Fuzzy Analysis of Groundwater Resources Including Incomplete Data of Aquifer Spatial Extent

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1. SUMMARY

A major problem in the simulation of water resources is the uncertainty of the various parameters. Fuzzy analysis is a powerful tool for dealing with such problems involving uncertain data. Unlike surface water bodies systems, the exact boundaries of groundwater bodies are often not known. A methodology for the simulation of aquifers with vague spatial extent is introduced in this paper, and an application example is presented.

2. INTRODUCTION

Fuzzy logic owes its origins to ancient Greek philosophy and especially to Aristotle and the other philosophers who preceded him, like Parminides (around 400 B.C.). In their efforts to devise a concise theory of logic, they suggested that every proposition must either be True or False. Heraclitus proposed that things could be simultaneously "True and not True". It was Plato who laid the foundation for what would become fuzzy logic, indicating that there was a third region, beyond True and False, where these opposites "tumbled about". The mathematics of fuzzy set theory and the extension fuzzy logic, were first described in 1965 by Lotfi A. Zadeh in his original work "Fuzzy Sets". This paper will present the foundation of fuzzy systems, along with example in one-dimensional groundwater flow problem.

3. A BRIEF DESCRIPTION OF FUZZY SET THEORY

Fuzziness, as handled in fuzzy logic, can refer to various types of vagueness and uncertainty but particularly to the vagueness related to human linguistics and thinking, differing from the uncertainty of Probabilistic Theory /9/.

Definition 1. Fuzzy set.

If X is a collection of objects denoted generically by x, then a fuzzy set $\bar{A}$ in X is a set of ordered pairs: $\bar{A}=\left\{ \left(x, \mu_{\bar{A}}(x)\right) \mid x \in X \right\}$, where $\mu_{\bar{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\bar{A}$ that maps X to the membership space M (When M contains only the two points 0 and 1, $\bar{A}$ is no fuzzy set.) /10/.

Definition 2. α-level cut.

The α-level cut (α-level set) of the fuzzy subset A is the set of those elements, which have at least α membership: $A_\alpha = \left\{x \mid \mu_A(x) > \alpha\right\}$. If $A'_\alpha = \left\{x \mid \mu_A(x) > \alpha\right\}$ is called "strong α-level cut".

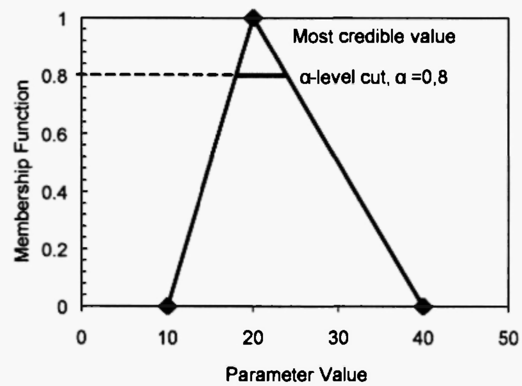

Fig. 1: A fuzzy number and an α-level cut

Definition 3. Convex fuzzy set.

A fuzzy set is convex if: $\mu_A\left(\lambda x_1 + (1-\lambda)x_2\right) > \min\left\{\mu_A(x_1), \mu_A(x_2)\right\}, x_1, x_2 \in X, \lambda \in [0,1]$

Also a fuzzy set can be convex if all α-level sets are convex.

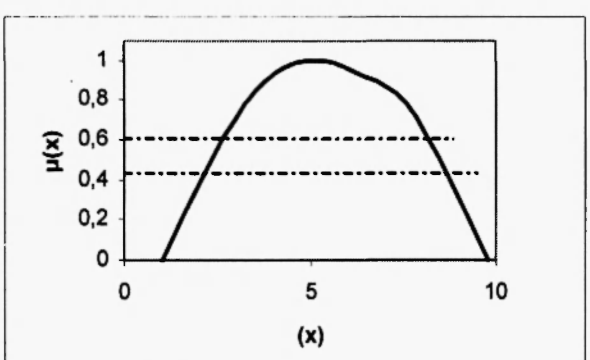

Fig. 2: Convex fuzzy set

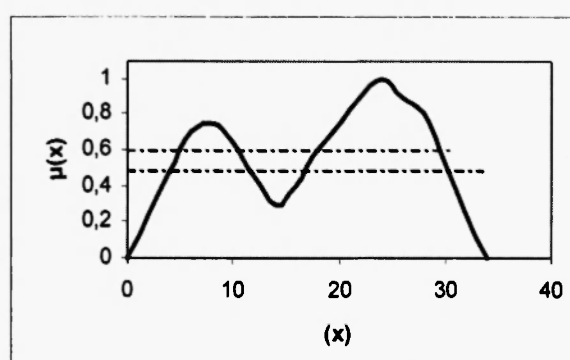

Fig. 3: Nonconvex fuzzy set

Definition 4. Fuzzy numbers.

