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*Research Article*

# New Stability Criteria for Event-Triggered Nonlinear Networked Control System with Time Delay

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This note focuses on the stability and stabilization problem of nonlinear networked control system with time delay. To alleviate the burden of transformation channel and shorten the dynamic process simultaneously, an improved event-triggered scheme is proposed. This paper employs an improved time delay method to enhance the performance and reduce the delay upper bound conservatism. Less conservative stability criteria related to the order $N$ are derived by establishing an augmented Lyapunov-Krasovskii functional manufactured for the use of Bessel-Legendre inequality. In addition, an event-triggered controller is designed for nonlinear networked control system with time delay. At last, numerical examples are proposed to verify the effectiveness of the new method.

## 1. Introduction

In recent years, the networked control has been applied to the practical control process [1]. Compared with the point to point control method, the networked control system has better reliability and can reduce power requirements, operation cost [2, 3]. There are different kinds of networked control systems, such as centralized networked control system [4], decentralized networked control system [5], distributed networked control system [6], and wireless networked control system [7]. In practice, the performance of networked control system is always influenced by the uncertainties and disturbance and nonlinear factors. Nonlinear networked control system's asymptotic behavior has been researched in [8]. Stability of nonlinear networked control system has been studied in [9]. The literature [10, 11] have investigated robust stability of nonlinear networked control system with uncertainties. This paper also takes into account nonlinearity to augment the performance of networked control system.

For traditional networked control system [12, 13], transmitting all sampled packets into the network is not always necessary from the point of view of the limited network channel resource under the time-triggered scheme. Thus, the event-triggered scheme (ETS) is proposed to reduce the burden of channel. Under the event-triggered condition, stochastic stability of nonlinear system was studied in [14]. The fault detection issue for nonlinear discrete-time networked systems was discussed in [15]. In recent years, researchers have improved event-triggered scheme continuously to adjust to various networked environments. A periodic ETS was proposed to overcome the shortcoming needing extra hardware to check triggering condition instantaneously [16]. For wireless sensor networks, the decentralized ETS was developed to better save the channel resource [17]. In order to shorten the dynamic process, an improved static ETS was researched in [18] which can increase the frequency of transmission at initial times. Furthermore, dynamic ETS was proposed by introducing a dynamic variable in triggering condition [19]. In this paper, we will put forward a new improved static ETS for the nonlinear networked control system with time delay to decrease the burden of channel and improve system dynamics.

At the same time, time delay problem has been widely investigated in the practical control system [20–24]. Variable time delay problems appear in control system [25–27]. For example, the stability problems of delay neural networks was studied in [28]. Based on the stochastic process, the random delay was researched in [29, 30]. Robust $H_\infty$ stability of time delay system was researched widely [31–33]. Reference [34] studied a network-induced delay to deal with the network transmission delay problem. The distributed delay was developed for a class of neural network control system. However, in this improved event-triggered networked control system, the time delay problems still have a lot of room for improvement [35–38].

For the sake of reducing the conservatism of stability criterion of time delay systems, a series of technical approaches have been proposed. Before listing these approaches, we state that the conservatism of stability criteria mainly results from the estimation gap of the integral terms expressed as $\int_{t-d_2}^{t} \dot{x}^T(s)R\dot{x}(s)ds$ in the derivative of Lyapunov-Krasovskii functional. To study stability of system, model transformation approach was used in [39]. The stability criteria obtained by model transformation approach have large conservatism. To decrease the conservatism of stability criteria, a free weighting matrix method which can remedy the drawback of model transformation was proposed in [40]. However, free weighting matrix method will increase the decision variables, which makes the computation complex. To overcome this point and better estimate the integral terms, the Jensen inequality was used widely [41]. Afterwards, Wirtinger-based inequality which is considered a tighter method than Jensen inequality for estimation of the integral term was developed and employed in various systems [42]. In recent years, many researchers have improved the Wirtinger-based inequality approach, such as free-matrix-based integral inequality [43], auxiliary function-based integral inequality [44]. In this note, we will make use of a new integral inequality called Bessel-Legendre inequality together with reciprocally convex combination lemma to research the stability of nonlinear networked control system with time delay under improved event-triggered scheme.

As is well known, there is a quadratic integral term $\int_{t-d_2}^{t}\int_{\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta$ in Lyapunov-Krasovskii functional, which means that there will be a term $\int_{t-d_2}^{t} \dot{x}^T(s)R\dot{x}(s)ds$ in the derivative of Lyapunov-Krasovskii functional. We apply the Bessel-Legendre inequality to estimate $\int_{t-d_2}^{t} \dot{x}^T(s)R\dot{x}(s)ds$ and obtain that

$$\int_{t-d_2}^{t} \dot{x}^T(s)R\dot{x}(s)ds \geq d_2\phi_N^T\overline{R}_N\phi_N, \tag{1}$$

where $\phi_N = (1/d_2)\int_{-d_2}^{0} \mathbb{L}_N((s+d_2)/d_2)x(s)ds$, $\overline{R}_N = \operatorname{diag}(R, 3R, \ldots, (2N+1)R)$ and $\mathbb{L}_N$ is Legendre polynomial matrix. This inequality provides a tighter bound on this specific term, which makes the obtained stability condition less conservative. In addition, we will construct an appropriate Lyapunov-Krasovskii functional manufactured for the use of Bessle-Legendre inequality.

The main contributions of this paper are summarized as follows. Firstly, an improved ETS is put forward for nonlinear networked control system in this paper to reduce transmission load of channel by decreasing the number of signal transmission. The triggering parameter in this improved scheme is time-varying to achieve the situation that transmission frequency at the beginning instants is higher than at the other times, which can shorten the dynamic process of system effectively. Secondly, less conservative stability criteria subject to the order $N$ are obtained by employing the Bessel-Legendre inequality and introducing a Legendre-based Lyapunov-Krasovskii functional. Conservatism will be reduced with the increase of the $N$. Furthermore, a controller is designed for event-triggered nonlinear networked control system with time delay.

The rest of the paper is summarized as follows. Section 2 gives the considered nonlinear networked control system and puts forward an improved ETS. In Section 3, less conservative stability criteria are derived via Bessel-Legrendre inequality method. Section 4 designs a controller for the system in this paper. To verify the effectiveness of results, numerical examples are shown in Section 5. Finally, conclusions are summarized in Section 6.

Notations: In this paper, symbol $T$ denotes the transpose. The $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ matrices. The set $\mathbb{S}^n(\mathbb{S}_+^n)$ means the set of the symmetric (positive definite) matrices of $\mathbb{R}^{n\times n}$. Furthermore, $He(A) = A + A^T$. For matrices $A \in \mathbb{R}^{u\times v}$ and $B \in \mathbb{R}^{m\times n}$, their Kronecker product is a matrix in $\mathbb{R}^{um\times vn}$ denoted as $A \otimes B$. The $\mathbb{N}$ denotes non-negative integer. $I_n$ denotes the identity matrix with $n\times n$ dimensions. $I_{nN}$ denotes the identity matrix with $n(N+1)\times n(N+1)$ dimensions.

## 2. Problem Formulation

In this paper, it is assumed that the networked control system has nonlinear function which satisfies the Lipschitz condition and the system state is fully observable. Thus, in this section, we establish the system model as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_\tau f(x(t)) + Bu(t), \\ z(t) &= Cx(t), \\ x(t) &= \phi(t), \quad t \in [-d_2, 0), \end{aligned} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state; $z(t) \in \mathbb{R}^q$ is the controlled output; $u(t) \in \mathbb{R}^m$ denotes the control input; $A$, $A_\tau$, $B$, $C$ are real constant matrices; $\phi(t)$ denotes the initial condition function; $d_2$ is a positive scalar. Furthermore, $f(x(t))$ is nonlinear function and satisfies

$$|f(x) - f(y)| \leq |F(x-y)|, \tag{3}$$

where $F$ is a known constant matrix.

