

Performance analysis for two-way lossy-forward relaying with random Rician K-factor

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Abstract: A novel lossy-forward (LF) cooperative communication scheme based on the decode-and-forward (DF) protocol allowing source-relay link errors is proposed recently. The LF relaying scheme always allows the relay to forward the decoded information to the destination even though it may contain errors. The knowledge of the correlation between received information sequences is exploited in the decoding process at the destination to improve the system performance. In this paper, we derive exact expressions of the outage probability upper bound for a two-way LF relay system over Rician fading channels with random K-factor. The K-factor follows empirical distributions derived from measurement data. It is found that the two-way relaying with LF is superior to that with DF scheme in terms of the outage performance.

Keywords: outage probability, two-way relaying, lossy-forward, random K-factor, multiple access channel (MAC)

Classification: Wireless Communication Technologies

References

- [1] M. Chen and A. Yener, "Power allocation for F/TDMA multiuser two-way relay networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 546–551, Feb. 2010. DOI:10.1109/TWC.2010.02.090336
- [2] K. Anwar and T. Matsumoto, "Accumulator-assisted distributed turbo codes for relay systems exploiting source-relay correlation," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1114–1117, 2012. DOI:10.1109/LCOMM.2012.050112.120629
- [3] S. Qian, J. He, M. Juntti, and T. Matsumoto, "Fading correlations for wireless cooperative communications: Diversity and coding gains," *IEEE Access*, Apr. 2017. DOI:10.1109/ACCESS.2017.2699785
- [4] S. Zhu, T. S. Ghazaany, S. M. R. Jones, R. A. Abd-Alhameed, J. M. Noras, T. V. Buren, J. Wilson, T. Suggett, and S. Marker, "Probability distribution of Rician K-factor in urban, suburban and rural areas using real-world captured data," *IEEE Trans. Antennas Propagat.*, vol. 62, no. 7, pp. 3835–3839, July 2014. DOI:10.1109/TAP.2014.2318072
- [5] X. Zhou, M. Cheng, X. He, and T. Matsumoto, "Exact and approximated outage probability analyses for decode-and-forward relaying system allowing intra-link

errors,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 7062–7071, Dec. 2014. DOI:10.1109/TWC.2014.2354337

1 Introduction

Two-way (TW) relaying for bidirectional communications has attracted great interests due to its spectral efficiency gain compared to one-way relaying. Some relaying protocols, e.g., amplify-and-forward (AF), compress-and-forward (CF), and decode-and-forward (DF), have been considered for their utilization in TW relaying [1]. In [2], a novel lossy-forward (LF) relaying scheme, which was developed based on the DF relaying, has been proposed. The basic idea behind the LF relaying is the relay system, as a whole, can be seen as a distributed turbo code, and hence it can achieve turbo-cliff-like bit-error-rate (BER) performance in additive white Gaussian noise (AWGN) channels [3]. Prior studies on the LF relaying only focus on one-way relaying, and TW with the LF relaying has not been considered. In this paper, we study a TW relaying transmission network, where the LF relaying is performed at the relay node, referred to as (TW-LF) relaying. The channels experience the Rician fading with random K-factor following the probability distributions estimated in [4] based on measurement data. The outage probability upper bound of the TW-LF relaying is theoretically derived by a multi-fold integral over the achievable rate region determined by the theorem of source coding with side information. It is shown that the outage performance averaged over distribution of random K-factor can be worse than that with fixed distribution mean K-factor.

2 System model

The system model is shown in Fig. 1(a), where two nodes, A and B, exchange their independent and identically distributed (i.i.d.) binary information with the help of a relay node R. During the first time slot, A and B encode their own information sequences \mathbf{u}_A and \mathbf{u}_B separately, and send them to R simultaneously over a multiple access channel (MAC). \mathbf{u}_A and \mathbf{u}_B are also broadcast to B and A, respectively. During the second time slot, R decodes the information from A and B, and performs the exclusive-or (XOR) operation on the decoded binary information sequences $\hat{\mathbf{u}}_A$ and $\hat{\mathbf{u}}_B$. Then, the relay re-encodes, and broadcasts the resulting sequence to A and B in a broadcast channel. We consider successive interference cancellation is adopted and therefore each node can obtain its desired signal only.

