

Conservative evaluation of the uncertainty in the LAGEOS-LAGEOS II Lense-Thirring test

Research Article

Lorenzo Iorio*

INFN-Sezione di Pisa, Viale Unità di Italia 68, 70125 Bari (BA), Italy

Received 10 December 2008; accepted 12 March 2009

Abstract:

We deal with the test of the general relativistic gravitomagnetic Lense-Thirring effect currently being conducted in the Earth's gravitational field with the combined nodes Ω of the laser-ranged geodetic satellites LAGEOS and LAGEOS II. One of the most important sources of systematic uncertainty on the orbits of the LAGEOS satellites, with respect to the Lense-Thirring signature, is the bias due to the even zonal harmonic coefficients J_ℓ of the multipolar expansion of the Earth's geopotential which account for the departures from sphericity of the terrestrial gravitational potential induced by the centrifugal effects of its diurnal rotation. The issue addressed here is: are the so far published evaluations of such a systematic error reliable and realistic? The answer is negative. Indeed, if the difference ΔJ_ℓ among the even zonals estimated in different global solutions (EIGEN-GRACE02S, EIGEN-CG03C, GGM02S, GGM03S, ITG-Grace02, ITG-Grace03s, JEM01-RL03B, EGM2008, AIUB-GRACE01S) is assumed for the uncertainties δJ_ℓ instead of using their more-or-less calibrated covariances σ_{J_ℓ} , it turns out that the systematic error $\delta\mu$ in the Lense-Thirring measurement is about 3 to 4 times larger than in the evaluations so far published based on the use of the covariances of one model at a time separately, amounting up to 37% for the pair EIGEN-GRACE02S/ITG-Grace03s. The comparison among the other recent GRACE-based models yields bias as large as about 25–30%. The major discrepancies still occur for J_4 , J_6 and J_8 , which are just to which the zonals the combined LAGEOS/LAGEOS II nodes are most sensitive.

PACS (2008): 04.; 04.80.-y; 04.80.Cc

Keywords: experimental tests of gravitational theories • satellite orbits • harmonics of the gravity potential field
© Versita Warsaw and Springer-Verlag Berlin Heidelberg.

1. Introduction

In the weak-field and slow motion approximation, the Einstein field equations of general relativity get linearized to a form resembling Maxwell's equations of electromagnetism. Thus, a gravitomagnetic field, induced by the

off-diagonal components g_{0i} , $i = 1, 2, 3$ of the space-time metric tensor related to the mass-energy currents of the source of the gravitational field, arises [1]. It affects in several ways the motion of, e.g., test particles and electromagnetic waves [2]. Perhaps the most famous gravitomagnetic effects are gyroscope precession [3, 4] and the

*E-mail: lorenzo.iorio@libero.it

Lense-Thirring¹ precessions [6] of the orbit of a test particle, both occurring in the field of a central slowly rotating mass like a planet.

The measurement of gyroscope precession in the Earth's gravitational field has been the goal of the dedicated space-based GP-B mission² [7, 8] launched in 2004; its data analysis is still in progress.

In this paper we critically discuss some issues concerning the test of the Lense-Thirring effect performed with the LAGEOS and LAGEOS II terrestrial artificial satellites³ [9] tracked with the Satellite Laser Ranging (SLR) technique [10].

The authors of Refs. [11, 12] proposed measuring the Lense-Thirring nodal precession of a pair of counter-orbiting spacecraft in terrestrial polar orbits and equipped with drag-free apparatus. A somewhat equivalent, cheaper version of such an idea was put forth in Ref. [13] whose author suggested to launch a passive, geodetic satellite in an orbit identical to that of the LAGEOS satellite apart from the orbital planes which should have been displaced by 180 deg apart. The measurable quantity was, in this case, the sum of the nodes of LAGEOS and of the new spacecraft, later named LAGEOS III, LARES, WEBER-SAT, in order to cancel the confounding effects of the multipoles of the Newtonian part of the terrestrial gravitational potential (see below). Although extensively studied by various groups [14–16], such an idea has not been implemented for a long time. Recently, the Italian Space Agency (ASI) has approved this project and should launch a VEGA rocket for this purpose at the end of 2009–beginning of 2010 (<http://www.asi.it/en/activity/cosmology/lares>). For recent updates of the LARES/WEBER-SAT mission, including recently added additional goals in fundamental physics and related criticisms, see Refs. [17–24].

