

Mathematical Model of Vane Compressors for Computer Simulation of Automotive Air Conditioning Cycle*

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Computer simulation of refrigerating cycles for automotive air conditioners is effective in predicting performance of the cycle. A proper mathematical model of a compressor is necessary to predict the performance exactly. In this study, the mathematical model of a vane compressor for the automotive air conditioning cycle is developed. The model consists of two control volumes, a cylinder and a rear case, and a compressor body. It takes account of the thermal effect of lubricating oil and heat transfer between refrigerant and the compressor body. Validity of the model is confirmed by comparison between experimental results and calculated ones. The transient behavior in the cycle simulation depends on the compressor modeling noticeably.

Key Words: Compressor, Refrigeration, Air Conditioning, Vane Compressor, Simulation, Mathematical Model, Transient Behavior

1. Introduction

Computer simulation of vapor compression refrigerating cycles such as in automotive air conditioners is effective in predicting performance of the cycle, and several simulation models have been developed^{(1)–(6)}. Most of these conventional simulations, however, used an ideal or very simple compressor model since they mainly focused on a modeling of heat exchangers. To predict the performance of the cycle exactly, a proper mathematical model of the compressor is needed which takes account of heat transfer between refrigerant and the compressor body^{(4)–(6)} and influences of lubricating oil such as the thermal effect^{(7),(8)}.

In this study, a mathematical model of a vane compressor for the automotive air conditioning cycle

is developed. It takes account of the heat transfer in the compressor and the thermal effect of lubricating oil, and it can easily be integrated into the cycle simulation. Validity of the model is confirmed by comparisons between experimental results and calculated ones. By integration of two types of compressor model into the cycle simulations, the influence of propriety of the compressor modeling on that result is discussed.

2. Theoretical Analysis

2.1 Outline of compressor

Figure 1 shows a schematic view of the vane compressor which is the subject of modeling in this study. The lubricating oils from two sources, one of which is sucked in together with refrigerant gas and the other of which leaks from sliding portions into the cylinder chamber of the compressor, mix with the refrigerant gas in the cylinder. The mixture of refrigerant gas and oil is compressed in the cylinder, and discharged into a rear case through a discharge valve. After the mixture is separated in the rear case, the refrigerant gas is delivered into a condenser while the oil is fed back into the compressor for lubrication and

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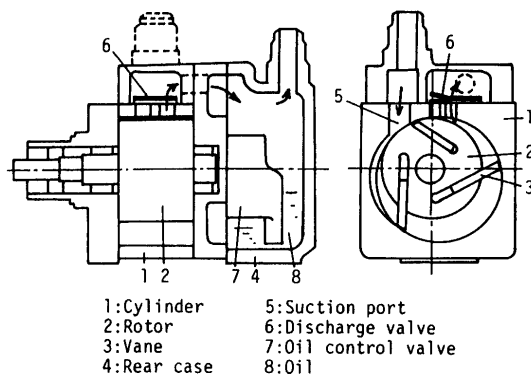


Fig. 1 Schematic view of vane compressor

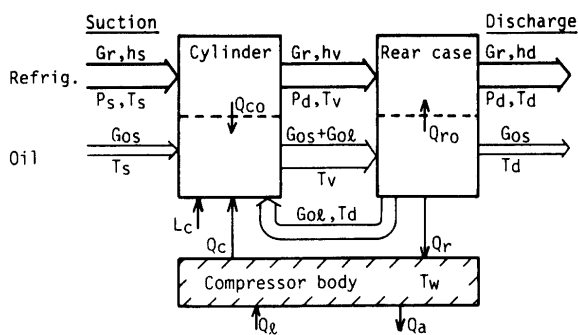


Fig. 2 Physical model of vane compressor

seal.

2.2 Compressor model

Figure 2 shows a physical model of the vane compressor shown in Fig. 1. The model consists of two control volumes (i.e., the cylinder chamber and the rear case chamber) and a compressor body. We neglect the change of mass of refrigerant gas existing in the control volumes since it is small compared with mass flow rate in and out through the control volumes. It is assumed that temperatures of the refrigerant and the oil change homogeneously.

