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Research Article

An Efficient Siphon-Based Deadlock Prevention Policy for a Class of Generalized Petri Nets

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We propose a new deadlock prevention policy for an important class of resource allocation systems (RASs) that appear in the modeling of flexible manufacturing systems (FMSs). The model of this class in terms of generalized Petri nets is, namely, S^4PR. On the basis of recent structural analysis results related to the elementary siphons in generalized Petri nets on one hand and an efficient deadlock avoidance policy proposed for the class of conjunctive/disjunctive (C/D) RASs on the other hand, we show how one can generate monitors to be added to a net system such that all its strict minimal siphons are max$'$-controlled and no insufficiently marked siphon is generated. Thereby, a new, simple, and more permissive liveness-enforcing supervisor synthesis method for S^4PR is established.

1. Introduction

A flexible manufacturing system (FMS) is characterized by flexibility, concurrent operations, and mainly automated elements, such as production controllers, machines, automated guided vehicles, and conveyors. In an FMS, raw parts are processed in a preestablished sequence to compete for a limited number of system resources. Deadlocks may occur when some processes keep waiting indefinitely for other processes to release resources, which can lead to catastrophic results in highly automated systems. One way of dealing with deadlock problems is to model an FMS with Petri nets [1]. Deadlock prevention is considered to be one of the most effective methods in deadlock control [2–9], which is usually achieved by either designing an effective system or using an off-line mechanism to control the requests for resources to ensure that deadlocks never occur in a system. To achieve this purpose, monitors and related arcs are added to the net system. One of the most interesting past developments is the use of structural objects to design liveness-enforcing Petri net supervisors [10–16]. Above all, the concept of elementary siphons provides an efficient and effective avenue for designing structurally simple supervisors [17, 18]. Elementary siphons have been maturely applied for a class of ordinary Petri nets such as S^3PR [19], as well as some classes of generalized ones [16]. Since a siphon is a set of places that does not carry the weight information and the complex allocations of shared resources in a generalized Petri net, elementary siphons in [17] are not well suitable for generalized Petri nets. By fully investigating the topological structure and the requirements of multiple resource types of S^4PR [20–22], the concept of augmented siphons is recently proposed in [23, 24]. Indeed, since the role of weight of arcs in determining the liveness of generalized Petri nets cannot be neglected, the notion of elementary siphons is redefined by considering augmented siphons, from which a compact and suitable set of elementary siphons can be obtained.

For automated operation of modern technological systems that involve resource sharing, deadlock avoidance is also an essential control requirement. Broadly speaking, a deadlock avoidance policy tries to restrict the operation of an FMS to its reachable and safe sub-state-space. It is worth noting that we can translate the enforcement of liveness into a forbidden state problem in essence. Mutual exclusion constraints are a natural way of expressing the concurrent use of a finite number of resources, shared among different processes. In the framework of Petri nets formalisms, the work in [25] defines a generalized mutual exclusion constraint (GMEC) as a condition that limits a weighted sum of tokens contained in a subset of places. Based on this concept, the problem of forbidden state specification can be represented by GMECs. Many constraints that deal with exclusions between states and events can be transformed into the form of GMECs.

The work in [26] generalizes the deadlock avoidance policy (DAP) of conjunctive/disjunctive resource upstream neighbourhood (C/D RUN) for resource allocation systems with multiple resource acquisitions and flexible routings, namely, S^4PR, and this policy is of polynomial complexity. Motivated by the DAP of C/D RUN policy, a deadlock prevention policy is developed in this work by combining this method with the concept of augmented and elementary siphons in S^4PR net. First, the concept of augmented siphons is proposed. Among augmented siphons, a set of improved elementary siphons can be derived. After that, we obtain a set of linear inequality constraints expressed by state vectors from elementary siphons. After modifying them by the proposed policy, we find a set of GMECs expressed by marking vectors. Then monitors are added to the plant model such that the elementary siphons in S^4PR are all max$'$-controlled and no insufficiently marked siphon is generated due to the addition of the monitors. Finally, it can usually lead to a highly permissive liveness-enforcing supervisor by using the elementary siphon-based deadlock control policy.

The rest of the paper is organized as follows. Section 2 introduces the definition of S^4PR. Section 3 elaborates the concept of augmented siphons and the method of deriving a set of elementary siphons in S^4PR. The deadlock control policy for S^4PR is proposed in Section 4, and a typical example is introduced to show the applicability and efficiency of the proposed method, while Section 5 concludes the paper. Some basics of Petri nets and elementary siphons used throughout this paper are listed in the Appendix.

2. S^4PR

This section gives the definition of a class of generalized Petri nets, namely, S^4PR [13, 26, 27]. Note that, in [26], an S^4PR is called S^3PGR^2: the definition of these two subclasses of nets are in fact identical. S^4PR nets include some well known classes of Petri nets such as S^3PR, ES^3PR, and WS^3PR. Indeed, an S^4PR concerns the modeling of concurrently cyclic sequential processes sharing common resources where an operation place can use simultaneously multiple resources of different types. Also sequential processes (state machines) mean that an operation place can be shared (flexible routings) but assembly and disassembly operations, for which synchronization is required, cannot be considered.

Definition 1. An S^4PR net is a generalized and self-loop free net $N = \bigcirc_{i=1}^{n} N_i = (P, T, F, W)$, where

(1) $N_i = (P_{A_i} \cup \{p_i^0\} \cup P_{R_i}, T_i, W_i)$, $i \in \mathbb{N}_n = \{1, \ldots, n\}$;

(2) $P = P_A \cup P_0 \cup P_R$ is a partition such that (i) $P_A = \bigcup_{i=1}^{n} P_{A_i}$ is called the set of operation places, where $P_{A_i} \neq \emptyset$ and $P_{A_i} \cap P_{A_j} = \emptyset$, $\forall i \neq j$ $(i, j \in \mathbb{N}_n)$; (ii) $P_R = \{r_1, r_2, \ldots, r_m \mid m \in \mathbb{N}^+\}$ is called the set of resource places; (iii) $P_0 = \bigcup_{i=1}^{n} \{p_i^0\}$ is called the set of idle places; and (iv) the output transitions of idle places are called source transitions;

(3) $T = \bigcup_{i=1}^{n} T_i$ is called the set of transitions, where $\forall i, j \in \mathbb{N}_n$, $i \neq j$, $T_i \neq \emptyset$, and $T_i \cap T_j = \emptyset$;

(4) $W = W_A \cup W_R$, where $W_A : ((P_A \cup P_0) \times T) \cup (T \times (P_A \cup P_0)) \rightarrow \{0, 1\}$ such that $\forall i, j \in \mathbb{N}_n$, $j \neq i$, $((P_{A_j} \cup \{p_j^0\}) \times T_i) \cup (T_i \times (P_{A_j} \cup \{p_j^0\})) \rightarrow \{0\}$, and $W_R : (P_R \times T) \cup (T \times P_R) \rightarrow \mathbb{N}$;

(5) $\forall i \in \mathbb{N}_n$, the subset $\overline{N}_i$ generated by $P_{A_i} \cup \{p_i^0\} \cup T_i$ is a strongly connected state machine such that every cycle contains p_i^0;

(6) $\forall r \in P_R$, there exists a unique minimal P-invariant $I_r \in \mathbb{N}^{|P|}$ such that $\{r\} = \|I_r\| \cap P_R$, $P_0 \cap \|I_r\| = \emptyset$, $P_A \cap \|I_r\| \neq \emptyset$, and $I_r(r) = 1$, where $\mathbb{N}^{|P|}$ is a set of P-dimensional nonnegative integer vectors. Furthermore, $P_A = \bigcup_{r \in P_R} (\|I_r\| \setminus \{r\})$;

(7) N is strongly connected.

In the special case where S^4PR net corresponds to an asymmetric-choice (AC) net with non-blockingness ($\forall p \in P$, $\min_{t \in {}^\bullet p}\{W(t, p)\} \geq \min_{t \in p^\bullet}\{W(p, t)\}$) and homogeneous valuation ($\forall p \in P$, $\forall t, t' \in p^\bullet$, $W(p, t) = W(p, t')$), then it is well known that liveness property is equivalent to controlled-siphon property [2]. In this paper we extend this structural liveness characterization by dealing with S^4PR nets in the general case.