A fuzzy number *M* is a convex normalized set *M* of the real line $\Re$ such that

1. There exists one $x_0 \in \Re$ with $\mu_M(x_0) = 1$ (x_0 is called the mean value of *M*)
2. $\mu_M(x) = 1$ is piecewise continuous.

Nowadays, this definition is very often modified. For the sake of simplicity, trapezoidal membership functions are often used. A triangular fuzzy number is a special case of this. When using fuzzy set theory to solve real problems of realistic size, more efficiency is to use a special type of fuzzy numbers, the LR-type /10/.

Definition 5. LR-type.

A fuzzy number *M* is of LR-type if there exist reference functions L (for left), R (for right), and scalars $\alpha>0$, $\beta>0$ with

$$\mu_M(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & For\ x < m \\ R\left(\frac{x-m}{\beta}\right) & For\ x > m \end{cases}$$

m is a real number called the mean value of *M*, and α and β are called the left and right spreads respectively.

Strictly speaking, these special cases of fuzzy numbers are fuzzy intervals. So every a-level cut actually gives an interval number. Using various a-level cuts we can construct a fuzzy number in discrete form. Finally, if we want to use fuzzy sets in applications, we will have to deal with interval number operations.

If * is one of the symbols +, -, ·, /, we define arithmetic basic operations on interval number by $[a,b]*[c,d]=\{x*y|a<x<b, c<y<d\}$ except that we do not define [a, b]/[c, d] if $0 \in [c, d]$.

Specifically,

[a, b]+[c, d] = [a + c, b + b]

[a, b]-[c, d] = [a - d, b - c]

[a, b] · [c, d] = [min(ac, ad, bc, db), max(ac, ad, bc, db)]

[a, b]/[c, d] = [a, b] · [1/d, 1/c] if $0 \notin [c, d]$.

4. WATER RESOURCES MANAGEMENT WITH UNCERTAIN PDE COEFFICIENTS

The behavior of water systems is usually simulated by using partial differential equations.

The exact value of various coefficients (transmissivity, dispersion, etc.) of the PDEs is often not exactly known.

The classical approach in dealing with this type of uncertainty is the use of a probabilistic approach like the Monte Carlo methods /1/. However this method is difficult to apply in real world problems: The probabilistic characteristics of the input data are rarely known. A huge number of simulations are necessary and the organization of the output data is a tricky issue.

For these reasons many authors propose the use of fuzzy arithmetic to deal with uncertainties occurring in water management problems /3,4,5,6,8/.

In this approach, uncertain or vague parameters are defined as fuzzy numbers. Unlike the classical definition of fuzzy theory, the membership function denotes if a certain value of a given parameter is likely to occur, not the degree of certainty.

A fuzzy analysis approach of water management problems, involves usually the consideration of several α-level cuts, and an explicit scheme approach for the PDE's discretization /3,6,8/.

Several application examples of this approach are presented in the literature, including uncertainty in transmissivities /4, 5/, porosities and dispersivities /3/, and deoxygenation rate coefficient /8/.

5. A METHODOLOGY FOR TREATING SPATIAL IMPRECISION WITH FUZZY SET ANALYSIS

For the simulation of incomplete data concerning the spatial extent of aquifers, we may introduce a fuzzy number for the description of the aquifer length (Figure 4), or the evaluation of the possibility that a discrete area belongs to a given aquifer (see Figures 5, 6).