2.2.1 Cylinder In the cylinder, the suction refrigerant (mass flow rate: G_r , pressure: P_s , temperature: T_s), the oil which circulates in the cycle with the refrigerant (mass flow rate: G_{os} , temperature: T_s) and the oil which leaks into compression chambers (mass flow rate: G_{ol} , temperature: T_d) receive compression power, L_c , and heat flow, Q_c , from the compressor body, and they are discharged into the rear case in uniform conditions (pressure: P_d , temperature: T_v). The equation of energy for the refrigerant in the cylinder is expressed as follows:

$$h_v = h_s + (L_c + Q_c - Q_{co})/G_r \quad (1)$$

where h_s : specific enthalpy of the suction refrigerant, h_v : specific enthalpy of refrigerant at an exit of the cylinder. Q_{co} is heat flow transferred from the refrigerant to the oil in the cylinder and expressed as

follows:

$$Q_{co} = c_o [G_{os}(T_v - T_s) + G_{ol}(T_v - T_d)] \quad (2)$$

where c_o is specific heat capacity of the oil. In Eq. (1), compression power, L_c , is derived from the following equations:

$$\begin{aligned} L_c &= \eta_m L_{in} = \eta_m (G_r \Delta h / \eta_t) \\ &= \eta_m (\eta_v V_{th} N / v_s) (\Delta h / \eta_t) \end{aligned} \quad (3)$$

where L_{in} : shaft input power, Δh : enthalpy difference when refrigerant is compressed isentropically, V_{th} : theoretical suction volume, N : rotational speed of the compressor, v_s : specific volume of the suction refrigerant, η_m : mechanical efficiency, η_t : total efficiency and η_v : volumetric efficiency. On the other hand, the heat flow, Q_c , is derived from the following equation:

$$Q_c = \alpha_c A_c (T_w - T_{cr}) \quad (4)$$

where α_c : heat transfer coefficient in the cylinder, A_c : heat transfer area in the cylinder, T_w and T_{cr} : representative temperatures of the compressor body and the refrigerant in the cylinder. Substituting Eqs. (2) - (4) into Eq. (1), we obtain an equation for the discharge temperature, T_v , and the enthalpy, h_v , at the exit of the cylinder and solve the equation by using a thermophysical correlation between temperature and enthalpy. If the discharge temperature is lower than saturation temperature at discharge pressure, the discharge condition is regarded as a wet one. Then the discharge temperature is made equal to the saturation temperature, and the quality of the discharge refrigerant at the exit of the cylinder is calculated from the enthalpy, h_v , derived from Eqs. (1) - (4).

2.2.2 Rear case In the rear case, the refrigerant and the oil release heat flow, Q_r , to the compressor body and exchange heat flow, Q_{ro} , with each other. Equations of energy for the refrigerant and the heat flows in the rear case are expressed as follows:

$$h_d = h_v - (Q_r - Q_{ro})/G_r \quad (5)$$

$$Q_r = \alpha_r A_r (T_{rr} - T_w) \quad (6)$$

$$Q_{ro} = C_o (G_{os} + G_{ol}) (T_v - T_d) \quad (7)$$

where h_d : specific enthalpy of discharge refrigerant, α_r : heat transfer coefficient in the rear case, A_r : heat transfer area in the rear case, T_{rr} : representative temperature of the refrigerant in the rear case. Equations (5) - (7) are solved in the same manner as Eqs. (1) - (4).

2.2.3 Compressor body Assuming that the compressor body has a uniform temperature, time derivative of representative temperature, T_w , of the compressor body is expressed as follows by taking account of the energy balance:

$$dT_w/dt = (Q_i - Q_c + Q_r - Q_a)/C \quad (8)$$

where t : time, C : overall heat capacity of the compressor body, Q_i : heat flow equivalent to loss power, Q_a : heat flow released to ambient air. The heat flows

are expressed as follows :

$$Q_l = L_{in}(1 - \eta_m) \quad (9)$$

$$Q_a = \alpha_a A_a (T_w - T_a) \quad (10)$$

where α_a and A_a : heat transfer coefficient and heat transfer area on outer wall of the compressor, T_a : ambient temperature.