In the nonlinear networked control system, there exists a phenomenon of transmitting some unnecessary sampling data during the transmission from the sensor to the controller. In order to improve the networked control system transmission performance, this paper will propose an improved event-triggered mechanism to reduce the load of the network transmission. Next, we will build an event-triggered generator for the nonlinear networked control

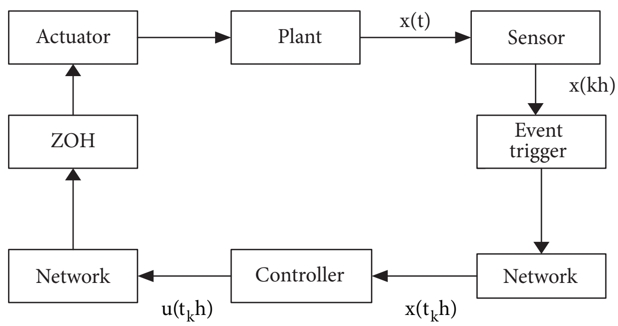


Figure 1: Schematic of networked control system with ETS.

system. It is assumed that the sampling sequence is $\mathbb{S}_s = \{0, h, 2h, \dots, nh\}$. Suppose $t_k h$ is the current released time and $t_{k+1}h$ is the next released time. In addition, $t_{k+1}h = t_k h + nh$, where $nh$ is the release interval of the transmitted data.

Although the data is released at $t_k h$, it will arrive at actuator at $t_k h + \hat{\tau}_k$ instant resulting from the existence of time delay $\hat{\tau}_k \in [0, \overline{\tau})$, $k \in \{0, 1, 2, \cdots\}$, scalar $\overline{\tau} > 0$.

Next, based on [18, 35] and the diagram of event-triggered networked control system in Figure 1, a network time delay model for the nonlinear networked control system can be constructed. Suppose that

$$\rho_k = \min\{j \mid t_k h + \hat{\tau}_k + jh \ge t_{k+1}h + \hat{\tau}_{k+1},\ j = 0, 1, 2, \dots\}. \tag{4}$$

The interval $[t_k h + \hat{\tau}_k, t_{k+1}h + \hat{\tau}_{k+1})$ can be rewritten as

$$[t_k h + \hat{\tau}_k, t_{k+1}h + \hat{\tau}_{k+1}) = \bigcup_{j=1}^{\rho_k} \Pi_j, \tag{5}$$

where

$$\Pi_j = [t_k h + \hat{\tau}_k + (j-1)h, t_k h + \hat{\tau}_k + jh), \quad j = 1, 2, \dots, \rho_k - 1, \tag{6}$$

$$\Pi_{\rho_k} = [t_k h + (\rho_k - 1)h + \hat{\tau}_k, t_{k+1}h + \hat{\tau}_{k+1}).$$

$$d(t) = \begin{cases} t - t_k h, & t \in \Pi_1 \\ t - t_k h - h, & t \in \Pi_2 \\ \vdots & \vdots \\ t - t_k h - (\rho_k - 1)h, & t \in \Pi_{\rho_k} \end{cases} \tag{7}$$

$$e_k(t) = \begin{cases} 0, & t \in \Pi_1 \\ x(t_k h) - x(t_k h + h), & t \in \Pi_2 \\ \vdots & \vdots \\ x(t_k h) - x(t_k h + (\rho_k - 1)h), & t \in \Pi_{\rho_k} \end{cases} \tag{8}$$

where $0 < \hat{\tau}_k \le d(t) \le \overline{\tau} + h$. We set the $d_2 = \overline{\tau} + h$, then $0 < \hat{\tau}_k \le d(t) \le d_2$.

For the $t \in [t_k h + \hat{\tau}_k, t_{k+1}h + \hat{\tau}_{k+1})$, the event-triggered condition is

$$e_k^T(t)\,\Omega e_k(t) \le \sigma_k(t)\, x^T(t - d(t))\,\Omega x(t - d(t)), \tag{9}$$

$\Omega \in \mathbb{S}_+^n$, $\sigma_k(t)$ is a time-varying function as

$$\sigma_k(t) = \sigma_{kj}, \quad t \in \Pi_j, \ j = 1, 2, \dots, \rho_k, \tag{10}$$

where

$$\sigma_{k(j+1)} = \overline{\sigma} + \frac{e_{kj}^T e_{kj}}{\lambda + e_{kj}^T e_{kj}}\left(\sigma_{kj} - \overline{\sigma}\right), \tag{11}$$

$$e_{kj} = x(t_k h) - x(t_k h + (j-1)h), \quad j = 1, 2, \dots, \rho_k, \tag{12}$$

where known constants $\overline{\sigma} \in [0, 1]$, $\lambda > 0$, $0 < \sigma_{k1} < \overline{\sigma}$, and $\sigma_{k1}$ is the initial value of $\sigma_{kj}$. It is easy to see that the $\sigma_k(t)$ is monotonically increasing and has an upper bound $\overline{\sigma}$.

*Remark 1.* The sampled data will be transmitted when condition (9) is not satisfied. It is noticed that the $\sigma_k(t)$ in the improved event-triggered scheme (9) is time-varying and meets (10), which can increase the triggering time at the initial times to optimize the dynamic process of the system in this paper.

Therefore, according to formulae (2)-(9), we have

$$u(t_k h) = Kx(t_k h) = Ke_k(t) + Kx(t - d(t)), \tag{13}$$

where $t \in [t_k h + \hat{\tau}_k, t_{k+1}h + \hat{\tau}_{k+1})$, $K \in \mathbb{R}^{m\times n}$ represents networked controller gain, then the system model can be rewritten as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_\tau Fx(t) + BKx(t - d(t)) \\ &\quad + BKe_k(t), \\ z(t) &= Cx(t), \\ x(t) &= \phi(t), \quad [-d_2, 0), \end{aligned} \tag{14}$$

where

$$\begin{gathered} 0 < \hat{\tau}_k \le d(t) \le d_2, \\ d_m \le \dot{d}(t) \le d_M, \end{gathered} \tag{15}$$

$d_m, d_M$ are known constants, $d_m < 0$, $d_M > 0$.

For analyzing the stability and stabilization problem of nonlinear networked control system conveniently, we will give a definition and some lemmas.

*Definition 2* (see [24]). For given scalars $i, j \in \mathbb{N}$, the Legendre polynomial considered over the interval $\mu \in [0, 1]$ is

$$L_i(\mu) = (-1)^i \sum_{j=0}^{i} p_j^i \mu^j, \tag{16}$$

where $p_j^i = (-1)^j \binom{i}{j}\binom{i+j}{j}$, and $\binom{k}{l}$ means $k!/(k-l)!l!$.

Define that

$$\mathbb{L}_N(\mu) := [L_0(\mu) I_n, \ldots, L_N(\mu) I_n]^T \quad (17)$$

is a polynomial matrix with $(N+1)n \times n$ dimensions, where the integers $N \geq 0$, $n > 0$.

