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Extension to de Broglie formula of Quantum Mechanics

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ABSTRACT:

Planck-Einstein's $E = \hbar\omega$ and de Broglie's $\mathbf{p} = \hbar\mathbf{k}$ are two fundamental equations of quantum physics. But vacuum Cherenkov radiation (VCR) is convincing evidence that the momentum of a single photon should be $\mathbf{p} = \hbar\boldsymbol{\beta}$ where the phase constant $\boldsymbol{\beta}$ takes the place of the wave vector $\mathbf{k}$. Apart from the momentum, the operator $-i\hbar\nabla$ is associated to $m_0 c$. The acousto-optic effect and anomalous VCR can be applied to test this conjecture.

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1. INTRODUCTION

The key of quantum mechanics is the Hamiltonian in terms of the momentum operator $\hat{p} \to -i\hbar\nabla$ which associated with de Broglie formula $p = \hbar k$. There are lots of experimental verifications for $p = \hbar k$ [1]

Traveling waves	**Stationary waves**
Compton effect, 1923	Density of states
Davisson and Germer 1927, Thomson 1928	Bohr-Sommerfeld quantization
Estermann and Stern, 1930	

In de Broglie's theory, the phase velocity is $V_p = \dfrac{\omega}{k}$ and group velocity is $V_g = \dfrac{d\omega}{dk}$. Nevertheless, they are $V_p = \dfrac{\omega}{\beta}$ and $V_g = \dfrac{d\omega}{d\beta}$ in microwave electronics where the phase constant β is a component of k and the phase is changed merely in this direction. The wave with a phase velocity $\dfrac{\omega}{\beta} < c$ is called slow wave [2][3] and $\dfrac{\omega}{\beta} > c$ [4][5] is fast wave. Concretely,

	Dispersion relation	**Phase velocity**
Free space	$\omega = kc$	$\dfrac{\omega}{k} = c$
Slow wave	$\omega = kc = \sqrt{\beta^2 c^2 - \tau^2 c^2} < \beta c$ (τ -transverse eigenvalue) [2][3]	$\dfrac{\omega}{\beta} < c$
Fast wave	$\omega = kc = \sqrt{\beta^2 c^2 + \omega_c^2} > \beta c$ (ω_c -cutoff frequency) [4][5]	$\dfrac{\omega}{\beta} > c$

Then whether the momentum should be $p = \hbar\beta$ under the circumstances?

2. CASIMIR EFFECT

The ratio of the Poynting vector(energy flux density) $\mathbf{S}=\mathbf{E}\times\mathbf{H}$ to momentum density $\mathbf{g}=\varepsilon_0\mathbf{E}\times\mathbf{B}$ of an electromagnetic field in vacuum is a constant [6][7]

.
$$\frac{\mathbf{S}}{\mathbf{g}}=\frac{\mathbf{E}\times\mathbf{H}}{\varepsilon_0\mathbf{E}\times\mathbf{B}}=\frac{1}{\varepsilon_0\mu_0}=c^2 \tag{1}$$

An EM field consists of photons so the relation between the energy and momentum of a single photon should be

$$\frac{\frac{\overline{w}}{N}}{\frac{\overline{g}}{N}}=\frac{\overline{w}}{\overline{g}}=\frac{\frac{\overline{S}}{\overline{g}}}{\frac{\overline{S}}{\overline{w}}}=\frac{c^2}{\frac{\overline{S}}{\overline{w}}}=\frac{c^2}{V_e} \tag{2}$$

where w is the energy density, N is the number density of photons and $V_e=\frac{\overline{S}}{\overline{w}}$ is the energy velocity of this field. In a hollow waveguide, the energy velocity is $c\sqrt{1-\frac{\omega_c^2}{\omega^2}}<c$ [8] and the ratio of energy to momentum equals

$$\frac{c^2}{V_e}=\frac{c^2}{c\sqrt{1-\frac{\omega_c^2}{\omega^2}}}=\frac{c}{\sqrt{1-\frac{\omega_c^2}{\omega^2}}}=V_p=\frac{\omega}{\beta}=\frac{\hbar\omega}{\hbar\beta} \tag{3}$$

In the Casimir effect [9]~[12], two metal plates can be regarded as a one-dimensional rectangular waveguide and experiments demonstrate the Planck-Einstein relation $E=\hbar\omega$ in this structure. Consequently, the momentum of a photon in the waveguide should be $\hbar\beta$ rather than $\hbar k$.

