$$\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \text{ is to } OB\,.\,OC. \quad \text{Thus } 6 \text{ vol. } OPQR = \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \times OA.$$

But writing equation to plane through P, Q, R, and putting $y=0$, $z=0$, we get

$$\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} \times OA = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \equiv (x_1y_2z_3), \text{ suppose.}$$

Now, since any tetrahedron $PQRS$ is equal to the algebraic sum of the four tetrahedra whose common vertex is O, and bases the faces of $PQRS$, we have

$$6 \text{ vol. } PQRS = (x_2y_3z_4) - (x_1y_3z_4) + (x_1y_2z_4) - (x_1y_2z_3)$$
$$= \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}, \text{ the required expression.}$$

When the axes are oblique, we multiply this expression by $2n$, where

$$4n^2 = 1 - \Sigma \cos^2 \alpha + 2\Pi \cos \alpha,$$

α, β, γ being the angles between the co-ordinate axes.

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643. [K[1]. 9. b.] The following construction in Practical Solid seems worth noting. It has useful applications in Crystallography.

Let $OAMBLCHK$ be a rectangular parallelopiped. It is required to find the point at which the normal through O to the plane ABC meets the face $CHKL$. Draw perprs. from O to CA, CB meeting CH, CL in D, E. Complete the rectangle $DCEF$. Then OF is perpr. to pl. ABC.

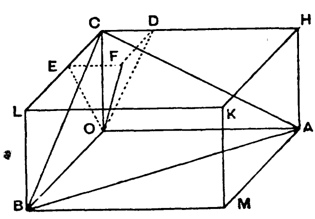

FIG. 1.—The parallelopiped in oblique parallel perspective.

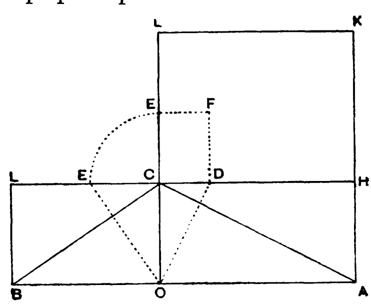

FIG. 2.—The planes $COBL$, $CLKH$ rabatted into the plane of the paper, that of $OAHC$, round CO, CH.

If OC is the axial unit along OZ, and if OA, OB along OX, OY contain m and n units respectively, then F is the gnomonic projection of the pole of pl. ABC $\left(\dfrac{x}{m}+\dfrac{y}{n}+z=1\right)$ with respect to the unit sphere with centre O. In this case the construction admits of simplification.

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644. [K[1]. 1. c.] *A Problem.*

ABC is an isosceles triangle. $B=C=80°$. CF at 30° to AC cuts AB in F. BE at 20° to AB cuts AC in E. Prove $B\hat{E}F=30°$. E. M. LANGLEY.