**Lemma 3** (see [28], reciprocally convex inequality). *Let integer $n > 0$ and $R_1$, $R_2$ be in $\mathbb{S}_+^n$. If there exist $X_1$, $X_2$ in $\mathbb{S}^n$ and $Y_1$, $Y_2$ in $\mathbb{R}^{n\times n}$ such that*

$$\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} - \alpha \begin{bmatrix} X_1 & Y_1 \\ Y_1^T & 0 \end{bmatrix} - (1-\alpha) \begin{bmatrix} 0 & Y_2 \\ Y_2^T & X_2 \end{bmatrix} \geq 0 \quad (18)$$

*holds for $\alpha = 0, 1$, then the following inequality*

$$\begin{bmatrix} \frac{1}{\alpha} R_1 & 0 \\ 0 & \frac{1}{1-\alpha} R_2 \end{bmatrix} \geq \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} + (1-\alpha) \begin{bmatrix} X_1 & Y_2 \\ Y_2^T & 0 \end{bmatrix} + \alpha \begin{bmatrix} 0 & Y_1 \\ Y_1^T & X_2 \end{bmatrix} \quad (19)$$

*holds for all $\alpha \in (0,1)$.*

**Lemma 4** (see [3]). *$\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ are given constant matrices, where $\mathcal{S}_1^T = \mathcal{S}_1$, $\mathcal{S}_2 = \mathcal{S}_2^T$. If and only if*

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3 \\ \mathcal{S}_3^T & -\mathcal{S}_2 \end{bmatrix} < 0$$
$$\text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^T & \mathcal{S}_1 \end{bmatrix} < 0, \quad (20)$$

*then we have $\mathcal{S}_1 + \mathcal{S}_3^T \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$.*

**Lemma 5** (see [42]). *For any matrix $R \in \mathbb{S}_+^n$, integer $N \geq 0$, time functions a, b, $a < b$, and a function $x$ in $\mathscr{L}_2([a,b]) \longrightarrow \mathbb{R}^n$), the inequality*

$$\int_a^b x^T(s) R x(s)\, ds \geq (b-a) \phi_N^T \overline{R}_N \phi_N \quad (21)$$

*holds, where*

$$\phi_N = \frac{1}{b-a} \int_a^b \mathbb{L}_N\left(\frac{s-a}{b-a}\right) x(s)\, ds$$
$$\overline{R}_N = \operatorname{diag}(R, 3R, \ldots, (2N+1)R). \quad (22)$$

**Lemma 6** (see [29]). *For any given positive matrices $L > 0$, $Y > 0$, if the following inequality holds*

$$(L - Y) Y^{-1} (L - Y) > 0, \quad (23)$$

*then we have*

$$L Y^{-1} L > 2L - Y. \quad (24)$$

*Remark 7.* For the networked control system, we have designed an improved event-triggered generator in this section. Under the nonlinear function and improved event-triggered condition, the time delay problem will be reconsidered. Based on above preliminaries, we will give the related stability analysis in the next section.

## 3. Stability Criteria

In this section, let us investigate the stability problem of nonlinear networked control system. Compared with the previous networked control system, we will employ the Bessel-Legendre inequality method to reduce the delay upper bound conservatism of the nonlinear networked control system with time delay. At first, the relevant properties of the Legendre polynomials will be introduced. For any given matrix $R \in \mathbb{S}_+^n$, it holds that

$$\int_0^1 \mathbb{L}_N(\mu) R^{-1} \mathbb{L}_N^T(\mu)\, d\mu = \overline{R}_N^{-1}, \quad (25)$$

where

$$\overline{R}_N = \operatorname{diag}\{R, 3R, \ldots, (2N+1)R\}$$

$$\mathbb{L}_N(1) = \begin{bmatrix} I_n \\ I_n \\ \vdots \\ I_n \end{bmatrix} := 1_N,$$
$$\mathbb{L}_N(0) = \begin{bmatrix} I_n \\ -I_n \\ \vdots \\ (-1)^N I_n \end{bmatrix} := \overline{1}_N. \quad (26)$$

According to the properties of the orthogonal polynomials, the Legendre polynomials satisfy

$$\frac{d\mathbb{L}_N(\mu)}{d\mu} = \overline{\Gamma}_N \mathbb{L}_N(\mu) = \Gamma_N \mathbb{L}_{N-1}(\mu),$$
$$\frac{d(\mu \mathbb{L}_N(\mu))}{d\mu} = \mathbb{L}_N(\mu) + \Theta_N \mathbb{L}_N(\mu), \quad (27)$$

where $\overline{\Gamma}_N = [\Gamma_N \quad 0_{n(N+1),n}]$, $\Gamma_N = \gamma_N \otimes I_n$ and $\Theta_N = \theta_N \otimes I_n$, matrices $\gamma_N \in \mathbb{R}^{(N+1)\times N}$ and $\theta_N \in \mathbb{R}^{(N+1)\times(N+1)}$ are defined as

$$\gamma_N(k,i) = \begin{cases} 0 & \text{if } k \geq i, \\ (2k-1)\left(1 - (-1)^{i+k}\right) & \text{if } k < i, \end{cases}$$
$$\theta_N(k,i) = \begin{cases} 0 & \text{if } k \geq i, \\ k & \text{if } k = i, \\ (2k-1)\left(1 - (-1)^{i+k}\right) & \text{if } k < i. \end{cases} \quad (28)$$

**Theorem 8.** *For given $N \in \mathbb{N}$, the system (14) is stable if there exist any matrices $P_N \in \mathbb{S}_+^{(2N+3)n}$, $Q_1 \in \mathbb{S}_+^n$, $Q_2 \in \mathbb{S}_+^n$, $R \in \mathbb{S}_+^n$,*

$\Omega \in \mathbb{S}_+^n$, *and* $Y_1 \in \mathbb{R}^{(N+2)n\times(N+2)n}$, $Y_2 \in \mathbb{R}^{(N+2)n\times(N+2)n}$ *such that the following inequality holds for all* $(d(t), \dot{d}(t)) \in \mathscr{H}$:

$$\Xi = \begin{bmatrix} \Xi_{N0}\left(d(t), \dot{d}(t)\right) & W_N^T \mathbb{Y}_1 & W_N^T \mathbb{Y}_2 \\ * & -\dfrac{d_2 - d(t)}{d_2}\widetilde{R}_{N+1} & 0 \\ * & * & -\dfrac{d(t)}{d_2}\widetilde{R}_{N+1} \end{bmatrix} \leq 0, \quad (29)$$

*where*

$$\begin{aligned}
&\mathscr{H} = \mathscr{C}o\left\{(0,0), (0, d_M), (d_2, 0), (d_2, d_m)\right\}, \\
&H_N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{1}_N & 0 & 0 & \overline{\Gamma}_N - \Theta_N & 0 \\ 0 & -\overline{1}_N & 0 & 0 & 0 & \Theta_N \end{bmatrix}, \\
&J_N = \begin{bmatrix} A + A_\tau F & BK & BK & 0 & 0 & 0 \\ 1_N & -\overline{1}_N & 0 & 0 & -\overline{\Gamma}_N & 0 \\ 0 & 1_N & 0 & -\overline{1}_N & 0 & -\overline{\Gamma}_N \end{bmatrix}, \\
&W_N = \begin{bmatrix} 1_{N+1} & -\overline{1}_{N+1} & 0 & 0 & -\Gamma_{N+1} & 0 \\ 0 & 1_{N+1} & 0 & -\overline{1}_{N+1} & 0 & -\Gamma_{N+1} \end{bmatrix}, \\
&\Lambda_N = \begin{bmatrix} \overline{R}_{N+1} & 0 \\ 0 & \overline{R}_{N+1} \end{bmatrix} + \frac{d_2 - d(t)}{d_2}\begin{bmatrix} \overline{R}_{N+1} & Y_2 \\ Y_2^T & 0 \end{bmatrix} \\
&\quad + \frac{d(t)}{d_2}\begin{bmatrix} 0 & Y_1 \\ Y_1^T & \overline{R}_{N+1} \end{bmatrix}, \\
&\mathbb{Y}_1 = \begin{bmatrix} \dfrac{d_2 - d(t)}{d_2} Y_1 \\ 0 \end{bmatrix}, \\
&\mathbb{Y}_2 = \begin{bmatrix} 0 \\ \dfrac{d(t)}{d_2} Y_2^T \end{bmatrix}, \\
&F_N = \left[A + A_\tau F \;\; BK \;\; BK \;\; 0 \;\; 0 \;\; 0\right], \\
&\widetilde{R}_{N+1} = \operatorname{diag}\left(R, 3R, \ldots, (2N+1)R, (2N+3)R\right), \\
&G_N(d(t)) \\
&\quad = \begin{bmatrix} I_n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d(t) I_{nN} & 0 \\ 0 & 0 & 0 & 0 & 0 & (d_2 - d(t)) I_{nN} \end{bmatrix}, \\
&\Sigma_N\left(\dot{d}(t)\right) = \operatorname{diag}\left(Q_1, -\left(1 - \dot{d}(t)\right)(Q_1 - Q_2)\right. \\
&\quad \left. + \sigma_k(t)\,\Omega, -\Omega, -Q_2, 0, 0\right), \\
&\Xi_{N0}\left(d(t), \dot{d}(t)\right) \\
&\quad = He\left(G_N^T(d(t)) P_N \left(J_N + \dot{d}(t) H_N\right)\right) \\
&\quad + \Sigma_N\left(\dot{d}(t)\right) + d_2^2 F_N^T R F_N - W_N^T \Lambda_N W_N.
\end{aligned} \quad (30)$$