Among scenarios involving *existing* orbiting bodies, the idea of measuring the Lense-Thirring node rate with the just launched LAGEOS satellite, along with the other SLR targets orbiting at that time, was proposed in Ref. [25]. Tests have been effectively performed using the LAGEOS and LAGEOS II satellites [26], according to a strategy involving a suitable combination of the nodes of both satellites and the perigee ω of LAGEOS II [27]. This was done to reduce the impact of the most relevant source of systematic bias, viz. the mismodelling in the even ($\ell = 2, 4, 6 \dots$)

zonal ($m = 0$) harmonics J_ℓ of the multipolar expansion of the Newtonian part of the terrestrial gravitational potential:⁴ they account for non-sphericity of the terrestrial gravitational field induced by centrifugal effects of the Earth's diurnal rotation. The even zonals affect the node and the perigee of a terrestrial satellite with secular precessions which may mimic the Lense-Thirring signature. The three-elements combination used allowed for removing the uncertainties in J_2 and J_4 . In Ref. [28] a $\approx 20\%$ test was reported by using the⁵ EGM96 [29] Earth gravity model; subsequent analyses showed that such an evaluation of the total error budget was overly optimistic in view of the likely unreliable computation of the total bias due to the even zonals [30–32]. An analogous, huge underestimation turned out to hold also for the effect of non-gravitational perturbations [33] like direct solar radiation pressure, the Earth's albedo, various subtle thermal effects depending on the physical properties of the satellites' surfaces and their rotational state [31, 34–40], which the perigees of LAGEOS-like satellites are particularly sensitive to. As a result, the realistic total error budget in the test reported in Ref. [28] might be as large as 60–90% or even more (by considering EGM96 only). The observable used in Ref. [9] with the GRACE-only EIGEN-GRACE02S model [41] and in Ref. [42] with other global terrestrial gravity solutions was the following linear combination⁶ of the nodes of LAGEOS and LAGEOS II, explicitly computed in Ref. [44] following the approach proposed in Ref. [27]:

$$f = \dot{\Omega}^{\text{LAGEOS}} + c_1 \dot{\Omega}^{\text{LAGEOS II}}, \quad (1)$$

where

$$\begin{aligned} c_1 &\equiv -\frac{\dot{\Omega}_2^{\text{LAGEOS}}}{\dot{\Omega}_2^{\text{LAGEOS II}}} \\ &= -\frac{\cos i_{\text{LAGEOS}}}{\cos i_{\text{LAGEOS II}}} \left(\frac{1 - e_{\text{LAGEOS II}}^2}{1 - e_{\text{LAGEOS}}^2} \right)^2 \left(\frac{a_{\text{LAGEOS II}}}{a_{\text{LAGEOS}}} \right)^{7/2}. \end{aligned} \quad (2)$$

¹ According to an interesting historical analysis recently performed in Ref. [5], it would be more correct to speak about an Einstein-Thirring-Lense effect.

² See <http://einstein.stanford.edu/>

³ LAGEOS was put into orbit in 1976, followed by its twin LAGEOS II in 1992.

⁴ The relation among the even zonals J_ℓ and the normalized gravity coefficients $\bar{C}_{\ell 0}$ is $J_\ell = -\sqrt{2\ell+1} \bar{C}_{\ell 0}$.

⁵ Contrary to the subsequent CHAMP/GRACE-based models, EGM96 relies upon multidecadal tracking of SLR data of a constellation of geodetic satellites including LAGEOS and LAGEOS II as well; thus the possibility of a sort of a *a priori* 'imprinting' of the Lense-Thirring effect itself, not solved-for in EGM96, cannot be neglected.

⁶ See also Refs. [31, 32, 43].

The coefficients $\dot{\Omega}_\ell$ of the aliasing classical node precessions [45] $\dot{\Omega}_{\text{class}} = \sum_\ell \dot{\Omega}_\ell J_\ell$ induced by even zonals have been analytically worked out in, e.g., Ref. [30]; a, e, i are the satellite's semimajor axis, eccentricity and inclination, respectively and yield $c_1 = 0.544$ for Eq. (2). The Lense-Thirring signature of Eq. (1) amounts to 47.8 milliarcseconds per year (mas yr⁻¹). The combination of Eq. (1) allows, by construction, to remove the aliasing effects due to the static and time-varying parts of the first even zonal J_2 . The nominal bias (computed with the estimated values of J_ℓ , $\ell = 4, 6, \dots$) due to the remaining higher degree even zonals would amount to about 10^5 mas yr⁻¹; the need of a careful and reliable modeling of such an important source of systematic bias is, thus, quite apparent. Conversely, the nodes of the LAGEOS-type spacecraft are affected by the non-gravitational accelerations $\approx 1\%$ of the Lense-Thirring effect [36–40]. For a comprehensive, up-to-date overview of the numerous and subtle issues concerning the measurement of the Lense-Thirring effect see Ref. [46].

