Research Article

Yu Jiang* and Hengnian Li Equilibria and orbits in the dynamical environment of asteroid 22 Kalliope

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Abstract: This paper studies the orbital dynamics of the potential of asteroid 22 Kalliope using observational data of the irregular shape. The zero-velocity surface are calculated and showed with different Jacobian values. All five equilibrium points are found, four of them are outside and unstable, and the other one is inside and linearly stable. The movement and bifurcations of equilibrium points during the variety of rotation speed and density of the body are investigated. The Hopf bifurcations occurs during the variety of rotational speed from ω =1.0 ω_0 to 0.5 ω_0 , and the Saddle-Node bifurcation occurs during the variety of rotation of an unstable periodic orbit shows that the gravitational field of Kalliope is strongly perturbed.

Keywords: equilibria; periodic orbits; zero velocity surfaces; asteroids; 22 Kalliope

1 Introduction

The exploration of asteroids in our Solar System make the study of the orbital environment around asteroids to be significant (Marchis et al. 2012). Several papers have already been interested in these problems (Jain and Sinha 2014; Aljbaae et al. 2017). Scheeres et al. (1996) presented the Jacobi integral for particles orbiting around asteroid. Using the Jacobi integral, it is possible to determine zero-velocity surfaces for the possible motion of a particle. Werner and Scheeres (1997) presented the polyhedral model method to calculate the gravitation of the irregular-shaped asteroids. Yu and Baoyin (2012a) used the Jacobi integral presented in Scheeres et al. (1996) and discussed zero-velocity surfaces in detail. Yu and Baoyin (2012b) presented a hierarchical grid searching method to calculate periodic orbits in the potential of asteroids; the gravitational field model of asteroids is built using the polyhedral model method. Jiang et al. (2014) derived the linearised equations and characteristic equations for the motion of particles relative to the equilibrium point around asteroids. Using the characteristic equations, eigenvalues, topological cases, and stability

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of equilibrium points can be calculated. Yang *et al.* (2018) expanded the linearized equations to artificial equilibrium points.

Considering a simplified model of the asteroid, one can study the dynamics of either a gyrostat orbiting in the restricted three-body problem or the rotating mass dipole to help understand the complicated dynamical behaviors in the potential of asteroids. Vera (2009) studied the rotational Poisson dynamics of a gyrostat around an Eulerian equilibrium point and presented the nonlinear stability of the equilibrium point in the three-body problem. Guirao and Vera (2010) presented the linear stability of Lagrangian equilibrium points in the three-body problem. These literatures use geometric-mechanic methods to establish the Hamiltonian, total angular momentum, Poisson tensor, vectorial equations of the motion, as well as the condition of relative equilibria of the system. Wang and Xu (2014) used the second degree harmonic coefficients to model the gravitational field of asteroids, and the geometric-mechanic methods to analyze the nonlinear stability of a spacecraft placed at relative equilibria of the gravitational field of second-degree spheric harmonics functions. Zhang and Zhao (2015) used the same model of Wang and Xu (2014), i.e. the seconddegree spheric harmonics function, and discussed the attitude stability of a dual-spin spacecraft placed at relative equilibria of the gravitational field. Yang et al. (2015) and Zeng et al. (2016) applied the linearised equations and characteristic equations from Jiang et al. (2014) to the rotating mass dipole. Yang et al. (2015) calculated stable regions of

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equilibrium points around the rotating mass dipole. Zeng *et al.* (2018) used a dipole segment model to simulate the gravitational field of (8567) 1996 HW1.

However, the simplified model cannot model the dynamical behaviors caused by the irregular shape of asteroids. The polyhedral model method can be used to calculate the irregular shape and gravitation of asteroids. Chanut et al. (2015) used the polyhedral model method (Werner and Scheeres 1997) and the characteristic equations (Jiang et al. 2014) to calculate the eigenvalues and stability of equilibrium points around asteroid 216 Kleopatra. They conclude that among three inner equilibrium points of 216 Kleopatra, one is unstable, and the other two are linearly stable. Jiang et al. (2015) found collision and annihilation of equilibrium points as well as saddle-node bifurcations and saddlesaddle bifurcations for the relative equilibria around asteroids while the rotation speed varies. Yang *et al.* (2018) expanded the results of collision and annihilation to artificial equilibrium points. Scheeres et al. (2016) calculated dynamical structures and equilibrium points for asteroid 101955 Bennu with different density values; however, this literature didn't analyze the bifurcations of the equilibrium points.