$$(31)$$

*Proof.* Now we choose the Lyapunov-Krasovskii functional candidate to investigate nonlinear networked control system as

$$\begin{aligned} V(x(t), \dot{x}(t)) &= V_1(x(t), \dot{x}(t)) + V_2(x(t), \dot{x}(t)) \\ &\quad + V_3(x(t), \dot{x}(t)), \end{aligned} \quad (32)$$

$$\begin{aligned} V_1(x(t), \dot{x}(t)) &= \tilde{x}^T(t) P_N \tilde{x}(t), \\ V_2(x(t), \dot{x}(t)) &= \int_{t-d(t)}^{t} x^T(s) Q_1 x(s)\,ds \\ &\quad + \int_{t-d_2}^{t-d(t)} x^T(s) Q_2 x(s)\,ds, \\ V_3(x(t), \dot{x}(t)) &= d_2 \int_{t-d_2}^{t}\int_{\theta}^{t} \dot{x}^T(s) R \dot{x}(s)\,ds\,d\theta, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \tilde{x}^T(t) &= \left[x^T(t) \;\; d(t)\phi_{1,N}^T(t) \;\; (d_2 - d(t))\phi_{2,N}^T(t)\right], \\ \phi_{1,N}(t) &= \frac{1}{d(t)}\int_{t-d(t)}^{t} \mathbb{L}_N\left(\frac{s - t + d(t)}{d(t)}\right) x(s)\,ds, \\ \phi_{2,N}(t) &= \frac{1}{d_2 - d(t)}\int_{t-d_2}^{t-d(t)} \mathbb{L}_N\left(\frac{s - t + d_2}{d_2 - d(t)}\right) x(s)\,ds. \end{aligned} \quad (34)$$

Taking the derivative of $V(x(t), \dot{x}(t))$, we have

$$\begin{aligned} &\dot{V}(x(t), \dot{x}(t)) \\ &\quad = \dot{V}_1(x(t), \dot{x}(t)) + \dot{V}_2(x(t), \dot{x}(t)) \\ &\quad\quad + \dot{V}_3(x(t), \dot{x}(t)), \\ &\dot{V}_1(x(t), \dot{x}(t)) = He\left(\tilde{x}^T(t) P_N \dot{\tilde{x}}(t)\right), \\ &\dot{V}_2(x(t), \dot{x}(t)) \\ &\quad = x^T(t) Q_1 x(t) \\ &\quad\quad - \left(1 - \dot{d}(t)\right) x^T(t - d(t))(Q_1 - Q_2)\, x(t - d(t)) \\ &\quad\quad - x^T(t - d_2) Q_2 x(t - d_2), \\ &\dot{V}_3(x(t), \dot{x}(t)) \\ &\quad = d_2^2 \dot{x}^T(t) R \dot{x}(t) - d_2 \int_{t-d_2}^{t} \dot{x}(s) R \dot{x}(s)\,ds. \end{aligned} \quad (35)$$

Now, we define the

$$\begin{aligned} &\xi^T(t) \\ &= \left[x^T(t) \;\; x^T(t - d(t)) \;\; e_k^T(t) \;\; x^T(t - d_2) \;\; \phi_{1,N}^T(t) \;\; \Phi_{2,N}^T(t)\right], \end{aligned} \quad (36)$$

then due to the networked control system model (29), the derivative of $\tilde{x}(t)$ can be represented by $\dot{\tilde{x}}(t) = (J_N + \dot{d}(t)H_N)\xi(t)$, where matrices $J_N$ and $H_N$ are defined in (31). On the one hand, let us consider

$$\phi_N = \frac{1}{b-a}\int_a^b \mathbb{L}_N\left(\frac{s-a}{b-a}\right)x(s)\,ds, \tag{37}$$

where $a$ and $b$ are time functions. Now we set $s(\mu) = (b-a)\mu + a$ to get

$$\phi_N = \int_0^1 \mathbb{L}_N(\mu)\,x(s(\mu))\,d\mu, \tag{38}$$

and

$$\frac{d[(b-a)\phi_N]}{dt} = \dot{a}\psi_{1,N} + (\dot{b}-\dot{a})(\psi_{2,N}+\phi_N), \tag{39}$$

where

$$\begin{aligned}\psi_{1,N} &= (b-a)\int_0^1 \mathbb{L}_N(\mu)\,\dot{x}(s(\mu))\,d\mu,\\ \psi_{2,N} &= (b-a)\int_0^1 \mu\mathbb{L}_N(\mu)\,\dot{x}(s(\mu))\,d\mu.\end{aligned} \tag{40}$$

Applying integration by parts, we obtain

$$\begin{aligned}\psi_{1,N} &= 1_N x(b) - \overline{1}_N x(a) - \overline{\Gamma}_N\phi_N,\\ \psi_{2,N} &= 1_N x(b) - \Theta_N\phi_N - \phi_N.\end{aligned} \tag{41}$$

Adding these equations into (39), for all $t \in \mathbb{R}^+$,

$$\begin{aligned}&\frac{d[(b-a)\phi_N]}{dt}\\ &\quad= \dot{a}\left(1_N x(b) - \overline{1}_N x(a) - \overline{\Gamma}_N\phi_N\right)\\ &\qquad+ (\dot{b}-\dot{a})\left(1_N x(b) - \Theta_N)\phi_N - \phi_N + \phi_N\right)\\ &\quad= \dot{b}1_N x(b) - \dot{a}\overline{1}_N x(a) - \left(\dot{a}\overline{\Gamma}_N + (\dot{b}-\dot{a})\Theta_N\right)\phi_N.\end{aligned} \tag{42}$$

Now, let us analyze two cases that $(a, b) = (t-d(t), t)$ and $(a, b) = (t-d_2, t-d(t))$.

$$\begin{aligned}\dot{\tilde{x}} &= \left[\dot{x}^T(t)\quad \frac{d\left(d(t)\phi_{1,N}^T\right)}{dt}\quad \frac{d\left((d_2-d(t))\phi_{2,N}^T\right)}{dt}\right],\\ \dot{x}(t) &= A(t) + A_d x(t-d(t)),\end{aligned} \tag{43}$$

then we have

$$\begin{aligned}&\frac{d(d(t)\phi_{1,N})}{dt}\\ &\quad= 1_N x(t) - \left(1-\dot{d}(t)\right)\overline{1}_N x(t-d(t))\\ &\qquad- \left(\left(1-\dot{d}(t)\right)\overline{\Gamma}_N + \dot{d}(t)\Theta_N\right)\phi_{1,N},\\ &\frac{d((d_2-d(t))\phi_{2,N})}{dt}\\ &\quad= \left(1-\dot{d}(t)\right)1_N x(t-d(t)) - \overline{1}_N x(t-d_2)\\ &\qquad- \left(\overline{\Gamma}_N - \dot{d}(t)\Theta_N\right)\phi_{2,N}.\end{aligned} \tag{44}$$