2.3 Cylinder-rear case united model

In the above section, we describe the model with two control volumes (cylinder chamber and rear case chamber). On the other hand, a model which treats the cylinder chamber and the rear case chamber as one control volume is also derived. In the following, the former is referred to as a separated model and the latter is referred to as a united model. In the united model, heat transfer in the cylinder chamber and the rear case chamber with the oil which leaks into the cylinder from the sliding portion cancel each other. This is because the oil which leaks into the cylinder at discharge temperature exchanges the heat flow in the cylinder and the rear case and returns to the discharge temperature. The suction refrigerant and the suction oil receive the compression power and the heat flow from the compressor body, and they are discharged from the compressor at the discharge temperature, T_d , (enthalpy, h_d). Equations corresponding to Eqs. (1) and (2) are expressed as follows.

$$h_d = h_s + (L_c + Q_c - Q'_{co})/G_r \quad (1)'$$

$$Q'_{co} = c_o G_{os} (T_d - T_s) \quad (2)'$$

Time derivative of the representative temperature of the compressor body is expressed as follows.

$$dT_w/dt = (Q_l - Q_c - Q_a)/C \quad (8)'$$

In the above equations, the compression power, L_c , the heat flows, Q_c , Q_l and Q_a , and the mass flow rate, G_r , are derived in the same way as in section 2.2.

2.4 Simulation model of refrigerating cycle

In order to discuss the influence of the propriety of the compressor modeling on transient behavior of a refrigerating cycle, we integrate the compressor model into a simple simulation of the cycle. The simulation model using in this study consists of models of compressor, condenser, evaporator, expansion valve and line. See Ref. (9) in detail.

3. Experiment

Figure 3 shows a schematic view of the experimental setup. The vane compressor is connected to an experimental refrigerating cycle which consists of a double tube condenser, a double tube evaporator, a manual expansion valve and a rotameter. The working fluid of the cycle is CFC-12. The compressor is driven by an inverter-controlled induction motor, and a torque meter is inserted between them. Pressure transducers are mounted on suction and discharge lines. Temperatures of refrigerant in the compressor

are measured at a suction side, the exit of the cylinder and the exit of the rear case by thermocouples. Temperatures of the compressor body are measured at four points, three points on the cylinder wall and one point on the rear case, by the thermocouples. Heat load of the condenser (cold water) is controlled by an automatic water valve and that of the evaporator (hot water) is controlled by a manual one.

In experiments, pressures of suction and discharge, temperatures of the refrigerant and temperatures of the compressor body are recorded from start-up to steady-state operation of the compressor. Performance of the compressor and concentration of oil circulating in the cycle under steady-state operation are also measured at several rotational speeds. Specifications of the compressor are: theoretical suction volume: $V_{th} = 115 \text{ cm}^3$, overall heat capacity of the compressor body: $C = 2.4 \text{ kJ/K}$.

4. Results and Discussion

4.1 Experimental results

Figure 4 shows the transient behavior of the compressor measured under start-up operation at 1000 rpm. In this figure, symbols are; T_s : suction temperature, P_s : suction pressure (T'_s : saturated suction temperature), P_d : discharge pressure (T'_d :

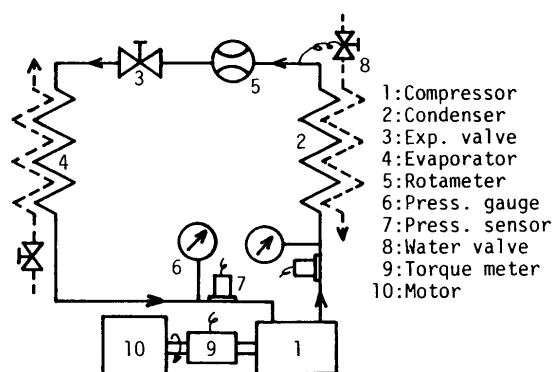


Fig. 3 Experimental setup

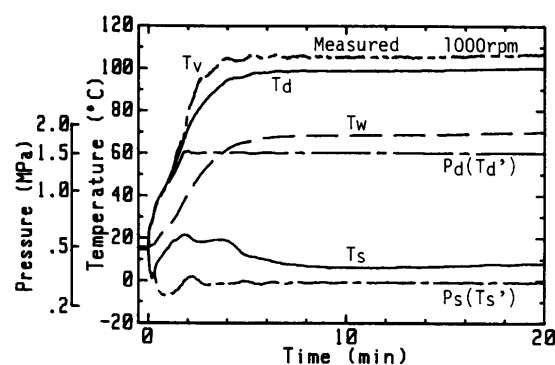


Fig. 4 Transient behavior measured at start-up operation

saturated discharge temperature), T_v and T_d : discharge temperatures at the exits of the cylinder and the rear case, T_w : temperature of the compressor body, which is an average of temperatures at four points where the thermocouples are mounted. During the first few minutes after the start-up, the condition of the discharge refrigerant is a saturated one ($T_v = T_d = T_d'$) though the suction refrigerant is superheated ($T_s > T_s'$), since the refrigerant is cooled by the cold compressor body. After this period, as the temperature of the compressor body rises, the refrigerant becomes superheated ($T_v > T_d'$) at the exit of the cylinder. The discharge pressure suddenly reaches a stable condition two minutes after the start-up. This is because the heat load of the condenser is controlled by the automatic water valve.