The channel coefficient $h_{i,j}$ between two nodes, i and j follows the Rician distribution with the K-factor $K_{i,j}$ ($i, j \in (A, B, R), i \neq j$). The joint probability density function (PDF) and joint cumulative distribution function (CDF) of the Rician distributed instantaneous signal-to-noise ratio (SNR) $\gamma_{i,j}$ and the random K-factor is denoted as $p_{\gamma_{i,j}}(\gamma_{i,j}, K_{i,j})$ and $F_{\gamma_{i,j}}(\gamma_{i,j}, K_{i,j}) = F_{\gamma_{i,j}}(\gamma_{i,j}|K_{i,j})p_{K_{i,j}}(K_{i,j})$, respectively. The PDF of K-factor $p_{K_{i,j}}(K_{i,j})$ is represented by either logistic distribution $p_{\log}(K)$ or normal distribution $p_{\text{nor}}(K)$, which are found to be well-fit models in [4].

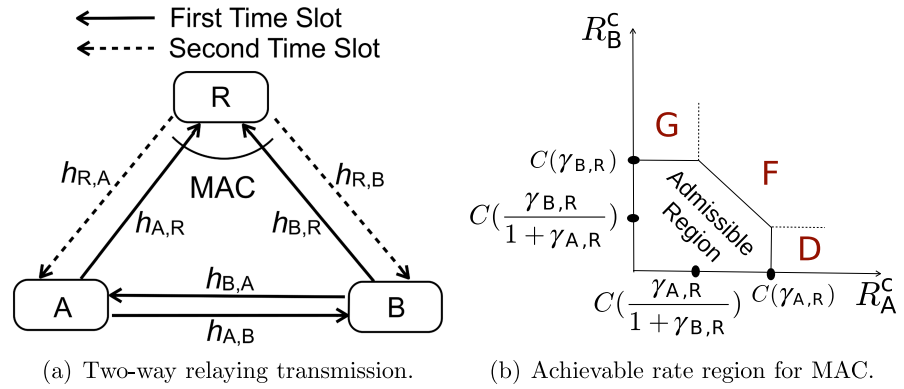


Fig. 1. (a) System model, and (b) rate region for A-R and B-R transmissions.

3 Outage probabilities

The two-way relay system fails in achieving reliable communication if at least one of destination nodes (A or B) is in outage. The overall outage event \mathcal{E}_O is, therefore, defined as $\mathcal{E}_O = \mathcal{E}_{OA} \cup \mathcal{E}_{OB}$, where \mathcal{E}_{OA} and \mathcal{E}_{OB} represent the outage event for A and B, respectively. With the symmetric system topology setup, we only focus on the derivation of \mathcal{E}_{OA} .

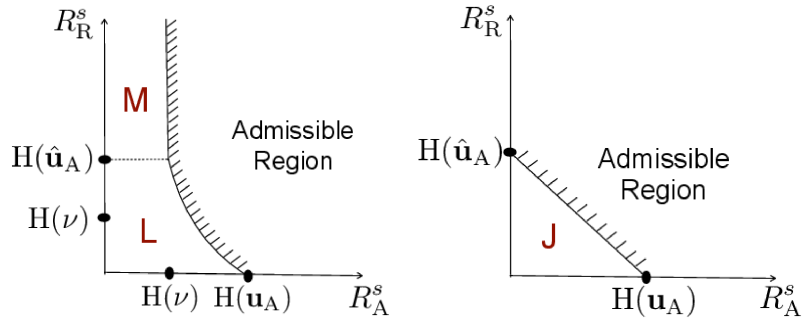
First, we calculate the probability of the outage event for \mathbf{u}_A and \mathbf{u}_B occurring at the first MAC time slot. It is obvious that for the first time slot transmission, the admissible region for {A,B}-to-R transmission is determined by the MAC rate region as shown in Fig. 1(b). The inadmissible MAC rate region can be divided into three sub-regions, D, F, and G, with P_D , P_F , and P_G denoting the probability that the rate pair (R_A^c, R_B^c) falls into the inadmissible regions D, F, and G in Fig. 1(b), respectively. R_x^c , $x \in (A, B)$, denotes the normalized spectrum efficiency of x-R channel.

From Fig. 1(b), it is found that P_F and P_D can be expressed as $P_F = \Pr[R_A^c > C(\frac{\gamma_{A,R}}{1 + \gamma_{B,R}}), R_B^c > C(\frac{\gamma_{B,R}}{1 + \gamma_{A,R}}), R_A^c + R_B^c > \log_2(1 + \gamma_{A,R} + \gamma_{B,R})]$ and $P_D = \Pr[R_A^c > C(\gamma_{A,R}), R_B^c \leq C(\frac{\gamma_{B,R}}{1 + \gamma_{A,R}})]$, where $C(\bullet) \triangleq \log_2(1 + \bullet)$.