The asteroid 22 Kalliope-Linus is one of the biggest main belt binary asteroid systems. The primary Kalliope was discovered in 1852, the secondary Linus was discovered in 2001 (Merline *et al.* 2001; Marchis *et al.* 2003). The mean diameter of the primary and the secondary are 166.2±2.8 km and 28±2 km, respectively (Descamps *et al.* 2008). The dynamics in the vicinity of Kalliope is important for the following reasons: first, 22 Kalliope-Linus is one of the biggest binary asteroidal systems in the Solar system; second, it is a large-size-ratio binary asteroidal system, which means the study of dynamics around Kalliope is useful for the understanding of the complicated dynamical behaviours of Linus relative to Kalliope.

This paper is organized as follows. In Section 2 we discussed the general properties of Kalliope, including the moments of inertia, zero-velocity surfaces, and equilibrium points. In Section 3, the movement of equilibrium points has been presented to clarify the variety of locations, topological cases, and number of equilibrium points. The Hopf bifurcation, which is related to the appearance or the disappearance of one or more periodic orbit families, has been found during the variety of rotational speed from ω =1.0 ω_0 to 0.5 ω_0 . The Saddle-Node bifurcation, which is related to the appearance of the disappearance or the disappearance or the disappearance or the disappearance equilibrium points, has been found during the variety of rotational speed from ω =1.0 ω_0 to 2.0 ω_0 . Section 4 deals with the stability and resonance of orbits around Kalliope. The perturbed non-

periodic orbit which is generated by a 2:1 resonant periodic orbit, is found to be also 2:1 resonant.

2 Gravitational Field and Equilibrium Points around Kalliope

In this section, we investigate the gravitational field, irregular shape, motion equations, zero-velocity surfaces, as well as equilibrium points around Kalliope. The gravitational force, zero-velocity surfaces, and equilibrium points are computed by integrating the irregular shape of the asteroid.

2.1 Calculate Parameters from the Shape Model of Kalliope

The bulk density of Kalliope is estimated to be 3.35 ± 0.33 g·cm⁻³ (Descamps *et al.* 2008), and the rotation period is 4.148 h (Laver *et al.* 2009; Johnston 2014; Sokova *et al.* 2014). We use the polyhedral model method (Werner and Scheeres 1997; Khushalani 2000) to calculate: the moments of inertia, the total mass, the asteroid's irregular shape and the gravitational potential of asteroid Kalliope. The gravitational potential and gravitational force are

$$\begin{cases} U = -\frac{1}{2}G\sigma \sum_{e \in edges} \mathbf{r}_{e} \cdot \mathbf{E}_{e} \cdot \mathbf{r}_{e} \cdot L_{e} \\ +\frac{1}{2}G\sigma \sum_{f \in faces} \mathbf{r}_{f} \cdot \mathbf{F}_{f} \cdot \mathbf{r}_{f} \cdot \omega_{f} \\ \nabla U = G\sigma \sum_{e \in edges} \mathbf{E}_{e} \cdot \mathbf{r}_{e} \cdot L_{e} \\ -G\sigma \sum_{f \in faces} \mathbf{F}_{f} \cdot \mathbf{r}_{f} \cdot \omega_{f} \end{cases}$$
(1)

where $G = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$ is the gravitational constant, σ represents the bulk density of the polyhedron body; r_e and r_f represent vectors from fixed points to points on the edge e and face f, respectively; E_e and F_f represent geometric parameters of edges and faces, respectively; L_e means the integration factor of the particle, and ω_f means the signed solid angle of the face. Figure 1 shows the 3D ployhedron model of asteroid Kalliope. Based on the calculation, the size of the body is 191.94×152.55×127.50km. The total mass is 5.1292×10^{18} kg. The moments of inertia are

$$I_{xx} = 0.82245 \times 10^{13} \text{ kg} \cdot \text{km}^2,$$

$$I_{yy} = 1.2486 \times 10^{13} \text{ kg} \cdot \text{km}^2,$$

and
$$I_{zz} = 1.4371 \times 10^{13} \text{ kg} \cdot \text{km}^2.$$



Figure 1. 3D ployhedron model of asteroid Kalliope, The shape was built by 2038 faces and 1021 vertexes.