On the other hand, for the function of $V_3(x(t), \dot{x}(t))$, we have

$$\begin{aligned}\int_{t-d_2}^t \dot{x}^T(s)R\dot{x}(s)\,ds &\geq \begin{bmatrix}\tilde{\phi}_{1,N+1}\\ \tilde{\phi}_{2,N+1}\end{bmatrix}^T\\ &\cdot\begin{bmatrix}d(t)\tilde{R}_{N+1} & 0\\ 0 & (d_2-d(t))\tilde{R}_{N+1}\end{bmatrix}\begin{bmatrix}\tilde{\phi}_{1,N+1}\\ \tilde{\phi}_{2,N+1}\end{bmatrix},\end{aligned} \tag{45}$$

where

$$\begin{aligned}\tilde{\phi}_{1,N+1} &= \frac{1}{d(t)}\int_{t-d(t)}^t \mathbb{L}_{N+1}\left(\frac{s-t+d(t)}{d(t)}\right)\dot{x}(s)\,ds,\\ \tilde{\phi}_{2,N+1} &= \frac{1}{d_2-d(t)}\int_{t-d_2}^{t-d(t)} \mathbb{L}\left(\frac{s-t+d_2}{d_2-d(t)}\right)\dot{x}(s)\,ds.\end{aligned} \tag{46}$$

Due to equation (27), we obtain

$$\begin{aligned}&\begin{bmatrix}d(t)\tilde{\phi}_{1,N+1}\\ (d_2-d(t))\phi_{2,N+1}\end{bmatrix}\\ &= \begin{bmatrix}1_{N+1}x(t) - \overline{1}_{N+1}x(t-d(t)) - \Gamma_{N+1}\phi_{1,N}\\ 1_{N+1}x(t-d(t)) - \overline{1}_{N+1}x(t-d(t)) - \Gamma_{N+1}\phi_{2,N}\end{bmatrix}\\ &= W_N\xi_N,\end{aligned} \tag{47}$$

where $W_N\xi_N$ is defined in (31).

According to Lemma 5 and Bessel-Legendre inequality, we have

$$\begin{aligned}&d_2\int_{t-d_2}^t \dot{x}^T(s)R\dot{x}(s)\,ds\\ &\geq \xi^T(t)W_N^T\begin{bmatrix}\frac{d_2}{d(t)}\tilde{R}_{N+1} & 0\\ 0 & \frac{d_2}{d_2-d(t)}\tilde{R}_{N+1}\end{bmatrix}W_N\xi(t).\end{aligned} \tag{48}$$

In addition, for any $Y_1, Y_2 \in \mathbb{R}^{(N+2)n\times(N+2)n}$, consider the matrix

$$\Lambda_{N0} = \begin{bmatrix}\frac{d_2-d(t)}{d_2}Y_1\tilde{R}_{N+1}^{-1}Y_1^T & 0\\ 0 & \frac{d(t)}{d_2}Y_2^T\tilde{R}_{N+1}^{-1}Y_2\end{bmatrix}. \tag{49}$$

By applying Lemma 3, we have

$$\begin{aligned}&d_2\int_{t-d_2}^t \dot{x}^T(s)R\dot{x}(s)\,ds\\ &\quad\geq \xi^T(t)W_N^T\left[\Lambda_N - \Lambda_{N0}\right]W_N\xi(t),\end{aligned} \tag{50}$$

where $\Lambda_N$ is defined in (31).

In addition, let us consider the following event-triggered condition of the networked control system

$$e_k^T(t)\Omega e_k(t) \leq \sigma_k(t)x^T(t-d(t))\Omega x(t-d(t)). \tag{51}$$

Let us add (51) into the derivative of $V(x(t), \dot{x}(t))$, then, we have

$$\begin{aligned} \dot{V}(x(t), \dot{x}(t)) &= He\left(\xi^T(t) G_N^T(d(t)) P_N \dot{\tilde{x}}(t)\right) \\ &\quad + \xi^T(t)\left[d_2^2 F_N^T R F_N + \Sigma_N\left(\dot{d}(t)\right)\right]\xi(t) \\ &\quad - d_2 \int_{t-d_2}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds, \end{aligned} \tag{52}$$

where $G_N$, $F_N$, $\Sigma_N(\dot{d}(t))$ are given in (31).

By the integral inequality method (50), the derivative of Lyapunov-Krasovskii can be rewritten as

$$\begin{aligned} \dot{V}(x(t), \dot{x}(t)) &\le \xi^T(t) \\ &\cdot \left[He\left(G_N^T(d(t)) P_N \left(J_N + \dot{d}(t) H_N\right)\right)\right. \\ &+ \Sigma_N\left(\dot{d}(t)\right) + d_2^2 F_N^T R F_N \\ &\left.- W_N^T\left[\Xi_N(d(t)) - \Xi_{N0}(d(t))\right] W_N\right]\xi(t) = \xi^T(t) \\ &\cdot \left(\Psi_{N0}\left(d(t), \dot{d}(t)\right) + W_N^T \Xi_{N0}(d(t)) W_N\right)\xi(t). \end{aligned} \tag{53}$$

Notice that the $\Psi_{N0}(d(t), \dot{d}(t)) + W_N^T \Xi_{N0}(d(t)) W_N$ is multi-affine about $d(t)$ and $\dot{d}(t)$, where $(d(t), \dot{d}(t)) \in \mathscr{H}$. Therefore, by the Schur's complement, if the matrix $\Xi < 0$, then $\dot{V}(x(t), \dot{x}(t)) < 0$ for $(d(t), \dot{d}(t)) \in \mathscr{H}$, the system is stable. This proof is completed. □

*Remark 9.* Notice that LMI (29) is considered satisfying $(d(t), \dot{d}(t)) \in \mathscr{H}$, where

$$\mathscr{H} = \mathscr{C}o\left\{(0,0), (0, d_M), (d_2, 0), (d_2, d_m)\right\}. \tag{54}$$

That is because the vertices $(0, d_m)$, $(d_2, d_M)$ are impossible to reach. In another words, at the lower bound of time delay 0, the derivative of time delay $\dot{d}(t)$ can not be negative; at the upper bound of time delay $d_2$, the derivative of time delay $\dot{d}(t)$ should be non-positive. Thus, we choose the allowable delay set as $\mathscr{H}$.

*Remark 10.* In this section, the stability problem of the nonlinear networked control system has been discussed. We employed the Bessel-Legendre inequality method to improve the Lyapunov-Krasovskii functional and obtained the delay upper bound. For the $V_1(x(t), \dot{x}(t))$, we take the terms $\phi_{1,N}(t)$ and $\phi_{2,N}(t)$ into $\tilde{x}(t)$ to consider the delay dependent condition. Furthermore, the $V_2(x(t), \dot{x}(t))$ will be zoomed by this integral method.

## 4. Stabilization of Networked Control System

In this section, we will deal with the stabilization problem of the event-triggered nonlinear networked control system with time delay. In order to more effectively control the system state and achieve a stable, fast and accurate networked control system, the state feedback controller will be designed. Next, the relevant stabilization theorem is given as follows.

