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SEPTEMBER 1972 MATT-921

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The Effect of Injection Fuel Energy on Fusion Reactor Stability

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ABSTRACT

The effect of injection fuel energy on the plasma dynamics stability of a stationary fusion reactor has been investigated, using a mathematical model based on the energy density and particle density of four plasma constituents. For this purpose, numerical calculations of eigenvalues have been made after linearization of a set of plasma equations. The result suggests that a neutral beam injection gun of moderate energy is favorable to realizing a stable plasma rather than a pellet injection method of very low energy. Further calculation has been made for studying the inner mechanism of plasma self-stabilization.

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1. INTRODUCTION

As one of the approaches to a fusion reactor dynamics problem, Mills[1] has made an investigation of time dependent behavior. Ohta, Yamato, and Mori[2] have shown the stability region of a reactor plasma using a rather simple model of one particle group. Since a stable plasma of high temperature and density is not yet available for study, it is inevitable that some uncertain factors must be incorporated in the plasma dynamics study. Nevertheless, it seems to be a matter of great interest and significance to make clear the stability conditions of a reactor plasma, using a more precise plasma model.

In this report we adopt a four group model of plasma particles to include the effect of fast α-particles produced by the D-T reaction and investigate the effect of neutral injection fuel energy on plasma stability. Although the results for the mathematical model of the plasma are subject, to some extent, to assumptions not yet confirmed by experiment, they may still suggest a right way to a fusion reactor.

2. PLASMA DYNAMICS MODEL

2.1. Assumptions

Prior to making up a mathematical model to investigate the plasma stability of a hypothetical stationary reactor, we first make a number of assumptions described in the following:

(1) The plasma model as shown in Fig. 1 consists of four groups of particles:

α_f , α-particles;
α , α-particles;
e , electrons;
I , fuel ions.

It should be noted here that the fast α-particles and slow α-particles are separated according to their energy, although they are quite the same species.

(2) Deuterium and tritium ions are classified into the same fuel ion group for simplicity of calculation, and are equal in density. They are fed from outside the plasma in the form of neutral injection fuel of the same mixture.

(3) Only the D-T reaction is considered.

(4) The fast α-particles are monoenergetic. Their energy is fixed at 3.52 MeV from the D-T reaction.

(5) The fast α-particles give their kinetic energy to the other groups of particles through Coulomb collision. After some number of collisions, they suddenly transfer into the group of the slow α-particles, carrying a decreased energy. Meanwhile, a fractional part of the fast α-particles flows out from the plasma.

(6) The three particle groups other than the fast α-particle group are assumed to have a Maxwellian distribution of velocity.

(7) The effects of neutrons, impurities, and cyclotron radiation loss are ignored. The magnetic field is fixed.

2.2. Energy Transfer Rate Between Particle Groups

The energy transfer rate of a single test particle to a group of Maxwellian distributed field particles has been well calculated by Butler and Buckingham[3] as follows:

$$\frac{d\mathcal{E}_T}{dt} = - \frac{8\sqrt{\pi} n_F}{m_F} \left(\frac{q_T \; q_F}{4\pi\varepsilon_o} \right)^2 \frac{\ln \Lambda}{w_t} F\left(\frac{v}{w_t} \right) \tag{1}$$

where

$$F(a) = \int_o^1 \exp(-\, a^2 x^2)dx - (1 + m_F/m_T) \exp(-\, a^2) \quad , \tag{2}$$

$$a = v/w_t \quad , \quad x = w/v \quad , \quad \text{and} \quad \ln \Lambda = 20 \; .$$

The symbols are defined as

q =	charge of a particle	(Coulomb)
m =	mass of a particle	(kg)
n =	particle density	($1/m^3$)
v =	velocity of a test particle	(m/sec)
w =	thermal velocity of a particle group	(m/sec)
$\mathcal{E}$ =	kinetic energy of a particle	(Joule)
$\ln \Lambda$ =	Coulomb logarithm .	

The suffixes F and T denote the field particle and test particle, respectively. The Coulomb logarithm is assumed to be a constant for simplicity of calculation. Equation (1) is postulated to apply to the calculation of an energy transfer rate between two groups of plasma particles, although its mathematical expression differs according to the distribution function of the test particles, and also to the mass ratio of the two groups concerned. Now we consider three cases in the following.

2.2.1. Test particle, α_f ; and field particle, e

Although the average energy of the electrons is much lower than that of the fast α-particles, the squared thermal velocity of the electron group is larger in the energy range of interest here because of the small mass ratio. The average energy of the electron group will be estimated at less than two times the fuel ion energy. Accordingly, Eq. (2) may be approximated under the conditions

$$m_e/m_{\alpha f} << 1$$

and

$$a^2 << 1$$

in the following form,

$$\begin{aligned} F(a) &\simeq (1 - a^2/3) - (1 + m_e/m_{\alpha f})(1 - a^2) \\ &\simeq (2/3)\ a^2 - m_e/m_{\alpha f} \end{aligned} \tag{3}$$

