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Mathematical Model for Thermal Shock Problem of a Generalized Thermoelastic Layered Composite Material with Variable Thermal Conductivity

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Abstract: One-dimensional generalized thermoelastic mathematical model with variable thermal conductivity for heat conduction problem is constructed for a layered thin plate. The basic equations are transformed by Laplace transform and solved by a direct method. The solution was applied to a plate of sandwich structure, which is thermally shocked, and traction free in the outer sides. The inverses of Laplace transforms are obtained numerically. The temperature, the stress and the displacement distributions are represented graphically.

Key words: thermoelasticity, generalized thermoelasticity, thermal shock, composite materials, variable thermal conductivity

Notation:

λ, μ	– Lame's constants	u_i	– components of displacement vector
ρ	– density	F_i	– body force vector
C_E	– specific heat at constant strain	K	– thermal conductivity
t	– time	κ	– diffusivity
T	– temperature	Q	– heat source
T_0	– reference temperature	q	– heat flux
θ	– temperature increment $\theta = \lvert T - T_0 \rvert$	τ_0	– relaxation time
σ_{ij}	– components of stress tensor	$i = 1, 2, 3$	– the cartesian coordinates
e_{ij}	– components of strain tensor		

1. INTRODUCTION

Lord and Shulman [1] obtained the governing equations of generalized thermoelasticity involving one relaxation time for isotropic homogeneous media, which is called the first generalization to the coupled theory of elasticity. These equations predict finite speeds of propagation of heat and displacement distribution. The corresponding equations for an isotropic case were obtained by Dhaliwal and Sherief [2]. Due to complexity of the governing equations and mathematical difficulties associated with their solution, several simplifications have been used. For example, some authors [3, 4] use the framework of coupled thermoelasticity, where the relaxation time is taken as zero resulting in a parabolic system of partial differential equations. The solution of this system exhibits infinite speed of propagation of heat signals contradictory to physical observation. Some other authors use still further going simplifications by ignoring the inertia effects in a coupled theory [5] or by neglecting the coupling effect.

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Müller [6], in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [7]. Green and Lindsay obtained an explicit version of the constitutive equations in [8]. These equations were also obtained independently by Suhubi [9]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Eraby and Suhubi [10] studied wave propagation in a cylinder. Ignaczak [11] and [12] studied a strong discontinuity wave and obtained a decomposition theorem.

It is usual to assume in thermal stress calculations that material properties are independent of temperature. However, significant variations occur over the working temperature range of the "engineering ceramics", particularly in the coefficient of thermal conductivity, K. Godfrey has reported decreases of up to 45 per cent in the thermal conductivity of various samples of silicon nitride (between 1 and 400°C). The question arises: what are the effects of these variations on the stress and displacement distributions in metal components [13].

Modern structural elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in an approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependent material properties must be taken into consideration in the thermal stress analysis of these elements.

This work deals with a plate consisting of layers of various materials, each of which is homogeneous and isotropic. When this plate, which is initially at rest and having a uniform temperature, is suddenly heated at the free surfaces, a heat flow occurs in the plate and change in thermal and the mechanical field is brought about.

2. THE GOVERNING EQUATIONS

We will consider the thermal conductivity variable as

$$K = K(\theta) = K_0\left(1 + K_1\,\theta\right), \tag{1}$$

where $\theta = (T - T_0)$ such that $|T - T_0|/T_0 << 1$, K is called the thermal conductivity, K_1 is a small value and K_0 is the thermal conductivity when K does not depend on temperature ($K_1 = 0$).

Now, we will consider the mapping

$$\vartheta = \frac{1}{K_0}\int_0^{\theta} K(\theta')\, d\theta', \tag{2}$$

where ϑ is a new function expressing the heat conduction.

By substitution from (1) in (2), we have

$$\vartheta = \frac{1}{K_0}\int_0^{\theta} K_0\left(1 + K_1\,\theta'\right) d\theta' .$$

Then, we have

$$\vartheta = \theta + \frac{K_1}{2}\theta^2 , \tag{3}$$

$$\vartheta_{,i} = \theta_{,i}\left(1 + K_1\,\theta\right), \tag{4}$$

$$K_0\,\vartheta_{,ii} = \left[K\,\theta_{,i}\right]_{,i} \tag{5}$$

and

$$K_0\,\dot{\vartheta} = K\,\dot{\theta} , \tag{6}$$

where $\dot{[*]} = \frac{\partial}{\partial t}[*]$ and $[*]_{,i} = \frac{\partial}{\partial x_i}[*]$.