2.2 Equations of Motion

For a test particle or satellite orbiting around an asteroid, the motion equations respect to the body-fixed frame can be stated as

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + \frac{\partial U(\boldsymbol{r})}{\partial \boldsymbol{r}} = 0, \qquad (2)$$

where r is the radius vector of the particle, the derivatives of r are with respect to the body-fixed frame, ω is the asteroid's rotational angular velocity relative to inertial space, and U(r) is the asteroid's gravitational potential.

The Jacobi integral H (Scheeres *et al.* 1996), the mechanical energy E (Jiang and Baoyin 2014), and the effective potential V (Yu and Baoyin 2012a,b) can be written as

$$H = \frac{1}{2} \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} - \frac{1}{2} (\boldsymbol{\omega} \times \boldsymbol{r}) \cdot (\boldsymbol{\omega} \times \boldsymbol{r}) + U(\boldsymbol{r}), \qquad (3)$$

$$E = \frac{1}{2} \boldsymbol{v}_I \cdot \boldsymbol{v}_I + U(\boldsymbol{r}), \qquad (4)$$

$$V(\mathbf{r}) = -\frac{1}{2} (\boldsymbol{\omega} \times \mathbf{r}) \cdot (\boldsymbol{\omega} \times \mathbf{r}) + U(\mathbf{r}).$$
 (5)

where $v_I = \dot{r} + \omega \times r$ is the particle velocity in the inertial space.

Figure 2 demonstrates the zero-velocity surfaces around Kalliope calculated by different values of Jacobi



Figure 2. Zero-velocity surfaces with different Jacobi integral values $(km^2 \cdot s^{-2})$, figure bar represents the location value of the point on z axis: (a) 0.0038; (b) 0.0041; (c) 0.0044.





Figure 3. Zero-velocity curves and projections of equilibrium points in the equatorial plane (z=0).

Table 1. Positions of the equilibrium points around 22 Kalliope

Equilibrium	x (m)	y (m)	z (m)	Effective
Points				Potential
				$(m^2 s^{-2})$
E1	133.617	-2.27372	0.945615	0.00431891
E2	-2.68949	122.435	0.454720	0.00407598
E3	-132.809	1.88359	1.30748	0.00430499
E4	1.44546	-123.804	0.471773	0.00409724
E5	-1.05405	0.661290	-0.746006	0.00702688

integral. The sawtooth shape of the boundary in Figure 2 appears because we use discrete grids to calculate the values on the zero-velocity surfaces. Figure 3 illustrates the zerovelocity curves and projections of equilibrium points in the equatorial plane of rotating Kalliope. From Figure 2, one can see that the zero-velocity surfaces with different values of Jacobi integral are different. The zero-velocity surfaces have two branches, north branch and south branch. From Figure 3, we know that there are five equilibrium points in the potential of Kalliope.

When the Jacobi integral is H=0.0038, the north branch and south branch are non-intersecting. When the Jacobi integral increases to H=0.0041, the north branch and south branch are intersected; the intersection region of the north branch and south branch are near the equilibrium points E2 and E4; the intersection region of the north branch and south branch can be seen in Figure 3; from Figure 3, we can see that there are two intersection regions, one region surrounds equilibrium point E2, the other one surrounds equilibrium point E4. When the Jacobi integral increases to H=0.0044, the north branch and south branch are also intersect; there are two intersection regions, the inner one surrounds equilibrium point E5 and the projection of the irregular body onto the equatorial plane, the outer one surrounds all of the five equilibrium points.

