Alternating Gradient Synchrotron Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York 11973

AGS Division Technical Note
No. 209

BUNCH-TO-BUNCH TRANSVERSE FEEDBACK FOR THE AGS

E. Raka

October 12, 1984

Abstract

Transverse feedback in both planes is necessary to control the resistive wall instablity that is present in the AGS at low energy. A bunch-to-bunch system will provide damping of this instability, some control of any coherent oscillations arising from injection errors when the Booster becomes operational, and also damping of any m = 0 head/tail instability that might occur in the neighborhood of the transition energy (≈ 8 GeV) while the sign of the chromaticity is being reversed. A broad band system using 50 ohm traveling wave deflectors, commercially available power amplifiers, and either analogue or digital signal processing is described.

I. Introduction

At present, damping of the resistive wall instability in the AGS is accomplished with a narrow band system using magnetic deflection provided by two turn coils in both planes.[1] Essentially, only the lowest frequency $(9-Q)f_o$, of the signal spectrum of the coherent oscillations of the twelve bunches is fed back to control the instability. In order to control injection errors with some bunches already in the machine, as will be the case with the Booster operation, or to control head/tail instabilities of individual bunches, a system that detects the oscillations of an inidividual bunch and corrects only that bunch is required. Such a system will, of course, also damp the coupled bunch instability that arises from the long-range wakefield part of the resistive wall impedance.

Injection errors if uncorrected will result in phase space dilution. In addition, the finite oscillation amplitudes will couple through the resistive wall impedance to the other bunches in the machine, thus possibly stimulating coherent growth. Bunch-to-bunch feedback is necessary to simultaneously control these effects. With the higher intensities expected when the Booster is used as an injector, i.e., 3-4 x 10^{13}/pulse, head/tail instabilities arising from short-range wake field impedances are to be expected in the AGS. In principle, these can be controlled by making the chromaticity ($\Delta Q/Q/\Delta p/p = \xi$) negative below the transition energy and positive above.[2] This will be accomplished by programming the two sets of twelve sextupoles in the AGS. However, in the region around transition where the sign of the chromaticity has to be reversed, the correction will not be ideal and one might expect individual bunches to become unstable. Again, a bunch-to-bunch feedback system will provide some degree of stabilization.

II. Damping Rate Requirements

Damping is obtained by measuring y the position of the center of charge of the beam and changing y' by means of a kicker whose deflection is ~ Δy, the deviation from the equilibrium orbit. One must have $n\pi/2$ (n odd) betatron wavelengths between the pickup and kicker to produce pure damping. We define

$$\varepsilon = \sqrt{\frac{\beta_k}{\beta_p}} \; \frac{\Delta p/p}{\Delta y/\beta_p}$$

which is a measure of the open loop gain of the feedback system. Here β_k is the beta function at the kicker, β_p at the pickup, and $\Delta p/p$ the realtive transverse momentum due to the kicker ($\Delta y' = \Delta p/p$). For ideal damping $a = a_o \exp(-\varepsilon N/2) = a_o \exp(-\varepsilon f_o t/2)$ where N is the number of revolutions and f_o the rotation frequency. Here we have assumed initial angle and position errors Δy_o, $\Delta y'_o$ such that $a_o = \sqrt{\Delta y_o^2 + (\Delta y'_o \beta)^2}$. Then $\Delta y = a_o \sin Q\,\omega_o t$ and the damping rate is $\varepsilon f_o/2$. The damping rate required to minimize dilution due to injection errors depends upon the betatron tune spread in the beam. This will be discussed later.