3. NORMAL VCR

Cherenkov radiation is generally emitted by charged particles pass through the dielectric medium at a speed V faster than the phase velocity $\frac{1}{\sqrt{\varepsilon\mu}}<c$. Owing to wave-particle duality, this effect can also be deduced from quantum mechanics [13]. Moreover, it occurs in vacuum ($\frac{\omega}{k}=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$) provided that the phase velocity is decreased by slow wave systems [14] and we use the quantum theory to study too. Consider charged particles moving near the surface of a periodic structure [2][3] (Fig.1)

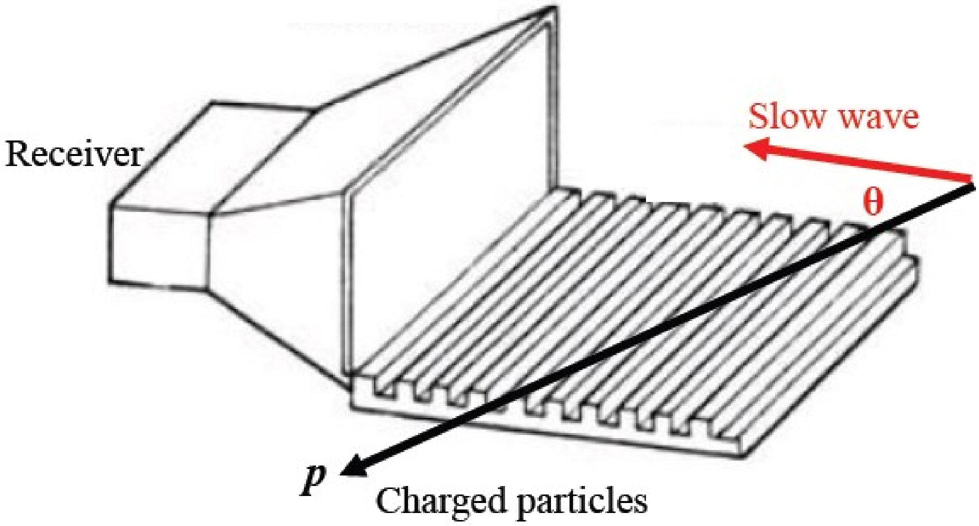

Figure 1. Corrugated planar conductor

Assume the momentum of a photon is $\hbar\mathbf{k}$ (Fig.2),

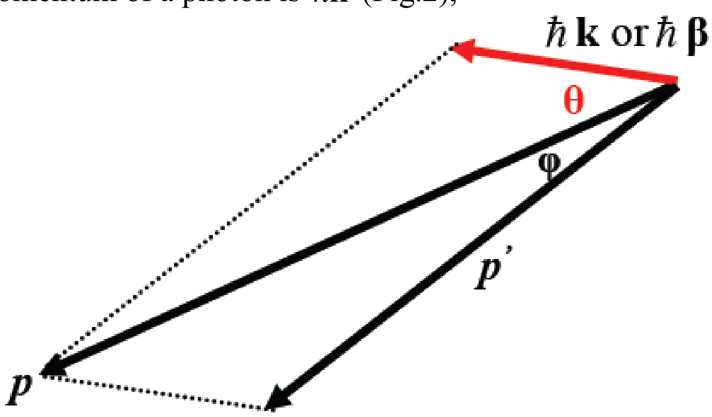

Figure 2. Parallelogram law

Momentum conservation (Fig.2):