in which the suffixes e and αf denote the electron and fast α-particle groups, respectively, and $m_{\alpha f} = m_\alpha$. Thus, the energy transfer rate is represented from Eq. (1) as

$$\begin{aligned} \left\langle \frac{dE_{\alpha f}}{dt} \right\rangle &= \frac{1}{\int f(v)dv} \int \left(\frac{dE_{\alpha f}}{dt} \right) f(v)dv \\ &\simeq dE_{\alpha f}/dt \\ &\simeq - A_e n_e\ (E_{\alpha f} - E_e) \quad , \end{aligned} \tag{4}$$

where

$$A_e = \frac{4(3\pi)^{1/2}}{m_{\alpha f}\ m_e}\left(\frac{q_{\alpha f}\ q_e}{4\pi\ \varepsilon_o}\right)^2 \frac{\ln \Lambda}{(E_e/m_e)^{3/2}} \quad , \tag{5}$$

f(v) is the distribution function of the test particles in velocity space, and the symbol E denotes an average energy of particles. Especially, when the particles are Maxwellian distributed, E is equal to (3/2)kT in which k is the Boltzmann constant and T the temperature.

2.2.2. Test particle, α_f; and field particles, α and I

In contrast to case (1), the fast α-particle group is much larger in thermal velocity than each of the two field particle groups because of their comparable masses so that Eq. (2) is differently approximated under the condition

$$a^2 >> 1$$

in the form of

$$F(a) \simeq \sqrt{\pi}/2a - 1/2a^2 \exp(-a^2) - (1 + m_F/m_T) \exp(-a^2) \quad . \tag{6}$$

The second and third terms on the right-hand side of Eq. (6) may be ignored because of their remarkably small exponential multiplicand, and Eq. (6) is reduced to

$$F(a) \simeq \sqrt{\pi}/2a \quad . \tag{7}$$

Thus, we have the following expression for this case,

$$\left\langle \frac{dE_{\alpha f}}{dt} \right\rangle \simeq - A_F\ n_F\ E_{\alpha f} \quad , \tag{8}$$

$$A_F = \frac{2\sqrt{2}\pi}{m_{\alpha f} m_F} \left(\frac{q_{\alpha f} q_F}{4\pi \ \varepsilon_o}\right)^2 \frac{\ln \Lambda}{(E_{\alpha f}/m_{\alpha f})^{3/2}} \tag{9}$$

and the suffix F denotes the α and I groups of field particles according to each case.

The mass of a combined fuel ion is defined as

$$m_I \equiv 2 \ m_D m_T/(m_D + m_T) \qquad .$$

2.2.3. Test particle, e; and field particles, α and I. Test particle, α; and field particle, I

Since every group of particles under consideration is assumed to have a Maxwellian distribution, the energy transfer rate is obtained by averaging Eq. (1) over the velocity of test particles so that

$$\left\langle \frac{dE_T}{dt} \right\rangle = \frac{1}{\pi^{3/2} v_t^3} \int_o^\infty \left(\frac{dE_T}{dt}\right) \exp\left(-\frac{v^2}{v_t^2}\right) dv \quad , \tag{10}$$

where v_t is the thermal velocity of test particles. By substituting Eq. (1) into Eq. (10), we can derive the well-known relation[4] in the following

$$\left\langle \frac{dE_T}{dt} \right\rangle = - B_{TF} n_F (E_T - E_F) \quad , \tag{11}$$

where

$$B_{TF} = \frac{4\sqrt{2}\pi}{m_T m_F} \left(\frac{q_T q_F}{4\pi\varepsilon_o}\right)^2 \frac{\ln \Lambda}{[(E_T/m_T) + (E_F/m_F)]^{3/2}} \quad . \tag{12}$$

2.3. Energy and Particle Losses

2.3.1. Confinement time

Many things have been said concerning confinement time. Nevertheless, the state of the art of plasma physics has not led to a definite mathematical expression for the confinement time. However, Yoshikawa[5] has shown that there exists good agreement between the neoclassical confinement time and experimental results. Ohta et.al.[2] have shown that the classical type is more limiting than the Bohm type for plasma dynamics stability. Here, we adopt the former expression of the confinement time given by

$$\tau_j = G \frac{\sqrt{E_j}}{\Sigma n_i} \quad , \qquad [i = \alpha_f, \alpha, \text{ and } I] \quad , \tag{13}$$

where

$$\Sigma n_i = n_{\alpha f} + n_\alpha + n_I \tag{14}$$

and the coefficient G is so determined as to attain a plasma equilibrium, and assumed to be common to all species of ions. On the other hand, the confinement time of the electrons must be determined from the plasma equations as stated later.

2.3.2. Bremsstrahlung radiation loss

A bremsstrahlung radiation loss originates from the acceleration of an electron in the field of an ion, and is given by

$$W_b = 4.8 \times 10^{-37}\, n_e\, (T_e)^{1/2} \sum_i Z_i^2 n_i \quad , \qquad (\mathrm{Watts/m^3})$$

$$[i = \alpha_f, \alpha, \text{ and } I] \quad , \tag{15}$$

where T_e is the electron temperature in keV, and Z the ionic charge number. On the other hand, we may ignore, for simplicity of calculation, the cyclotron radiation loss[6] which has been predicted to be less than the bremsstrahlung loss for most operating states of a fusion reactor plasma.