Here, we will address the following questions:

- Has the systematic error due to the competing secular node precessions induced by the static part of the even zonal harmonics been realistically evaluated so far in literature? (Section 2)
- Are other approaches to extract the gravitomagnetic signature from the data feasible? (Section 3)

2. The systematic error of gravitational origin

The realistic evaluation of the total error budget of such a test raised a lively debate [47–53], mainly focussed on the impact of the static and time-varying parts of the Newtonian component of the Earth's gravitational potential through the aliasing secular precessions induced on a satellite's node. A common feature of all the competing evaluations so far published is that the systematic bias due to the static component of the geopotential was calculated always by using the released (more or less accurately calibrated) covariances σ_{J_ℓ} of one Earth gravity model solution at a time for the uncertainties δJ_ℓ in the even zonal harmonics, yielding a percentage error particular to each model.

Since it is always difficult to reliably calibrate the formal, statistical uncertainties in the estimated zonals of the covariance matrix for a global solution, it is much more realistic and conservative to instead take the dif-

Table 1. Impact of the mismodelling in the even zonal harmonics on $f_\ell = \left| \dot{\Omega}_\ell^{\text{LAGEOS}} + c_1 \dot{\Omega}_\ell^{\text{LAGEOS II}} \right| \Delta J_\ell$, $\ell = 4, \dots, 20$, in mas yr⁻¹. Recall that $J_\ell = -\sqrt{2\ell+1} \bar{C}_{\ell 0}$; for the uncertainty in the even zonals we have taken here the difference $\Delta \bar{C}_{\ell 0} = \left| \bar{C}_{\ell 0}^{(X)} - \bar{C}_{\ell 0}^{(Y)} \right|$ between the model X = EIGEN-CG03C [57] and the model Y = EIGEN-GRACE02S [41]. EIGEN-CG03C combines data from CHAMP (860 days out of October 2000 to June 2003), GRACE (376 days out of February to May 2003, July to December 2003 and February to July 2004) and terrestrial measurements; EIGEN-GRACE02S is based on 110 days (out of August and November 2002 and April, May and August 2003) of GRACE-only GPS-GRACE high-low satellite-to-satellite data, on-board measurements of non-gravitational accelerations, and especially GRACE intersatellite tracking data. $\sigma_{X/Y}$ are the covariance calibrated errors for both models. Values of f_ℓ smaller than 0.1 mas yr⁻¹ have not been quoted. The Lense-Thirring precession of the combination of Eq. (1) amounts to 47.8 mas yr⁻¹. The percent bias $\delta\mu$ has been computed by normalizing the linear sum of f_ℓ , $\ell = 4, \dots, 20$ to the Lense-Thirring precession. The discrepancies between the models are significant since $\Delta \bar{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, \dots, 16$.

ℓ	$\Delta \bar{C}_{\ell 0}$ (EIGEN-CG03C -EIGEN-GRACE02S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	1.96×10^{-11}	1.01×10^{-11}	7.3
6	2.50×10^{-11}	4.8×10^{-12}	5.4
8	4.9×10^{-12}	3.3×10^{-12}	0.2
10	3.7×10^{-12}	3.4×10^{-12}	-
12	2.5×10^{-12}	2.3×10^{-12}	-
14	6.1×10^{-12}	2.1×10^{-12}	-
16	2.1×10^{-12}	1.7×10^{-12}	-
18	6×10^{-13}	1.7×10^{-12}	-
20	1.7×10^{-12}	1.7×10^{-12}	-
$\delta\mu = 27\%$ (SAV)		$\delta\mu = 19\%$ (RSS)	

ferences⁷ ΔJ_ℓ between the estimated even zonals for different pairs of Earth gravity field solutions as representative of the real uncertainty δJ_ℓ in the zonals [55]. In Tab. 1–Tab. 12 we present our results for the most recent GRACE-based models released so far by different institutions and retrievable on the Internet at⁸ <http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html>. The models used are EIGEN-GRACE02S [41] and EIGEN-CG03C [57] from GFZ (Potsdam, Germany), GGM02S [58] and GGM03S [56] from CSR (Austin, Texas), ITG-Grace02s [59] and ITG-Grace03 [60] from IGG (Bonn, Germany), JEM01-RL03B from JPL (NASA, USA), EGM2008 [61] from

⁷ See Fig. 5 of Ref. [54] for a comparison of the estimated \bar{C}_{40} in different models.

⁸ I thank J Ries, CSR, and M Watkins (JPL) for having provided me with the even zonals of the GGM03S [56] and JEM01-RL03B models.