$$\mathbf{p}-\hbar\mathbf{k}=\mathbf{p}' \tag{4}$$

$$p^2-2p\,\hbar k\cos\theta+\hbar^2k^2=p'^2 \tag{5}$$

$$\cos\theta=\frac{p^2+\hbar^2k^2-p'^2}{2p\,\hbar k} \tag{6}$$

Energy conservation:

$$E-\hbar\omega=E' \tag{7}$$

The energy of a charged particle is $\sqrt{p^2c^2+m_0^2c^4}$,

$$\sqrt{p^2c^2+m_0^2c^4}-\hbar\omega=\sqrt{p'^2c^2+m_0^2c^4} \tag{8}$$

$$p^2c^2+m_0^2c^4-2\hbar\omega E+\hbar^2\omega^2=p'^2c^2+m_0^2c^4 \tag{9}$$

$$p^2-\frac{2\hbar\omega E}{c^2}+\frac{\hbar^2\omega^2}{c^2}=p'^2 \tag{10}$$

Substituting Eq.(10) into Eq.(6),

$$\cos\theta=\frac{\frac{2\hbar\omega E}{c^2}-\frac{\hbar^2\omega^2}{c^2}+\hbar^2k^2}{2p\,\hbar k}=\frac{\frac{2\hbar\omega E}{c^2}}{2p\hbar k}=\frac{\frac{\omega}{k}}{\frac{pc^2}{E}} \tag{11}$$

Since $E=\sqrt{p^2c^2+m_0^2c^4}=\dfrac{m_0c^2}{\sqrt{1-\dfrac{V^2}{c^2}}}$ and $p=\dfrac{m_0V}{\sqrt{1-\dfrac{V^2}{c^2}}}$ of the charged particle,

$$\frac{pc^2}{E}=V \tag{12}$$

$$\cos\theta=\frac{\frac{\omega}{k}}{V}=\frac{c}{V}>1 \tag{13}$$

VCR is forbidden because of $\dfrac{\omega}{k}\equiv c$ in free space, fast wave structures [4][5] and slow wave systems [2][3] . However, people utilize slow wave devices in vacuum to generate VCR [14]. To explain this phenomenon, the momentum of a photon must be $\hbar\boldsymbol{\beta}$. Hence

$$\mathbf{p}-\hbar\boldsymbol{\beta}=\mathbf{p}' \tag{14}$$

$$\cos\theta=\frac{p^2+\hbar^2\beta^2-p'^2}{2p\,\hbar\beta} \tag{15}$$

$$\sqrt{p^2c^2+m_0^2c^4}-\hbar\omega=\sqrt{p'^2c^2+m_0^2c^4} \tag{8}$$

$$\cos\theta=\frac{\frac{2\hbar\omega E}{c^2}-\frac{\hbar^2\omega^2}{c^2}+\hbar^2\beta^2}{2p\,\hbar\beta}=\frac{\frac{\omega}{\beta}}{\frac{pc^2}{E}}+\frac{\hbar^2(\beta^2-\frac{\omega^2}{c^2})}{2p\,\hbar\beta} \tag{16}$$

The rightmost term including a reduced Planck constant $\hbar$ has no counter part in classical physics. In the main, it has negligible effect on the trajectory because $\hbar\beta$ of a photon is much less than the momentum p of the charged particle, i.e.

$$\cos\theta\approx\frac{\frac{\omega}{\beta}}{\frac{pc^2}{E}}=\frac{\frac{\omega}{\beta}}{V} \tag{17}$$

$$\cos\varphi=\frac{p^2+p'^2-\hbar^2\beta^2}{2pp'}\approx\frac{p^2+p'^2}{2pp'}\approx 1,\quad \varphi\approx 0 \qquad \text{(Fig.2)} \tag{18}$$

Now we interpret why VCR merely exists in a slow wave device under conditions of $\dfrac{\omega}{\beta}<V<c$ and cannot be found in free space or fast-wave systems ($\dfrac{\omega}{\beta}>c$). Besides mentioned periodic structures, VCR can be excited by a smooth surface such as the dielectric waveguide [15] and Sommerfeld wire [16]~[18]. They're useful to obtain THz waves.