Definition 2. Let $N = (P_A \cup P_0 \cup P_R, T, F, W)$ be an S^4PR net. An initial marking M_0 is acceptable for N if (1) $\forall i \in \mathbb{N}_n$, $M_0(p_i^0) > 0$, (2) $\forall p \in P_A$, $M_0(p) = 0$, and (3) $\forall r \in P_R$, $M_0(r) > \max_{p \in \|I_r\|} I_r(p)$.

Example 3. The net (N, M_0) shown in Figure 1(a) is an S^4PR (it is also an AC net but with no homogeneous valuation), where $P_A = \{p_2\text{–}p_6, p_8\text{–}p_{12}\}$ is the set of operation places, $P_0 = \{p_1, p_7\}$ is the set of idle places, and $P_R = \{p_{13}\text{–}p_{16}\}$ is the set of shared resource places. Each operation place in S^4PR can simultaneously require multiple units of different resource types. For instance, operation place p_2 needs one unit in place p_{13} and two in place p_{14} simultaneously for its operation. And the S^4PR net model consists of two parallel sequential processes as shown in Figure 1(b), that is, process 1: $P_{A_1} = \{P_2\text{–}P_6\}$ and process 2: $P_{A_2} = \{P_8\text{–}P_{12}\}$. These two parallel sequential processes compete for the limited resources represented by four resource places $p_{13}\text{–}p_{16}$. For example, the operation place p_2 in process 1 requests one unit in resource place p_{13}, whereas the resource units may be held by operation place p_{12} in process 2. The competition of the limited resources may lead to deadlocks of the net model, which is an undesired phenomenon and must be prevented by some effective instruments.

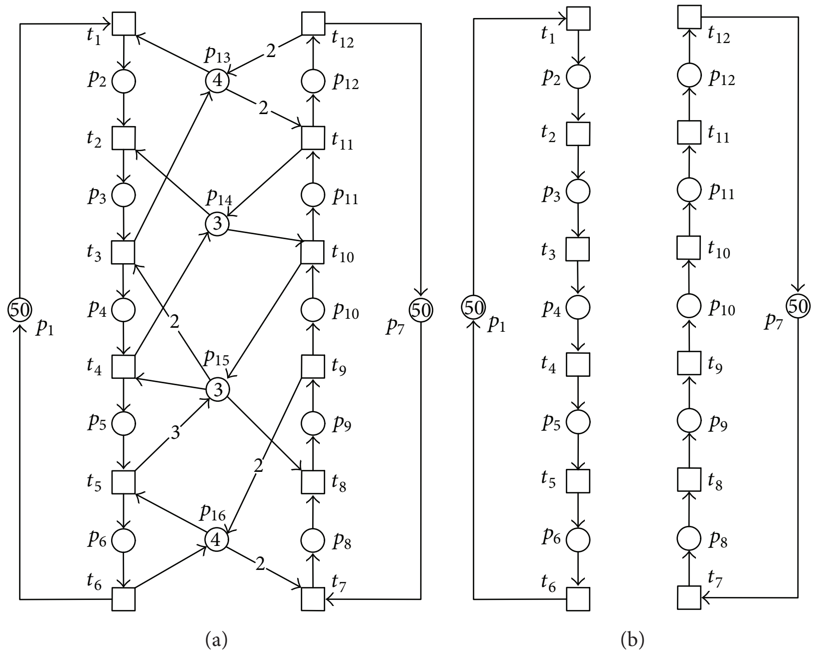

FIGURE 1: (a) An S^4PR model (N, M_0); (b) Processes 1 and 2.

$I_{p_1} = p_1+p_2+p_3+p_4+p_5+p_6$ and $I_{p_7} = p_7+p_8+p_9+p_{10}+p_{11}+p_{12}$ are the minimal P-semiflows associated with idle places p_1 and p_7. $I_{p_{13}} = p_2+p_3+2p_{12}+p_{13}$, $I_{p_{14}} = p_3+p_4+p_{11}+p_{14}$, $I_{p_{15}} = 2p_4+3p_5+p_9+p_{10}+p_{15}$, and $I_{p_{16}} = p_6+2p_8+2p_9+p_{16}$ are the minimal P-semiflows associated with resources p_{13}, p_{14}, p_{15}, and p_{16}, respectively. Let us consider the following acceptable initial marking $M_0 = 50p_1 + 50p_7 + 4p_{13} + 3p_{14} + 3p_{15} + 4p_{16}$. The net has seven strict minimal siphons (SMSs): $S_1 = \{p_3, p_4, p_{12}, p_{13}, p_{14}\}$, $S_2 = \{p_5, p_{11}, p_{14}, p_{15}\}$, $S_3 = \{p_5, p_9, p_{10}, p_{15}\}$, $S_4 = \{p_6, p_9, p_{10}, p_{15}, p_{16}\}$, $S_5 = \{p_5, p_{12}, p_{13}, p_{14}, p_{15}\}$, $S_6 = \{p_6, p_9, p_{11}, p_{14}, p_{15}, p_{16}\}$, and $S_7 = \{p_6, p_9, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$. The strict minimal siphons are closely related to the deadlocks of a net model. Once an SMS is insufficiently marked in an S^4PR, the net model will trap into deadlock.

3. Elementary Siphons in Generalized Petri Nets

3.1. Augmented and Elementary Siphons in S^4PR. The concept of elementary siphons was originally proposed in [17] for a class of ordinary Petri nets, S^3PR, and has been widely applied in ordinary Petri nets for designing liveness-enforcing supervisors. For more details, please refer to Appendix B: Elementary siphons. However, it still needs to be improved when a generalized Petri net is considered. In order to differentiate from the improved elementary siphons in this work, in what follows, let Π_{E_O} (resp., Π_{D_O}) denote elementary (resp., dependent) siphons defined in [17], which is called original elementary (resp., dependent) siphons in the rest of this paper.

For the S^4PR in Figure 1(a), by utilizing the concept of elementary siphons in [17], we have $[\lambda]$ and $[\eta]$ shown as follows:

$$[\lambda] = \begin{bmatrix} 0&0&1&1&0&0&0&0&0&0&0&1&1&1&0&0 \\ 0&0&0&0&1&0&0&0&0&0&1&0&0&1&1&0 \\ 0&0&0&0&1&0&0&0&1&1&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&1&1&0&0&0&0&1&1 \\ 0&0&0&0&1&0&0&0&0&0&0&1&1&1&1&0 \\ 0&0&0&0&0&1&0&0&1&0&1&0&0&1&1&1 \\ 0&0&0&0&0&1&0&0&1&0&0&1&1&1&1&1 \end{bmatrix}, \quad (1)$$

$$[\eta] = \begin{bmatrix} -1&0&1&0&0&0&0&0&0&-1&0&1 \\ 0&-1&-2&1&2&0&0&-1&0&1&0&0 \\ 0&0&-2&0&2&0&0&0&0&0&0&0 \\ 0&0&-2&-1&3&0&-2&0&2&0&0&0 \\ -1&-1&-1&1&2&0&0&-1&0&0&0&1 \\ 0&-1&-2&0&3&0&-2&0&1&1&0&0 \\ -1&-1&-1&0&3&0&-2&0&1&0&0&1 \end{bmatrix}. \quad (2)$$

It is easy to verify that $\text{Rank}([\eta]) = 5$, $\eta_{S_5} = \eta_{S_1} + \eta_{S_2}$, and $\eta_{S_7} = \eta_{S_1} + \eta_{S_6}$. It means that there are 5 original elementary siphons $\Pi_{E_O} = \{S_1\text{–}S_4, S_6\}$ and 2 original strongly dependent siphons $\Pi_{D_O} = \{S_5, S_7\}$.

For an S^4PR, the weight of an arc may be greater than one and an operation place can use simultaneously multiple types of resources. In this subsection, augmented siphons and improved elementary ones proposed for S^4PR in [23] are introduced. Since the weights information of arcs is vital for the liveness of generalized Petri nets and the permissive behavior of their corresponding liveness-enforcing supervisors, the notion of elementary siphons is redefined for S^4PR nets on the basis of augmented siphons. Consequently, the improved elementary siphons are compact and well suitable for S^4PR, which can lead to a structurally simple controlled system.