The heat equation reads

$$\left(K\,\theta_{,i}\right)_{,i} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[\frac{K}{\kappa}\dot{\theta} + \gamma T_0\,\dot{e}\right] \quad i = 1,2,3 , \tag{7}$$

where κ is the diffusivity.

Using Eqs. (5) and (6), the heat equations will take the form

$$\vartheta_{,ii} = \left[\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right]\left[\frac{\vartheta}{\kappa} + \frac{\gamma T_0}{K_0}e\right] \quad i = 1,2,3 . \tag{8}$$

The equations of motion are

$$\rho\,\ddot{u}_i = (\lambda + \mu)\,u_{j,ji} + \mu\,u_{i,jj} - \gamma\,\theta_{,i} + \rho F_i , \tag{9}$$

or we have

$$\rho\,\ddot{u}_i = (\lambda + \mu)\,u_{j,ji} + \mu\,u_{i,jj} - \gamma\frac{\vartheta_{,i}}{\left(1 + K_1\theta\right)} + \rho F_i .$$

By neglecting the small values and the body forces, we arrive at

$$\rho\,\ddot{u}_i = (\lambda + \mu)\,u_{j,ji} + \mu\,u_{i,jj} - \gamma\,\vartheta_{,i} . \tag{10}$$

The constitutive relation

$$\sigma_{ij} = 2\mu e_{ij} + \left(\lambda e_{kk} - \gamma\vartheta\right)\delta_{ij} . \tag{11}$$

After evaluation of ϑ, the temperature increment θ can be obtained by solving equation (3) to give

$$\theta = \frac{-1 + \sqrt{1 + 2K_1\,\vartheta}}{K_1} .$$

3. FORMULATION OF THE PROBLEM

The coordinate system is chosen that the x-axis is taken perpendicularly to the layer, and the y- and z-axes in parallel. We are dealing with one-dimensional generalized thermoelasticity with one relaxation time.

We consider that the displacement components for one dimension medium have the form

$$u_x = u(x,t), \quad u_y = u_z = 0 .$$

The strain components are

$$e = e_{xx} = \frac{\partial u}{\partial x}, \tag{12}$$

$$e_{yy} = e_{zz} = e_{xy} = 0 .$$

The heat equation

$$\frac{\partial^2 \vartheta}{\partial x^2} = \left[\frac{\partial}{\partial \tau} + \tau_0 \frac{\partial^2}{\partial \tau^2} \right] \left[\frac{\vartheta}{\kappa} + \frac{\gamma T_0 e}{K_0} \right]. \tag{13}$$

The constitutive relation takes the form

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \vartheta . \tag{14}$$

The equation of motion

$$\rho \ddot{u} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \vartheta}{\partial x}. \tag{15}$$

For simplicity we use the following non-dimensional variables [14]:

$$x = \left(\frac{\lambda + 2\mu}{\rho} \right)^{1/2} \frac{x'}{\kappa}, \quad u = \left(\frac{\lambda + 2\mu}{\rho} \right)^{1/2} \frac{u'}{\kappa},$$

$$t = \left(\frac{\lambda + 2\mu}{\rho} \right)^{1/2} \frac{t'}{\kappa}, \quad \beta = \left(\frac{\lambda + 2\mu}{\mu} \right)^{1/2},$$

$$\tau_0 = \left(\frac{\lambda + 2\mu}{\rho} \right) \frac{\tau_0'}{\kappa}, \quad \sigma = \frac{\sigma'}{\lambda + 2\mu}, \quad \vartheta = \frac{(3\lambda + 2\mu)\alpha_T}{(\lambda + 2\mu)} \vartheta'.$$

For convenience, we drop the primes.