4. ANOMALOUS VCR

In addition to normal VCR observed long ago[14], there should be anomalous VCR when the quantum correction plays a major role. For instance, the phase constant of a corrugated plane surface is [2][3]

$$\beta=\frac{\omega}{c}\sqrt{1+\tan^2\frac{\omega}{c}d}=\frac{\omega}{c\cos\frac{\omega}{c}d}>\frac{\omega}{c} \qquad (d\ \text{ corrugation depth}) \tag{19}$$

In event of $\dfrac{\omega}{c}d\to\dfrac{\pi}{2}$, $\hbar\beta>>\dfrac{\hbar\omega}{c}$ is of the same order of magnitude of p and Eq.(16) is now

$$\cos\theta \approx \frac{\frac{\omega}{\beta}}{V} + \frac{\hbar\beta}{2p} \tag{20}$$

Meanwhile, the phase velocity $\frac{\omega}{\beta}$ is very low and $\frac{\frac{\omega}{\beta}}{V} \approx 0$. Thus,

$$\cos\theta \approx \frac{\hbar\beta}{2p} \tag{21}$$

Namely radiation whose angular frequency is about $\frac{\pi c}{2d}$ has an enormous phase constant $\beta = \frac{2p\cos\theta}{\hbar} \to \infty$ and

$$\cos\varphi = \frac{p^2 + p'^2 - \hbar^2\beta^2}{2pp'} \approx \frac{p^2 + p'^2 - (2p\cos\theta)^2}{2pp'} \tag{22}$$

On the other hand, it is $p' \approx p$ due to Eq.(8) and $\hbar\omega \approx \hbar\frac{\pi c}{2d} << \sqrt{p^2c^2 + m_0^2c^4}$. Accordingly,

$$\cos\varphi \approx \frac{2p^2 - (2p\cos\theta)^2}{2p^2} = 1 - 2\cos^2\theta = -\cos 2\theta \tag{23}$$

$$\text{i.e.} \quad \varphi = \pi - 2\theta \tag{24}$$

That is to say, a small portion of charged particles are deflected through a large angle $\varphi = \pi - 2\theta$ (Fig.3)

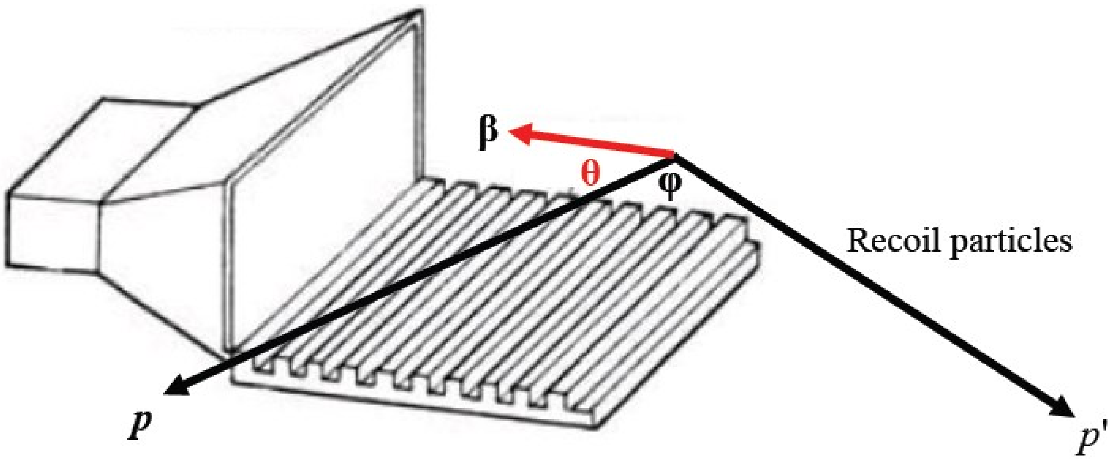

Figure 3. Anomalous VCR

In particular, the particle will be defected backwards ($\varphi \approx \pi$) if $\theta \approx 0$. Anomalous radiation is a purely quantum effect and cannot be predicted by classical electrodynamics.