We consider first the damping rate required to suppress the resistive wall instability on a 1 GeV flat top. The last measurement of the growth rate in the vertical plane[1] gave a maximum value of 500 sec^{-1} at a $\gamma = 1.65$ and an intensity > 6×10^{12} but with $\chi \cong \pi$. Here $\chi = \xi \omega_o Q \tau_\ell/\eta$ is the betatron phase shift from head to tail along a bunch whose length is τ_ℓ. $\xi = (\Delta Q/Q/\Delta p/p)$ is the chromaticity and Δp is the half momentum spread in a bunch. With $\gamma = 2.06$ but $\chi \approx 0^+$ and at five times the intensity, i.e. > 3×10^{13}, we calculate a growth rate of $2.3 \times 5 \times 500\ sec^{-1} \times 0.8 = 4600\ sec^{-1}$. This estimate is based on the analysis of Sacherer[3] and includes the contributions from the lines at $(-9 + Q)\omega_o$, $(-21 + Q)\omega_o$, $(3 + Q)\omega_o$, $(-33 + Q)\omega_o$, and $(15 + Q)\omega_o$ present in the spectrum of the coherent signal due to twelve equal bunches oscillation in the n = 1, m = 0 coupled bunch mode. We have assumed a Q = 8.75 and a bunch length $\cong$ one-half the bucket width and note that for $\chi = \pi$, the rate would be less by a factor of $\cong 1/2.3$ or 2000 sec^{-1}. For the n = 1, m = 1 mode, the growth rate at $\chi = 0$ will be less than for $\chi = \pi$ but the latter is still substantially less than for m = 0 and can be easily suppressed provided the system bandwidth is at least 15 MHz. The growth rates for n > 1, where $(2\pi n/M)$ is the phase shift from one bunch to the next bunch (M bunches $1 \geq n \geq M$) will also be smaller and hence damped if the n = 1 mode is damped.

In principle, the damping rate need only be greater than the growth rate but since the injection process will always result in a group of bunches having a finite initial amplitude or angle error, additional damping strength must be provided. Let us assume $\varepsilon = 0.023$, then since $f_o = 325$ kc at 1 GeV in the AGS, we can immediately write down the damping rate as $(325 \times 10^3 \text{ sec}^{-1} \times 0.023) \div 2 = 3737 \text{ sec}^{-1}$ for ideal feedback. The actual decay rate for injection errors (assuming equal damping rates for all $n = M$ modes) would be $(\chi = \pi)$ 1737 sec^{-1} for the last group of three Booster bunches. This assumes that $a_o < a_{max}$ where

$$a_{max} = \frac{\beta_p}{\varepsilon} \left(\frac{\Delta p}{p}\right)_{max}$$

with $(\Delta p/p)_{max}$ being the maximum linear value of $\Delta y'$ that the system can deliver. Here we have taken $\beta_p = \beta_k$ so that for $a(\Delta p/p)_{max} = 2.3 \times 10^{-6}$, i.e. $\Delta y' = 2.3 \times 10^{-6}$ rad, we find $a_{max} = 1.5$ mm at a $\beta = 15$ meters. For a_o larger than this the damping rate decreases and one can in principle find the value that leads to antidamping if the non-linear characteristics of the system (primarily the final amplifers that drive the deflectors) are known.[4]

We see, therefore, how the presence of injection errors can affect some of the parameters of the damping system. Let us return to the question of phase space dilution caused by these errors. Computer simulations have been carried out[5] for the case where the initial amplitude and phase distributions of 10^3 particles are Gaussian as well as the momentum and hence tune distribution. It was found that if $\varepsilon/\sigma_Q > 40$ then very little dilution occurs. Here σ_Q is the rms value of the tune distribution and all distributions were cutoff at 3.5 σ in the simulation. It was also shown that for a Gaussian distribution in tune (due to momentum spread) and equal initial position errors the coherent amplitude of the center of charge decays like a Gaussian. The time to decay to 1/e is called τ_c or the coherence time where $\tau_c = \sqrt{2}/\omega_o \sigma_Q$. Hence, if $\tau_d = 2/\epsilon f_o$ is the damping time, then $\tau_d \leq 0.22\ \tau_c$. is an equivalent requirement for minimal dilution. Now although the initial transverse phase space distribution may be Gaussian, the momentum distribution will most likely approach a parabolic one. Hence, the tune spread will also be parabolic with some half width at the base ΔQ. It can be shown[6] that the undamped coherent amplitude will decay as