Hence, we obtain

$$D^2 \vartheta = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) [\vartheta + \varepsilon e], \tag{16}$$

$$\sigma = \frac{\partial u}{\partial x} - \vartheta = e - \vartheta, \tag{17}$$

$$\ddot{u} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial \vartheta}{\partial x} = \frac{\partial e}{\partial x} - \frac{\partial \vartheta}{\partial x}. \tag{18}$$

By eliminating ϑ between Eqs. (17) and (18), we get

$$D^2 \sigma = \ddot{e}, \tag{19}$$

where

$$\varepsilon = \frac{(3\lambda + 2\mu)^2 \alpha_T^2 T_0 \kappa}{(\lambda + 2\mu) K_0}.$$

Taking Laplace transform as define

$$\overline{f}(s) = \int_0^\infty f(t) e^{-st} dt.$$

Then, Eqs. (16), (17) and (19) will take the forms

$$\left[D^2 - \left(s + \tau_0 s^2 \right) \right] \overline{\theta} = \varepsilon h^2 \overline{e}, \tag{20}$$

$$\sigma = \left(\overline{e} - \overline{\theta} \right), \tag{21}$$

$$D^2 \overline{\sigma} = s^2 \overline{e}, \tag{22}$$

where $D = \frac{\partial}{\partial x}$.

By eliminating $\overline{e}$, we get

$$\left[D^2 - s^2 \right] \overline{\sigma} = s^2 \overline{\theta}, \tag{23}$$

$$\left[D^2 - (\varepsilon + 1)\left(s + \tau_0 s^2 \right) \right] \overline{\theta} = \\ = \varepsilon \left(s + \tau_0 s^2 \right) \overline{\sigma}, \tag{24}$$

Using the above two equations, we obtain

$$\left(D^4 - L D^2 + M \right) \overline{\theta} = 0, \tag{25a}$$

$$\left(D^4 - L D^2 + M \right) \overline{\sigma} = 0, \tag{25b}$$

where

$$L = s^2 + \left(s + \tau_0 s^2 \right)(\varepsilon + 1) + \\ + \varepsilon \left(s + \tau_0 s^2 \right) s^2$$

and

$$M = s^2 \left(s + \tau_0 s^2 \right)(1 + \varepsilon).$$

4. APPLICATION

Considering a layered of sandwich structure such as shown in Fig. 1, where layers III and I are made from the same metal, and the layer II is a different metal. Layer II is put in the middle of the plate, and its thickness is a half of that of the plate.

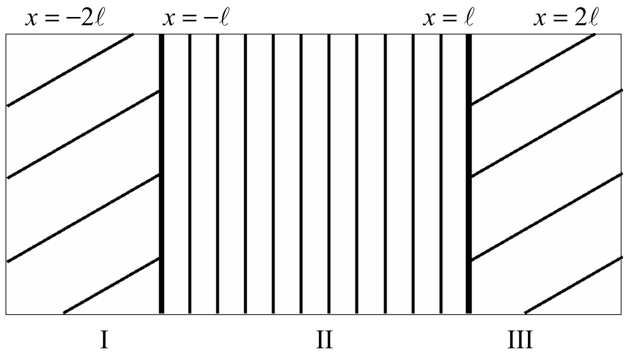

Fig. 1.

1) In region I where $-2\ell \le x \le -\ell$

The solution of the Eqs. (24) and (25) takes the form

$$\bar{\vartheta}^{\mathrm{I}} = A_1\left(k_1^2 - s^2\right)\cosh\left(k_1 x\right) + A_2\left(k_2^2 - s^2\right)\cosh\left(k_2 x\right), \tag{26}$$

$$\bar{\sigma}^{\mathrm{I}} = A_1 s^2 \cosh\left(k_1 x\right) + A_2 s^2 \cosh\left(k_2 x\right), \tag{27}$$

where the parameters k_1 and k_2 satisfy the equation

$$k^4 - L^{\mathrm{I}} k^2 + M^{\mathrm{I}} = 0,$$

where:

$$L^{\mathrm{I}} = s^2 + \left(s + \tau_0^{\mathrm{I}} s^2\right)\left(\varepsilon^{\mathrm{I}} + 1\right) + \varepsilon\left(s + \tau_0^{\mathrm{I}} s^2\right)s^2$$

$$M^{\mathrm{I}} = s^2\left(s + \tau_0^{\mathrm{I}} s^2\right)\left(1 + \varepsilon^{\mathrm{I}}\right).$$