Table 2. Impact of the mismodelling in the even zonal harmonics as solved for in X=GGM02S [58] and Y=ITG-Grace02s [59]. GGM02S is based on 363 days of GRACE-only data (GPS and intersatellite tracking, neither constraints nor regularization applied) spread between April 4, 2002 and Dec 31, 2003. The σ are formal for both models. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas, apart from $\ell = 12$ and $\ell = 18$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (GGM02S -ITG-Grace02s)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	1.9×10^{-11}	8.7×10^{-12}	7.2
6	2.1×10^{-11}	4.6×10^{-12}	4.6
8	5.7×10^{-12}	2.8×10^{-12}	0.2
10	4.5×10^{-12}	2.0×10^{-12}	-
12	1.5×10^{-12}	1.8×10^{-12}	-
14	6.6×10^{-12}	1.6×10^{-12}	-
16	2.9×10^{-12}	1.6×10^{-12}	-
18	1.4×10^{-12}	1.6×10^{-12}	-
20	2.0×10^{-12}	1.6×10^{-12}	-
$\delta\mu = 25\%$ (SAV)		$\delta\mu = 18\%$ (RSS)	

Table 3. Impact of the mismodelling in the even zonal harmonics as solved for in X=GGM02S [58] and Y=EIGEN-CG03C [57]. The σ are formal for GGM02S, calibrated for EIGEN-CG03C. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas.

ℓ	$\Delta\bar{C}_{\ell 0}$ (GGM02S -EIGEN-CG03C)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	1.81×10^{-11}	3.7×10^{-12}	6.7
6	1.53×10^{-11}	1.8×10^{-12}	3.3
8	1.5×10^{-12}	1.1×10^{-12}	-
10	4.9×10^{-12}	8×10^{-13}	-
12	8×10^{-13}	7×10^{-13}	-
14	7.7×10^{-12}	6×10^{-13}	-
16	3.8×10^{-12}	5×10^{-13}	-
18	2.1×10^{-12}	5×10^{-13}	-
20	2.3×10^{-12}	4×10^{-13}	-
$\delta\mu = 22\%$ (SAV)		$\delta\mu = 16\%$ (RSS)	

NGA (USA) and AIUB-GRACE01S [62] from AIUB (Bern, Switzerland). This approach was taken also in Ref. [27] with the JGM3 and GEMT-2 models. We included both the sum of the absolute values (SAV) of each mismodelled term and the square root of the sum of the squares (RSS) of each mismodelled term.

The systematic bias evaluated with a more realistic approach is about 3 to 4 times larger than one can obtain by only using this or that particular model. The scatter is still quite large and differs greatly from that 5 – 10% claimed in Ref. [9]. In particular, it appears that J_4 , J_6 , and to a lesser extent J_8 , the most relevant zonals for us owing

Table 4. Bias due to the mismodelling in the even zonals of the models X=ITG-Grace03s [60], based on GRACE-only accumulated normal equations from data out of September 2002-April 2007 (neither apriori information nor regularization used), and Y=GGM02S [58]. The σ for both models are formal. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas, apart from $\ell = 12$ and $\ell = 18$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (ITG-Grace03s-GGM02S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.58×10^{-11}	8.6×10^{-12}	9.6
6	1.39×10^{-11}	4.7×10^{-12}	3.1
8	5.6×10^{-12}	2.9×10^{-12}	0.2
10	1.03×10^{-11}	2×10^{-12}	-
12	7×10^{-13}	1.8×10^{-12}	-
14	7.3×10^{-12}	1.6×10^{-12}	-
16	2.6×10^{-12}	1.6×10^{-12}	-
18	8×10^{-13}	1.6×10^{-12}	-
20	2.4×10^{-12}	1.6×10^{-12}	-
$\delta\mu = 27\%$ (SAV)		$\delta\mu = 21\%$ (RSS)	

Table 5. Bias due to the mismodelling in the even zonals of the models X = GGM02S [58] and Y = GGM03S [56] retrieved from data spanning January 2003 to December 2006. The σ for GGM03S are calibrated. $\Delta\bar{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, 6$. (The other zonals are of no concern)

ℓ	$\Delta\bar{C}_{\ell 0}$ (GGM02S-GGM03S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	1.87×10^{-11}	1.25×10^{-11}	6.9
6	1.96×10^{-11}	6.7×10^{-12}	4.2
8	3.8×10^{-12}	4.3×10^{-12}	0.1
10	8.9×10^{-12}	2.8×10^{-12}	0.1
12	6×10^{-13}	2.4×10^{-12}	-
14	6.6×10^{-12}	2.1×10^{-12}	-
16	2.1×10^{-12}	2.0×10^{-12}	-
18	1.8×10^{-12}	2.0×10^{-12}	-
20	2.2×10^{-12}	1.9×10^{-12}	-
$\delta\mu = 24\%$ (SAV)		$\delta\mu = 17\%$ (RSS)	

to their effecta on the combination of Eq. (1), are the most uncertain ones, with discrepancies ΔJ_ℓ between different models generally larger than the sum of their covariances σ_{J_ℓ} whether calibrated or not.