The locations of equilibrium points can be obtained by solving the following equation and are presented in Table 1.

$$\nabla V(\mathbf{r}) = 0. \tag{6}$$

The equilibrium points are static relative to the body fixed frame of Kalliope, in other words, the equilibrium points are stationary orbits in the inertial reference frame. From Table 1, one can see that the equilibrium points are not in the equatorial plane, because Kalliope is not North-South symmetric. The values of effective potential at these five equilibrium points are different. The effective potential at E5 is the biggest, while at E2 is the smallest.

The eigenvalues of the equilibrium points (Jiang *et al.* 2014) can be obtained by solving the following equation

$$\lambda^{6} + \left(V_{xx} + V_{yy} + V_{zz} + 4\omega^{2}\right)\lambda^{4} +$$

$$\left(V_{xx}V_{yy} + V_{yy}V_{zz} + V_{zz}V_{xx} - V_{xy}^{2} - V_{yz}^{2} - V_{xz}^{2} + 4\omega^{2}V_{zz}\right)\lambda^{2} + \left(V_{xx}V_{yy}V_{zz} + 2V_{xy}V_{yz}V_{xz} - V_{xx}V_{yz}^{2} - V_{yy}V_{xz}^{2} - V_{zz}V_{xy}^{2}\right)$$

$$= 0$$

$$(7)$$

where
$$V_{rr} \stackrel{\Delta}{=} \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{xy} & V_{yy} & V_{yz} \\ V_{xz} & V_{yz} & V_{zz} \end{pmatrix}_{L}$$
 represents the Hessian

matrix of $V(\mathbf{r})$, λ is the eigenvalues, and $\omega = |\boldsymbol{\omega}|$.

Table 2 presented the eigenvalues of the equilibrium points around asteroid Kalliope. From the distribution of eigenvalues, one can confirm the stability of equilibrium points. Equilibrium points E1 and E3 possess a pair of real eigenvalues, one is positive and the other one is negative, which is in the form of $\pm \alpha$ ($\alpha \in \mathbb{R}$, $\alpha > 0$). While equilibrium points E2 and E4 possess a pair of complex eigenvalues, which is in the form of $\pm \sigma \pm i\tau$ ($\sigma, \tau \in \mathbb{R}; \sigma, \tau > 0$). In addition, equilibrium points E1 and E3 possess two pairs of purely imaginary eigenvalues, which is in the form of $\pm i\beta_i$ ($\beta_i \in \mathbb{R}, \beta_i > 0; j = 1, 2$); while equilibrium points E2 and E4 possess a pair of purely imaginary eigenvalues, which is in the form of $\pm i\beta_i$ ($\beta_i \in \mathbb{R}, \beta_i > 0; j = 1$). Equilibrium point E5 only has purely imaginary eigenvalues, i.e. three pairs of purely imaginary eigenvalues, which is in the form of $\pm i\beta_i$ ($\beta_i \in \mathbb{R}$, $\beta_i > 0$; j = 1, 2, 3). According to the values of eigenvalues, one can conclude that all the outside equilibrium points, i.e. E1-E4, are unstable. The

Equilibrium Points(×10 ⁻³ s ⁻¹)	λ_1	λ_2	λ3	λ_4	λ_5	λ_6
E1	0.483306i	-0.483306i	0.467954i	-0.467954i	0.313817	-0.313817
E2	0.431302i	-0.431302i	0.129738	0.129738	-0.129738	-0.129738
			+0.317589i	-0.317589i	+0.317589i	-0.317589i
E3	0.474876i	-0.474876i	0.464768i	-0.464768i	0.295689	-0.295689
E4	0.445327i	-0.445327i	0.118383	0.118383	-0.118383	-0.118383
			+0.303148i	-0.303148i	+0.303148i	-0.303148i
E5	1.31735i	-1.31735i	1.10542i	-1.10542i	0.454310i	-0.454310i

Table 2. Eigenvalues of the equilibrium points around 22 Kalliope

inner equilibrium point E5 is linearly stable. The value of effective potential can help to understand the stability of equilibrium point. The inner equilibrium point E5 has the biggest value of effective potential, *i.e.* 0.00702688. Using the value of effective potential to list the equilibrium point from large to small, we get E5, E1, E3, E4, as well as E2. Thus one can conclude that E2 is the most unstable equilibrium points.