2) In region II where $-\ell \le x \le \ell$

The solution of the Eqs. (24) and (25) takes the form

$$\begin{aligned}\bar{\vartheta}^{\mathrm{II}} &= B_1\left(p_1^2 - s^2\right)\cosh\left(p_1 x\right) + \\ &+ B_2\left(p_2^2 - s^2\right)\cosh\left(p_2 x\right),\end{aligned} \tag{28}$$

$$\bar{\sigma}^{\mathrm{II}} = B_1 s^2 \cosh\left(p_1 x\right) + B_2 s^2 \cosh\left(p_2 x\right), \tag{29}$$

where the parameters p_1 and p_2 satisfy the equation

$$p^4 - L^{\mathrm{II}} p^2 + M^{\mathrm{II}} = 0$$

and

$$L^{\mathrm{II}} = s^2 + \left(s + \tau_0^{\mathrm{II}} s^2\right)\left(\varepsilon^{\mathrm{II}} + 1\right) + \varepsilon\left(s + \tau_0^{\mathrm{II}} s^2\right)s^2$$

$$M^{\mathrm{II}} = s^2\left(s + \tau_0^{\mathrm{II}} s^2\right)\left(1 + \varepsilon^{\mathrm{II}}\right).$$

3) In region III where $\ell \le x \le 2\ell$

The solution of the Eqs. (24) and (25) takes the form

$$\begin{aligned}\bar{\vartheta}^{\mathrm{III}} &= c_1\left(k_1^2 - s^2\right)\cosh\left(k_1 x\right) + \\ &+ c_2\left(k_2^2 - s^2\right)\cosh\left(k_2 x\right),\end{aligned} \tag{30}$$

. $$\bar{\sigma}^{\mathrm{III}} = c_1 s^2 \cosh\left(k_1 x\right) + c_2 s^2 \cosh\left(k_2 x\right). \tag{31}$$

5. THE BOUNDARY CONDITIONS

(1) The thermal boundary conditions

$$\theta = \theta_0 H(t) \quad \text{for} \quad x = \pm 2\ell,$$

which takes the form

$$\bar{\vartheta} = \frac{\vartheta_0}{s} \quad \text{for} \quad x = \pm 2\ell, \tag{32}$$

where $\vartheta_0 = \theta_0 + \frac{K_1}{2}\theta_0^2$.

(2) The mechanical boundary conditions

$$\sigma = 0 \ \text{ for } \ x = \pm 2\ell,$$

which takes the form

$$\bar{\sigma} = 0 \ \text{ for } \ x = \pm 2\ell. \tag{33}$$

(3) The continuity conditions

(i) $\bar{\vartheta}^{\mathrm{I}} = \bar{\vartheta}^{\mathrm{II}}$ at $x = -\ell$, and $\bar{\vartheta}^{\mathrm{II}} = \bar{\vartheta}^{\mathrm{III}}$ at $x = \ell$,

(ii) $\bar{\sigma}^{\mathrm{I}} = \bar{\sigma}^{\mathrm{II}}$ at $x = -\ell$, and $\bar{\sigma}^{\mathrm{II}} = \bar{\sigma}^{\mathrm{III}}$ at $x = \ell$. (34)

Introducing the pervious conditions into Eqs. (14-19), we obtain

$$\begin{aligned}\bar{\vartheta}^{\mathrm{I}} = \bar{\vartheta}^{\mathrm{III}} &= \frac{\vartheta_0}{s\left(k_1^2 - k_2^2\right)} \times \\ &\times \left[\frac{\left(k_1^2 - s^2\right)}{\cosh\left(2\ell k_1\right)}\cosh\left(k_1 x\right) - \frac{\left(k_2^2 - s^2\right)}{\cosh\left(2\ell k_2\right)}\cosh\left(k_2 x\right)\right],\end{aligned} \tag{35}$$

$$\bar{\sigma}^{\mathrm{I}} = \bar{\sigma}^{\mathrm{III}} = \frac{s\vartheta_0}{\left(k_1^2 - k_2^2\right)}\left[\frac{\cosh\left(k_1 x\right)}{\cosh\left(2\ell k_1\right)} - \frac{\cosh\left(k_2 x\right)}{\cosh\left(2\ell k_2\right)}\right], \tag{36}$$