Definition 4 (see [23]). Let $N = (P_A \cup P_0 \cup P_R, T, F, W)$ be an S^4PR. For $r \in P_R$, $H(r) = \{p \mid p \in \|I_r\| \cap P_A\}$, the operation places that use r, is called the set of holders of r. Let place set $h_i(r) \subseteq H(r)$ be a subset of holders of r and $\bigcup h_i(r) = H(r)$. If $\forall p \in h_i(r)$, $\exists t \in (h_i(r))^{\bullet}$ and $p' \in h_i(r)$, such that $p^{\bullet} \cap {}^{\bullet}(p' \cup r) = t \neq \emptyset$, where $i \in \mathbb{N}^+$. Then $h_i(r)$ is called a subset of sequential holders of r, denoted as $h_i^s(r)$.

Definition 5 (see [23]). Let S be a siphon in an S^4PR $N = (P_A \cup P_0 \cup P_R, T, F, W)$ with $S = S^P \cup S^R$, where $S^R = S \cap P_R$ and $S^P = S \setminus S^R$. A multiset $\widetilde{S} = \sum_{p \in S} l(p)p$ is called an augmented version of S, where (1) $\forall p \in S^R$, $l(p) = 1$ and (2) $\forall p \in S^P$: (a) $p \in h^s(r)$, if $p^{\bullet} = t$, $t \in {}^{\bullet}r$, then $\exists A = \{p_i \mid p_i \prec_N p,\ p_i \in h^s(r),\ p_i \in S^P\} \cup \{p\}$, $\forall p \in A$, $l_r^t(p) = W(t, r)$; and (b) $l_r(p) = \sum_{t \in {}^{\bullet}r} l_r^t(p)$, $l(p) = \sum_{r \in S_R} l_r(p)$.

Note that a siphon S and its augmented version $\widetilde{S}$ are in one-to-one correspondence. In an S^4PR net, by considering the simultaneous requirements of multiple resources of different types by an operation place p, multiset $\widetilde{S}$ is introduced to represent the weighted relationship of p of holding and releasing resources in S. From Definition 5, $\widetilde{S} = \sum_{p \in S} l(p)p$, $\forall p \in S$, $l(p)$ denotes the coefficient of the places in an augmented siphon $\widetilde{S}$, which means that the support set of $\widetilde{S}$ is S; that is, $\|\widetilde{S}\| = S$. $\forall p \in S^R$, $l(p)$ always equals one; and $\forall p \in S^P$; the coefficient $l(p)$ is determined by the number of resource units held by the operation place p.

Example 6. For the net in Figure 1(a), take $S_2 = \{p_5, p_{11}, p_{14}, p_{15}\}$ as an example. For S_2, note that $H(p_{14}) = \{p_3, p_4, p_{11}\}$ with $h_1^s(p_{14}) = \{p_3, p_4\}$ and $h_2^s(p_{14}) = \{p_{11}\}$; $H(p_{15}) = \{p_4, p_5, p_9, p_{10}\}$ with $h_1^s(p_{15}) = \{p_4, p_5\}$ and $h_2^s(p_{15}) = \{p_9, p_{10}\}$. Thus, we have the following: (1) For $p_{14}, p_{15} \in P_R$, $l(p_{14}) = l(p_{15}) = 1$ and (2) for $p_5 \in P_A$, $p_5 \in h_1^s(p_{15})$, and $p_5^{\bullet} = t_5 \in {}^{\bullet}p_{15}$, then $l_{p_{15}}^{t_5}(p_5) = W(t_5, p_{15}) = 3$. For $p_{11} \in P_A$, $p_{11} \in h_2^s(p_{14})$ and $p_{11}^{\bullet} = t_{11} \in {}^{\bullet}p_{14}$, then $l_{p_{14}}^{t_{11}}(p_{11}) = W(t_{11}, p_{14}) = 1$. Hence, $l(p_5) = l_{p_{15}}(p_5) = l_{p_{15}}^{t_5}(p_5) = 3$ and $l(p_{11}) = l_{p_{14}}(p_{11}) = l_{p_{14}}^{t_{11}}(p_{11}) = 1$. As a result, we obtain $\widetilde{S}_2 = \{3p_5, p_{11}, p_{14}, p_{15}\}$. Similarly, we have $\widetilde{S}_1 = \{2p_3, p_4, 2p_{12}, p_{13}, p_{14}\}$, $\widetilde{S}_3 = \{3p_5, p_9, p_{10}, p_{15}\}$, $\widetilde{S}_4 = \{p_6, 3p_9, p_{10}, p_{15}, p_{16}\}$, $\widetilde{S}_5 = \{3p_5, 2p_{12}, p_{13}, p_{14}, p_{15}\}$, $\widetilde{S}_6 = \{p_6, 2p_9, p_{11}, p_{14}, p_{15}, p_{16}\}$, and $\widetilde{S}_7 = \{p_6, 2p_9, 2p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$.

Definition 7 (see [23]). Let $S = S^R \cup S^P$ be a siphon in an S^4PR N and let $\widetilde{S}$ be its augmented version. $[S] = \{p \mid p \in \|I_S\| \setminus S\}$ is called the complementary set of siphon S and $[\widetilde{S}] = I_S - \widetilde{S}$ is called the augmented complementary set of siphon S, where $I_S = \sum_{r \in S^R} I_r$.

Example 8. Take S_2 in the net N in Figure 1(a) as an example. Note that $I_{S_2} = I_{p_{14}} + I_{p_{15}} = p_3 + 3p_4 + 3p_5 + p_9 + p_{10} + p_{11} + p_{14} + p_{15}$. Thus, $[\widetilde{S}_2] = I_{S_2} - \widetilde{S}_2 = p_3 + 3p_4 + p_9 + p_{10}$.

Definition 9 (see [23]). Let $S \subseteq P$ be a subset of places in an S^4PR $N = (P, T, F, W)$. P-vector $\lambda_{\widetilde{S}}$ is called the augmented characteristic P-vector of $\widetilde{S}$ if $\forall p \in S$, $\lambda_{\widetilde{S}}(p) = l(p)$; otherwise $\lambda_{\widetilde{S}}(p) = 0$. $\eta_{\widetilde{S}} = [N]^T \lambda_{\widetilde{S}}$ is called the augmented characteristic T-vector of $\widetilde{S}$.

Definition 10 (see [23]). Let $N = (P, T, F, W)$ be an S^4PR with $|P| = m$ and $|T| = n$, and let $\Pi = \{S_1, S_2, \ldots, S_k\}$ be a set of siphons of N, where $m, n, k \in \mathbb{N}^+$. Let $\lambda_{\widetilde{S}_i}(\eta_{\widetilde{S}_i})$ be the augmented characteristic $P(T)$-vector of siphon S_i, $i \in \mathbb{N}_k$. $[\widetilde{\lambda}]_{k \times m} = [\lambda_{\widetilde{S}_1} \mid \lambda_{\widetilde{S}_2} \mid \cdots \mid \lambda_{\widetilde{S}_k}]^T$ and $[\widetilde{\eta}]_{k \times n} = [\widetilde{\lambda}]_{k \times m} \times [N]_{m \times n} = [\eta_{\widetilde{S}_1} \mid \eta_{\widetilde{S}_2} \mid \cdots \mid \eta_{\widetilde{S}_k}]^T$ are called the augmented characteristic P- and T-vector matrices of the siphons in N, respectively.

Definition 11 (see [23]). Let $[\widetilde{\eta}]$ be augmented characteristic T-vector matrix of the set of siphons $\Pi = \{S_1, S_2, \ldots, S_k\}$ in an S^4PR $N = (P, T, F, W)$:

(1) $\Pi_{E_A} = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of augmented elementary siphons in N if $\eta_{\widetilde{S}_\alpha}, \eta_{\widetilde{S}_\beta}, \ldots$, and $\eta_{\widetilde{S}_\gamma}$ ($\{\alpha, \beta, \ldots, \gamma\} \subseteq \mathbb{N}_k$) is a linearly independent maximal set of matrix $[\widetilde{\eta}]$.

(2) $S \notin \Pi_{E_A}$ is called a strongly augmented dependent siphon if $\eta_{\widetilde{S}} = \sum_{S_i \in \Pi_{E_A}} a_i \eta_{\widetilde{S}_i}$, where $a_i \geq 0$; $S \notin \Pi_{E_A}$ is called a weakly augmented dependent siphon if $\exists A, B \subset \Pi_{E_A}$ such that $A \neq \emptyset$, $B \neq \emptyset$, $A \cap B = \emptyset$, and $\eta_{\widetilde{S}} = \sum_{S_i \in A} a_i \eta_{\widetilde{S}_i} - \sum_{S_j \in B} a_j \eta_{\widetilde{S}_j}$, where $a_i, a_j \geq 0$.