$$\frac{3}{x^2}\left[\frac{\sin x}{x} - \cos x\right]$$

where $x = \omega_o t\ \Delta Q$ and that it will have a T_c (i.e., decay to 1/e of the initial amplitude) given by

$$T_c \cong \frac{0.92\ \pi}{\omega_o\ \Delta Q} = \frac{1.29}{\omega_o\ \sigma^*}$$

with $\sigma^* = \Delta Q/\sqrt{5}$ being the rms value for a parabolic distribution. Hence, in the absence of a simulation of damping in this case we assume that a $\tau_d \leq 0.2\ T_c$ would be required to minimize dilution. We should note that these results ignore the mixing due to synchrotron oscillations and hence are strictly valid only if all the times involved are short compared to a phase oscillation period.

Again, we take $\epsilon = 0.023$, $\beta_k = \beta_p = \bar{\beta} = R/Q = \cong 15$ meters in the AGS and consider injection at 1 BeV where p = 1.696 BeV/c. Taking $V_{rf} = 166.5$ kV so that the phase oscillation frequency is 2.3 kc and assuming a bunch area of $\cong$ 1 eV sec or a bunch length of 174°, we find a $\Delta p/p$ of 3.56×10^{-3} for the half momentum spread of the bunch. Then for the ideal damping rate in the absence of wall impedance effects, i.e. 3737 $\sec^{-1}$, we find $\Delta Q \leq 10^{-3}$ if dilution is to be minimal. This in turn means that $\xi = \Delta Q/Q/ \Delta p/p \cong 0.032$ and that $\chi \cong 0.32$, but for this low value of χ we expect that the resistive wall instability growth rate would be greater than the damping rate! Thus, we see that some dilution is unavoidable unless we were to make ε extremely large and hence also $(\Delta p/p)_{max}$ transverse very large.

Now the expected emittance from the Booster is less than 30 π μrad meters[7] at intensities of $1\text{-}2 \times 10^{13}$ protons in three bunches. The present AGS emittance at 1.5×10^{13} in twelve bunches is 80-100 π at full energy. Thus, dilution of a factor of two would result in no increase in aperture utilization. Tighter control of the operating point during acceleration, i.e. the betatron tune and chromaticity, should minimize any further dilution due to field non-linearities. In any event, if we assume a normalized emittance of 30 π μrad meters and a Gaussian distribution initially for 95% of the beam, then at a $\bar{\beta} = 15$ meters ($\beta\gamma = 1.8$) $a = \sqrt{\dfrac{30 \times 15}{1.8}} = 1.58$ cm or $\sigma = 6.45$ mm. It can be shown[8] that if σ is the rms width of a distribution and the beam is displaced an amount D then the new rms width is given by

$$\sigma_1^2 = \sigma_o^2 + \frac{D^2}{2}$$

The new distribution, asuming the resulting coherent oscillations are allowed to smear out, is not Gaussian and would have to be calculated by simulation in order to find the resulting dilution. However, if D = 1.5 mm, i.e. the a_{max} obtained above, then $\sigma_1^2 = 6.45^2 + 2.25/2 = 6.54^2$ which would indicate relatively small dilution. Also, one can easily calculate the area swept out in phase space by a beam of width 2.45 σ that is displaced by 0.233 σ at a β_{max} or β_{min} where $\alpha = 0$ (α, β, γ being the Courant-Snyder parameters). One finds that the total area is 1.2 times the initial area, again showing that the dilution resulting from such an injection error would be quite tolerable.

We conclude, therefore, that the required damping rate at 1 GeV should be about 4,000 sec^{-1} in both planes and that the machine chromaticity should be $\cong 0.3$ in order that the resistive wall instability be adequately suppressed. We will accept whatever dilution may result from injection errors, but require that the maximum excursion at a β average location be around 1.5 mm.