5. ACOUSTO-OPTIC INTERACTION

The light can be diffracted by sound waves through acousto-optic devices [19] (Fig.4).

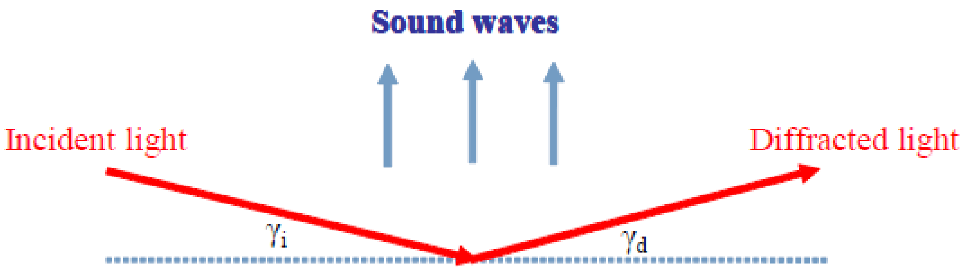

Figure 4. Bragg Diffraction

The momentum P_s of a phonon is much less that of the incident photon p_i and diffracted photon p_d (Fig.5).

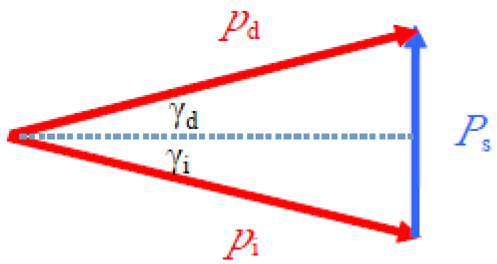

Figure 5. Momentum conservation

$$P_s << p_i \tag{25}$$

$$p_i \approx p_d \tag{26}$$

$$\text{Bragg angle} \quad \sin\gamma_i = \frac{P_s}{2p_i} \tag{27}$$

In unbonuded acousto-optic materials,

$$\sin\gamma_i = \frac{P_s}{2p_i} = \frac{\hbar K_s}{2\hbar k_i} = \frac{K_s}{2k_i} \qquad [19] \tag{28}$$

As bounded materials in different structures, there may be $P_s = \hbar\beta_s \neq \hbar K_s$ and $p_i = \hbar\beta_i \neq \hbar k_i$ for sound waves and EM waves.

6. PHYSICAL MEANING

The rectangular waveguide [4][5] can be regarded as an infinite square well [20] of photons and the wave function is

$$\Psi_{(x,y,z)} = 0 \qquad (|x| \geq \frac{a}{2},\ |y| \geq \frac{b}{2}) \tag{29}$$

Stationary wave equation:

$$\nabla^2\Psi + k^2\Psi = 0 \tag{30}$$

Waves propagate along the z -axis,

$$\Psi_{(x,y,z)} = \psi_{(x,y)} e^{i\beta z} \qquad (i = \sqrt{-1}\ ,\ \beta = k_z\ ) \tag{31}$$