With the random Rician K-factor, P_F and P_D are expressed as a set of integrals with respect to the PDFs of the instantaneous SNRs and K, as¹,

$$\begin{aligned}
 P_F = & \int_{K_{A,R}} \int_{K_{B,R}} \int_{\gamma_{A,R}=0}^{2^{R_A^c}-1} p_{\gamma_{A,R}}(\gamma_{A,R}|K_{A,R}) p_{\log}(K_{A,R}) p_{\log}(K_{B,R}) \\
 & \cdot F_{\gamma_{B,R}}[(2^{2R_B^c} - 1)(1 + \gamma_{A,R})|K_{B,R}] d\gamma_{A,R} dK_{A,R} dK_{B,R} + \int_{K_{A,R}} \int_{K_{B,R}} \\
 & \int_{\gamma_{A,R}=2^{R_A^c}-1}^{2^{R_B^c}(2^{R_A^c}-1)} p_{\gamma_{A,R}}(\gamma_{A,R}|K_{A,R}) p_{\log}(K_{A,R}) p_{\log}(K_{B,R}) \left\{ F_{\gamma_{B,R}}[(2^{R_A^c+R_B^c} \right. \\
 & \left. - 1 - \gamma_{A,R})|K_{B,R}] - F_{\gamma_{B,R}}\left[\frac{\gamma_{A,R}}{2^{R_A^c}-1} |K_{B,R}\right] \right\} d\gamma_{A,R} dK_{A,R} dK_{B,R}, \quad (1)
 \end{aligned}$$

¹We set the K-factor of the A-B (B-A) channel at $K_{A,B} = K_{B,A} = 0$, corresponding to Rayleigh fading. Therefore, the integrate over $K_{A,B}$ and $K_{B,A}$ can be ignored.



(a) Rate region for Case 1.

(b) Rate region for Case 3.

Fig. 2. Rate region for A-B and R-B transmissions.

$$P_D = \int_{K_{A,R}} \int_{K_{B,R}} \int_{\gamma_{A,R}=0}^{2^{R_A^c}-1} \int_{\gamma_{B,R}=(2^{R_B^c}-1)(1+\gamma_{A,R})}^{\infty} p_{\gamma_{A,R}}(\gamma_{A,R}|K_{A,R}) \cdot p_{\gamma_{B,R}}(\gamma_{B,R}|K_{B,R}) p_{\log}(K_{A,R}) p_{\log}(K_{B,R}) d\gamma_{A,R} d\gamma_{B,R} dK_{A,R} dK_{B,R}, \quad (2)$$

for the logistic-distributed K-factor. With the normal-distributed K-factor, the expressions of P_D and P_F are obtained by replacing $p_{\log}(K)$ with $p_{\text{nor}}(K)$ in Eq. (1) and Eq. (2). P_G can be calculated similarly as P_D in Eq. (2).

Based on the decoding results for \mathbf{u}_A and \mathbf{u}_B at R, the calculation of the outage probability of \mathcal{E}_{OA} , $\Pr(\mathcal{E}_{OA})$, can be further classified into three cases as: Case 1: ($\hat{\mathbf{u}}_A \neq \mathbf{u}_A$ and $\hat{\mathbf{u}}_B = \mathbf{u}_B$) or ($\hat{\mathbf{u}}_A = \mathbf{u}_A$ and $\hat{\mathbf{u}}_B \neq \mathbf{u}_B$) with which $\Pr(\text{Case 1}) = P_D + P_G$; Case 2: $\hat{\mathbf{u}}_A \neq \mathbf{u}_A$ and $\hat{\mathbf{u}}_B \neq \mathbf{u}_B$, $\Pr(\text{Case 2}) = P_F$; Case 3: $\hat{\mathbf{u}}_A = \mathbf{u}_A$ and $\hat{\mathbf{u}}_B = \mathbf{u}_B$, $\Pr(\text{Case 3}) = 1 - P_D - P_F - P_G$.

3.1 $\Pr(\mathcal{E}_{OA}|\text{Case 1})$

In Case 1, since the XOR-ed sequences are correlated with the original sources of A and B, the problem can be regarded as source coding with a helper. The admissible rate region in this case is illustrated in Fig. 2(a)². In this case, the outage probability becomes $\Pr(\mathcal{E}_{OA}|\text{Case 1}) = P_M + P_L$, where P_M and P_L are the probabilities that the source rate pair (R_A^s, R_R^s) falls into regions M and L in Fig. 2(a), respectively.