2.3.3. Slowing down rate of fast α-particle

To complete the plasma dynamics model, we must estimate the number of slowing down particles from the α_f-group to the α-group per unit time per unit volume so that both equations of the energy and density of the α_f-group may be compatible with each other under the condition of the fixed $E_{\alpha f}$. The rate of change of energy that a fast α-particle suffers through collisions with field particles of the group k is represented by

$$\left\langle \frac{dE_{\alpha f}}{dt} \right\rangle = -A_k n_k F_k(E_{\alpha f}, E_k) \quad , \tag{16}$$

where A_k is defined in Eqs. (5) and (9) for each group of the field particles, and F_k is a simple function of $E_{\alpha f}$ and E_k such as

$$F_k = E_{\alpha f} - E_e \quad , \qquad \text{for the electron group,}$$

$$F_k = E_{\alpha f} \quad , \qquad \text{for the } \alpha\text{- and I-groups .}$$

Let ν_k be the collision frequency between the two particle groups concerned, then the average change of energy of a fast α-particle through a single collision is given by

$$\begin{aligned} \left\langle \Delta E_{\alpha f} \right\rangle &= \frac{\langle dE_{\alpha f}/dt \rangle}{\nu_k} \\ &= -\frac{1}{\nu_k} A_k n_k F_k(E_{\alpha f}, E_k) \quad . \end{aligned} \tag{18}$$

In an average sense, the number of collisions experienced by a fast α-particle and needed for its retardation down to the energy level $a_k E_k$ may be given as

$$N_c \equiv - \frac{E_{\alpha f} - a_k E_k}{\langle \Delta E_{\alpha f} \rangle} \quad , \qquad (19)$$

where a_k is a multiplication constant to make zero the energy transfer rate. Thus the slowing down rate of the fast α-particle becomes

$$\begin{aligned} R_c &= \frac{n_{\alpha f}}{N_c} \nu_k \\ &= \frac{\langle \Delta E_{\alpha f} \rangle}{E_{\alpha f} - a_k E_k} \cdot \frac{n_{\alpha f} \langle dE_{\alpha f}/dt \rangle}{\langle \Delta E_{\alpha f} \rangle} \\ &= C_k \; n_{\alpha f} n_k \quad , \qquad (20) \end{aligned}$$

where

$$C_k = A_k \frac{F_k(E_{\alpha f}, E_k)}{E_{\alpha f} - a_k E_k} \quad . \qquad (21)$$

If the fast α-particles as test particles are monoenergetic and field particles are Maxwellian, the a_k would become slightly less than unity; $a_I \simeq 2/3$ for the fuel ion as field particles, $a_\alpha \simeq 3/4$ for the slow α-particle and $a_e \simeq 1$ for the electron as easily calculated from Eq. (2) by equating it to zero.[7] However, the a_k is assumed to be unity for simplicity of calculation. Their discrepancy due to the approximation will not result in a large error, since no more than a relatively small fraction of energy flows via the slow α-particle.

2.4. Dynamics Equations

We are now prepared to write down the dynamics equations, being based on a spatially averaged, one point model. The time derivatives of the energy density and particle density are described for each particle group in the following:

$$\frac{dn_{\alpha f}E_{\alpha f}}{dt} = \frac{1}{4}n_I^2 \langle\sigma v\rangle E_o - A_e n_{\alpha f} n_e (E_{\alpha f} - E_e) - A_\alpha n_{\alpha f} n_\alpha E_{\alpha f}$$

$$- A_I n_{\alpha f} n_I E_{\alpha f} - \sum_{k=\alpha,e,I} C_k n_{\alpha f} n_k (a_k E_k) - n_{\alpha f} E_{\alpha f}/\tau_{\alpha f} \quad , \tag{22}$$

$$\frac{dn_\alpha E_\alpha}{dt} = A_\alpha n_{\alpha f} n_\alpha E_{\alpha f} - B_{\alpha e} n_\alpha n_e (E_\alpha - E_e) - B_{\alpha I} n_\alpha n_I (E_\alpha - E_I)$$

$$+ \sum_{k=\alpha,e,I} C_k n_{\alpha f} n_k (a_k E_k) - n_\alpha E_\alpha/\tau_\alpha \quad , \tag{23}$$

$$\frac{dn_e E_e}{dt} = A_e n_{\alpha f} n_e (E_{\alpha f} - E_e) + B_{\alpha e} n_\alpha n_e (E_\alpha - E_e)$$

$$- B_{eI} n_e n_I (E_e - E_I) + W_b - n_e E_e/\tau_e \quad , \tag{24}$$

$$\frac{dn_I E_I}{dt} = A_I n_{\alpha f} n_I E_{\alpha f} + B_{\alpha I} n_\alpha n_I (E_\alpha - E_I) + B_{eI} n_e n_I (E_e - E_I)$$