**Theorem 11.** *For given integer scalar $N \ge 0$, scalar $\epsilon > 0$, system (14) with the feedback controller gain $K = -\epsilon B^{-1} \widehat{P}_1^{-1}$ is stable if there exist matrices $\widehat{P}_1 \in \mathbb{S}_+^n$, $\widehat{P}_2 \in \mathbb{S}_+^{(N+1)n}$, $\widehat{P}_3 \in \mathbb{S}_+^{(N+1)n}$, $Q_1, Q_2, R \in \mathbb{S}_+^n$, $\Omega \in \mathbb{S}_+^n$, and $Y_1, Y_2 \in \mathbb{R}^{(N+2)n \times (N+2)n}$, such that*

$$\Xi = \begin{bmatrix} \widetilde{\Xi}_{N0}\left(d(t), \dot{d}(t)\right) & W_N^T \mathbb{Y}_1 & W_N^T \mathbb{Y}_2 & d_2^2 \widehat{F}_N^T & -W_N^T \Lambda_N \\ * & -\dfrac{d_2 - d(t)}{d_2} \widetilde{R}_{N+1} & 0 & 0 & 0 \\ * & * & -\dfrac{d(t)}{d_2} \widetilde{R}_{N+1} & 0 & 0 \\ * & * & * & -2I_n + R & 0 \\ * & * & * & * & \Lambda_N \end{bmatrix} \le 0 \tag{55}$$

*holds for all $(d(t), \dot{d}(t)) \in \mathscr{H}$, where*

$$\mathscr{H} = \mathscr{C}o\left\{(0,0), (0, d_M), (d_2, 0), (d_2, d_m)\right\},$$

$$\widehat{F}_N = \left[A + A_\tau F \;\; \epsilon\left(-2I_n + \widehat{P}_1\right) \;\; \epsilon\left(-2I_n + \widehat{P}_1\right) \;\; 0 \;\; 0 \;\; 0\right],$$

$$W_N = \begin{bmatrix} 1_{N+1} & -\overline{1}_{N+1} & 0 & 0 & -\Gamma_{N+1} & 0 \\ 0 & 1_{N+1} & 0 & -\overline{1}_{N+1} & 0 & -\Gamma_{N+1} \end{bmatrix},$$

$$\Lambda_N = \begin{bmatrix} \overline{R}_{N+1} & 0 \\ 0 & \overline{R}_{N+1} \end{bmatrix} + \frac{d_2 - d(t)}{d_2} \begin{bmatrix} \overline{R}_{N+1} & Y_2 \\ Y_2^T & 0 \end{bmatrix} + \frac{d(t)}{d_2} \cdot \begin{bmatrix} 0 & Y_1 \\ Y_1^T & \overline{R}_{N+1} \end{bmatrix},$$

$$\widetilde{R}_{N+1} = \operatorname{diag}\left(R, 3R, \ldots, (2N+1)R, (2N+3)R\right),$$

$$G_N(d(t)) = \begin{bmatrix} I_n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d(t) I_{nN} & 0 \\ 0 & 0 & 0 & 0 & 0 & (d_2 - d(t)) I_{nN} \end{bmatrix},$$

$$\Sigma_N\left(\dot{d}(t)\right) = \operatorname{diag}\left(Q_1, -\left(1 - \dot{d}(t)\right)(Q_1 - Q_2) + \sigma_k(t)\Omega, -\Omega, -Q_2, 0, 0\right),$$

$$\widehat{\Xi}_{N0}\left(d(t),\dot{d}(t)\right) = He\left(G_N^T(d(t))\,\Xi_{N0}\left(\dot{d}(t)\right)\right) + \Sigma_N\left(\dot{d}(t)\right),$$

$$\Xi_{N0}\left(\dot{d}(t)\right) = \begin{bmatrix} \widehat{P}_1(A+A_\tau F) & -\epsilon I_n & -\epsilon I_n & 0 & 0 & 0 \\ \widehat{P}_2(1_N) & -\widehat{P}_2\overline{1}_N & 0 & 0 & -\widehat{P}_2\overline{\Gamma}_N & 0 \\ 0 & \widehat{P}_3 1_N & 0 & -\widehat{P}_3\overline{1}_N & 0 & -\widehat{P}_3\overline{\Gamma}_N \end{bmatrix} + \dot{d}(t)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \widehat{P}_2\overline{1}_N & 0 & 0 & \widehat{P}_2\overline{\Gamma}_N - \widehat{P}_2\Theta_N & 0 \\ 0 & -\widehat{P}_3\overline{1}_N & 0 & 0 & 0 & \widehat{P}_3\Theta_N \end{bmatrix},$$

$$\mathbb{Y}_1 = \begin{bmatrix} \dfrac{d_2-d(t)}{d_2}Y_1 \\ 0 \end{bmatrix},$$

$$\mathbb{Y}_2 = \begin{bmatrix} 0 \\ \dfrac{d(t)}{d_2}Y_2^T \end{bmatrix}, \tag{56}$$

*Proof.* According to Theorem 8 and Lemma 4, we have obtained the LMI as follows:

$$\Xi = \begin{bmatrix} \widehat{\Xi}_{N0}\left(d(t),\dot{d}(t)\right) & W_N^T\mathbb{Y}_1 & W_N^T\mathbb{Y}_2 & d_2^2F_N^T \\ * & -\dfrac{d_2-d(t)}{d_2}\widetilde{R}_{N+1} & 0 & 0 \\ * & * & -\dfrac{d(t)}{d_2}\widetilde{R}_{N+1} & 0 \\ * & * & * & -R^{-1} \end{bmatrix} \leq 0, \tag{57}$$

where $\widehat{\Xi}_{N0}(d(t),\dot{d}(t)) = He(G_N^T P_N(J_N+\dot{d}(t)H_N))+\Sigma_N(\dot{d}(t))+W_N^T\Lambda_N W_N$.

For the symmetric matrix $P_N$, we set

$$P_N = \begin{bmatrix} \widehat{P}_1 & 0 & 0 \\ 0 & \widehat{P}_2 & 0 \\ 0 & 0 & \widehat{P}_3 \end{bmatrix}, \tag{58}$$

where $\widehat{P}_1 \in \mathbb{S}_+^n$, $\widehat{P}_2 \in \mathbb{S}_+^{(N+1)n}$ and $\widehat{P}_3 \in \mathbb{S}_+^{(N+1)n}$. Then we have

$$G_N^T\Xi_{N0}\left(d(t),\dot{d}(t)\right) = G_N^T P_N\left(J_N+\dot{d}(t)H_N\right)$$

$$= G_N^T\begin{bmatrix} \widehat{P}_1 & 0 & 0 \\ 0 & \widehat{P}_2 & 0 \\ 0 & 0 & \widehat{P}_3 \end{bmatrix}\left(\begin{bmatrix} A+A_\tau F & BK & BK & 0 & 0 & 0 \\ 1_N & -\overline{1}_N & 0 & 0 & -\overline{\Gamma}_N & 0 \\ 0 & 1_N & 0 & -\overline{1}_N & 0 & -\overline{\Gamma}_N \end{bmatrix} + \dot{d}(t)\cdot\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{1}_N & 0 & 0 & \overline{\Gamma}_N-\Theta_N & 0 \\ 0 & -\overline{1}_N & 0 & 0 & 0 & \Theta_N \end{bmatrix}\right). \tag{59}$$

At the same time, we set the controller gain $K = -\epsilon B^{-1}\widehat{P}_1^{-1}$, then we obtain

$$G_N^T\Xi_{N0}\left(d(t),\dot{d}(t)\right) = G_N^T\left(\begin{bmatrix} \widehat{P}_1(A+A_\tau F) & -\epsilon I_n & -\epsilon I_n & 0 & 0 & 0 \\ \widehat{P}_2(1_N) & -\widehat{P}_2\overline{1}_N & 0 & 0 & -\widehat{P}_2\overline{\Gamma}_N & 0 \\ 0 & \widehat{P}_3 1_N & 0 & -\widehat{P}_3\overline{1}_N & 0 & -\widehat{P}_3\overline{\Gamma}_N \end{bmatrix} + \dot{d}(t)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \widehat{P}_2\overline{1}_N & 0 & 0 & \widehat{P}_2\overline{\Gamma}_N - \widehat{P}_2\Theta_N & 0 \\ 0 & -\widehat{P}_3\overline{1}_N & 0 & 0 & 0 & \widehat{P}_3\Theta_N \end{bmatrix}\right). \tag{60}$$