III. System Hardware

A. Deflection Components

In order to obtain the bandwidth required for bunch-to-bunch feedback, we propose to use 50 Ω traveling wave deflectors. These will be in the form of strip lines running along a vacuum chamber above and below and on both sides of the centerline. They will be fed by 50 Ω output impedance power amplifiers at the downstream end and terminated in 50 Ω loads at the upstream end. Now the force on a particle traveling along the center of this chamber due to a TEM wave moving in the opposite direction is $F = e \, \| \vec{E} + \beta c \vec{B} \|$ where βc is the particle velocity and E, B are the fields due to current I flowing in the strip line of impedance Z_c. We assume $\beta \approx 1$ so that $F = 2\ ecB$ (at 1 GeV this is in error by less than 7%) and following the analysis of J. Pellegrin[9] write this as $F = 2e \sqrt{Z_o P}\ k$. Here Z_o is the impedance of free space (377.5 Ω), P is the peak power at a frequency ω and $k(M^{-1})$ is a figure of merit of the structure. If $\emptyset$ is the angular width of the strip line, a the chamber radius and b the distance from the strip line to the center of the chamber, then it can be shown that

$$k = \frac{1}{\pi b} \sqrt{\frac{Z_o}{Z_c}} \left(1 - \frac{b^2}{a^2}\right) \frac{\sin \phi}{\phi}$$

If we take $a = 7.5$ cm, $b = 0.5$ cm, $\phi = 30°$, then $Z_c \cong (Z_o/\phi)\ln(a/b) = 50\ \Omega$ and $k = 1.68\ (M^{-1})$. Here the term $(1-b^2/a^2)$ represents the reduction in field at the center due to image currents in the vacuum chamber. Next, let us calculate the transverse momentum gained by a particle as it passes the deflector driven at a frequency ω. We put

$$F(x,t) = Ae^{i[\omega t - 2\pi x/\lambda]};\ x = -ct + \ell$$

(i.e. at $t = 0$ $F(\ell,0) = A \cos(\omega\ell/c)$), where ℓ is the length of the strip line and compute

$$\Delta p = \int_0^{\ell/c} F\, dt = A \int_0^{\ell/c} e^{i(\omega t - \frac{2\pi ct}{\lambda} - \frac{2\pi\ell}{\lambda})} dt = \frac{A}{\omega} \sin \frac{\omega\ell}{c}$$

Thus, as $\omega \to 0$ $\Delta p \to A\ \ell/c = A\ \Delta t$ while for ω such that $\ell = \lambda/2$, $\Delta p = 0$ and for $\ell \approx \lambda/4$ $\Delta p = 2A\Delta t/\pi$. While for $\omega > \pi c/\ell$ the Δp can be of opposite sign. We see that ℓ will be limited by the required bandwidth if not by mechanical considerations. If we choose $\ell = 1.5$ M then the first zero is at 100 MHz and the net deflection is $2/\pi \cong 0.64$ of its low frequency value at 50 MHz. Using the expression above for $A = F(0,0) = 2e \sqrt{Z_o P}\ 1.68(M^{-1})$ and assuming $P = 400$ watts we find

$$\Delta p = \frac{A\ell}{c} \sin \theta/\theta = F(0,0) \frac{\ell}{c} \frac{\sin \theta}{\theta} = 2\ e \sqrt{400 \times 377}\ \frac{1.5 \times 1.68}{c} \frac{\sin \theta}{\theta} =$$

$$1.96 \frac{\text{keV}}{c} \frac{\sin \theta}{\theta} \text{ where } \theta = \omega\ell/c.$$

Therefore, at a 1 GeV injection energy where p = 1.69 BeV/c we obtain for push/pull excitation of a pair of deflectors a $\Delta p/p = 2.32 \times 10^{-6}$ ($\sin \theta/\theta$) or 2.2×10^{-6} at 16.67 MHz. This is, of course, the value of $(\Delta p/p)_{max}$ assumed for linear operation of the feedback loop in the preceeding section.