$$+ SE_s - \frac{1}{2} n_I^2 \langle\sigma v\rangle E_I - n_I E_I/\tau_I \quad , \tag{25}$$

$$\frac{dn_{\alpha f}}{dt} = \frac{1}{4} n_I^2 \langle\sigma v\rangle - \sum_{k=\alpha,e,I} C_k n_{\alpha f} n_k - n_{\alpha f}/\tau_{\alpha f} \quad , \tag{26}$$

$$\frac{dn_\alpha}{dt} = \sum_{k=\alpha,e,I} C_k n_{\alpha f} n_k - n_\alpha/\tau_\alpha \quad , \tag{27}$$

$$\frac{dn_e}{dt} = S - n_e/\tau_e \quad , \tag{28}$$

$$\frac{dn_I}{dt} = S - \frac{1}{2} n_I^2 \langle\sigma v\rangle - n_I/\tau_I \quad , \tag{29}$$

where

E_o ≡ initial energy of $E_{\alpha f}$ from the D-T reaction (Joule)

E_s ≡ injection fuel energy (Joule)

S ≡ injection fuel flow rate $m^{-3}\ sec^{-1}$

In a bit of manipulation, the time derivatives of the average energy are derived for each particle group such that

$$\frac{dE_\alpha}{dt} = \frac{1}{n_\alpha}[A_\alpha n_{\alpha f} n_\alpha E_{\alpha f} - B_{\alpha e} n_\alpha n_e (E_\alpha - E_e) - B_{\alpha I} n_\alpha n_I (E_\alpha - E_I) - \sum_k C_k n_{\alpha f} n_k (E_\alpha - a_k E_k)] \quad , \tag{30}$$

$$\frac{dE_e}{dt} = \frac{1}{n_e}[A_e n_{\alpha f} n_e (E_{\alpha f} - E_e) + B_{\alpha e} n_\alpha n_e (E_\alpha - E_e) - B_{eI} n_e n_I (E_e - E_I) - W_b - SE_e] \quad , \tag{31}$$

$$\frac{dE_I}{dt} = \frac{1}{n_I}[A_I n_{\alpha f} n_I E_{\alpha f} + B_{\alpha I} n_\alpha n_I (E_\alpha - E_I) + B_{eI} n_e n_I (E_e - E_I) + SE_s - SE_I] \quad , \tag{32}$$

and

$$E_{\alpha f} = E_o \tag{33}$$

where

$$A_\alpha = \frac{2\sqrt{2}\pi}{m_{\alpha f} m_\alpha} \left(\frac{q_{\alpha f} q_\alpha}{4\pi\ \varepsilon_o} \right)^2 \frac{\ln\Lambda}{(E_{\alpha f}/m_{\alpha f})^{3/2}} \quad , \tag{34}$$

$$A_e = \frac{4(3\pi)^{1/2}}{m_{\alpha f} m_e} \left(\frac{q_{\alpha f} q_e}{4\pi\varepsilon_o} \right)^2 \frac{\ln\Lambda}{(E_e/m_e)^{3/2}} \quad , \tag{35}$$

$$A_I = \frac{2\sqrt{2}\pi}{m_{\alpha f} m_I} \left(\frac{q_{\alpha f} q_I}{4\pi\varepsilon_o} \right)^2 \frac{\ln\Lambda}{(E_{\alpha f}/m_{\alpha f})^{3/2}} \quad , \tag{36}$$

$$C_\alpha = A_\alpha \frac{E_{\alpha f}}{E_{\alpha f} - a_\alpha E_\alpha} \quad , \tag{37}$$

$$C_e = A_e \quad , \tag{38}$$

$$C_I = A_I \frac{E_{\alpha f}}{E_{\alpha f} - a_I E_I} \quad . \tag{39}$$

Equation (33) results from the relation

$$n_{\alpha f} \frac{dE_{\alpha f}}{dt} = \frac{d(n_{\alpha f} E_{\alpha f})}{dt} E_{\alpha f} \frac{dn_{\alpha f}}{dt} = 0 \quad . \tag{40}$$

This is consistent with the initial assumption that the fast α-particle group has a constant average energy of 3.52 MeV. Furthermore, we must take into consideration the neutrality condition of the plasma which imposes a constraint among the particle densities such that

$$n_e = n_I + 2(n_\alpha + n_{\alpha f}) \tag{41}$$

which leads to the following relation that, keeping in mind the neutrality of injection fuel, the leakage of plasma particles must be also neutral as a whole: that is

$$\frac{n_e}{\tau_e} = \frac{n_I}{\tau_I} + 2\left(\frac{n_\alpha}{\tau_\alpha} + \frac{n_{\alpha f}}{\tau_{\alpha f}}\right) \quad . \tag{42}$$

From the dynamics equations described above, it can be easily seen that the rate of change of their time behavior is proportional to a plasma density, or fuel ion density n_I.