Thanks to Lemma 6, we replace $-R^{-1}$ with $-2I_n + R$; on the other hand, define

$$F_N = \left[A + A_\tau F \quad \epsilon\left(-2I_n + \widehat{P}_1\right) \quad 0 \quad 0 \quad 0\right]. \tag{61}$$

Combining equations (58), (59), (60) and (61), we obtain the following inequality:

$$\begin{bmatrix} \widehat{\Xi}_{N0}\left(d(t),\dot{d}(t)\right) & W_N^T\mathbb{Y}_1 & W_N^T\mathbb{Y}_2 & d_2^2F_N^T \\ * & -\dfrac{d_2-d(t)}{d_2}\widetilde{R}_{N+1} & 0 & 0 \\ * & * & -\dfrac{d(t)}{d_2}\widetilde{R}_{N+1} & 0 \\ * & * & * & -2I_n+R \end{bmatrix} \leq 0. \tag{62}$$

In addition, apply the Schur's complement to (62) again. Then, linear matrix inequality is obtained in Theorem 11. This proof is completed. □

## 5. Numerical Examples

*Example 1.* In order to understand the applicability of the system more clearly, and demonstrate the effectiveness of Bessel-Legendre inequalities method, we give Example 1. First of all, let us consider the system model as follows:

$$\dot{x}(t) = Ax(t) + A_\tau Fx(t) + Bu(t), \tag{63}$$

and the system relevant parameters are given as

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$

Table 1: Delay upper bound $d_2$ for different $\dot{d}(t)(d_M = -d_m = \dot{d}(t))$.

| Method \ $\dot{d}(t)$ | 0.1 | 0.3 | 0.5 | 0.9 |
|---|---|---|---|---|
| Theorem 8, N=3 | 4.7526 | 3.01780 | 3.0472 | 2.9856 |
| Theorem 8, N=2 | 3.2546 | 2.3136 | 2.1797 | 2.0732 |
| Theorem 8, N=1 | 2.8572 | 2.6095 | 2.1472 | 2.0153 |
| Theorem 8, N=0 | 2.3544 | 1.1498 | 1.0755 | 1.0526 |
| Theorem 1 of [45] | 2.1403 | 1.0653 | 0.8671 | 0.7644 |
| Theorem 1 of [46] | 0.7522 | 0.7197 | 0.6496 | 0.4768 |
| Theorem 1 of [13] | 1.2890 | 1.0126 | 0.9775 | 0.6324 |

$$A_\tau = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$B = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix},$$
$$F = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix}. \tag{64}$$

For testifying the less conservatism of the obtained stability condition in this paper, the comparison with other paper's results about the upper bound of delay is given in Table 1. From Table 1, for $N = 0$, $\dot{d}(t)$ is taken different values as 0.1, 0.3, 0.5, 0.9, respectively, then the obtained upper bound of time delays are 2.3544, 1.1498, 1.0755, 1.0526, respectively, which are all larger than the values in [13, 45, 46] for the corresponding values of $\dot{d}(t)$. Obviously, for this nonlinear networked control system, the conservatism of the stability criteria derived by using Bessel-Legendre inequality method in this paper has been greatly reduced. Furthermore, the obtained stability criteria are related to the order $N$. As we can see in Table 1, the upper bound of time delay increases with $N$ increasing. In other words, the larger $N$, the lower conservatism.

Next, we propose Example 2 to investigate the triggering performance of nonlinear networked control system under the improved event-triggered scheme.

*Example 2.* This example concerns the parameters of nonlinear networked control system as follows:

$$A = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ -1 & 0 & 0 & 1 \\ 0.5 & 0 & -3 & 0.7 \end{bmatrix},$$
$$A_\tau = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0.5 & 0 & -3 & 1 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ -0.01 \end{bmatrix},$$
$$F = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -0.3 & 0 & 0 \\ 0 & 2.0 & -0.6667 & 0.6667 \\ 0 & -2.7 & 0.1 & -0.8 \end{bmatrix}. \tag{65}$$

Set $\sigma_{k1} = 0.01$, $\overline{\sigma} = 0.6$, $h = 0.22$, $\dot{d}(t) = 0.1$. Under the improved event-triggered scheme, the obtained controller gains $K$ and triggering parameters $\Omega$ at $t = 0, t = 1, \ldots, t = 30$ are shown in Table 2.

In this improved event-triggered scheme, the parameter $\sigma_k(t)$ is time-varying and satisfies (10), which can change the release rate at different times while saving the network transmission resource. When $h = 0.22$, $\lambda = 0.01$, the variation of $\sigma_k(t)$ and release instants are shown in Figure 2. In Figure 2, there are only 29 sampled signals which takes 21.3% of the whole sampled signals need to be sent out when $t \in [0, 30]$. In addition, from Figure 2, we can see that the parameter $\sigma_k(t)$ varies from 0.01 to 0.4 and the frequency of triggering in initial times is higher than other times, which can shorten the dynamic process.

To further illustrate the advantage of our improved static event-triggered scheme, we compare it with the general static event-triggered scheme in which the parameter $\sigma_k(t)$ is time invariant. We set $h = 0.12$, then the results are shown in Figures 3 and 4. Obviously, the release times in initial times in Figure 4 are more than those in Figure 3 and there are more packets that can be transmitted in initial times.

The state response of the event-triggered nonlinear networked control system is shown in Figure 5. At 3 seconds, the system achieves steady.

## 6. Conclusions

In this paper, we discussed the stability and stabilization problem of the event-triggered nonlinear networked control

Table 2: The controller gains $K$ and triggering parameters $\Omega$ at different times.

| t | 0 | 1 | … | 30 |
|---|---|---|---|---|
| $\Omega$ | $\begin{bmatrix} 4.57 & 1.06 & 4.46 & 0.49 \\ 1.06 & 6.19 & 0.14 & 0.95 \\ 4.46 & 0.14 & 9.79 & 3.70 \\ 0.49 & 0.95 & 3.70 & 9.65 \end{bmatrix}$ | $\begin{bmatrix} 4.47 & 0.15 & 1.67 & 1.33 \\ 0.15 & 7.62 & 0.13 & 0.95 \\ 1.67 & 0.13 & 8.89 & 0.40 \\ 1.33 & 0.95 & 0.40 & 7.23 \end{bmatrix}$ | … | $\begin{bmatrix} 9.77 & 0.13 & 0.22 & 0.36 \\ 0.13 & 5.14 & 0.20 & 0.94 \\ 0.22 & 0.20 & 6.94 & 0.18 \\ 0.36 & 0.94 & 0.18 & 5.53 \end{bmatrix}$ |
| K | $\begin{bmatrix} 6.2667 \\ 5.6134 \\ 2.0162 \\ 4.8761 \end{bmatrix}^T$ | $\begin{bmatrix} 4.4035 \\ 5.1434 \\ 6.1441 \\ 6.5962 \end{bmatrix}^T$ | … | $\begin{bmatrix} 3.3367 \\ 2.4931 \\ 5.0829 \\ 8.9023 \end{bmatrix}^T$ |