(3) Let Π (resp., Π_{D_A}) be the set of strict minimal siphons (resp., augmented dependent siphons); we have $\Pi = \Pi_{E_A} \cup \Pi_{D_A}$.

Example 12. The net N in Figure 1(a) has 7 strict minimal siphons; we have obtained augmented versions of all 7 SMS by Definition 5. Accordingly, the corresponding $[\widetilde{\lambda}]$ and $[\widetilde{\eta}]$ are shown as follows:

$$[\widetilde{\lambda}] = \begin{bmatrix} 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$[\widetilde{\eta}] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & -2 & 3 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 3 & 0 & -2 & 2 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 3 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 & 3 & 0 & -2 & 1 & 0 & 1 & 0 & 0 \\ -1 & -1 & -1 & 0 & 3 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{3}$$

It is easy to verify that $\mathrm{Rank}([\widetilde{\eta}]) = 5$, $\eta_{\widetilde{S}_6} = \eta_{\widetilde{S}_2} + \eta_{\widetilde{S}_4} - \eta_{\widetilde{S}_3}$, and $\eta_{\widetilde{S}_7} = \eta_{\widetilde{S}_4} + \eta_{\widetilde{S}_5} - \eta_{\widetilde{S}_3}$. It means that 5 augmented elementary siphons $\Pi_{E_A} = \{S_1\text{–}S_5\}$ and 2 augmented dependent siphons $\Pi_{D_A} = \{S_6, S_7\}$ can be obtained based on the concept of augmented siphons.

Definition 13 (see [23]). Let Π_{E_O} (resp., Π_{D_O}) be a set of original elementary (resp., dependent) siphons and let Π_{E_A} (resp., Π_{D_A}) be a set of augmented elementary (resp., dependent) siphons in an S^4PR N. $\Pi_E = \Pi_{E_O} \cap \Pi_{E_A}$ (resp., $\Pi_D = \Pi_{D_O} \cup \Pi_{D_A}$) is called the set of elementary (resp., dependent) siphons of N.

Lemma 14 (see [23]). *Let Π be a set of SMS in an S^4PR. Then $\Pi = \Pi_E \cup \Pi_D$ and $\Pi_E \cap \Pi_D = \emptyset$, where $\Pi_E = \Pi_{E_O} \cap \Pi_{E_A}$ and $\Pi_D = \Pi_{D_O} \cup \Pi_{D_A}$.*

Example 15. For the net in Figure 1(a), we have $\Pi_{E_O} = \{S_1\text{–}S_4, S_6\}$, $\Pi_{D_O} = \{S_5, S_7\}$, $\Pi_{E_A} = \{S_1\text{–}S_5\}$, and $\Pi_{D_A} = \{S_6, S_7\}$. By Definition 13, $\Pi_E = \Pi_{E_O} \cap \Pi_{E_A} = \{S_1\text{–}S_4\}$ and $\Pi_D = \Pi_{D_O} \cup \Pi_{D_A} = \{S_5\text{–}S_7\}$ can be obtained. It is obvious that $\Pi_E \cup \Pi_D = \Pi = \{S_1\text{–}S_7\}$ is true. We have 4 elementary siphons and 3 dependent siphons finally.

3.2. Controllability of Siphons. The cs-property [2] is an important concept in liveness-enforcement for a generalized Petri net. The work in [2] provides the max-controlled condition of siphons that may overly restrict the permissive behavior of the supervisor. In order to reduce this restriction, a max$'$-controlled condition of siphons for generalized Petri net was first proposed in [28]. In this subsection, the formal definitions of max$'$-controlled siphons and the controllability of siphons are presented.

Definition 16 (see [28]). Let (N_1, M_{0_1}) be a subnet of (N, M_0) with $N_1 = (P_1, T_1, F_1, W_1)$ and $N = (P, T, F, W)$. Place $p \in P$ is called an input place of N_1 if $p \notin P_1$ and $\exists t \in T_1$, $p \in {}^\bullet t$ (i.e., $p \in {}^\bullet T_1$).

Let $N = (P, T, F, W)$ be a PN and let $S \subseteq P$ be a subset of places. The subnet generated by $X = S \cup {}^\bullet S$ is denoted by N_S, where $N_S = (S, {}^\bullet S, F_X, W_X)$. For convenience, the set of input places of N_S is called the set of input places of S, denoted as P_S^{in}.

For the net in Figure 1(a), take $S_2 = \{p_5, p_{11}, p_{14}, p_{15}\}$ as an example. $X = S_2 \cup {}^\bullet S_2$; for the subnet N_{S_2}, we have $P_{S_2}^{\mathrm{in}} = \{p_2, p_8\}$.

Definition 17 (see [23]). Let S be a siphon of a well initially marked S^4PR (N, M_0). S is said to be max$'$-marked at marking $M \in R(N, M_0)$ if (1) $\exists p \in S^P$ such that $M(p) \geq 1$ or (2) $\exists p \in S^R$ such that $M(p) \geq \max_{t \in (p^\bullet \cap ([S] \cap P_S^{\mathrm{in}})^\bullet)}\{W(p, t)\}$.

Lemma 18 (see [23]). *Let S be a siphon in an S^4PR (N, M_0) and let $M \in R(N, M_0)$ be a marking. S is max$'$-marked at M if $M(S) > \varpi(S)$, where $\varpi(S) = \sum_{p \in S^R}(\max_{t \in (p^\bullet \cap ([S] \cap P_S^{\mathrm{in}})^\bullet)}\{W(p, t)\} - 1)$.*

Definition 19. Let S be a siphon of a well initially marked S^4PR (N, M_0). S is said to be max$'$-controlled if S is max$'$-marked at any reachable marking M, $\forall M \in R(N, M_0)$.

Example 20. For the net shown in Figure 1(a), take $S_4 = \{p_6, p_9, p_{10}, p_{15}, p_{16}\}$ as an example. Note that $[S_4]^\bullet = \{t_4, t_5, t_8\}$; we have $p_{15}^\bullet \cap ([S_4] \cap P_{S_4}^{\mathrm{in}})^\bullet = \{t_8\}$ and $p_{16}^\bullet \cap ([S_4] \cap P_{S_4}^{\mathrm{in}})^\bullet = \{t_5\}$. Thus $\varpi(S_4) = \sum_{p \in S_{4R}}(\max_{t \in (p^\bullet \cap ([S] \cap P_S^{\mathrm{in}})^\bullet)}\{W(p, t)\} - 1) = 0$; that is, S_4 is max$'$-controlled if $\forall M \in R(N, M_0)$, $M(S_4) > 0$.

Theorem 21 (see [28]). *Let (N, M_0) be a well initially marked S^4PR. N is live if every siphon in N is* max$'$*-controlled.*

4. Deadlock Prevention Policy for S^4PR

An S^4PR is a subclass of sequential resource allocation systems, which can be defined by a set of resource types $P_R = \{r_i \mid i \in \mathbb{N}_m\}$ and a set of job processes $J = \{J_j \mid j \in \mathbb{N}_n\}$. Each resource r_i is characterized by its capacity C_{r_i}, a finite positive integer, which stands for the maximum number of parts that can contemporaneously hold in r_i. Each job type J_j is defined by a set of operations $J_j = \{p_{j,k} \mid k \in \mathbb{N}_{l_j},\ l_j \in \mathbb{N}^+\}$, which is partially ordered through a set of precedence constraints. l_j is the number of operation places in J_j.

An algebraic polynomial deadlock avoidance policy is proposed by Park and Reveliotis [26] for the class of conjunctive/disjunctive RASs, which can be represented as a polynomially sized set of linear inequalities in the state vector:

$$A_p \cdot q \leq f_p, \tag{4}$$

where A_p is an incidence matrix. Each row of A_p can be associated with a subset of process stages $JT^{(i)} = \{JT_{jk} \mid A_{[i,(j,k)]} = 1\}$. $A_{[i,\cdot]} \cdot q$ counts the number of operations in state q, which execute stages in $JT^{(i)}$. An algebraic policy can be expressed by the condition that a state q is admissible if the number of operations in state q in process stage $JT^{(i)}$ does not exceed the policy-defined bound $f_p[i]$ for every process stage subset $JT^{(i)}$. Hence, the sequential resource allocation can be managed reasonably to guarantee the absence of deadlock states and processes by considering the RUN policy; that is, it requires that a state is admissible if the number of jobs in the upstream neighbourhood of each resource r_i does not exceed its buffering capacity C_{r_i}.