P_M and P_L can then be written as [5]

$$P_M = \int_{K_{A,R}} \int_{K_{B,R}} \int_{\gamma_{A,R}=0}^1 F_{\gamma_{A,B}}(2^{H[v]} - 1) [1 - F_{\gamma_{R,B}}(1, K_{R,B})] \cdot p_{\gamma_{A,R}}(\gamma_{A,R}|K_{A,R}) p_{\log}(K_{A,R}) p_{\log}(K_{B,R}) d\gamma_{A,R} dK_{A,R} dK_{B,R}, \quad (3)$$

$$P_L = \int_{K_{A,R}} \int_{K_{B,R}} \int_{\gamma_{A,R}=0}^1 \int_{\gamma_{R,B}=0}^1 F_{\gamma_{A,B}}(2^{H[\chi]} - 1) \cdot p_{\gamma_{R,B}}(\gamma_{R,B}|K_{R,B}) p_{\log}(K_{R,B}) d\gamma_{R,B} d\gamma_{A,R} dK_{A,R} dK_{B,R}, \quad (4)$$

where $H[x] = -x \log_2 x - (1-x) \log_2 (1-x)$, with its inverse function $H^{-1}[x]$, and $\rho_i(\gamma_{i,j}) = H^{-1}[1 - C(\gamma_{i,j})]$.

²Let $v = \rho_A(\gamma_{A,R}) * \rho_B(\gamma_{B,R})$ and $\chi = v * \rho_R(\gamma_{R,B})$, where $\rho_i(\gamma_{i,j})$, $i, j \in \{A, B, R\}$ is the error rate of the corresponding i - j link. The operation $a * b$ is defined as $a(1-b) + b(1-a)$.

3.2 $\Pr(\mathcal{E}_{OA}|\text{Case 2})$

In this case, we ignore the influence of the helper, and the outage probability only depends on the direct A-B link transmission, as

$$\Pr(\mathcal{E}_{OA}|\text{Case 2}) = \int_{\gamma_{A,B}=0}^{2^{R_A^s}-1} p_{\gamma_{A,B}}(\gamma_{A,B}) d\gamma_{A,B}. \quad (5)$$

Note that in Case 2 we do not consider the impact of the errors occurred in the A-R and B-R links, even though the XOR-ed $\hat{\mathbf{u}}_A \oplus \hat{\mathbf{u}}_B$ still may make contribution for recovering \mathbf{u}_A at B. Therefore, the overall outage probability calculated in this paper upper bounds the performance of the practical signaling schemes.

3.3 $\Pr(\mathcal{E}_{OA}|\text{Case 3})$

For Case 3, the admissible rate region of R_A^s and R_R^s is shown in Fig. 2(b). The outage occurs when (R_A^s, R_R^s) falls into the inadmissible rate region J in Fig. 2(b), with probability $\Pr(\mathcal{E}_{OA}|\text{Case 3}) = P_J$. Therefore, we have

$$P_J = \int_{K_{A,R}} \int_{K_{R,B}} \int_{\gamma_{R,B}=0}^1 F_{\gamma_{A,B}}(2^{1-C(\gamma_{R,B})} - 1) [1 - F_{\gamma_{A,R}}(1, K_{A,R})] \\ \cdot p_{\gamma_{R,B}}(\gamma_{R,B}|K_{R,B}) p_{\log}(K_{A,R}) p_{\log}(K_{R,B}) d\gamma_{R,B} dK_{R,B} dK_{A,R}. \quad (6)$$