$$\bar{\vartheta}^{\mathrm{II}} = \frac{\left(p_1^2 - s^2\right)\vartheta_0}{2s\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh\left(\ell p_1\right)} \times$$
$$\times \left[\frac{k_1^2 - p_2^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_2^2}{\sinh\left(\ell k_2\right)}\right]\cosh\left(p_1 x\right) +$$
$$- \frac{\left(p_2^2 - s^2\right)\vartheta_0}{2s\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh\left(\ell p_2\right)} \times$$
$$\times \left[\frac{k_1^2 - p_1^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_1^2}{\sinh\left(\ell k_2\right)}\right]\cosh\left(p_2 x\right), \tag{37}$$

$$\bar{\sigma}^{\mathrm{II}} = \frac{s\vartheta_0}{2\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh\left(\ell p_1\right)} \times$$
$$\times \left[\frac{k_1^2 - p_2^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_2^2}{\sinh\left(\ell k_2\right)}\right]\cosh\left(p_1 x\right) +$$
$$- \frac{s\vartheta_0}{2\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh\left(\ell p_2\right)} \times$$
$$\times \left[\frac{k_1^2 - p_1^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_1^2}{\sinh\left(\ell k_2\right)}\right]\cosh\left(p_2 x\right). \tag{38}$$

We can get the displacement by using Eq. (4), such that

$$\bar{u} = \frac{1}{s^2} D\bar{\sigma},$$

so, we obtain

$$\bar{u}^{\mathrm{I}} = \bar{u}^{\mathrm{III}} = \frac{\vartheta_0}{s\left(k_1^2 - k_2^2\right)}\left[\frac{k_1 \sinh\left(k_1 x\right)}{\cosh\left(2\ell k_1\right)} - \frac{k_2 \sinh\left(k_2 x\right)}{\cosh\left(2\ell k_2\right)}\right] \tag{39}$$

and

$$\bar{u}^{\mathrm{II}} = \frac{p_1 \vartheta_0}{2s\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh(\ell p_1)} \times$$
$$\times \left[\frac{k_1^2 - p_2^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_2^2}{\sinh\left(\ell k_2\right)}\right]\sinh\left(p_1 x\right) +$$
$$- \frac{p_2 \vartheta_0}{2s\left(k_1^2 - k_2^2\right)\left(p_1^2 - p_2^2\right)\cosh(\ell p_2)} \times \tag{40}$$
$$\times \left[\frac{k_1^2 - p_1^2}{\sinh\left(\ell k_1\right)} - \frac{k_2^2 - p_1^2}{\sinh\left(\ell k_2\right)}\right]\sinh\left(p_2 x\right).$$

6. THE INVERSION OF THE LAPLACE TRANSFORM

In order to invert the Laplace transform in Eqs. (35-40), we adopt a numerical inversion method based on a Fourier series expansion [15].

By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t) = \frac{e^{ct}}{t_1}\left[\frac{1}{2}\bar{f}(c) + \operatorname{Re}\sum_{k=1}^{N}\bar{f}\left(c + \frac{ik\pi}{t_1}\right)\exp\left(\frac{ik\pi t}{t_1}\right)\right],$$
$$0 < t < 2t,$$

where N is a sufficiently large integer representing the number of terms in the truncated Fourier series, chosen in the form

$$\exp(ct)\operatorname{Re}\left[\bar{f}\left(c + \frac{iN\pi}{t_1}\right)\exp\left(\frac{iN\pi t}{t_1}\right)\right] \le \varepsilon_1,$$

where ε_1 is a prescribed small positive number that corresponds to the degree of accuracy required. The parameter c is a positive free parameter that must be greater than the real part of all the singularities of $\bar{f}(s)$.

The optimal choice of c was obtained according to the criteria described in [15].

7. NUMERICAL RESULTS

The copper material and the type 316 stainless steel are chosen for the purposes of numerical evaluations [16, 17].