3 Movement of Equilibrium Points during the Variety of Parameters

When the parameters of the asteroid vary, the gravitational field and dynamical environment vary. For the equilibrium points, bifurcations may occur (Jiang et al. 2015, 2016; Wang et al. 2016). The Hopf bifurcation of equilibrium points occurs when the purely imaginary eigenvalues of the equilibrium points produce or disappear (see Figure 4). In Figure 4, before the Hopf bifurcation, there are three families of periodic orbits around the equilibrium points; after the Hopf bifurcation, there is only one family of periodic orbits around the equilibrium points. The Hopf bifurcation of the equilibrium points is related to the appearance or the disappearance of one or more periodic orbit families. More detailed contents of Hopf bifurcations about equilibrium points around asteroids can be seen in Jiang et al. (2016). Yang et al. (2018) expanded the theory of Hopf bifurcation for asteroidal equilibrium points in Jiang et al. (2016) to artificial equilibrium points of asteroids.



Figure 4. The Hopf bifurcation of equilibrium points

The Hopf bifurcation of the equilibrium points has no relationship with the appearance or the disappearance equilibrium points. Another kind of bifurcation, which is named as Saddle-Node bifurcation, is related to the appearance or the disappearance equilibrium points. For the Saddle-Node bifurcation, the number of non-degenerate equilibrium points will change during the variety of the parameters of the asteroids. Two different equilibrium points corresponding to Case 1 and Case 2 collide and vanish during the parameter variety. Case 1 represents the distribution of eigenvalues are $\pm i\beta_j$ ($\beta_j \in \mathbb{R}, \beta_j > 0; j = 1, 2, 3$), Case 2 represents the distribution of eigenvalues are $\pm \alpha_j$ ($\alpha_j \in \mathbb{R}, \alpha_j > 0, j = 1$) and $\pm i\beta_j$ ($\beta_j \in \mathbb{R}, \beta_j > 0; j = 1, 2$), Case 5 represents the distribution of eigenvalues are $\pm i\beta_j$ ($\beta_j \in \mathbb{R}, \beta_j > 0, j = 1$) and $\pm \sigma \pm i\tau$ ($\sigma, \tau \in \mathbb{R}; \sigma, \tau > 0$).

First, we consider the movement of equilibrium points during the variety of rotation speed. Figure 5 shows the locations of projections of equilibrium points in the equatorial plane during the variety of rotation speed. Let ω_0 be the rotation speed of Kalliope, *i.e.* $\omega_0 = \frac{2\pi}{4.148} h^{-1}$. When $\omega = 0.5\omega_0$, there are five equilibrium points, the locations of the equilibrium points are different from the locations when $\omega = 1.0\omega_0$. When $\omega = 2.0\omega_0$, there are only three equilibrium points left. Searching from $\omega = 0.5\omega_0$ to $\omega = 2.0\omega_0$, we find that when $\omega = 1.908\omega_0$, two equilibrium points collide with each other. After that, if $\omega > 1.908\omega_0$, these two equilibrium points left.

Now, we consider the movement of equilibrium points during the variety of density. Figure 6 illustrates the locations of projections of equilibrium points in the equatorial plane during the variety of density. In Figure 6, two

cases are plotted, one is
$$\begin{cases} \omega = 1.0\omega_0 \\ \rho = 1.1\rho_0 \end{cases}$$
, the other one is

 $\begin{cases} \omega = 1.0\omega_0 \\ \rho = 0.9\rho_0 \end{cases}$. For both of these two cases, the locations of equilibrium points has not so much change with the case of $\begin{cases} \omega = 1.0\omega_0 \\ \rho = 1.0\rho_0 \end{cases}$. Table 3 presents the topological cases



Figure 5. The locations of projections of equilibrium points in the equatorial plane during the variety of rotation speed. (a) $\omega = 0.5\omega_0$, $\rho = 1.0\rho_0$; (b) $\omega = 2.0\omega_0$, $\rho = 1.0\rho_0$; (c) $\omega = 1.908\omega_0$, $\rho = 1.0\rho_0$

