2}{2}+(\sin i\Delta\Omega)^2\right]}. \tag{4}$$

According to, e.g., Milani *et al.* [17], the perturbing acceleration $\boldsymbol{A}_{\rm drag}$ due to atmospheric drag can be written in the form

$$\boldsymbol{A}_{\rm drag} = -\frac{1}{2}ZC_{\rm D}\frac{S}{M}\rho v\boldsymbol{v}, \tag{5}$$

where S/M is the spacecraft cross sectional area (perpendicular to the velocity) divided by its mass, $C_{\rm D}$ is the drag coefficient, ρ is the atmospheric density (assumed to be constant over one orbital revolution), $\boldsymbol{v}$ is the satellite velocity in a planetocentric, non-rotating frame of reference and Z is a corrective coefficient accounting for the fact that the atmosphere is not at rest, but rotates with angular velocity $\omega_{\rm A}$ more or less rigidly with the planet ($Z \approx 1$ for polar orbits [17]). While the drag shift on the node vanishes when averaged over one orbital period T, the same is not true for the inclination: indeed, it turns out [17]

$$\langle\Delta i\rangle_T \approx \pi\left(\frac{A_{\rm drag}}{n^2 a}\right)\frac{\omega_{\rm A}}{n}+\mathcal{O}(e), \tag{6}$$

where $n = \sqrt{GM/a^3}$ is the Keplerian mean motion. As a result, the orbital plane tends to approach the planet's equator. The term in brackets is the ratio of the drag force to the Newtonian monopole. As usual, in perturbation theory, a refers to the unperturbed reference ellipse. Thus, the out-of-plane drag shift is from (4)

$$\langle\Delta N_{\rm drag}\rangle \approx a\frac{\langle\Delta i_{\rm drag}\rangle}{\sqrt{2}}. \tag{7}$$

In the following I will assume that $\omega_{\rm A} \approx \omega_{\rm Mars} = 7.10 \times 10^{-5}$ s^{-1}. Let us see what happens in the (unlikely) worst-case $A_{\rm drag} \approx 10^{-11}$ m s^{-2}; it turns out that

$$\langle\Delta N_{\rm drag}\rangle_T \sim 1\times 10^{-5}\ \text{m}. \tag{8}$$

But $A_{\rm drag}$ is not constant over time spans of days or years [18], so such an effect is not a concern here. Even if this was not so, the assumption of $\approx 10\%$ mismodeling in drag –which is, in fact, modeled by Konopliv *et al.* in Ref. [3], mapped onto about 5 yr, would give a $\approx 0.7\%$ uncertainty.

Finally, Krogh [7] remarks that a decrease in the averages of the RMS orbit overlaps occurred in view of constantly improved modeling [15, 19]. He does not, however, recognize that improved modeling of non-gravitational forces acting on MGS (introduced in Ref. [3] with respect to previous works [15, 19] in which the Lense-Thirring effect was not modelled as well), *only had a relevant effect on the along-track RMS overlap differences* (a factor of 10 better than in Ref. [15, 19]), *not the normal ones* (just a factor of 2 better than in Ref. [15, 19]).

Moreover, if the relativistic signature was removed (or not present at all) so that the determined out-of-plane RMS overlap differences were only (or mainly) due to other causes such as mismodeling or unmodeling in the non-gravitational forces, it is difficult to understand why the along-track RMS overlap differences (middle panel of Figure 3 of Ref. [3]) have almost the same magnitude, since the along-track component of the MGS orbit is affected much more by the non-gravitational accelerations (e.g. the atmospheric drag) than the out-of-plane forces.

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