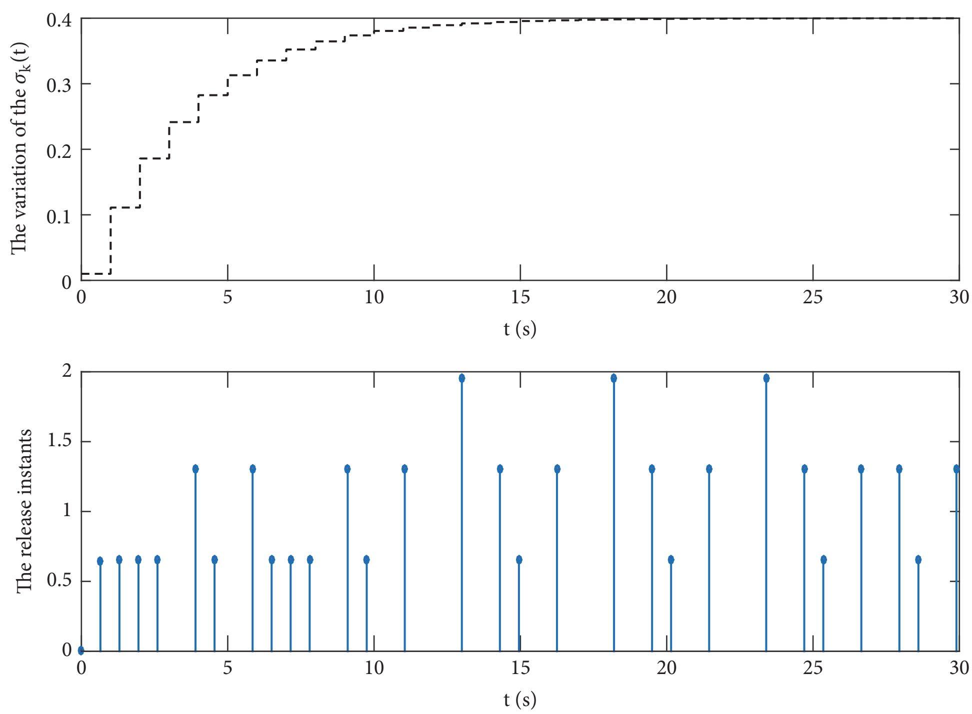


Figure 2: The variation of $\sigma_k(t)$ and release instants, h=0.22.

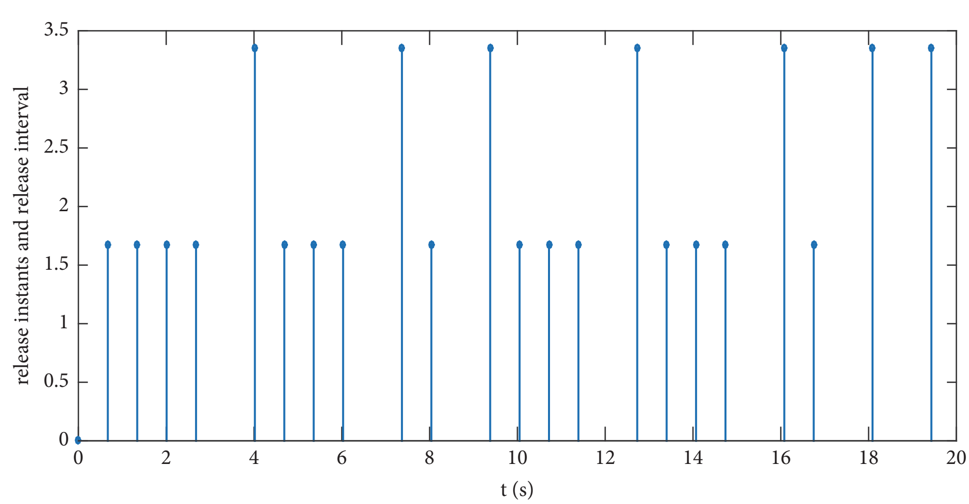


Figure 3: Improved static event-based release interval and release instants, h=0.12.

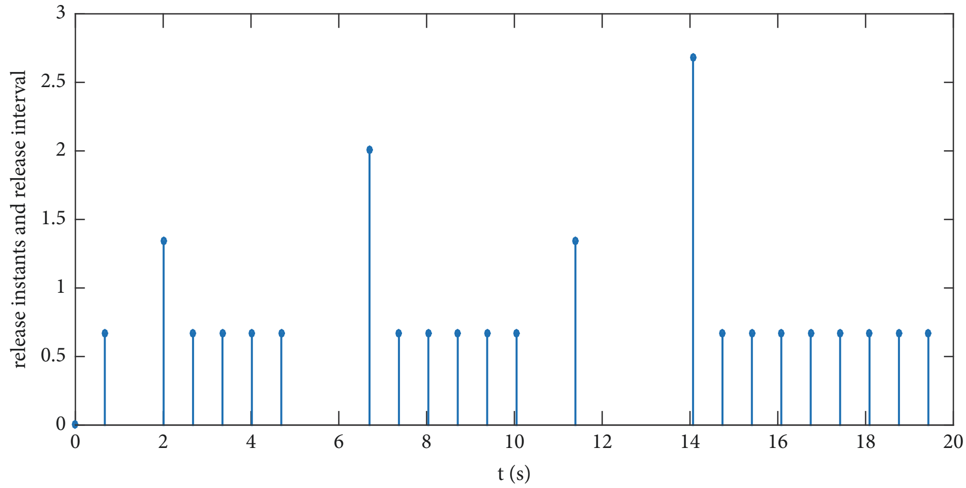


FIGURE 4: Static event-based release interval and release instants, h=0.12.

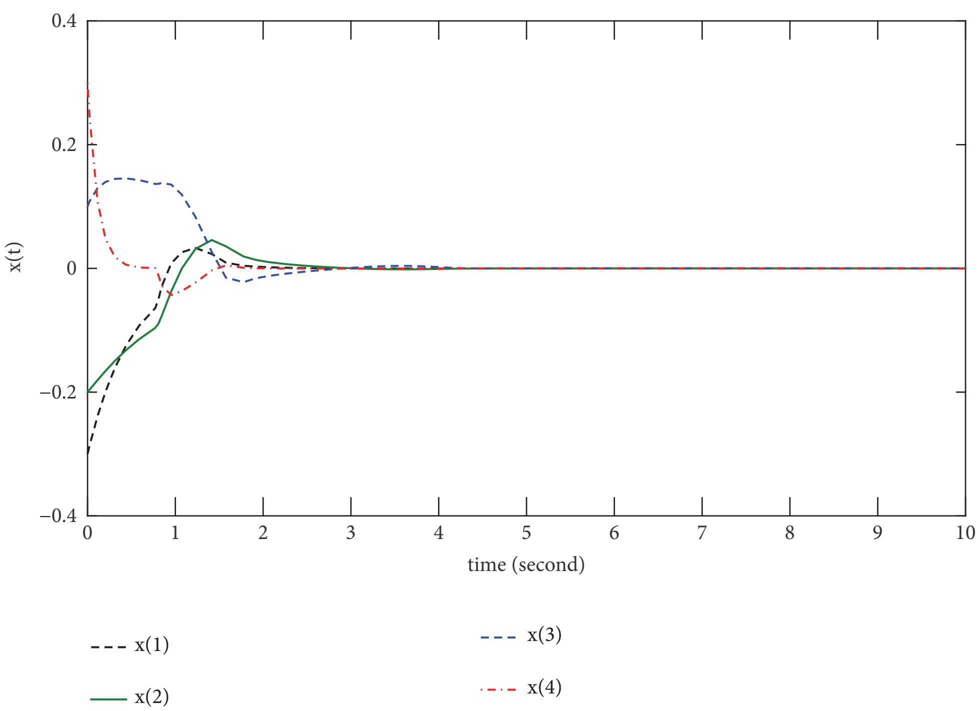


FIGURE 5: State response of the event-trigger nonlinear networked control system.

system with time varying delay. Firstly, on one hand, compared with the previous investigation of networked control system, this paper utilizes the Bessel-Legendre inequality method to reduce the conservatism of the system delay upper bound. On the other hand, by designing an appropriate state feedback controller, the stabilization problem of the nonlinear networked control system has been solved. Secondly, an improved ETS was put forward to reduce transmission load and make the system has a better dynamic process. Finally, two simulation examples have been shown to verify the effectiveness of the improved time delay method and ETS. Further study can be concentrated on the discrete time matters for the networked control system and finite time issues for nonlinear networked control switch system under an improved ETS.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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