Definition 22 (see [25]). Let (N, M_0) be a net system with place set P. A GMEC (l, b) defines a set of legal markings: $\mathcal{M}(l, b) = \{M \in \mathbb{N}^{|P|} \mid l^T M \leq b\}$, where $l : P \to \mathbb{N}$ is a weighting vector, $\mathbb{N}^{|P|}$ is a set of $|P|$-dimensional nonnegative integer vectors, and $b \in \mathbb{N}^+ = \{1, 2, \ldots\}$ is called the constraint constant.

The markings in $\mathbb{N}^{|P|}$ that are not in $\mathcal{M}(l, b)$ are called forbidden markings with respect to constraint (l, b).

Definition 23. A set of GMEC (L, B) with $L = [l_1 \mid l_2 \mid \cdots \mid l_m]$ and $B = (b_1, b_2, \ldots, b_m)$ defines a set of legal markings $\mathcal{M}(L, B) = \{M \in \mathbb{N}^{|P|} \mid L^T M \leq B\} = \bigcap_{i=1}^{m} \mathcal{M}(l_i, b_i)$.

Definition 24. Let (N, M_0) be an S^4PR with $N = (P_0 \cup P_A \cup P_R, T, F, W)$ and let (l, b) be a GMEC; the monitor that enforces this constraint is a new place V to be added to the net system (N, M_0). The resulting system is denoted as (N^c, M_0^c) with additional structure $N_V = (V, T_V, F_V, W_V)$. We assume that there are no self-loop containing V in N_V, and the initial marking M_0 satisfies the constraint (l, b). Then N_V will have incidence matrix: $[N_V] = -l^T \cdot [N]$. F_V can be uniquely determined by N_V, and $M_{0V}(V) = b - l^T$.

Definition 25. Let $r \in P_R$ be a resource in N. $H(r) = \{p \mid p \in \|I_r\| \cap P_A\}$ is the set of holders of r. Then $\forall r_i \in P_R$, $\sum_{p \in \{r_i\} \cup H(r_i)} I_{r_i}(p) M(p) = M_0(r_i) \equiv C_{r_i}$.

Theorem 26. *Let (N, M_0) be an S^4PR with $N = (P_A \cup P_0 \cup P_R, T, F, W)$ and let S be a strict minimal siphon of N. Siphon S is* max$'$*-marked if $\forall M \in R(N, M_0)$, where $M(S) \geq \xi_S$, $\xi_S > \varpi(S)$.*

Proof. From Lemma 18, S is max$'$-controlled if $\forall M \in R(N, M_0)$, $M(S) > \varpi(S)$. As a result, it is easy to see if $\forall M \in R(N, M_0)$, $M(S) \geq \xi_S$, where $\xi_S > \varpi(S)$, S is max$'$-marked. □

In order to develop a liveness-enforcing supervisor for any given S^4PR $(N, M_0) = (P_A \cup P_0 \cup P_R, T, F, W, M_0)$, we can add a monitor for every elementary siphon S of N, which imposes the linear inequality:

$$\lambda_S^T \cdot M \geq \xi_S, \tag{5}$$

where $\lambda_{\widetilde{S}}$ is an augmented characteristic vector of $\widetilde{S}$, M is the marking of net N, and ξ_S is a control depth variable such that $\varpi(S) < \xi_S < M_0(S)$. Since $\widetilde{S} + [\widetilde{S}] = I_S$ is the support of a P-invariant of N, we can conclude that $M(\widetilde{S}) + M([\widetilde{S}]) = M_0(\widetilde{S})$. From Definition 5, $M_0(S) = M_0(\widetilde{S}) = M(\widetilde{S}) + M([\widetilde{S}]) \geq M(S) + M([\widetilde{S}])$; that is, $M_0(S) - M([\widetilde{S}]) \geq M(S)$. As a result, the satisfaction of (5) can be ensured by satisfying the following inequality:

$$l_p^T \cdot M([S]) \leq M_0(S) - \xi_S, \tag{6}$$

where $\forall p \in [S_j]$, $l_p(p) := [\widetilde{S}_j](p)$; otherwise $l_p(p) := 0$.

According to Theorem 26 and Definition 24, an elementary siphon S of an S^4PR cannot be insufficiently marked after the addition of its corresponding monitor based on (6). The controllability of dependent siphons of N can be ensured by changing the control depth variables of its related elementary siphons. That is to say, all strict minimal siphons in N can be controlled. However, this can generate new insufficiently marked control-induced siphons. It is necessary for us to modify (6) to get some new constraints. Consequently, we can find that (6) is of the same type as (4). Hence the RUN policy [26] is considered, and the sequential resource allocation can be managed reasonably to guarantee the absence of deadlock states in the resulting net. In what follows, we utilize the polynomial-complexity DAP, which is an effective modification to C/D-RAS of the RUN DAP. Some notations are defined as follows.

Definition 27. Let $(N, M_0) = (P_A \cup P_0 \cup P_R, T, F, W, M_0)$ be an S^4PR. Suppose that a monitor R_i^* is added for each elementary siphon S_i with $M_0(R_i^*) = M_0(S_i) - \xi_{S_i}$; then R_i^* is called a virtual resource of (N, M_0).

Definition 27 defines the right-hand side of (6) by a virtual resource R_i^*. Each virtual resource relates to the resources included in its corresponding elementary siphon S_i but does not exist in net N actually. The number of virtual resources is equal to that of elementary siphons. Virtual resources R_i^* can serve as the temporary buffer in a system, and a state is admissible if the number of jobs in the upstream neighborhood of virtual resources R_i^* (denoted as $P_{\text{un}}(R_i^*)$) does not exceed its buffering capacity $M_0(R_i^*)$, where $P_{\text{un}}(R_i^*)$ is defined below.

Definition 28. Let $R^*(p_{j,k})$ be the virtual resource that support the execution of operation $p_{j,k}$ and let $\text{EP}(p_{j,k-q}, p_{j,k})$ be an elementary path from $p_{j,k-q}$ to $p_{j,k}$, where $j \in \mathbb{N}_n$, $k \in \mathbb{N}_{|J_j|}$, $q = \{1, 2, \ldots, k-1\}$, and J is the set of job processes in an S^4PR. $P_{\text{un}}(R_i^*) = \{p_{j,k} \mid R_i^* \in R^*(p_{j,k})\} \cup \{p_{j,k-q} \mid R_i^* \in R^*(p_{j,k-q+1}) \wedge O[R^*(p_{j,k-q})] \leq O(R_i^*)\}$ is called a set of job operations in the upstream neighborhood of resource place R_i^*, where $O(\cdot) : R^* \to \mathbb{N}_m$ is any partial order defined on R^*, and $i \in \mathbb{N}_m$.

Definition 29. Let $o_i \equiv O(R_i^*)$, $O : R^* \to \mathbb{N}_m$, be any partial order imposed on the virtual resource set R^*. Given an operation place $p \in P_A$, let

$$\rho_p^{\max} = \begin{cases} \max\left\{o_i \mid a_p[i] > 0,\ i \in \mathbb{N}_m\right\} & \text{if } \exists r \in R^*,\ p \in \|H(r)\| \\ 0, & \text{otherwise,} \end{cases}$$
$$\rho_p^{\min} = \begin{cases} \min\left\{o_i \mid a_p[i] > 0,\ i \in \mathbb{N}_m\right\} & \text{if } \exists r \in R^*,\ p \in \|H(r)\| \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Also, let $L_p = \{q \mid q \in p^{\bullet\bullet} \cap P_A \wedge \rho_q^{\max} = \min_{v \in p^{\bullet\bullet} \cap P_A} \rho_v^{\max}\}$. By convention, $L_p = \emptyset$ if $p^{\bullet\bullet} \cap P_0 \neq \emptyset$. Then we have the following:

(1) The neighborhood set N_p of $p \in P_A$ is defined recursively by the following equation:

$$N_p = \{p\} \cup \left\{ q \mid q \in \bigcup_{v \in L_p} N_v \wedge \rho_p^{\min} \leq \rho_q^{\max} \right\}. \quad (8)$$

(2) $\forall p \in P_A$, the adjusted resource allocation requirement $\widehat{a}_p$, is a m-dimensional nonnegative vector under partial order $O(\cdot)$ (resource ordering), which is given by the following expression:

$$\widehat{a}_p[i] = \begin{cases} \max\left\{a_q[i] \mid q \in N_p\right\}, & \text{if } o_i \geq \rho_p^{\min},\ i \in \mathbb{N}_m \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

The policy-imposed constraint on the system operation is expressed by the requirement that no virtual resource is overallocated with respect to the adjusted operation requirements specified by (9). Actually, the set of linear inequality constraints of elementary siphons can be written in the following matrix form:

$$\widehat{A}_p \cdot M_P \leq f_p, \quad (10)$$

where the column vector in $\widehat{A}_p$ corresponding to an operation place p is $\widehat{a}_p$, vector M_P is the restriction of marking M to operation places, and f_p is the capacity vector of virtual resources; that is, $f_p[i] = M_0(R_i^*)$, $i \in \mathbb{N}_m$.