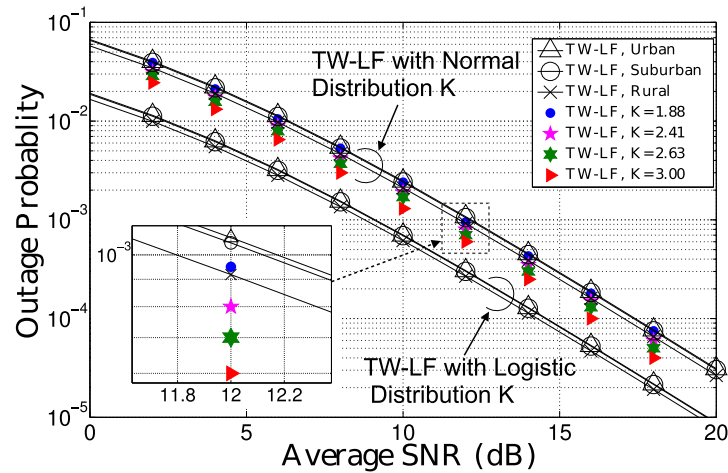
The outage probability of \mathcal{E}_{OA} is given as $\Pr(\mathcal{E}_{OA}) = \Pr(\mathcal{E}_{OA}|\text{Case 1}) \Pr(\text{Case 1}) + \Pr(\mathcal{E}_{OA}|\text{Case 2}) \Pr(\text{Case 2}) + \Pr(\mathcal{E}_{OA}|\text{Case 3}) \Pr(\text{Case 3})$. The overall outage probability of the TW-LF can be expressed as $\Pr(\mathcal{E}_O) = 1 - [1 - \Pr(\mathcal{E}_{OA})] \cdot [1 - \Pr(\mathcal{E}_{OB})]$, assuming that the outage occurrence for A and B are independent. $\Pr(\mathcal{E}_{OB})$ can be obtained by following the similar calculations as those for $\Pr(\mathcal{E}_{OA})$.

4 Numerical results

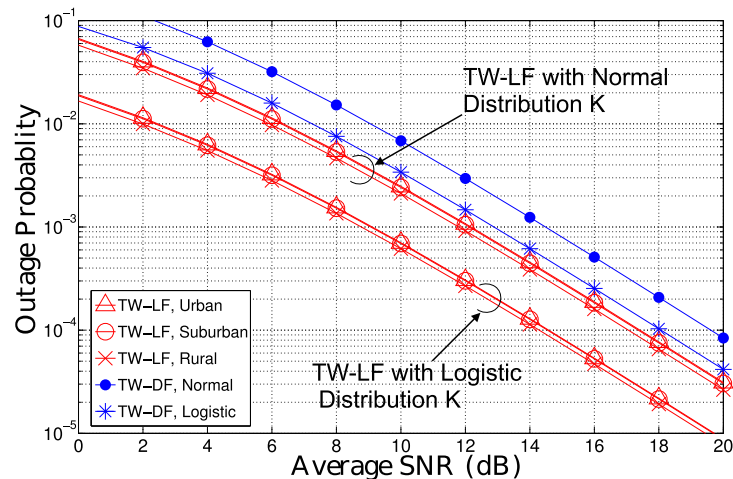
The outage curves of TW-LF with random and fixed K-factor are demonstrated in Fig. 3(b). The mean μ and variance σ^2 of the normal and logistic distributions follow the measurement data from urban, suburban, and rural areas [4], as shown in Fig. 3(a). It can be found from Fig. 3(b) that the outage curves with the Rician K-factor following the normal distribution are close to those with fixed K with the values being approximately equal to the measured mean value of the normal distribution. However, the outage probability with the Rician K-factor following the logistic distribution is far apart from those with the mean of the logistic distribution. It is also found that the outage performance with fixed K-factor (distribution means) are slightly better than those averaged over the distributions of random K-factor.

Fig. 3(c) compares the outage performance of the TW-LF and TW-DF relaying. With the TW-DF relaying, the relay keeps silent if error is detected after decoding. Since the outage performance of TW-DF in urban, suburban, and rural areas are almost the same, we only plot the outage curves of the TW-DF relaying in rural area with Rician K-factor following the normal and logistic distributions. It is found that the outage probability of TW-LF is lower to that of TW-DF with either normal- or logistic-distributed K-factor.

	Urban Area		Suburban Area		Rural Area	
	μ (dB)	σ^2 (dB)	μ (dB)	σ^2 (dB)	μ (dB)	σ^2 (dB)
$\rho_{\text{hor}}(K)$	1.88	4.13	2.41	3.84	2.63	3.82
$\rho_{\text{og}}(K)$	1.85	4.15	2.6	3.88	3	3.88

(a) Measurement data of mean μ and variance σ^2 .

(b) With fixed and random K-factor.



(c) With TW-LF and TW-DF relaying.

Fig. 3. (a) Measurement data, and comparison of the outage probability of the TW-LF with random K-factor with (b) fixed K-factor, and (c) TW-DF.

5 Conclusion

We analyzed the outage probability of the TW-LF relaying with random K-factor of the Rician fading channels. The numerical result revealed that the outage probability derived assuming normal-distributed random K-factor matches that derived assuming the constant distribution mean K-factor. We also found that this is not the case with the logistic distribution. However, the outage performance averaged over distribution of random K-factor is worse than that with fixed distribution mean K-factor. Moreover, compared to TW-DF, the TW-LF relaying is found to achieve lower outage probability.