Now, we have also assumed that $Q_{x,y} \cong 8.75$ so that the pickups and deflectors could be located in the same 10' straight section of the AGS. This arrangement requires a one-turn delay between these elements so that the necessary odd multiple of a quarter betatron wavelength occur from displacement to deflection. We will discuss the delay question later, but here we are concerned with the low frequency requirement for the power amplifiers. Since f_o = 325 kc at 1 GeV, the lowest frequency line in the bunched beam spectrum for coherent transverse osicllations will be 0.25 f_o or 81.25 kc. Hence, the amplifier bandwidth must extend below this value to at least 40 kc. This makes the minimum requirement 200 watts of linear CW power between 40 kc and 15 MHz for each of four amplifiers.

B. Signal Delay System

If the feedback is to be analogue in character, then the required one-turn delay must be provided by coaxial cables of high quality. A binary switching system similar to those employed on the NAL Booster,[10] or the CERN Booster[11] will be necessary. At 1 GeV with $\beta = 0.875$, the one-turn delay is $T_i = 12/3.9 \times 10^6 = 3.0769$ μsec while the one-turn delay at maximum energy is $T_f = 12/4.457 \times 10^6 = 2.692$ μsec so that $\Delta T = 389.5$ nanoseconds. If the minimum delay including all the electronics is made to equal T_f, then a six-bit system employing seven cables whose lengths are $2^m T_o$, $0 \geq m \geq 6$ with $T_o = 6$ nanoseconds will do the job. By counting a reference oscillator and the rf accelerating frequency, one can generate a six-bit word that determines which cables are to be used at a given time.[11] In principle then with six nanosecond steps, one should be able to adjust the switching such that the delay error is $\leq \pm 3$ nsec over the required frequency swing. This means that at a frequency ω, the phase error due to the cable delay will be at most $\omega\delta T$ or at 15 MHz, $\pm 16.2°$.

However, if digital processing of the pickup signals is used, then it should be possible to use digital delay techniques similar to those employed in the NAL main ring[12] to insure the proper phase relation between the amplitude of the displacement and the amplitude of the kick. One would use a clock running at qf_{rf} where q is 20-25. Then the number of clock pulses required could be programmed with f_{rf}. The kick would be a square wave of duration greater than the bunch width so that an error $\delta T = 1/qf_{rf}$ would not affect the result. Digitization of the input would be at f_{rf} and the amplitude of the square wave would be controlled accordingly. A five-bit system would be the minimum necessary while eight bits should be more than adequate.

C. Signal Processing Circuits

For the pure analogue option, one must have two circuits between the pickup electrodes and the amplifiers. The first is usually called a closed orbit suppressor. Its need arises from the fact that in general the beam orbit error at the PUE location is usually not zero. Hence, when one takes the difference signal from a pair of electrodes, there is a component at f_{rf} and its harmonics that is independent of coherent oscillations about the closed orbit. Because the voltage gain required for strong damping can be quite large, i.e. 40-60 db, very small orbit errors can saturate the final amplifers. In some systems the residual error even after suppression is the determining factor in the power amplifier rating. Suppression is usually achieved by sensing the average orbit offset and feeding back this information to control the differential gain of the system. In the CERN Booster[11] rejections of 60 db or more have been achieved. One also can try to keep the orbit error small by magnetic corrections.[10]

The second circuit required is a filter to control the loop gain. For ideal damping, the overall system should introduce a pure delay so that the phase shift is linear over the required bandwidth. Thus, the filter should roll off the gain by 40 db or more, but with a linear phase characteristic. A unit similar to that used on the CERN Booster[11] would be satisfactory, i.e. a ninth order equi-ripple filter with a 3 db frequency of 15 MHz and $\geq$ 40 db attenuation at 45 MHz. In order for the filter to determine the loop characteristics, the bandwidth of all the other components shold be at least 50 MHz. This will then insure that when the phase shift deviates by more than $\pm$ 90° from linearity, the gain will be of the order of unity.