Theorem 30. *Let (N, M_0) be an S^4PR and let (N^c, M_0^c) be the net resulting from adding monitors for elementary siphons only by Definition 24. (N^c, M_0^c) is a live controlled system if the following linear programming problem (LPP) has a feasible solution:*

$$\begin{aligned} \min \quad & \sum_{S_i \in \Pi_E} \widehat{\xi}_{S_i} \\ \text{s.t.} \quad & M\left(\left[\widetilde{S}_j\right]\right) > M_0\left(S_j\right) - \varpi\left(S_j\right), \quad \forall S_j \in \Pi_D; \\ & \widehat{A}_p \cdot M_P \leq f_p - \widehat{\xi}_p, \quad \forall S_i \in \Pi_E; \\ & I_r^T M = I_r^T M_0 = M_0(r), \quad \forall r \in P_R, \end{aligned} \quad (11)$$

where $\widehat{\xi}_p[i] + \varpi(S_i) + 1 = \xi_{S_i}$ and I_r is the P-invariant of r.

Proof. From Theorem 26 and Definition 29, all elementary siphons are max$'$-controlled and no control-induced siphon is generated due to the addition of monitors. If the LPP has a feasible solution, it means that all dependent siphons are controlled by properly setting the control depth variables of the elementary siphons. As a result, all siphons are max$'$-controlled, which indicates that the controlled system (N^c, M_0^c) is live. □

Based on the discussion above, an algorithm of designing liveness-enforcing supervisor for S^{4}PR based on improved elementary siphons is developed as in Algorithm 1.

Theorem 31. *Let (N, M_0) be a marked S^4PR net and let S_i be an elementary siphon of N. S_i is* max$'$*-controlled after adding monitor V_i by Algorithm 1 (line (14)).*

Proof. For Algorithm 1 (line (14)), the monitor V_i adding for siphon S_i is according to the Definition 24. And by Theorem 30, S_i is max$'$-controlled by the control of S_i. □

Theorem 32. *Let (N, M_0) be a marked S^4PR net and let Π_E be the set of elementary siphons of N. (N^c, M_0^c) is live if the set of monitors V is added by Algorithm 1.*

Proof. This result follows from the fact that all siphons are max$'$-controlled and no new non-max$'$-controlled siphon is generated in (N^c, M_0^c). By Theorem 21, (N^c, M_0^c) is live. □

For Algorithm 1, a complete siphon enumeration is needed, and the time complexity of computing all SMSs in the worst case is exponential with the number of nodes of the net system, that is, $O(a2^n)$, where a is the number of arcs and $n = |P| + |T|$ is the number of nodes of the nets. Therefore, the temporal complexity of Algorithm 1 is exponential with the scale of the Petri net model in the worst case.

Example 33. For the net in Figure 1(a), we have four elementary siphons: $S_1 = \{p_3, p_4, p_{12}, p_{13}, p_{14}\}$, $S_2 = \{p_5, p_{11}, p_{14}, p_{15}\}$, $S_3 = \{p_5, p_9, p_{10}, p_{15}\}$, and $S_4 = \{p_6, p_9, p_{10}, p_{15}, p_{16}\}$. Their augmented versions are $\widetilde{S}_1 = \{2p_3, p_4, 2p_{12}, p_{13}, p_{14}\}$, $\widetilde{S}_2 = \{3p_5, p_{11}, p_{14}, p_{15}\}$, $\widetilde{S}_3 = \{3p_5, p_9, p_{10}, p_{15}\}$, and $\widetilde{S}_4 = \{p_6, 3p_9, p_{10}, p_{15}, p_{16}\}$. Moreover, their augmented complementary sets are as follows: $[\widetilde{S}_1] = p_2 + p_{11}$, $[\widetilde{S}_2] = p_3 + 3p_4 + p_9 + p_{10}$, $[\widetilde{S}_3] = 2p_4$, and $[\widetilde{S}_4] = 2p_4 + 3p_5 + 2p_8$.

Then we can find the virtual resources for this system: $M_0(R_1^*) = M_0(S_1) - 2 = 5$, $M_0(R_2^*) = M_0(S_2) - 1 = 5$, $M_0(R_3^*) = M_0(S_3) - 1 = 2$, and $M_0(R_4^*) = M_0(S_4) - 1 = 6$. Therefore we obtain the initial set of linear inequality constraints expressed by state vector q:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 2 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot M \leq \begin{pmatrix} 5 \\ 5 \\ 2 \\ 6 \end{pmatrix}, \quad (12)$$

where M denotes the state vector defined by the operation place set, P_A, expressed by the following vector:

$$\left[M(p_2)\ \ M(p_3)\ \cdots\ M(p_6)\ \ M(p_8)\ \cdots\ M(p_{12})\right]^T. \quad (13)$$

We consider the net in Figure 1(a) under the virtual resource orders $o_1 = O(R_1^*) = 2$, $o_2 = O(R_2^*) = 1$, $o_3 = O(R_3^*) = 1$, and $o_4 = O(R_4^*) = 1$. From Definition 29, the neighborhood sets associated with the operation places $p \in P_A$ can be computed starting from the terminal operation places according to the partial order mentioned above.

Input: an S^4PR $(N, M_0) = (P_A \cup P_0 \cup P_R, T, F, W, M_0)$
Output: a live controlled system (N^c, M_0^c)
(1) begin;
(2) Compute the set of SMS and their corresponding augmented siphons
(3) Find the set of elementary siphons $\Pi_E = \{S_1, S_2, \ldots, S_m\}$, dependent siphons $\Pi_D = \{S_{m+1}, \ldots, S_n\}$, and the set of virtual resources $R^* = \{R_1^*, R_2^*, \ldots, R_m^*\}$.
(4) **for** ($j = 1$ to m) **do**
(5) Compute the augmented complementary set of elementary siphons $\Pi_E^c := \{[\widetilde{S}_1], [\widetilde{S}_2], \ldots, [\widetilde{S}_m]\}$,.
(6) **end for**
(7) Compute the set of linear inequality constraints expressed by state vectors from Π_E^c according to (4), where A_p is an incidence an incidence matrix, binary matrix q restricts the PN marking vector to its components corresponding to places $p \in P_A$, and $\forall p \in [\widetilde{S}_i]$, $a_p[i] := [\widetilde{S}_i](p)$, otherwise $a_p[i] := 0$; $f_p[i] := M_0(R_i^*) = M_0(S_i) - \xi_{S_i}$, $\xi_{S_i} := \varpi(S_i) + 1$.
(8) According to Definition 29, modify the matrix A_p to be matrix $\widehat{A}_p$.
(9) Get the set of GMEC expressed by the marking vector according to (10), that is, $\widehat{A}_p \cdot M_P \le f_p$, where vector M_P is the restriction of marking M to operation places.
(10) **if** ($\Pi_D \neq \emptyset$) **then**
(11) Find a set of $\xi_{S_1}, \ldots, \xi_{S_m}$ for elementary siphons by solving the LPP according to Theorem 30, where $\xi_{S_i} = \widehat{\xi}_p[i] + \varpi(S_i) + 1$.
(12) Get the adjusted GMEC $\widehat{A}_p \cdot M_P \le \widehat{f}_p$, where $\widehat{f}_p[i] := M_0(S_i) - \xi_{S_i}$.
(13) **end if**
(14) According to Definition 24 and the above set of GMEC, add the set of monitors for the corresponding Π_E to the net N, and the obtained controlled system is denoted by (N^c, M_0^c).
(15) output (N^c, M_0^c)
(16) end.

Algorithm 1: A deadlock prevention policy for S^4PR based on improved elementary siphons.