4.2 Calculating parameters

We need to consider compressor performance and the heat transfer coefficient for calculation with the compressor model. We, therefore, discuss the calculating parameters in the following.

4.2.1 Compressor performance For values of the volumetric efficiency, η_v , the mechanical efficiency, η_m and the total efficiency, η_t , we use values measured at standard operating condition at each rotational speed. The circulating oil concentration is set at 2% of the mass flow rate of refrigerant, based on a sampling measurement. The mass flow rate of the leakage oil is assumed to be 5 g/s, based on an analysis of leakage flow through the clearance between a rotor and a side plate⁽¹⁰⁾.

4.2.2 Heat transfer capacity Concerning the heat transfer in the compressor, it is difficult to determine the heat transfer area and the heat transfer coefficient due to the complicated flow path and the existence of oil. Thus, we use the product of the heat transfer coefficient, α , and the heat transfer area, A ,⁽⁵⁾ which is referred to as heat transfer capacity, αA , in the following. The values of the heat transfer capacity are obtained from Eqs. (4), (6) and (10) by substituting the heat flows, Q_c , Q_r and Q_a , which are estimated from Eqs. (1), (5) and (8) by using the performance measured under standard operating conditions.

The values depend on the representative temperatures of the compressor and the refrigerant. Figure 5 shows changes in the suction temperature, T_s , the discharge temperature at the exit of the cylinder, T_v , the temperature of the compressor body (average of the temperatures at four measuring points), T_w , which are shown in Fig.4, the temperature of the state after the refrigerant and the oil receive the compression power, T_c (referred to as the temperature after compression), and the mean value of the suction

temperature and the temperature after compression, $(T_s + T_c)/2$. Comparing the discharge temperature at the exit of the cylinder, T_v , with the temperature after compression, T_c , it is found that the refrigerant is cooled by the cold compressor body just after the start-up ($T_c > T_v$), and conversely heated by the compressor body mainly in the suction process at steady-state operation ($T_c < T_v$). The relationship between the mean temperature, $(T_s + T_c)/2$, and the temperature of the compressor body, T_w , shown in Fig.5 satisfies the relationship of the heat flows described in the above. They, therefore, can reasonably be used as the representative temperatures of the refrigerant in the cylinder, T_{cr} , and the compressor body, T_w , respectively. In the rear case chamber, the difference between temperatures at the exits of the cylinder and the rear case is small, and we use the discharge temperature at the exit of the cylinder, T_v , as the representative temperature of the refrigerant in the rear case, T_{rr} .

Figure 6 shows the heat transfer capacity in the cylinder, in the rear case and on the outer wall of the compressor. The heat transfer capacity in the cylinder increases with rotational speed due to the influence of flow velocity. On the other hand, the value in the rear case is almost constant because the heat