Our approach is valid also for all of the tests performed so far with the LAGEOS and LAGEOS II satellites. Another possible strategy, that takes into account the scatter among the various solutions, is to compute mean and standard deviation of the entire set of values of the even zonals for the models considered so far, degree by degree, and then to take the standard deviations as representative of the uncertainties δJ_ℓ , $\ell = 4, 6, 8, \dots$. This yields $\delta\mu = 15\%$, slightly larger than that recently obtained in Ref. [42]. But in evaluating mean and standard deviation for each even

Table 6. Bias due to the mismodelling in the even zonals of the models X = EIGEN-GRACE02S [41] and Y = GGM03S [56]. The σ for both models are calibrated. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas apart from $\ell = 14, 18$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (EIGEN-GRACE02S -GGM03S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.00×10^{-11}	8.1×10^{-12}	7.4
6	2.92×10^{-11}	4.3×10^{-12}	6.3
8	1.05×10^{-11}	3.0×10^{-12}	0.4
10	7.8×10^{-12}	2.9×10^{-12}	0.1
12	3.9×10^{-12}	1.8×10^{-12}	-
14	5×10^{-13}	1.7×10^{-12}	-
16	1.7×10^{-12}	1.4×10^{-12}	-
18	2×10^{-13}	1.4×10^{-12}	-
20	2.5×10^{-12}	1.4×10^{-12}	-
$\delta\mu = 30\%$ (SAV)		$\delta\mu = 20\%$ (RSS)	

Table 7. Bias due to the mismodelling in the even zonals of the models X = JEM01-RL03B, based on 49 months of GRACE-only data, and Y = GGM03S [56]. The σ for GGM03S are calibrated. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas apart from $\ell = 16$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (JEM01-RL03B-GGM03S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	1.97×10^{-11}	4.3×10^{-12}	7.3
6	2.7×10^{-12}	2.3×10^{-12}	0.6
8	1.7×10^{-12}	1.6×10^{-12}	-
10	2.3×10^{-12}	8×10^{-13}	-
12	7×10^{-13}	7×10^{-13}	-
14	1.0×10^{-12}	6×10^{-13}	-
16	2×10^{-13}	5×10^{-13}	-
18	7×10^{-13}	5×10^{-13}	-
20	5×10^{-13}	4×10^{-13}	-
$\delta\mu = 17\%$ (SAV)		$\delta\mu = 15\%$ (RSS)	

Table 8. Bias due to the mismodelling in the even zonals of the models X = JEM01-RL03B and Y = ITG-Grace03s [60]. The σ for ITG-Grace03s are formal. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas.

ℓ	$\Delta\bar{C}_{\ell 0}$ (JEM01-RL03B -ITG-Grace03s)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.68×10^{-11}	4×10^{-13}	9.9
6	3.0×10^{-12}	2×10^{-13}	0.6
8	3.4×10^{-12}	1×10^{-13}	0.1
10	3.6×10^{-12}	1×10^{-13}	-
12	6×10^{-13}	9×10^{-14}	-
14	1.7×10^{-12}	9×10^{-14}	-
16	4×10^{-13}	8×10^{-14}	-
18	4×10^{-13}	8×10^{-14}	-
20	7×10^{-13}	8×10^{-14}	-
$\delta\mu = 22\%$ (SAV)		$\delta\mu = 10\%$ (RSS)	

Table 9. Aliasing effect of the mismodelling in the even zonal harmonics estimated in the X=ITG-Grace03s [60] and the Y=EIGEN-GRACE02S [41] models. The covariance matrix σ for ITG-Grace03s are formal, while the ones of EIGEN-GRACE02S are calibrated. $\Delta\bar{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, \dots, 20$, apart from $\ell = 18$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (ITG-Grace03s -EIGEN-GRACE02S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.72×10^{-11}	3.9×10^{-12}	10.1
6	2.35×10^{-11}	2.0×10^{-12}	5.1
8	1.23×10^{-11}	1.5×10^{-12}	0.4
10	9.2×10^{-12}	2.1×10^{-12}	0.1
12	4.1×10^{-12}	1.2×10^{-12}	-
14	5.8×10^{-12}	1.2×10^{-12}	-
16	3.4×10^{-12}	9×10^{-13}	-
18	5×10^{-13}	1.0×10^{-12}	-
20	1.8×10^{-12}	1.1×10^{-12}	-
$\delta\mu = 37\%$ (SAV)		$\delta\mu = 24\%$ (RSS)	