For job $J_1 = \{p_2, p_3, p_4, p_5, p_6\}$, we have

$\rho_{p_6}^{\max} = \rho_{p_6}^{\min} = 0$, $L_{p_6} = \emptyset$ by $(p_6^\bullet)^\bullet \cap P^0 \neq \emptyset$, and $N_{p_6} = \{p_6\}$;

$\rho_{p_5}^{\max} = \rho_{p_5}^{\min} = 1$, $L_{p_5} = \{p_6\}$, and $N_{p_5} = \{p_5\}$;

$\rho_{p_4}^{\max} = \rho_{p_4}^{\min} = 1$, $L_{p_4} = \{p_5\}$, and $N_{p_4} = \{p_4, p_5\}$;

$\rho_{p_3}^{\max} = \rho_{p_3}^{\min} = 1$, $L_{p_3} = \{p_4\}$, and $N_{p_3} = \{p_3, p_4, p_5\}$;

$\rho_{p_2}^{\max} = \rho_{p_2}^{\min} = 2$, $L_{p_2} = \{p_3\}$, and $N_{p_2} = \{p_2\}$.

For job $J_2 = \{p_8, p_9, p_{10}, p_{11}, p_{12}\}$, we have

$\rho_{p_{12}}^{\max} = \rho_{p_{12}}^{\min} = 0$, $L_{p_{12}} = \emptyset$, and $N_{p_{12}} = \{p_{12}\}$;

$\rho_{p_{11}}^{\max} = \rho_{p_{11}}^{\min} = 2$, $L_{p_{11}} = \{p_{12}\}$, and $N_{p_{11}} = \{p_{11}\}$;

$\rho_{p_{10}}^{\max} = \rho_{p_{10}}^{\min} = 1$, $L_{p_{10}} = \{p_{11}\}$, and $N_{p_{10}} = \{p_{10}, p_{11}\}$;

$\rho_{p_9}^{\max} = \rho_{p_9}^{\min} = 1$, $L_{p_9} = \{p_{10}\}$, and $N_{p_9} = \{p_9, p_{10}, p_{11}\}$;

$\rho_{p_8}^{\max} = \rho_{p_8}^{\min} = 1$, $L_{p_8} = \{p_9\}$, and $N_{p_8} = \{p_8, p_9, p_{10}, p_{11}\}$.

The adjusted virtual resource allocation requirements can be found as follows: $\widehat{a}_{p_2} = (1,0,0,0)^T$, $\widehat{a}_{p_3} = (0,3,2,3)^T$, $\widehat{a}_{p_4} = (0,3,2,3)^T$, $\widehat{a}_{p_5} = (0,0,0,3)^T$, $\widehat{a}_{p_6} = (0,0,0,0)^T$, $\widehat{a}_{p_8} = (1,1,0,2)^T$, $\widehat{a}_{p_9} = (1,1,0,0)^T$, $\widehat{a}_{p_{10}} = (1,1,0,0)^T$, $\widehat{a}_{p_{11}} = (1,0,0,0)^T$, and $\widehat{a}_{p_{12}} = (0,0,0,0)^T$. Thus the modified set of linear inequality constraints expressed by state vector $\widehat{A}_p \cdot M_P \le f_p$ is shown below:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 3 & 3 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 & 0 & 2 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot M \le \begin{pmatrix} 5 \\ 5 \\ 2 \\ 6 \end{pmatrix}. \quad (14)$$

According to Algorithm 1, a feasible solution of the LPP according to Theorem 30, $\widehat{\xi}_p[1] = \widehat{\xi}_p[2] = \widehat{\xi}_p[3] = \widehat{\xi}_p[4] = 0$, can be found and $\xi_{S_i} = \varpi(S_i) + 1$. The adjusted GMEC is consistent with $\widehat{A}_p \cdot M_P \le f_p$.

Four constraints need to add four monitors V_1, V_2, V_3, and V_4 to the original net system. It can be indicated that all siphons are max$'$-controlled. The resulting net system (N^c, M_0^c) is live after adding four monitors as shown in Table 1.

The example is also studied by applying control polices reported in other papers. Table 2 shows their comparison that is done in terms of resultant supervisor's structural complexity and behavior permissiveness by using policies among [13, 23] and the proposed method.

Next, another S^4PR example shown in Figure 2, which is not an AC net and without homogeneous valuation, is used to further illustrate the applicability and efficiency of

Table 1: Monitors V_i added for the net shown in Figure 1(a).

V_i	${}^{\bullet}V_i$	$V_i^{\bullet}$	$M_0^c(V_i)$
V_1	t_2, t_{11}	t_1, t_7	5
V_2	$3t_4, t_{10}$	$3t_2, t_7$	5
V_3	$2t_4$	$2t_2$	2
V_4	$3t_5, 2t_8$	$3t_2, 2t_7$	6

Table 2: The performance comparison of supervisors for the S^4PR in Figure 1(a) due to different policies.

Parameter	The method in [13]	The method in [23]	The proposed method
Permissive behavior	676	1723	2032
Number of monitors	5	4	4
Number of arcs	19	14	14

Table 3: Monitors V_i added for the net shown in Figure 2.

V_i	${}^{\bullet}V_i$	$V_i^{\bullet}$	$M_0^c(V_i)$
V_1	$3t_3, 3t_5, t_{11}$	$3t_1, t_9$	4
V_2	t_6, t_{11}	t_5, t_9	4
V_3	$2t_7, t_{10}$	$2t_6, t_9$	3

Table 4: The performance comparison of supervisors for the S^4PR in Figure 2 due to different policies.

Parameter	The method in [13]	The method in [23]	The proposed method
Permissive behavior	456	1158	1453
Number of monitors	5	3	3
Number of arcs	26	13	13

the proposed method. By using Algorithm 1, the resulting live controlled system (N^c, M_0^c) with three monitors is shown in Table 3. Similarly, Table 4 shows the comparison of control performances among [13, 23] and the proposed method.

From this two case studies, it can be concluded that the liveness-enforcing supervisors synthesised by the proposed method are more structurally simple, and the final controlled net system can be obtained with more permissive behavior than some other elementary siphon-based approaches. Tracing to the essential reasons, the advantage of the proposed method is that the number of the elementary siphons explicitly controlled is more smaller by introducing the concept of augmented siphons, which can significantly reduce the structural complexity of the supervisors. Meanwhile, the constraints imposed by the monitors are less restrictive on account of combining the DAP of RUN policy with improved elementary siphons in an S^4PR net.

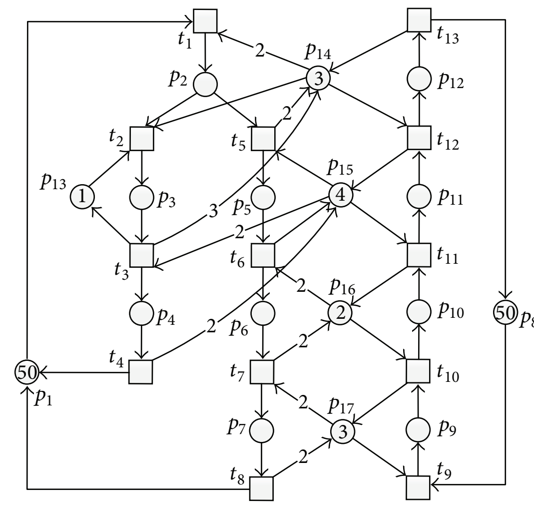

Figure 2: An S^4PR net system (N, M_0).

5. Conclusion

This paper presents an elementary siphon-based deadlock prevention method for a class of generalized Petri nets, called S^4PR that can deal with the modeling of concurrently cyclic sequential processes sharing common resources, where deadlocks are caused by the insufficiently marked siphons in their Petri net models. Generally, all insufficiently marked siphons are divided into elementary siphons and dependent ones by using the concept of augmented siphons in this work. From the former, a set of linear inequality constraints expressed by state vectors is obtained. Then we utilize a deadlock control method that combines the DAP of RUN policy with elementary siphons theory to ensure that all siphons are max$'$-controlled after adding a monitor for each elementary siphon. Consequently, the resulting net system by using the proposed method is live. The major advantage of this new Petri nets based deadlock prevention method as a liveness-enforcing supervisor synthesis is that a small number of monitors are added leading to a more permissive behavior. Since a complete siphon enumeration is computationally expensive, our future work will be guided to develop an approach without a complete siphon enumeration.

Appendices

In this part, the fundamental concepts and some of the notations of Petri nets and the definitions of elementary siphons involved in this work are recalled to make paper self-contained. For more details, please refer to [17, 29].