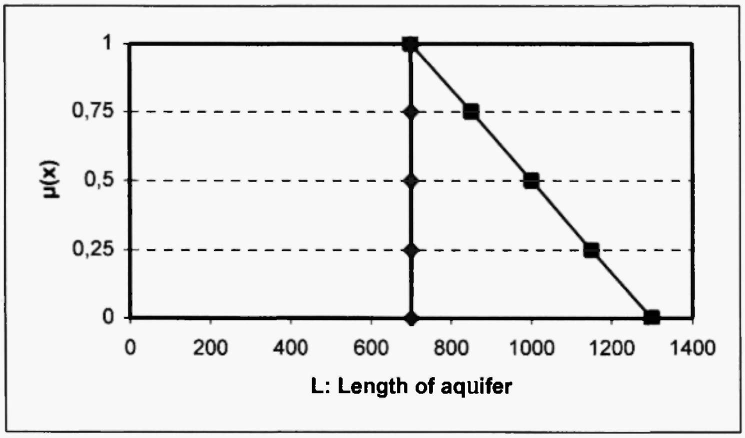

Fig. 4: Fuzzy number describing aquifer length

Fig. 5: Membership function levels of aquifer extent

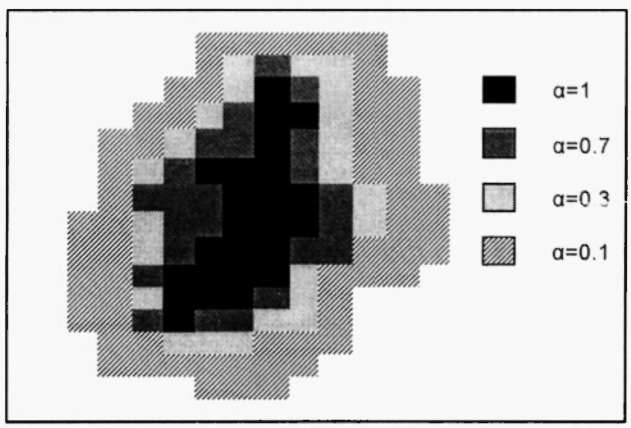

Fig. 6: Membership function levels of aquifer extent

The former definition may be used for in combination with analytical solutions, while the latter for a numerical analysis approach.

To solve problems involving uncertainty in spatial extent of aquifers, a similar methodology to that presented in the previous chapter may be used: Different α-levels cuts of the imprecise parameters are considered and interval analysis techniques are used. It is worth mentioning that for situations in which data imprecision lays exclusively in the knowledge of boundaries positions, the problem may be solved by a classical (crisp) mathematical approach.

6. AN APPLICATION EXAMPLE

We examine the hydraulic head variation in the following confined aquifer /2/. A stream at $x=l$ is the first boundary. The aquifer is limited by a no-flow boundary at the distance $x=\phi$. The aquifer is initially in equilibrium with the river $h=h_0$. At time $t=0$ the water level in the river becomes $h=0$.

The mathematical expression for the above conditions is:

$h(x,0) = h_0,$	$0 < x < l,$	(initial)
$h(l,t) = 0,$	$t > 0,$	(first boundary with prescribed head)
$\left(\dfrac{\partial h}{\partial x}\right)_{x=\phi} = 0,$	$t > 0,$	(second no-flow boundary)

The behavior of the hydraulic head is described by the above expression:

$$h = h_0 \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \exp\left[-\frac{(2n+1)^2 \pi^2 Tt}{4Sl^2}\right] \cos\frac{(2n+1)\pi x}{2l}$$

where t denotes time, x the distance from the origin, T the transmissivity, S the storativity and l the aquifer length.

We suppose that data concerning transmissivity and aquifer length are uncertain. We suppose that they can be expressed in verbal and fuzzy form as follows:

"The transmissivity is approximately 450m^2/d, and is certainly above 100m^2/d but not greater than 500m^2/d".

"The aquifer length is at least 700m but not greater than 1300m. The most likely value is 700m"

We further assume that S=0.0001.

The analytical solution is solved at five different α-level cuts (0.0,0.25,0.5,0.75,1.0). The values for the piezometric head h at x=500m for t=0,05d and t=0.1d are presented in figure (7a,b).

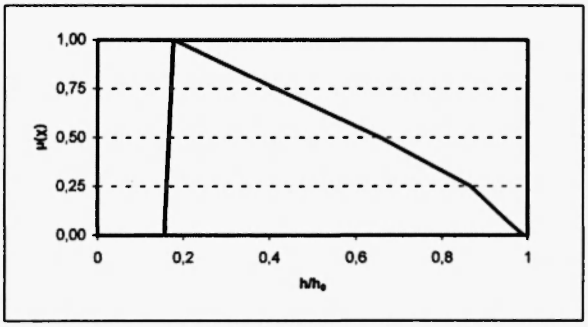

Fig. 7a: Fuzzy representation x=500m, t=0.05 days

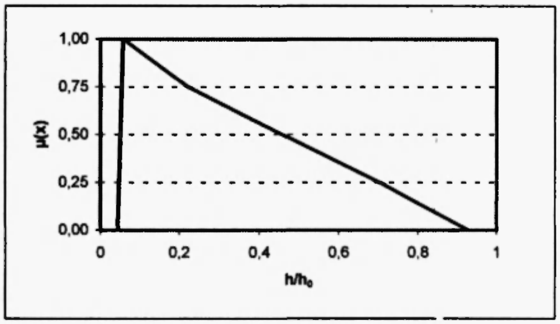

Fig. 7b: Fuzzy representation x=500m, t=0.1 days

7. CONCLUSIONS AND PERSPECTIVES

Fuzzy set logic and water management using fuzzy arithmetics are presented. An original methodology for dealing with problems in which the exact position of the aquifer boundaries is unknown is developed, and an application example is presented. Differences in fuzzy approximation of coefficient imprecision and aquifer extension are emphasized. Numerical codes for 1D- and 2D fuzzy simulation, including spatial uncertainty, are currently under development in our laboratory.

8. REFERENCES

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