Table 10. Impact of the mismodelling in the even zonal harmonics estimated in the X=EGM2008 [61] and the Y=EIGEN-GRACE02S [41] models. The covariance matrix σ are calibrated for both EGM2008 and EIGEN-GRACE02S. $\Delta\bar{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, \dots, 20$, apart from $\ell = 18$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (EGM2008-EIGEN -GRACE02S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.71×10^{-11}	8.3×10^{-12}	10.0
6	2.35×10^{-11}	4.1×10^{-12}	5.0
8	1.23×10^{-11}	2.7×10^{-12}	0.4
10	9.2×10^{-12}	2.9×10^{-12}	0.1
12	4.1×10^{-12}	1.9×10^{-12}	-
14	5.8×10^{-12}	1.8×10^{-12}	-
16	3.4×10^{-12}	1.5×10^{-12}	-
18	5×10^{-13}	1.5×10^{-12}	-
20	1.8×10^{-12}	1.5×10^{-12}	-
$\delta\mu = 33\%$ (SAV)		$\delta\mu = 23\%$ (RSS)	

zonals, the authors of Ref. [42] also used global gravity solutions like EIGEN-GL04C and EIGEN-GL05C which include data from the LAGEOS satellite itself; this may likely have introduced a sort of favorable *a priori* “imprint” of the Lense-Thirring effect itself. Moreover, the authors of Ref. [42] gave only a RSS evaluation of the total bias.

We must also remember to add the further bias due to the cross-coupling between J_2 and the orbit inclination, evaluated to be about 9% in Ref. [52].

Table 11. Bias due to the mismodelling in the even zonals of the models X = JEM01-RL03B, based on 49 months of GRACE-only data, and Y = AIUB-GRACE01S [62]. The latter one was obtained from GPS satellite-to-satellite tracking data and K-band range-rate data out of the period January 2003 to December 2003 using the Celestial Mechanics Approach. No accelerometer data, no de-aliasing products, and no regularisation was applied. The σ for AIUB-GRACE01S are formal. $\Delta\bar{C}_{\ell 0}$ are always larger than the linearly added sigmas.

ℓ	$\Delta\bar{C}_{\ell 0}$ (JEM01-RL03B –AIUB-GRACE01S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.95×10^{-11}	2.1×10^{-12}	11
6	3.5×10^{-12}	1.3×10^{-12}	0.8
8	2.14×10^{-11}	5×10^{-13}	0.7
10	4.8×10^{-12}	5×10^{-13}	–
12	4.2×10^{-12}	5×10^{-13}	–
14	3.6×10^{-12}	5×10^{-13}	–
16	8×10^{-13}	5×10^{-13}	–
18	7×10^{-13}	5×10^{-13}	–
20	1.0×10^{-12}	5×10^{-13}	–
$\delta\mu = 26\%$ (SAV)		$\delta\mu = 23\%$ (RSS)	

Table 12. Bias due to the mismodelling in the even zonals of the models X = EIGEN-GRACE02S [41] and Y = AIUB-GRACE01S [62]. The σ for AIUB-GRACE01S are formal, while those of EIGEN-GRACE02S are calibrated. $\Delta\bar{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, 6, 8, 16$.

ℓ	$\Delta\bar{C}_{\ell 0}$ (EIGEN-GRACE02S –AIUB-GRACE01S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr ⁻¹)
4	2.98×10^{-11}	6.0×10^{-12}	11.1
6	2.29×10^{-11}	3.3×10^{-12}	5.0
8	1.26×10^{-11}	1.9×10^{-12}	0.4
10	6×10^{-13}	2.5×10^{-12}	–
12	5×10^{-13}	1.6×10^{-12}	–
14	5×10^{-13}	1.6×10^{-12}	–
16	2.9×10^{-12}	1.4×10^{-12}	–
18	6×10^{-13}	1.4×10^{-12}	–
20	2×10^{-13}	1.5×10^{-12}	–
$\delta\mu = 34\%$ (SAV)		$\delta\mu = 25\%$ (RSS)	

3. A new approach to extract the Lense-Thirring signature from the data

The technique adopted so far by the authors of Ref. [9] and Ref. [42] to extract the gravitomagnetic signal from the LAGEOS and LAGEOS II data is described in detail in Refs. [54, 63]. The Lense-Thirring force is not included in the dynamical force models used to fit the satellites' data. In the data reduction process no dedicated gravito-

magnetic parameter is estimated, contrary to e.g. station coordinates, state vector, satellites' drag coefficients C_D and C_R , etc.; its effect is retrieved with a sort of post-post-fit analysis in which the time series of the computed⁹ “residuals” of the nodes with the difference between the orbital elements of consecutive arcs, combined with Eq. (1), is fitted with a straight line.

In order to enforce the reliability of the ongoing test it would be desirable to follow other approaches as well. For instance, the gravitomagnetic force could be modelled in terms of a dedicated solve-for parameter (not necessarily the usual PPN γ one) which could be estimated in the least-squares sense along with all the other parameters usually determined, and the resulting correlations among them could be inspected. Or, one could consider the changes in the values of the complete set of the estimated parameters with and without the Lense-Thirring effect.

A first, tentative step towards the implementation of a similar strategy with the LAGEOS satellites in term of the PPN parameter γ has been recently taken in Ref. [64].