Separation of variables

$$\frac{d^2X}{dx^2} + k_x^2 X = 0 \tag{32}$$

$$\frac{d^2}{dy^2} Y + k_y^2 Y = 0 \tag{33}$$

On account of the boundary condition (29),

$$k_x = l_x \frac{\pi}{a} \qquad (l_x = 0,1,2,3......) \tag{34}$$

$$k_y = l_y \frac{\pi}{b} \qquad (l_y = 0,1,2,3......) \tag{35}$$

$$k_x^2 + k_y^2 = \pi^2\left(\frac{l_x^2}{a^2} + \frac{l_y^2}{b^2}\right) = \frac{\omega_c^2}{c^2} \qquad [4][5] \tag{36}$$

Transverse components of the momentum of a single photon are not

$$p_x = \hbar k_x \tag{37}$$

$$p_y = \hbar k_y \tag{38}$$

$$\sqrt{p_x^2 + p_y^2} = \sqrt{\hbar^2\left(k_x^2 + k_y^2\right)} = \frac{\hbar\omega_c}{c} \tag{39}$$

otherwise the total momentum of this photon should be

$$\sqrt{p_z^2 + p_x^2 + p_y^2} = \sqrt{\hbar^2\beta^2 + \hbar^2\left(k_x^2 + k_y^2\right)} = \hbar\sqrt{\beta^2 + \frac{\omega_c^2}{c^2}} = \hbar k \tag{40}$$

Ratio of energy to momentum

$$\frac{\hbar\omega}{\hbar k} = \frac{\omega}{k} = c \tag{41}$$

Eqs.(37)~(39) are inconsistent with the result of classical electrodynamics that transverse components of the Poynting vector and momentum density are zero [21].To resolve this question, p_x and p_y have to be zero although k_x and k_y could be non-zero. Therefore,

$$\text{Wave number} \qquad k = \sqrt{\beta^2 + k_x^2 + k_y^2} = \sqrt{\beta^2 + \frac{\omega_c^2}{c^2}} = \frac{\omega}{c} > \beta \tag{42}$$

$$\text{Momentum} \qquad p = \sqrt{p_x^2 + p_y^2 + p_z^2} = \sqrt{\hbar^2\beta^2 + 0 + 0} = \hbar\beta < \hbar k \tag{43}$$

Ratio of energy to momentum

$$\frac{\hbar\omega}{\hbar\beta} = \frac{\omega}{\beta} = V_p > c \tag{44}$$

Operators $-i\hbar\frac{\partial}{\partial x}$ and $-i\hbar\frac{\partial}{\partial y}$ result in rest mass multiplied by c.

$$\hbar k_x = \hbar\, l_x \frac{\pi}{a} = m_{0x}c \tag{45}$$

$$\hbar k_y = \hbar\, l_y \frac{\pi}{b} = m_{0y}c \tag{46}$$

$$l_x \neq 0 \text{ and } l_y \neq 0 \qquad \text{—}TM \text{ modes} \quad [22] \tag{47}$$

$$l_x = 0,\ l_y \neq 0 \text{ or } l_x \neq 0,\ l_y = 0 \quad \text{—}TE \text{ modes} \quad [22] \tag{48}$$

$$\omega_c = \sqrt{k_x^2 + k_y^2}\,c = \pi c\sqrt{\frac{l_x^2}{a^2} + \frac{l_y^2}{b^2}} = \frac{m_0c^2}{\hbar} \qquad [23] \tag{49}$$

A non-zero rest mass of *TE* and *TM* wave is not surprising, if we think of longitudinal oscillations $\mathbf{B}_z \neq 0$ and $\mathbf{E}_z \neq 0$ in the direction of wave propagation. The quantity $m_0 = \frac{\hbar\omega_c}{c^2}$ derives from massless Maxwell's equations and the value is zero in an infinitely large waveguide ($a \to \infty$, $b \to \infty$) equivalent to free space. This has nothing to do with massive Proca's equations [24] whose rest mass of a photon is still nonzero even in free space and does not violate any fundamental physical law such as gauge invariance and Coulomb's inverse square law [25].

