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644. A Problem

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$$\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \text{ is to } OB \cdot OC. \text{ Thus 6 vol. } OPQR = \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \times OA.$$

But writing equation to plane through  $P, Q, R$ , and putting  $y=0, z=0$ , we get

$$\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \times OA = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \equiv (x_1 y_2 z_3), \text{ suppose.}$$

Now, since any tetrahedron  $PQRS$  is equal to the algebraic sum of the four tetrahedra whose common vertex is  $O$ , and bases the faces of  $PQRS$ , we have

$$\begin{aligned} 6 \text{ vol. } PQRS &= (x_2 y_3 z_4) - (x_1 y_3 z_4) + (x_1 y_2 z_4) - (x_1 y_2 z_3) \\ &= \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}, \text{ the required expression.} \end{aligned}$$

When the axes are oblique, we multiply this expression by  $2n$ , where

$$4n^2 = 1 - \sum \cos^2 \alpha + 2\prod \cos \alpha,$$

$\alpha, \beta, \gamma$  being the angles between the co-ordinate axes.

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643. [K<sup>1</sup>. 9. b.] The following construction in Practical Solid seems worth noting. It has useful applications in Crystallography.

Let  $OAMBLCHK$  be a rectangular parallelepiped. It is required to find the point at which the normal through  $O$  to the plane  $ABC$  meets the face  $CHKL$ . Draw perprs. from  $O$  to  $CA, CB$  meeting  $CH, CL$  in  $D, E$ . Complete the rectangle  $DCEF$ . Then  $OF$  is perpr. to pl.  $ABC$ .

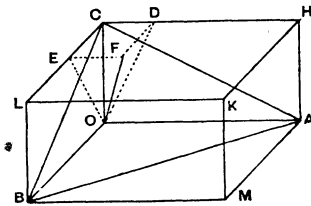


FIG. 1.—The parallelepiped in oblique parallel perspective.

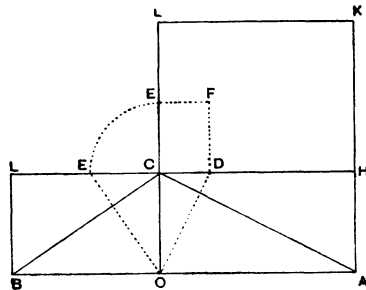


FIG. 2.—The planes  $COBL, CLKH$  rabatted into the plane of the paper, that of  $OAHK$ , round  $CO, CH$ .

If  $OC$  is the axial unit along  $OZ$ , and if  $OA, OB$  along  $OX, OY$  contain  $m$  and  $n$  units respectively, then  $F$  is the gnomonic projection of the pole of pl.  $ABC$   $\left(\frac{x}{m} + \frac{y}{n} + z = 1\right)$  with respect to the unit sphere with centre  $O$ . In this case the construction admits of simplification.

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$ABC$  is an isosceles triangle.  $B = C = 80^\circ$ .  $CF$  at  $30^\circ$  to  $AC$  cuts  $AB$  in  $F$ .  $BE$  at  $20^\circ$  to  $AB$  cuts  $AC$  in  $E$ . Prove  $\angle BEF = 30^\circ$ .

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