3. PLASMA STABILITY

3.1. Stability Calculation

The primary objective of the plasma stability calculation is to make clear the effect of the fuel ion temperature and injection fuel energy on plasma stability. Here we take E_I and E_s as given variables along with n_I. Steady state solutions are numerically calculated from Eqs. (26) through (32), and Eq. (41) instead of Eq. (28) for the combinations of the given variables by use of an iterative computer program. Then, a set of six differential equations from Eqs. (26) through (32) except Eq. (28) are linearized in terms of fractional variables normalized by the respective steady state values such that

$$\frac{d\vec{x}}{dt} = \underline{\underline{A}}\,\vec{x} + \vec{B}_s\, r_s \tag{43}$$

where

$$\vec{x} = \begin{bmatrix} \Delta E_{\alpha}/E_{\alpha o} \\ \Delta E_{e}/E_{eo} \\ \Delta E_{I}/E_{Io} \\ \Delta n_{\alpha f}/n_{\alpha fo} \\ \Delta n_{\alpha}/n_{\alpha o} \\ \Delta n_{I}/n_{Io} \end{bmatrix} \tag{44}$$

$$\vec{B}_s = \begin{bmatrix} 0 \\ -S_o/n_{eo} \\ (S_o/n_{Io})(E_s-E_{Io})/E_{Io} \\ 0 \\ 0 \\ S_o/n_{Io} \end{bmatrix} \tag{45}$$

$$\underline{\underline{A}} = \begin{bmatrix} a_{11}, a_{12}, \cdots a_{16} \\ a_{21}, a_{22}, \cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ a_{61} \ \cdots\cdots\cdots a_{66} \end{bmatrix} \tag{45A}$$

$$r_s = \Delta S/S_o \tag{46}$$

in which the symbol Δ and suffix o denote the fractional and steady state components of each variable, respectively. Thus the dynamic stability of the plasma can be examined from the eigenvalues of $\underline{\underline{A}}$, of which the most dominant real part is plotted in

Fig. 2 as a function of the fuel ion temperature T_I in keV. Its imaginary part is zero within computer accuracy in the range of T_I shown in Fig. 2.

For the operating condition of

$$T_{Io} = 13 \text{ keV} \quad , \quad E_s = 1 \text{ eV} \quad , \text{ and } n_{Io} = 5 \times 10^{20} \text{ m}^{-3} \quad ,$$

we get

$$\frac{n_{Ioo}}{n_{Io}} \sigma = 0.265 \quad \text{sec}^{-1}$$

where

$$n_{Ioo} = 10^{20} \quad \text{m}^{-3} \quad .$$

Thus the dominant real part becomes 1.32 sec^{-1}, hence lying in the unstable region of Fig. 2. This instability is not affected by n_I, but remarkably improved by the increase of T_I and/or E_s. For E_s of 25 keV and 30 keV, for example, the plasma is stable throughout the range of T_I in question whenever plasma equilibrium exists. In the lower range of T_I, the plasma equilibrium, stable or unstable, is impossible to obtain by the iterative computer program. The threshold value, although not necessarily clear because of numerical calculation, corresponds to the so-called ignition point of a plasma. In the higher range, the calculation is limited by the validity of the dynamics equations because we ignored the effect of cyclotron radiation loss which will be dominant in the range higher than 100 keV of T_I or so.[6]

From a practical point of view, it is preferable to operate a fusion reactor at a lower ion temperature in the stable region.

Such operation would be possible by choosing E_s near to and above 20 keV. This result tells us the importance of developing a neutral fuel injection unit of high energy and large current. However, it does not seem desirable to set E_s too high since it would move both the ignition point and the operating temperature to higher levels.

3.2. Inner Mechanism of Instability

To look at the inner mechanism of plasma instability, let us open the closed loop of the energy flow diagram at the feeding point of the fast α-particles as shown in Fig. 3. Suppose that a hypothetical external source feeds fast α-particles instead of the D-T reaction, of which the feeding rate $S_{\alpha fo}$ in a steady state is equal to that produced by the D-T reaction, thereby retaining the steady state unchanged. On the other hand, the fast α-particles produced as a result of D-T reaction go out of the plasma, hence playing no role in the plasma dynamics. Now we consider the response of the reaction rate R defined as

$$R = \frac{1}{4} n_I^2 \langle \sigma v \rangle \qquad m^{-3}\ sec^{-1}$$

to the fractional variations of the fast α-particle injection rate $\Delta S_{\alpha f}$ around a steady state value $S_{\alpha fo}$. Since the neutral fuel injection rate S is assumed fixed, the dynamics equation is written in matrix form as

$$\frac{d\vec{x}}{dt} = \underline{\underline{A}}_{\alpha f}\ \vec{x} + \vec{B}_{\alpha f}\ r_{\alpha f} \tag{47}$$

where

$$\vec{B}_{\alpha f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ S_{\alpha fo}/n_{\alpha fo} \\ 0 \\ 0 \end{bmatrix} , \tag{48}$$

$$r_{\alpha f} = \Delta S_{\alpha f}/S_{\alpha fo} \quad . \tag{49}$$

The elements of the matrix $\underline{\underline{A}}_{\alpha f}$ are equal to those of $\underline{\underline{A}}$ except that

$$(a_{43})_{\alpha f} = a_{43} - \frac{1}{n_{\alpha fo}} R_o c_{fo} \tag{50}$$

and

$$(a_{46})_{\alpha f} = a_{46} - \frac{1}{n_{\alpha fo}} 2R_o \tag{51}$$

where

$$R_o = \frac{1}{4} n_I^2 \langle \sigma v \rangle \Big|_o \tag{52}$$

and

$$c_{fo} = \frac{\partial \langle \sigma v \rangle / \partial T_I}{\langle \sigma v \rangle / T_I} \Big|_o \quad . \tag{53}$$