Table 3. Topological cases of the equilibrium points around 22 Kalliope with different parameters

Equilibrium Points	$\omega = 1.0 \omega_0, \rho = 1.0 \rho_0$	ω =0.5 ω_0 , ρ =1.0 ρ_0	$\omega = 1.0 \omega_0, \rho = 0.9 \rho_0$	$\omega = 1.0 \omega_0, \rho = 1.1 \rho_0$
E1	Case 2	Case 2	Case 2	Case 2
E2	Case 5	Case 1	Case 5	Case 5
E3	Case 2	Case 2	Case 2	Case 2
E4	Case 5	Case 1	Case 5	Case 5
E5	Case 1	Case 1	Case 1	Case 1

of the equilibrium points around Kalliope with different parameters. From Table 3, one can see that the two cases of density variety have no influence on the topological cases of the equilibrium points. In Table 3, there are three different topological cases, more detailed topological cases about minor celestial bodies can be found in Wang *et al.* (2014).

When $\begin{cases} \omega = 1.0\omega_0 \\ \rho = 1.0\rho_0 \end{cases}$, the topological cases for E1-E5 are Case 2, Case 5, Case 2, Case 5, and Case 1 by turn. When $\begin{cases} \omega = 1.0\omega_0 \\ \rho = 1.1\rho_0 \end{cases} \text{ and } \begin{cases} \omega = 1.0\omega_0 \\ \rho = 0.9\rho_0 \end{cases}$, the number of equilibrium points is also five, and the topological cases for E1-E5 have no change with the parameters of $\begin{cases} \omega = 1.0\omega_0 \\ \rho = 1.0\rho_0 \end{cases}$

However, when $\begin{cases} \omega = 1.908\omega_0\\ \rho = 1.0\rho_0 \end{cases}$, two equilibrium points, E3 and E5 collide; when $\begin{cases} \omega > 1.908\omega_0\\ \rho = 1.0\rho_0 \end{cases}$, E3 and E5 are

annihilated, and there are only three equilibrium points left, which are E1, E2, and E4. Let $\rho = 1.0\rho_0$, and ω change from $\omega = 1.0\omega_0$ to $\omega = 2.0\omega_0$, then the topological cases of equilibrium points remains no changed, only the number of equilibrium points changed from 5 to 3. Before the collision of E3 and E5, E3 belongs to Case 2 while E5 belongs to Case 1, which means the Saddle-Node bifurcation occurs during the collision.

Let $\rho = 1.0\rho_0$, and ω changes from $\omega = 1.0\omega_0$ to $0.5\omega_0$, then the number of equilibrium points remains unchanged, but the topological cases of two equilibrium points changed. The topological cases of E2 and E4 change from Case 5 to Case 1, which mean that the Hopf bifurcations occurs during the variety from $\omega = 1.0\omega_0$ to $0.5\omega_0$.

4 Orbits in the Potential of Kalliope

To study the orbit stability, we calculated several periodic orbits with different shape and stability. We chose one unstable periodic orbit, and gave a small perturbation to get a new orbit; then we integrated the orbit, and showed the orbit relative to the body-fixed frame and inertia system to help understand the stability mechanism of orbits around Kalliope.

4.1 Periodic Orbits

The dynamical equation can be expressed in the form of

$$\dot{\boldsymbol{X}} = \boldsymbol{f}\left(\boldsymbol{X}\right), \qquad (8)$$

where **X** represents the position and velocity of a particle in the body-fixed frame of the asteroid. Let $\nabla f := \frac{\partial f(z)}{\partial z}$, p is a periodic orbit, then the state transition matrix of the