If digitization is used, then one can eliminate the closed orbit suppressor circuit and the filter. However, one then needs twelve separate band pass filters that allow only the $\|-9 + Q\| f_o$ frequency in the output, i.e. from $(0.05 - 0.45) f_o$. These would have to be tuned over a narrow range since f_o varies from 325 to 371 kc during acceleration from 1 GeV to 30 GeV. Such a filter is already in use in the new tune[6] measurement system. The outputs are digitized at the rotation frequency and stored sequentially in a "circular" memory.[12] As mentioned above, the memory output would be used to control the amplitude of a square wave generator operating at f_{rf} whose bandwidth is at least 50 MHz. In principle, the gain of this system would fall off as 1/f, i.e. 20 db/decade due to the frequency spectrum of an ideal square wave, with no phase shift. Again, in order to insure that only one circuit element determines the loop characteristics, the band pass of the power amplifiers should also be greater than 50 MHz. The effective damping rate for this type of feedback is calculated in the appendix.

Now a digital system would work only with bunched beams and hence in order to control the debunched beam for SEB operation, some elements of an analogue system would still be required. One would bypass the $\|-9+Q\|f_o$ filters and go directly to the power amplifiers through a wideband filter and a fixed delay cable, cable switching and closed orbit suppression not being necessary.

The choice of one system over the other will require more detailed study of the requirements in manpower and costs. For both systems, however, the power amplifiers must have quite wide bandwidths, i.e. at least 40 kc to 50 MHz or more. We note that the required power is ~ $1/k^2$ and hence is very sensitive to the ratio (b/a) for the strip line. Since the size of the injected bunched beam from the Booster will be considerably smaller than the present 200 MeV beam, it should be possible to place the strip line deflectors closer to the vacuum chamber centerline. This will increase k and reduce the peak power required for the same ε and $(\Delta p/p)_{max}$. If both horizontal and vertical plates are to be at the same location, then the effect of one set on the field from the other pair must be considered when they get closer together. Hence, optimization of the kicker design should also be undertaken (see the Appendix).

IV. Cost and Manpower Requirements

Since the exact type of system to be used remains to be determined, detailed cost and construction schedules are not possible. However, a rough overall estimate can be made. At present, power amplifiers in the 150-200 watt range with 10 kc-220 MHz bandwidth are about $10,000 each. Hence, we assume $50,000 for four, plus a spare. Based on estimates made for the CBA, we take $10,000 for the cost of deflectors, cables, and terminations. We further assume another $40,000 for the low-level electronics and hence arrive at a total of $100,000 for the entire system. This is half of the estimate given in the AGSII Task Force Report[13] but there, 1 kW amplifiers were assumed. Also based on a preliminary estimate made for a similar system to be used in the CBA, we project about 2 manyears of engineering and development, and 2 manyears for fabrication, testing, and installation.

Appendix

Damping Rate for Square Kicker Pulse

We again use the results of Sacherer[3] and write

$$\Delta\omega_m \sim \frac{i}{1+m} \sum \frac{Z_\perp(\omega_p)\, h_m(\omega_p-\omega_\xi)}{\sum_p h_m(\omega_p-\omega_\xi)}$$

where the growth rate $1/\tau = -I_m \Delta\omega$ and $Z_\perp$ is the transverse impedance at the frequency $\omega_p = (p + Q)\omega_o$, $-\infty < p < \infty$ for independent bunch motion or $p = n + kM$, $-\infty < k < \infty$ for M bunches oscillating in the coupled bunch mode n such that $2\pi \| n \| /M$ is phase shift from bunch to bunch. Here $h_M(\omega) = \| \tilde{p}_m(\omega) \|^2$ with $\tilde{p}_m(\omega)$ being the Fourier transform of the within the bunch oscillation $p_m(t)$. $\omega_\xi = \chi/\tau_\ell$ as defined earlier. We take the $m = 0$ mode where

$$\tilde{p}_o(\omega) = \frac{2 \cos(\tau_\ell \omega/2)}{1 - (\tau_\ell \omega/\pi)^2}$$

and assume that the damping rate can be calculated in the same way if we know the equivalent $Z_\perp(\omega_p)$ introduced by the feedback loop. For $Z_\perp$ we use the expression due to Hereward given in Reference 3,

$$Z_\perp(\omega) = j \frac{\int_o^{2\pi R} [E+vxB]_\perp \, ds}{\beta I \tilde{\Delta}}$$