In sum, the dispersion relation of electromagnetic waves in a hollow waveguide

$$\omega^2 = \underbrace{\beta^2 c^2 + \omega_c^2}_{k^2 c^2} \tag{50}$$

corresponds to the relativistic mechanical equation

$$E^2 = p^2 c^2 + m_0^2 c^4 \tag{51}$$

Total energy

$$E = \hbar\omega \tag{52}$$

Momentum

$$p = \hbar\beta \tag{53}$$

Rest mass

$$m_0 = \frac{\hbar\omega_c}{c^2} \tag{54}$$

Mechanical velocity

$$V = c\sqrt{1-\frac{\omega_c^2}{\omega^2}} < c \tag{55}$$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{\hbar\omega_c}{\frac{\omega_c}{\omega}} = \hbar\omega \tag{56}$$

$$p = \frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{\frac{\hbar\omega_c}{c^2} c\sqrt{1-\frac{\omega_c^2}{\omega^2}}}{\frac{\omega_c}{\omega}} = \hbar\beta \tag{57}$$

Phase velocity

$$V_p = \frac{\omega}{\beta} = \frac{E}{p} = \frac{c}{\sqrt{1-\frac{\omega_c^2}{\omega^2}}} > c \tag{58}$$

Group velocity

$$V_g = \frac{d\omega}{d\beta} = c\sqrt{1-\frac{\omega_c^2}{\omega^2}} \tag{59}$$

Energy velocity [8]

$$V_e = c\sqrt{1-\frac{\omega_c^2}{\omega^2}} < c \tag{60}$$

$$V_p \cdot V = V_p \cdot V_e = V_p \cdot V_g = c^2 \tag{61}$$

By comparison, the dispersion relation of high frequency EM waves in plasma is [26]

$$\omega^2 = k^2 c^2 + \omega_p^2 \qquad (\omega_p = \sqrt{\frac{NZe^2}{\varepsilon_o m^*}} \text{ plasma frequency}) \tag{62}$$

Obviously, it is a good model to study de Broglie's theory because $E = \hbar\omega$ and $p = \hbar\mathrm{k}$ are both valid.

Total energy

$$E = \hbar\omega \tag{52}$$

Momentum

$$p = \hbar k \tag{63}$$

Rest mass

$$m_0 = \frac{\hbar\omega_p}{c^2} \tag{64}$$

Mechanical velocity

$$V = c\sqrt{1-\frac{\omega_p^2}{\omega^2}} < c \tag{65}$$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{\hbar\omega_p}{\frac{\omega_p}{\omega}} = \hbar\omega \tag{66}$$

$$p = \frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{\frac{\hbar\omega_p}{c^2} c\sqrt{1-\frac{\omega_p^2}{\omega^2}}}{\frac{\omega_p}{\omega}} = \hbar k \tag{67}$$

Phase velocity

$$V_p = \frac{\omega}{k} = \frac{E}{p} = \frac{c}{\sqrt{1-\frac{\omega_p^2}{\omega^2}}} > c \tag{68}$$

Group velocity

$$V_g = \frac{d\omega}{dk} = c\sqrt{1-\frac{\omega_p^2}{\omega^2}} \tag{69}$$

Energy velocity [27]

$$V_e = c\sqrt{1-\frac{\omega_p^2}{\omega^2}} < c \tag{70}$$

$$V_p \cdot V = V_p \cdot V_e = V_p \cdot V_g = c^2 \tag{71}$$

The opinion is wrong that the frequency of de Broglie's matter wave has no physical effects [28][29] and only the frequency difference is important [30].

7. CONCLUSION

de Broglie's $\mathbf{p} = \hbar\mathbf{k}$ is a special case of $\mathbf{p} = \hbar\boldsymbol{\beta}$. $\left[q, -i\hbar\frac{\partial}{\partial q}\right]$ equals $i\hbar$ indeed but the canonical commutation relation $[q, \hat{p}] = i\hbar$ is invalid because the operator $-i\hbar\frac{\partial}{\partial q}$ is not always a momentum.

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