4. Conclusions

In this paper we have shown how the so far published evaluations of the total systematic error in the Lense-Thirring measurement with the combined nodes of the LAGEOS satellites due to the classical node precessions induced by the even zonal harmonics of the geopotential are likely optimistic. Indeed, they are all based on the use of elements from the covariance matrix, more or less reliably calibrated, of various Earth gravity model solutions used one at a time separately in such a way that the model X yields an error of $x\%$, the model Y yields an error $y\%$, etc. Instead, comparing the estimated values of the even zonals for different pairs of models allows for a much more realistic evaluation of the real uncertainties in our knowledge of the static part of the geopotential. As a consequence, the bias in the Lense-Thirring effect measurement is about three to four times larger than that so far claimed, amounting to tens of parts per cent (37% for the pair EIGEN-GRACE02S and ITG-GRACE03s, about 25–30% for the other most recent GRACE-based solutions).

Finally, we have pointed out the need of following different strategies in extracting the Lense-Thirring pattern from the data; for instance by explicitly modelling it in fitting

⁹ The expression “residuals of the nodes” is used, strictly speaking, in an improper sense because the Keplerian orbital elements are not directly measured quantities.

the SLR data of LAGEOS and LAGEOS II, and estimating the associated solve-for parameter in a least-square sense along with the other parameters usually determined.

References

- [1] B. Mashhoon, In: L. Iorio (Ed.), *The Measurement of Gravitomagnetism: A Challenging Enterprise* (NOVA, Hauppauge, 2007) 29
- [2] M. L. Ruggiero, A. Tartaglia, *Nuovo Cimento B* 117, 743 (2002)
- [3] G. E. Pugh, WSEG Research Memorandum No. 11, 1959
- [4] L. Schiff, *Phys. Rev. Lett.* 4, 215 (1960)
- [5] H. Pfister, *Gen. Relat. Gravit.* 39, 1735 (2007)
- [6] J. Lense, H. Thirring, *Phys. Z.* 19, 156 (1918)
- [7] C. W. F. Everitt, In: B. Bertotti (Ed.), *Proc. Int. School Phys. "Enrico Fermi" Course LVI* (New Academic Press, New York, 1974) 331
- [8] C. W. F. Everitt et al., In: C. Lämmerzahl, C. W. F. Everitt, F. W. Hehl (Eds.), *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space* (Springer, Berlin, 2001) 52
- [9] I. Ciufolini, E. C. Pavlis, *Nature* 431, 958 (2004)
- [10] M. R. Pearlman, J. J. Degnan, J. M. Bosworth, *Adv. Space Res.* 30, 135 (2002)
- [11] R. A. Van Patten, C. W. F. Everitt, *Phys. Rev. Lett.* 36, 629 (1976)
- [12] R. A. Van Patten, C. W. F. Everitt, *Celest. Mech. Dyn. Astr.* 13, 429 (1976)
- [13] I. Ciufolini, *Phys. Rev. Lett.* 56, 278 (1986)
- [14] J. C. Ries, R. J. Eanes, M. M. Watkins, B. Tapley, *Joint NASA/ASI Study on Measuring the Lense-Thirring Precession Using a Second LAGEOS Satellite CSR-89-3*, The University of Texas at Austin: Center for Space Research (Austin, TX, 1989)
- [15] I. Ciufolini et al., *LARES Phase A*, University La Sapienza (Rome, Italy, 1998)
- [16] L. Iorio, D. M. Lucchesi, I. Ciufolini, *Classical Quant. Grav.* (19), 4311 (2002)
- [17] D. M. Lucchesi, A. Paolozzi, In: *XVI Congresso Nazionale AIDAA*, 24-28 Sept. 2001, Palermo, Italy
- [18] L. Iorio, *Classical Quant. Grav.* 19, L175 (2002)
- [19] I. Ciufolini, arXiv:gr-qc/0412001v3
- [20] I. Ciufolini, arXiv:gr-qc/0609081v1
- [21] L. Iorio, *Journal of Cosmology and Astroparticle Physics* 7, 8 (2005)
- [22] L. Iorio, *Planet. Space Sci.* 55, 1198 (2007)
- [23] L. Iorio, *Europhys. Lett.* 80, 40007 (2007)
- [24] L. Iorio, *Adv. Space Res.* 43, 1148 (2009)
- [25] L. Cugusi, E. Proverbio, *Astron. Astrophys.* 69, 321 (1978)
- [26] I. Ciufolini, D. M. Lucchesi, F. Vespe, A. Mandiello, *Nuovo Cimento A* 109, 575 (1996)
- [27] I. Ciufolini, *Nuovo Cimento A* 109, 1709 (1996)
- [28] I. Ciufolini, E. C. Pavlis, F. Chieppa, E. Fernandes-Vieira, J. Pérez-Mercader, *Science* 279, 2100 (1998)
- [29] F. G. Lemoine et al., *NASA/TP-1998-206861* (1998)
- [30] L. Iorio, *Celest. Mech. Dyn. Astr.* 86, 277 (2003)
- [31] J. C. Ries, R. J. Eanes, B. D. Tapley, In: R. J. Ruffini, C. Sigismondi (Eds.), *Nonlinear Gravitodynamics. The Lense-Thirring Effect* (World Scientific, Singapore, 2003) 201
- [32] J. C. Ries, R. J. Eanes, B. D. Tapley, G. E. Peterson, In: R. Noomen, S. Klosko, C. Noll, M. Pearlman (Eds.), *Proc. 13th Int. Laser Ranging Workshop*, NASA CP (2003-212248 (NASA Goddard, Greenbelt, 2003)
- [33] A. Milani, A. M. Nobili, P. Farinella, *Non-gravitational perturbations and satellite geodesy* (Adam Hilger, Bristol, 1987)
- [34] P. Inversi, F. Vespe, *Adv. Space Res.* 14, 73 (1994)
- [35] F. Vespe, *Adv. Space Res.* 23, 699 (1999)
- [36] D. M. Lucchesi, *Planet. Space Sci.* 49, 447 (2001)
- [37] D. M. Lucchesi, *Planet. Space Sci.* 50, 1067 (2002)
- [38] D. M. Lucchesi, *Geophys. Res. Lett.* 30, 1957 (2003)
- [39] D. M. Lucchesi, *Celest. Mech. Dyn. Astr.* 88, 269 (2004)
- [40] D. M. Lucchesi et al., *Planet. Space Sci.* 52, 699 (2004)
- [41] Ch. Reigber et al., *J. Geodyn.* 39 1 (2005)
- [42] J. C. Ries, R. J. Eanes, M. M. Watkins, In: S. Schillak (Ed.), *16th Int. Laser Ranging Workshop*, 13-17 Oct. 2008, Poznań, Poland
- [43] E.C. Pavlis, In: R. Cianci, R. Collina, M. Francaviglia, P. Fré (Eds.), *Recent Developments in General Relativity: Proc. 14th SIGRAV Conf. on General Relativity Gravitational Physics*, 18-22 Sept. 2000, Genova, Italy (Springer, Milan, 2002) 217
- [44] L. Iorio, A. Morea, *Gen. Relat. Gravit.* 36, 1321 (2004)
- [45] W. M. Kaula, *Theory of Satellite Geodesy* (Blaisdell, Waltham, 1966)
- [46] L. Iorio (Ed.), *The Measurement of Gravitomagnetism: A Challenging Enterprise* (NOVA, Hauppauge, 2007)
- [47] I. Ciufolini, E. C. Pavlis, *New Astron.* 10, 636 (2005)
- [48] I. Ciufolini, E. C. Pavlis, R. Peron, *New Astron.* 11, 527 (2006)
- [49] L. Iorio, *New Astron.* 10, 603 (2005)
- [50] L. Iorio, *J. Geodesy* 80, 128, (2006)
- [51] L. Iorio, *Gen. Relat. Gravit.* 38, 523 (2006)
- [52] L. Iorio, *Planet. Space Sci.* 55, 503 (2007)
- [53] D. M. Lucchesi, *Int. J. Mod. Phys. D* 14, 1989 (2005)
- [54] D. M. Lucchesi, *Adv. Space Res.* 39, 1559 (2007)
- [55] F. J. Lerch et al., *J. Geophys. Res.* 99(B2), 2815 (1994)