Equations (50) and (51) are used to remove the effect of the fast α-particle injection due to the D-T reaction from the original coefficient matrix $\underline{\underline{A}}$ in Eq. (43). By taking the Laplace transformation of Eq. (47), we get the following state variable in matrix form

$$\vec{X}(s) = (s\,\underline{\underline{I}} - \underline{\underline{A}}_{\alpha f})^{-1}\, \vec{B}_{\alpha f}\, r_{\alpha f} \tag{54}$$

where s and $\underline{\underline{I}}$ are the Laplacian operator and unit matrix, respectively. Thus, the average energy and density of the fuel ions are given by

$$\frac{\Delta E_I}{E_{Io}} = [(s\underline{\underline{I}} - \underline{\underline{A}}_{\alpha f})^{-1}]_{EI} \frac{S_{\alpha fo}}{n_{\alpha fo}} \frac{\Delta S_{\alpha f}}{S_{\alpha fo}} \tag{55}$$

and

$$\frac{n_I}{n_{Io}} = [(s\underline{\underline{I}} - \underline{\underline{A}}_{\ f})^{-1}]_{nI} \frac{S_{\alpha fo}}{n_{\alpha fo}} \frac{\Delta S_{\alpha f}}{S_{\alpha fo}} \quad . \tag{56}$$

Meanwhile, fractional variations of the reaction rate in response to $\Delta S_{\alpha f}$ can be written as

$$\Delta R = R_o \left(2\frac{\Delta n_I}{n_{Io}} + c_{fo} \frac{\Delta E_I}{E_{Io}} \right) \quad . \tag{57}$$

Substituting Eqs. (55) and (56) into Eq. (57), we get the open loop transfer function as follows:

$$G_R(s) = \frac{R/R_o}{S_{\alpha f}/S_{\alpha fo}} = [2Q_{nI}(s) + c_{fo}Q_{EI}(s)] \frac{S_{\alpha fo}}{n_{\alpha fo}} \tag{58}$$

in which $Q_{EI}(s)$ and $Q_{nI}(s)$ denote the bracketed terms in Eqs. (55) and (56), respectively. It should be noted that $R_o = S_{\alpha fo}$, and the magnitude of the transfer function at zero frequency, $|G_R(0)|$, is independent of the given fuel ion density. With the substitution of imaginary frequency, $j\omega$, for s, the numerical calculation of Eq. (58) shows that instability, if any, can occur at zero frequency. This is compatible with the numerical calculation that the imaginary part of the dominant eigenvalue is zero. Therefore, the point of interest is to know the magnitude of $G_R(s)$ at zero frequency.

Figure 4 shows the variation of the magnitude, $|G_R(0)|$, as a function of T_I at $E_s = 1$ eV along with the curves of

$$Q_{EI}(0)\ ,\quad c_{fo}Q_{EI}(0)\ ,\quad Q_{nI}(0)\ ,\ \text{and}\ 2Q_{nI}(0)\qquad ,$$

each multiplied by $S_{\alpha fo}/n_{\alpha fo}$. From the figure we can see that $Q_{EI}(0)(S_{\alpha fo}/n_{\alpha fo})$ is superior in magnitude to $2Q_{nI}(0)(S_{\alpha fo}/n_{\alpha fo})$, but is largely affected by c_{fo} throughout the range of T_I in question. The magnitude of the gain, $|G\ (0)|$, if larger than or equal to unity, indicates the plasma to be unstable, otherwise stable. This threshold value of unity corresponds to zero value of the real part of the dominant eigenvalue. c_{fo} and $S_{\alpha fo}/n_{\alpha fo}$ are decreasing functions with respect to T_I, while Q_{EI} and Q_{nI} are increasing functions. Therefore, the plasma stability depends on the balance of these functions. Shown in Fig. 5 is another example at $E_s = 25$ keV in which c_{fo} plays an important role in concert with $S_{\alpha fo}/n_{\alpha fo}$ for the stabilization of the plasma, that is, for making $|G_R(0)|$ less than unity. c_{fo} is plotted in Fig. 6, being manipulated from the reactivity data of the D-T reaction.[1]

4. DISCUSSION

The energy of a neutral injection fuel has been pointed out to be effective for the stabilization of the plasma in a stationary reactor. From the viewpoint of thermal stability, the equilibrium solution of the plasma has shown no instability within its possible range of operation, if the injection fuel energy is properly chosen. Accordingly, it seems to be advantageous to develop a neutral beam injection gun of large current and of energy in the neighborhood of 25 keV. An excessively high temperature of the fuel gun, however, results in an increase in the plasma temperature, and this is unfavorable, from the economic and technical points of view, for maintaining a stationary plasma. On the other hand, a pellet injection method as fuel feeding may be unfavorable for realizing a stationary plasma, if it is used only by itself, because its very low temperature requires high plasma temperature of 40 keV or more in order to keep the plasma in a steady state.