Here I is the dc component of the beam current and Δ the oscillation amplitude. In general,

$$\Delta \sim p_m(t)e^{j\omega\xi t + j2\pi kQ}$$

where k is an integer that counts the revolutions of the bunch. That is ΔI represents the oscillating part of the beam current that has a discrete spectrum given by the frequencies ωp. The center of this spectrum is shifted by ω_ξ and for the case of m = 0, the first zeros of the unshifted spectrum are at $\omega = \pm 3\pi/\tau_\ell$.

The term in brackets in the integral is just the transverse field at a frequency ω seen by the beam. Thus, the amplitude of the kick at the frequencies ω_p divided by amplitude of the component of the oscillating beam current at the same frequency is a measure of the transverse impedance introduced by the feedback. For a square wave at f_{rf} modulated by the signals arising from the coupled bunch mode n = 1, one has the product (for 8.5 < Q < 9)

$$\text{Cos }(9-Q)\omega_o t \left[1 + \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{-1^k}{(2k+1)} \text{Cos }(2k+1)\omega_{rf} t\right]$$

whose spectrum contains the frequencies $(9-Q)\omega_o$ and $(2k+1)f_{rf} \pm (9-Q)f_o$, k = 0, 1, 2, ... For mode n = 1 and M = 12, the bunch spectrum will contain the frequencies f_p or $\mp kf_{rf} - (9-Q)f_o$, k = 0, 1, 2, etc. f_{rf} = 12 f_o. The negative frequencies represent slow waves that contribute to growth while a positive frequency contributes damping. That is in the sum for $\Delta\omega_m$, $Z_\perp$ of a negative frequency is negative etc. In the case of feedback, the impedance is positive for all frequencies such that the phase difference from ideal damping is < ± 90°. We note that for symmetric square wave damping there ar no contributions for the lines around even multiples of f_{rf}.

For a single bunch oscillating independently and square wave damping, one replaces ω_{rf} by ω_o in the above expressions. We have caclulated the effective damping rate for the M = 12, n = 1 coupled bunch mode assuming the bunch width is one half the bucket width and square wave feedback. We ignore the sin θ/θ factor in the kicker response and consider only those lines k = 0, 1 for two cases $\chi = 0$, $\chi = \pi$.

For $\chi = 0$, we obtain 0.81 and for $\chi = \pi$, 0.7 relative to one for an ideal analogue system with a flat response over the same spectrum. Thus, the gain of the square wave system, that is the amplitude of the deflection pulse for a given amplitude of the filtered $(9-Q)f_o$ signal, would have to be increased to provide the same damping as an ideal analogue system with a given gain at $(9-Q)f_o$. In principle, for a bunched beam and $\chi \leq \pi$, the m = 0 coupled bunch mode could be damped with a system whose bandwidth was $\leq 3/2\ \tau_{\ell}$. For frequencies greater than this, as long as the gain decreased monotonically, the phase difference from ideal damping could be greater than 90° since the negative contribution of these lines to the overall damping rate would be quite small. One could increase the overall gain slightly to make up for any reduction due to these contributions. The limit would, of course, depend upon noise, residual closed orbit errors if an analogue system is used, and saturation due to injection errors if they are present.

Kicker Optimization

As mentioned above, we wish to increase the ratio a/b where a is the vacuum chamber radius and b is the radius of the strip line. In order to keep the characteristic impedance constant at 50 Ω, one then has to also increase ϕ the angle subtended by the strip line. The simple expression for Z_c is no longer accurate enough and detailed field calculations are necessary. Using the program POISSON the NSLS Group[14] has calculated Z_c for various geometries as well as the electric field pattern for a given potential on the strip line. Z_c is obtained by putting a potential on a conductor at the center, calculating the field lines, and then integrating over a path enclosing the strip line.