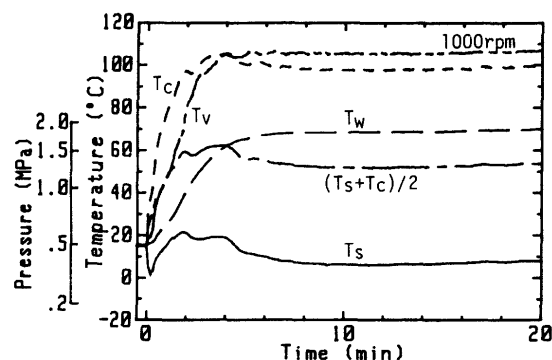


Fig. 5 Representative temperatures of refrigerant and compressor body

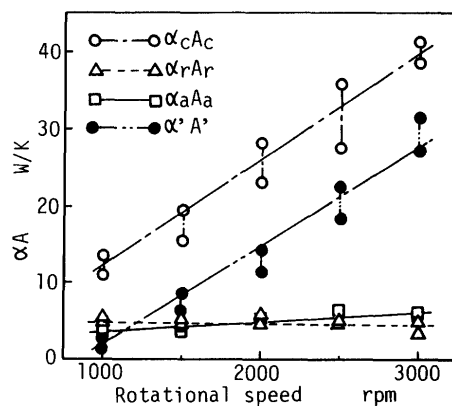


Fig. 6 Heat-transfer capacity

transfer in the rear case is governed by oil scattering in the rear case. The heat transfer capacity on the outer wall of the compressor is also constant. The reference value of the heat transfer capacity, $\alpha'A'$, in the case of the cylinder-rear case united model is smaller than that of the separated model, since the heat flow corresponding to the temperature difference at the exits of the cylinder and the rear case is ignored. In the case taking no account of the thermal effect of oil, moreover, the heat transfer capacity (not shown in Fig.6) is smaller than that in the case taking account of the oil influence, because the heat flow for temperature change of oil is neglected.

4.3 Calculated results

Figures 7 - 10 show calculated results corresponding to the experimental result in Fig. 4. They are calculated from the mathematical model of the compressor by using the experimental values of suction and discharge pressures, P_s and P_d , and suction temperature, T_s , in Fig. 4 as boundary condition. Figure 7 is the result which does not take the influence of oil into consideration. The calculated discharge temperatures, T_v and T_d , after the start-up in Fig. 7 are extremely high as compared with the measured ones due to the small heat-transfer capacity. Figure 8 is the result calculated with the united model. The calculated discharge temperatures after the start-up

are also high and the temperature change of the compressor body is quite different from the measured ones. These models are not suitable for expressing the transient behavior of the compressor after the start-up.

Figure 9 is the result calculated from the separated model. Though the calculated discharge temperatures rise almost in the same manner as the measured ones, they are somewhat higher just after the start-up. This is because a large quantity of oil circulates with refrigerant in a practical cycle due to a foaming phenomenon under the start-up operation, and the practical discharge temperatures are lowered by this oil. Since the foaming phenomenon is active at the wet discharge condition⁽¹¹⁾, therefore, Fig. 10 is the result under the assumption that the circulating oil concentration is high (30%) when the discharge condition is saturated and it decreases toward a stable condition (2%) in a certain time (2 minutes at 1000 rpm) from when the discharge condition begins to be superheated. In this case the calculated temperatures are in good agreement with the measured ones (Fig. 4) from the start-up to the stable operation. A good agreement between an experimental result and a calculated one under higher rotational speed is also obtained by taking account of the transient concentration of the circulating oil (Figures are not shown). To