We have assumed the confinement time to be of classical type for plasma particles. The results of the numerical calculation of plasma dynamics stability might differ, to some extent, if we had assumed the Bohm type.

ACKNOWLEDGMENTS

The author would like to express his deepest thanks to Drs. R. G. Mills, F. H. Tenney, and J. G. Gorman for their instructions and discussions in making up the plasma dynamics model.

This work was supported in part by U. S. Atomic Energy Commission Contract AT(11-1)-3073. Use was made of computer facilities supported in part by National Science Foundation Grant NSF-GP579.

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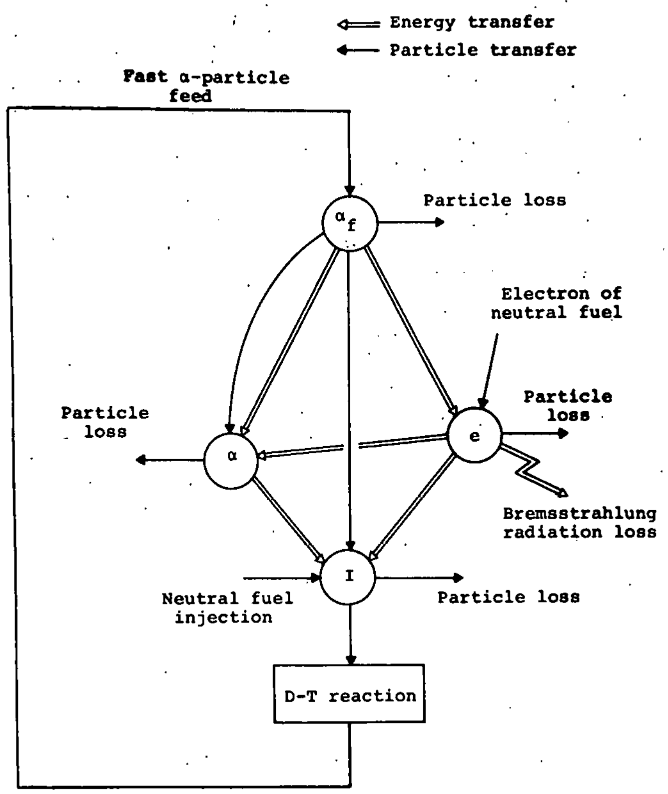

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Fig. 1. Energy and particle flow diagram of the plasma dynamics model.

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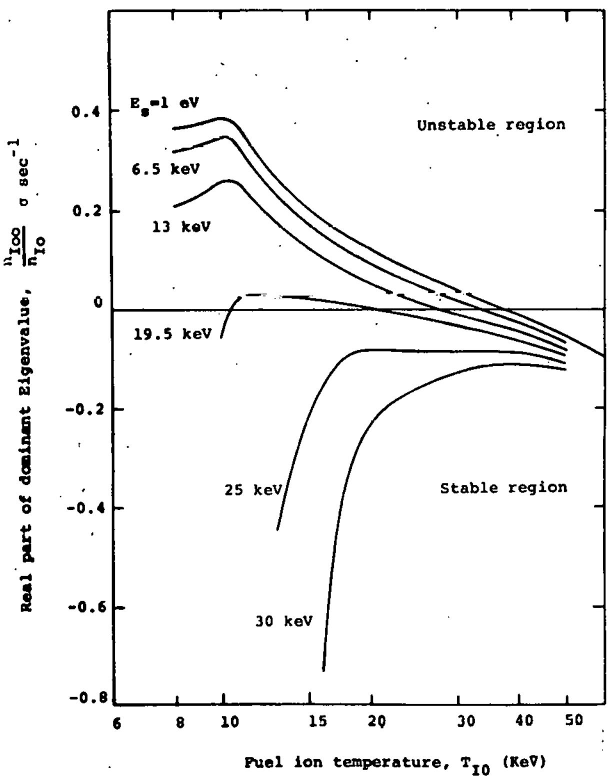
Fig. 2. Dynamic stability of a stationary plasma as a function of the fuel ion temperature with E_s as a parameter. $n_{Ioo} = 10^{20}\ m^{-3}$.



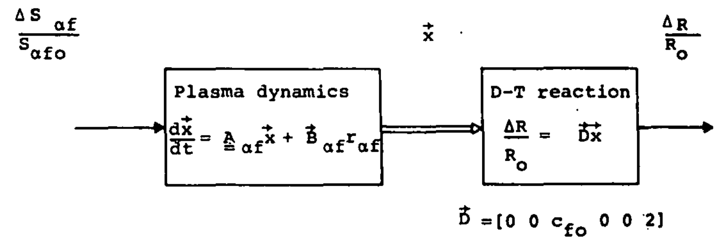


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Fig. 3. Signal flow diagram of open loop plasma dynamics.