Table 1. Materials parameters

Parameter	Copper I, III	Stainless steel, II
α_T	$17.8 \times 10^{-6}\ \mathrm{K}^{-1}$	$17.7 \times 10^{-6}\ \mathrm{K}^{-1}$
ρ	$8954\ \mathrm{kg \cdot m^{-3}}$	$7970\ \mathrm{kg \cdot m^{-3}}$
C_E	$383.1\ \mathrm{m^2 \cdot K^{-1} \cdot s^{-2}}$	$561\ \mathrm{m^2 \cdot K^{-1} \cdot s^{-2}}$
K_0	$386\ \mathrm{kg \cdot m \cdot K^{-1} \cdot s^{-3}}$	$19.5\ \mathrm{kg \cdot m \cdot K^{-1} \cdot s^{-3}}$
T_0	293 K	293 K
μ/λ	0.497425	0.700680
τ_0	0.02 s	0.01 s
ε	0.0150	0.0141
K_1	−0.1	−0.15

The computations were carried out for the value of time $t = 0.3$ and for the length $\ell = 1$ (unit length).

The numerical values of the temperature, displacement and stress are represented graphically, where the solid line in the temperature distribution shows the case when the thermal conductivity is constant, while the doted line illustrates the case of thermal conductivity variable.

8. CONCLUSIONS

1. Figure 2 shows the temperature field distribution with respect to the dimension x and we can see that, for the three layers the temperature increases when the thermal conductivity is constant and decreases when it is variable. The differences between the two plates appears to be more significant in the central layers than in the surface layers.

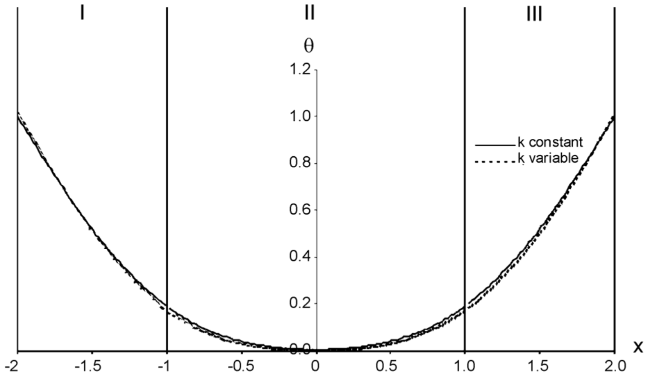

Fig. 2. The temperature distribution

2. Figure 3 shows the displacement field distribution with respect to the dimension x and we can see that, for the three layers the absolute value of the displacement increases when the thermal conductivity is constant and decreases when it is variable.

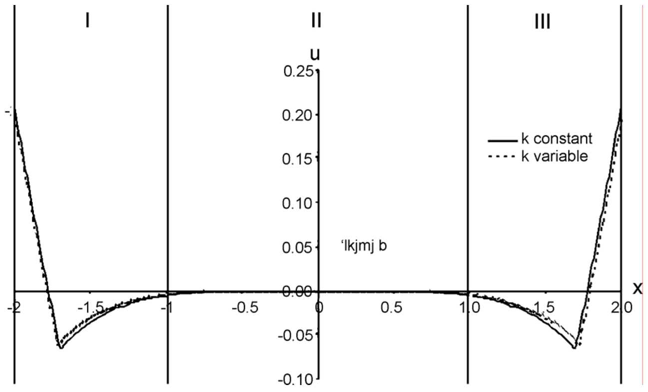

Fig. 3. The displacement distribution

3. Figure 4 shows the stress field distribution with respect to the dimension x and we can see that, for the three layers the absolute value of the displacement increases when the thermal conductivity is constant and decreases when it is variable.

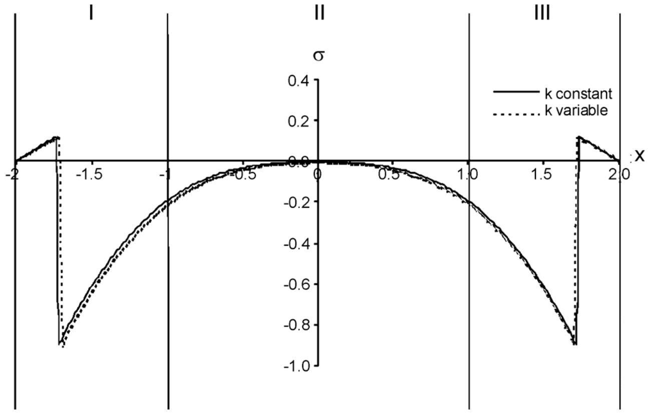

Fig. 4. The stress distribution

The three curves, demonstrate that the effect of the thermal conductivity on all the fields and in different materials is clear and we have to take it into account in any analysis of heat conduction.

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