Figure 6. The locations of projections of equilibrium points in the equatorial plane during the variety of density. (a) $\omega = 1.0 \omega_0$, $\rho = 1.1 \rho_0$; (b) $\omega = 1.0 \omega_0, \rho = 0.9 \rho_0$

periodic orbit can be written as

$$\Phi(t) = \int_{0}^{t} \frac{\partial f}{\partial z}(p(\tau)) d\tau, \qquad (9)$$

and the monodromy matrix of the periodic orbit reads

$$M = \Phi(T) \,. \tag{10}$$

Periodic Orbits	Positions	Velocities	Periods
1	-0.756782677849E+00	-0.288331158837E+01	2.01130
	-0.293991497801E+00	0.601757700255E+01	
	-0.246728394912E-01	0.375001934784E+01	
2	-0.867816335357E+00	0.218894918579E+01	2.00324
	0.439533548181E+00	0.613667996010E+01	
	0.251267167656E+00	-0.320140007283E+01	
3	0.774918083694E-02	-0.657493241866E+01	2.01795
	-0.783732336822E+00	0.714382217819E+00	
	-0.540864433670E-01	-0.375947419773E+01	

Table 4. The initial positions and velocities of periodic orbits in the body-fixed frame of Kalliope

Table 5. Floquet multipliers of periodic orbits in the potential of Kalliope

Orbits	λ_1, λ_2	λ_3, λ_4	λ_5, λ_6
1	0.97928002656787±0.20251805762761i	1.51531143129183, 0.65993028255734	1, 1
2	0.82144085002134± 0.57029328425729i	1.42824105924636, 0.70016180059605	1, 1
3	0.02010082348863±0.99979858341518i	1, 1	1, 1

Eigenvalues of the matrix M, which are also the Floquet multipliers of the periodic orbit p, can determine the stability of this orbit (Ni *et al.* 2016). The periodic orbit p has 6 Floquet multipliers, at least two of them are equal to 1.

The hierarchical grid searching method can be applied to calculate periodic orbits around asteroids (Yu and Baoyin 2012b). The method defines a section plane, which is perpendicular with the periodic orbit, to help to search the periodic orbit. The periodic orbits are calculated through five parameters, including the Jacobian constant *I*, the location (u, v) of the periodic orbit intersects at the section plane, as well as the azimuthal angle(α , β) of the section plane in the body-fixed frame. With this method, we calculated three 2:1 resonant periodic orbits. The 2:1 resonant means that the ratio of the period of the periodic orbit and the rotation period of the asteroid is 2:1. Figure 6 shows these three periodic orbits in the potential of the asteroid Kalliope. Define the length unit for motion around Kalliope to be 191.942955 km, and the time unit to be 4.148 h. Table 4 gives the initial positions and velocities of these periodic orbits in the body-fixed frame of Kalliope. The time unit to be 4.148 h, thus the period of periodic orbit 1 in Table 4 is 4.148h×2.01130=8.3428724h. Table 5 presents Floquet multipliers of these periodic orbits.

From Table 5 and the distribution of Floquet multipliers showed in Figure 7, one can see that the third periodic orbit has 4 Floquet multipliers equal to 1, there are other two Floquet multipliers in the unit circle. Although the geometrical shapes of the first and second periodic orbits are different, the distributions of Floquet multipliers of them are the same. Both of them have two Floquet multipliers in the unit circle, and two Floquet multipliers in the x axis. From the distribution of Floquet multipliers, we know that the periodic orbits in Figure 7(a) and 7(b) are unstable, while the periodic orbit in Figure 7(c) is stable. Thus, one can conclude that the unstable resonant periodic orbits and stable resonant periodic orbits are coexisting in the potential of Kalliope.

4.2 Perturbation of the Periodic Orbit

In this section, we consider the perturbation of the periodic orbit. We choose the first periodic orbit in the above section to study. The perturbation of this unstable periodic orbit in the potential of the asteroid Kalliope is shown in Figure 8.