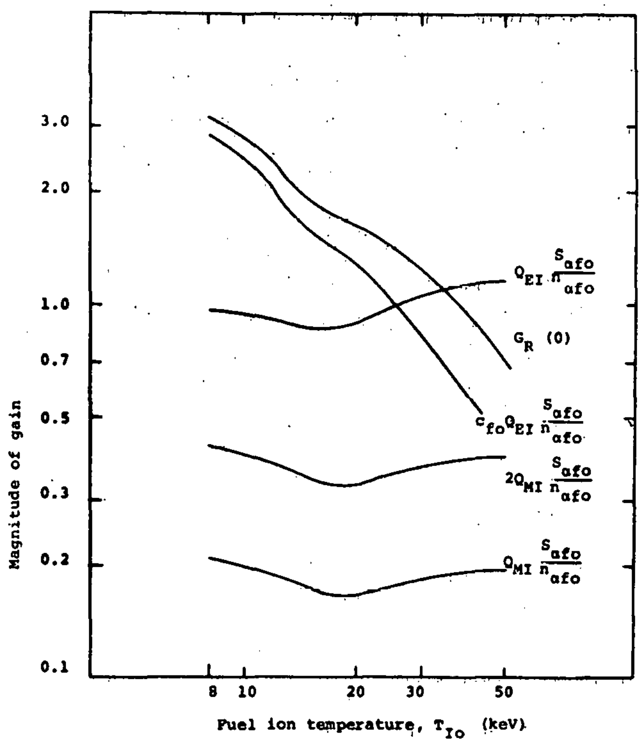

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Fig. 4. Variations of the magnitude of open loop plasma dynamics gains as a function of the fuel ion temperature at E_s = 1 eV.

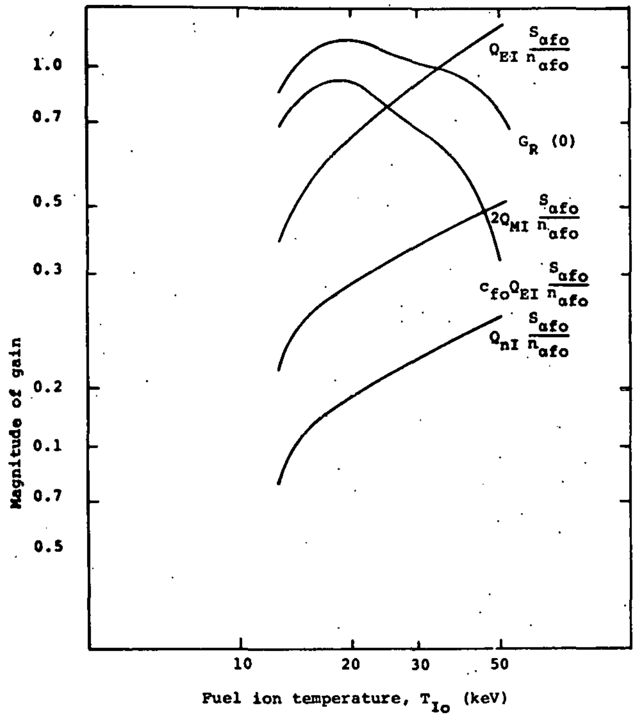

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Fig. 5. Variations of the magnitude of open loop plasma dynamics gains as a function of the fuel ion temperature at E_s = 25 keV.

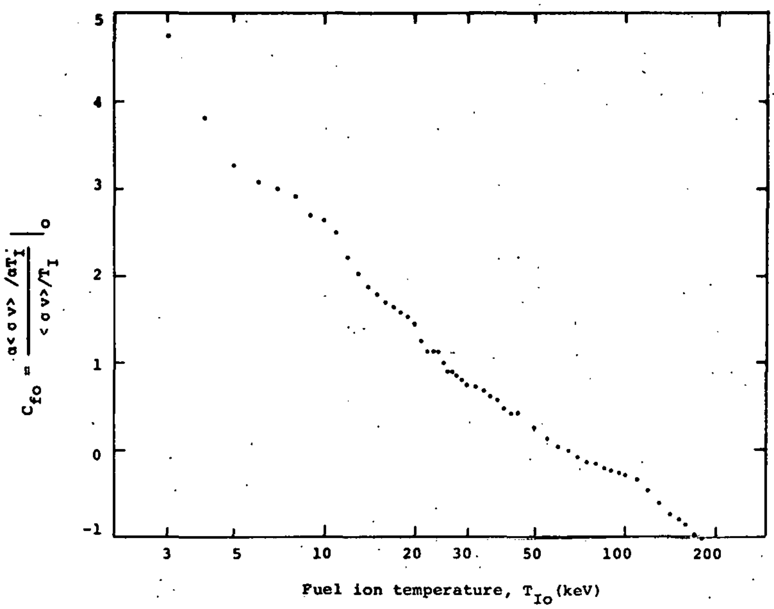

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Fig. 6. Variation of c_{fo} calculated from the reactivity of D-T reaction.