- [56] B. D. Tapley et al., In: American Geophysical Union, Fall Meeting 2007, abstract #G42A-03
- [57] C. Förste et al., In: EGU General Assembly 2005, 24-29 April 2005, Vienna, Austria
- [58] B. D. Tapley et al. J. Geodesy 79, 467 (2005)
- [59] T. Mayer-Gürr, A. Eicker, K.-H. Ilk, In: 1st Int. Symp. of the International Gravity Field Service "GRAVITY FIELD OF THE EARTH", 28 August-1 September 2006, Istanbul, Turkey
- [60] T. Mayer-Gürr, In: Joint Int. GSTMDFG SPP Symp., 15-17 Oct. 2007, Potsdam, Germany
- [61] N. K. Pavlis et al., In: 2008 General Assembly of the European Geosciences Union, 13-18 April 2008, Vienna, Austria
- [62] A. Jäggi, G. Beutler, L. Gmervart, In: IAG Symposium on "Gravity, Geoid, Earth Observation 2008", 23-27 June 2008, Chania, Greece
- [63] D. M. Lucchesi, G. Balmino, Planet. Space Sci. 54, 581 (2006)
- [64] L. Combrinck, In: S. Schillak (Ed.), 16th Int. Laser Ranging Workshop, 13-17 Oct. 2008, Poznań, Poland