We can scale the results of one such calculation to obtain an estimate of how much larger K can be made for a reasonable value of a/b. In particular for a/b = 1.457 with $\phi = 110°$, $Z_c \cong 50$ and if a = 10.2 cm, the field at the center is 8.2 volts/meter for one volt on the strip line. Now we take a = 7.5 cm so that b = 5.15 cm, ϕ remains the same as does Z_c but the field is now 8.2 x (10.2/7.5) = 11.15 volts/meter for one volt on the plate. Thus for $V = \sqrt{400 \times 50} = 141.4$ volts and $\ell = 1.5$ M, we obtain 2.365 keV/c from the electric field or 4.73 keV/c for the total Δp for one plate (assuming $\sin\theta/\theta = 1$). This is 2.41 times greater than the 1.96 keV/c obtained earlier. Of course, since $\phi = 110°$, only one set of deflectors can be located in a given space. Hence, for a given total length, the gain is only 20%. However, if we assume that each unit can be one meter in length, then they could be placed in the upstream and downstream ends of a ten-foot straight section or located separately in five-foot straight sections. For $\ell = 1$ M we would have 3.15 keV/c per plate for 400 watts peak power.

Now let us assume further that the deflecting unit is at a β_{min} = 10.5 M and that the available peak power is 300 watts, then Δp = 3.15 keV/c x 0.866 x 0.837 = 2.285 keV/c. At a β_{min} a beam of 60 $\pi \times 10^{-6}$ rad M normalized emittance, would have a size at 1 GeV/c of a = $\sqrt{60 \times 10.5/1.8}$ = 1.87 cm. Hence, at such a location there would be 5.15 - 1.87 = 3.28 cm half aperture available for closed orbit errors, energy spread, etc. Even at a β = 15 M there would be 2.9 cm available. This amount of space should be adequate for the assumed beam size (including a factor of two dilution at injection) and thus a choice of parameters close to those give here would provide additional margin for either power, damping rate, or injection error amplitude requirements.

References

1. E. Raka. The Transverse Coherence Dampers, AGS Technical Note 118, March 1976.

2. J. Gareyte. Head Tail Instabilities in the CERN PS, Proceedings of the IX International Conf. on High Energy Accelerators, SLAC, 1974, pg. 341.

3. F. Sacherer. Transverse Bunched Beam Instabilities, Theory, Ibid, pg. 347.

4. K. Wille. Calculation of a Nonlinear Transverse Feedback System, Internal Report DESYPET-79/01.

5. E. Keil, W. Schnell, P. Strolin. Feedback Damping of Horizontal Beam Transfer Errors, CERN 69-27, ISR Division.

6. E. Raka. On Line Tune Measurements in the AGS, AGS Technical Note 185, December 1982.

7. Y.Y. Lee, private communication.

8. H. Koziol. Beam Blow-up by Repetitive Pulsing of the Q-Kicker, CERN MPS/BR Note/79-15.

9. J-L Pellegrin. Design of an Electrode System for Beam Transverse Excitation, SLAC SPEAR-190 PEP120, August 1975.

10. C. Ankenbrandt, et al. Suppression of Transverse Instabilities by Fast Feedback in the Fermilab Booster, IEEE Trans. Nucl. Sci., NS-24, 1698-1700 (1977).

11. C. Carter, et al. The Transverse Feedback System for the CERN PS Booster, IEEE Trans. Nucl. Sci., NS-28, 2270 (1981).

12. R. Biner, G. Tool. A Variable Length Time Delay for 50 MHz Sampled Analogue Signals, IEEE Trans. Nucl. Sci., NS-22, 1509 (1975).

13. Report of the AGSII Task Force, BNL, February 15, 1984, pg. 210.

14. J. Galayda, private communication.