The initial position and velocity of the particle is set to be

 $\begin{cases} \boldsymbol{R} = [-0.75674687339285 \\ -0.29397758864848 \\ -0.02467167218436]^{\mathrm{T}} \\ \boldsymbol{V} = [-2.88317517483105 \\ 6.01729230249248 \\ 3.74984192913419]^{\mathrm{T}} \end{cases}$

The initial position and velocity of the particle is generated by the periodic orbit 1 in Table 4. We first generate a uniformly distributed random number in [-1, 1]. Denote the random number as *Rand* (k), where k represents the k-th generation. Then we use the following equations to generate the initial position and velocity of a new orbit.

$$\begin{cases} \mathbf{R}(k) = \mathbf{R}(k) \cdot (1.0 + 0.001 \cdot Rand(k)) \\ \mathbf{V}(k) = \mathbf{V}(k) \cdot (1.0 + 0.001 \cdot Rand(k+3)) \end{cases}$$
 (11)



Figure 7. Periodic orbits in the potential of the asteroid Kalliope in the body-fixed frame. (a) 3D plot of the first unstable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (b) 3D plot of the second unstable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distribution of Floquet multipliers; (c) 3D plot of a stable periodic orbit with 2:1 resonance and the distributi



Figure 8. Perturbation of a periodic orbit in the potential of the asteroid Kalliope. (a) 3D plot of the orbit in the body-fixed frame. (b) 3D plot of the orbit in the inertia system. (c) Mechanical energy of a particle in the periodic orbit. (d) Jacobian of a particle in the periodic orbit. The unit of time axis is rotation period of the asteroid

The totally integral time of the orbit we calculated is 746640s, which equals 25 times of the orbit period of the periodic orbit 1. From Figure 8, one can see that the shape of the orbit in the body-fixed frame is different from the shape of it in the inertia system. From the 3D plot of the orbit in the inertial system, we know the orbit is in a strong perturbed environment. The mechanical energy has quasi-periodic variety, while the Jacobian is conservative. Compared Figure 8(a) with Figure 7(a), one can see that the approximate shape of these two orbits looks like when the integration time is small. This is because the orbit in Figure 8(a) is generated by a small error of initial values from the orbit in Figure 7(a). However, the orbit variety in Figure 8(a) increases fast, this indicates that the orbit is unstable, which also illustrates the instability of the periodic orbit in Figure 7(a) in a new point of view.

From Figure 8(c), one can see that the mechanical energy changes suddenly when the time goes through an orbit period of periodic orbit 1. Thus, the mechanical energy has 25 sudden changes. Between the adjacent two sudden changes of the mechanical energy, the time continues about 8.3h, which is about 2 times of the rotation period of the asteroid. This implies the perturbed orbit is also a 2:1 resonant orbit, although it is not a periodic orbit. In addition, to consider the value of the mechanical energy, one can see

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that the mechanical energy has five periods, each period equal about 10 times of the rotation period of the asteroid.

5 Conclusions

In this paper, we stated the moments of inertia, zerovelocity surfaces, equilibrium points of Kalliope. Resonant periodic orbits and perturbations of the periodic orbit in the potential of Kalliope are also investigated. There are five equilibrium points around Kalliope, four of them are outside and unstable, the other one is inside and linearly stable. By changing the rotation speed and density of the body, we studied the movement of equilibrium points.

Let the density unchanged, and the rotation speed vary change from $\omega = 1.0\omega_0$ to $2.0\omega_0$, then two equilibrium points E3 and E5 will collide and annihilate when the rotation speed $\omega = 1.908\omega_0$. The number of equilibrium points change from five to three. The Saddle-Node bifurcation occurs during the collision of E3 and E5. Let ω change from $\omega = 1.0\omega_0$ to $0.5\omega_0$, then the Hopf bifurcations of equilibrium points E2 and E4 occurs, and the topological cases of E2 and E4 change from Case 5 to Case 1. Let the rotation speed unchanged, and the density vary from $\rho = 0.9\rho_0$ to $\rho = 1.1\rho_0$; then the number and topological cases of equilibrium points remains unchanged; however, the locations of equilibrium points vary.

Three resonant periodic orbits are calculated around Kalliope. The unstable resonant periodic orbits and stable resonant periodic orbits are coexisting in the potential of Kalliope. We choose an unstable resonant periodic orbit to study the motion of the orbit with perturbation. The orbit with a small perturbation is no longer a periodic orbit. It is not a closed orbit, but also a 2:1 resonant orbit. The figure of the orbit indicates that the gravitational field of Kalliope is strongly perturbed. The Jacobian of the orbit has a small variation relative to a constant while the mechanical energy of the orbit varies quasi-periodic.

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