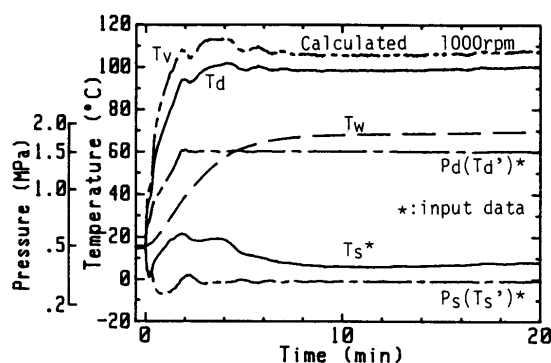


Fig. 7 Calculated result with no heat exchange with oil

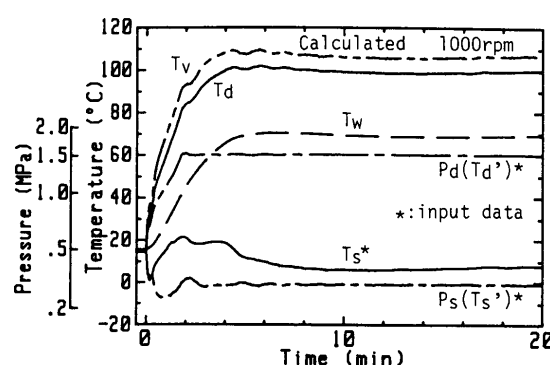


Fig. 9 Calculated result with constant oil ratio

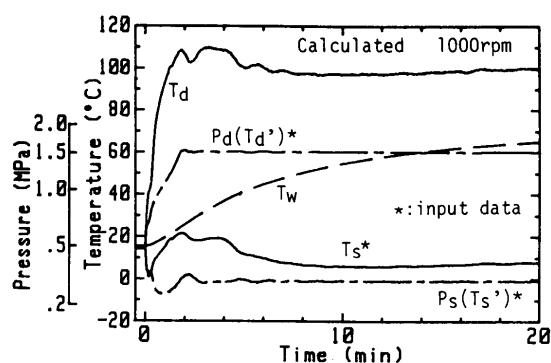


Fig. 8 Calculated result with cylinder-rear case united model

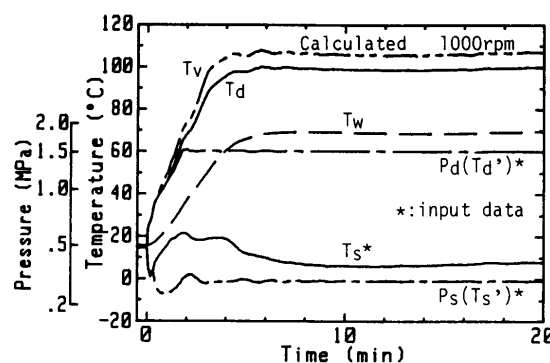


Fig. 10 Calculated result with variable oil ratio

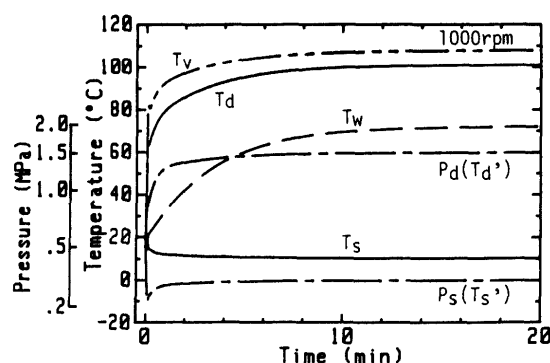


Fig. 11 Result of cycle simulation calculated with no heat exchange with oil

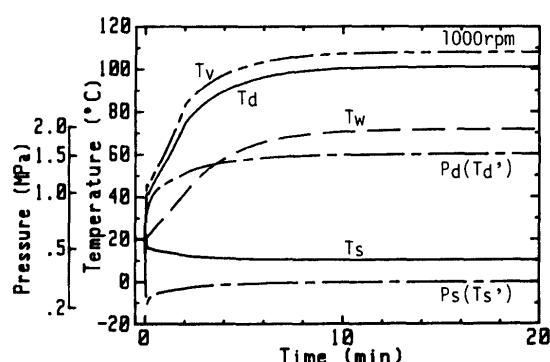


Fig. 12 Result of cycle simulation calculated with variable oil ratio

predict the transient behavior correctly after the start-up, therefore, it is necessary to estimate accurately the transient concentration of the circulating oil as well as the heat transfer phenomenon.

4.4 Results of cycle simulation

In this section we compare results of the cycle simulations⁽⁹⁾ in which different types of compressor models are integrated, and discuss the influence of the compressor modeling on the transient behavior of the cycle. Figure 11 is a result of the cycle simulation whose compressor model takes no account of the influence of oil, and Fig. 12 is a result calculated by using the same compressor model as that used in Fig. 10. Specifications of the cycle employed in the simulation are those for a small-class passenger car, and the opening of the expansion valve is constant. Similar to the results calculated by the compressor model only (Fig. 7 and 10), the discharge temperature in Fig. 11 rises steeply after the start-up as compared with that in Fig. 12. Discharge pressure in Fig. 11 also rises steeply, reflecting the higher discharge temperature. As shown in these figures, the transient behavior in the cycle simulation depends on the compressor modeling noticeably. Pressures in Fig. 11 and 12 change more steeply than that in the experimental result (Fig. 4).

This is because the thermal capacity of air-cooled heat exchangers assumed in the simulation is smaller than that of the double tube heat exchangers used in the experiment. This simulation takes no account of the influence of the oil circulating with refrigerant on heat transfer in the heat exchanger. It is necessary, however, to take account of its influence as well as the influence of the oil in the compressor modeling.

5. Conclusions

A mathematical model of the vane compressor which consists of the cylinder, the rear case and the compressor body is developed in this study. It takes account of the thermal effect of oil and the heat transfer. The calculated results are found to be in good agreement with the experimental ones by using the compressor performance and the heat-transfer capacity estimated from the steady-state operating condition. It is found that estimation of the transient concentration of circulating oil after the start-up is necessary, and the cylinder-rear case united model cannot express the transient behavior of the compressor well. The transient behavior in the cycle simulation depends on the compressor modeling noticeably.

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