A. Basics of Petri Nets

A Petri net, as a graphical and mathematical model, is a directed bipartite graph, which consists of a net structure and an initial marking. A formal definition is given as follows [29].

Definition 34. A generalized Petri net structure is a four-tuple $N = (P, T, F, W)$, where P and T are finite and nonempty sets. P is the set of places and T is the set of transitions with $P \cap T = \emptyset$. $F \subseteq (P \times T) \cup (T \times P)$ is called flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. $W : F \to \mathbb{N}$ is a mapping that assigns a weight to any arc: $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$, otherwise, where $x, y \in P \cup T$ and $\mathbb{N} = \{0, 1, 2, \ldots\}$. $N = (P, T, F, W)$ is called an ordinary net, denoted as $N = (P, T, F)$, if $\forall (x, y) \in F$, $W(x, y) = 1$.

A net $N = (P, T, F, W)$ is pure (self-loop free) if $\forall x, y \in P \cup T$, $W(x, y) > 0$ implies $W(y, x) = 0$. A pure net N can be alternatively represented by its incidence matrix $[N]$, which is a $|P| \times |T|$ integer matrix with $[N](p, t) = W(t, p) - W(p, t)$. For a place p (resp., transition t), its incidence vector, a row (resp., column) in $[N]$, is denoted as $[N](p, \cdot)$ (resp., $[N](\cdot, t)$).

A marking M of N is a mapping from P to $\mathbb{N}$. $M(p)$ denotes the number of tokens contained in place p, which is marked by M if $M(p) > 0$. Let $S \subseteq P$ be a set of places. $M(S)$ denotes the sum of tokens contained in S at marking M, where $M(S) = \sum_{p \in S} M(p)$. (N, M_0) is called a net system, and M_0 is called an initial marking of N. For economy of space, we use $\sum_{p \in P} M(p)p$ to denote vector M.

Let $x \in P \cup T$ be a node of net $N = (P, T, F, W)$. The preset of x is defined as ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$, while the postset of x is defined as $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$. This notation can be extended to a set of nodes as follows: given $X \subseteq P \cup T$, ${}^\bullet X = \bigcup_{x \in X} {}^\bullet x$, and $X^\bullet = \bigcup_{x \in X} x^\bullet$.

A transition $t \in T$ is enabled at a marking M if $\forall p \in {}^\bullet t$, $M(p) \geq W(p, t)$, which is denoted as $M[t\rangle$; when fired in a usual way, it gives a new marking M' such that $\forall p \in P$, $M'(p) = M(p) - W(p, t) + W(t, p)$, which is denoted as $M[t\rangle M'$. Marking M' is said to be reachable from M if there exists a sequence of transitions $\sigma = t_0 t_1 \cdots t_n$ and markings $M_1, M_2, \ldots$, and M_n such that $M[t_0\rangle M_1 [t_1\rangle M_2 \cdots M_n [t_n\rangle M'$ holds. The set of markings reachable from M in N is denoted as $R(N, M)$. The set of all reachable markings for a Petri net N with initial marking M_0 is denoted by $R(N, M_0)$.

A transition $t \in T$ is live under M_0 if $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$. (N, M_0) is live if $\forall t \in T$, t is live under M_0. (N, M_0) is deadlock-free if $\forall M \in R(N, M_0)$, $\exists t \in T$, and $M[t\rangle$ hold. A Petri net N contains a deadlock if there is a marking $M \in R(N, M_0)$ at which no transition is enabled. Such a marking is called a dead marking. Deadlock situations are a result of inappropriate resource allocation policies or exhaustive use of some or all resources. Liveness of a Petri net means that, for each marking $M \in R(N, M_0)$ reachable from M_0, it is finally possible to fire any transition t in the Petri net through some firing sequence. This means if a Petri net is live, then it has no deadlock. (N, M_0) is bounded if $\exists k \in \mathbb{N}^+$, $\forall M \in R(N, M_0)$, $\forall p \in P$, and $M(p) \leq k$ hold. Boundedness is used to identify the existence of overflows in the modeled system. (N, M_0) is said to be reversible, if, for each marking $M \in R(N, M_0)$, M_0 is reachable from M. A marking M' is said to be a home state, if, for each marking $M \in R(N, M_0)$, M' is reachable from M.

A P-vector is a column vector $I : P \to \mathbb{Z}$ indexed by P and a T-vector is a column vector $J : T \to \mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the set of integers. I^T and $[N]^T$ are the transposed versions of a vector I and a matrix $[N]$, respectively. P-vector I is a P-invariant (place invariant) if $I \neq \mathbf{0}$ and $I^T[N] = \mathbf{0}^T$. P-invariant I is said to be a P-semiflow if no element of I is negative. $\| I \|^+ = \{p \mid I(p) > 0\}$ denotes the positive support of P-invariant I, while $\| I \|^- = \{p \mid I(p) < 0\}$ denotes the negative support of I. An invariant is called minimal when its support is not a strict superset of the support of any other, and the greatest common divisor of its elements is one. If I is a P-invariant of (N, M_0) then $\forall M \in R(N, M_0)$, $I^T M = I^T M_0$.

A nonempty set $S \subseteq P$ is a siphon if ${}^\bullet S \subseteq S^\bullet$. $S \subseteq P$ is a trap if $S^\bullet \subseteq {}^\bullet S$. A siphon (resp., trap) is minimal if there is no siphon (resp., trap) contained in it as a proper subset. A minimal siphon S is said to be strict if ${}^\bullet S \subset S^\bullet$. A siphon remains empty once it loses all tokens. A trap remains marked once it is marked. Let S be a siphon in a net (N, M_0). S is said to be max-marked at a marking $M \in R(N, M_0)$ if $\exists p \in S$ such that $M(p) \geq \max_{p^\bullet}$. A siphon is said to be max-controlled if it is max-marked at any reachable marking.

B. Elementary Siphon

Elementary and dependent siphons were first proposed in [17] and are essential to the development of a structurally simple liveness-enforcing monitor-based supervisor.

Let $S \subseteq P$ be a subset of places of Petri net $N = (P, T, F, W)$. P-vector λ_S is called the characteristic P-vector of S if $\forall p \in S$, $\lambda_S(p) = 1$; otherwise $\lambda_S(p) = 0$. $\eta_S = [N]^T \lambda_S$ is called the characteristic T-vector of S, where $[N]^T$ is the transpose of incidence matrix $[N]$.

Let $N = (P, T, F, W)$ be a net with $|P| = m$, $|T| = n$, and let $\Pi = \{S_1, S_2, \ldots, S_k\}$ be a set of siphons of N, where $m, n, k \in \mathbb{N}^+ = \{1, 2, 3, \ldots\}$. Let λ_{S_i} (resp. η_{S_i}) be the characteristic P-vector (resp., T-vector) of siphon S_i, where $i \in \mathbb{N}_k = \{1, \ldots, k\}$. $[\lambda]_{k \times m} = [\lambda_{S_1} \mid \lambda_{S_2} \mid \cdots \mid \lambda_{S_k}]^T$ and $[\eta]_{k \times n} = [\lambda]_{k \times m} \times [N]_{m \times n} = [\eta_{S_1} \mid \eta_{S_2} \mid \cdots \mid \eta_{S_k}]^T$ are called the characteristic P- and T-vector matrices of the siphons in N, respectively.

Definition 35 (see [17]). Let $\eta_{S_\alpha}, \eta_{S_\beta}, \ldots$, and η_{S_γ} ($\{\alpha, \beta, \ldots, \gamma\} \subseteq \mathbb{N}_k$) be a linearly independent maximal set of matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons in N. $S \notin \Pi_E$ is called a strongly dependent siphon if $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$, where $a_i \geq 0$. $S \notin \Pi_E$ is called a weakly dependent siphon if $\exists A, B \subset \Pi_E$ such that $A \neq \emptyset$, $B \neq \emptyset$, $A \cap B = \emptyset$, and $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_j \in B} a_j \eta_{S_j}$, where $a_i, a_j \geq 0$.

Elementary and dependent siphons defined in Definition 35 are originally proposed in [17] and further clarified in [18]. In order to differentiate from the augmented elementary ones proposed in [23], elementary siphons defined in [17] are called the original elementary siphons (dependent siphons) in this paper and denoted as Π_{E_O} (resp., Π_{D_O}), which is called the set of original elementary (resp., original dependent) siphons.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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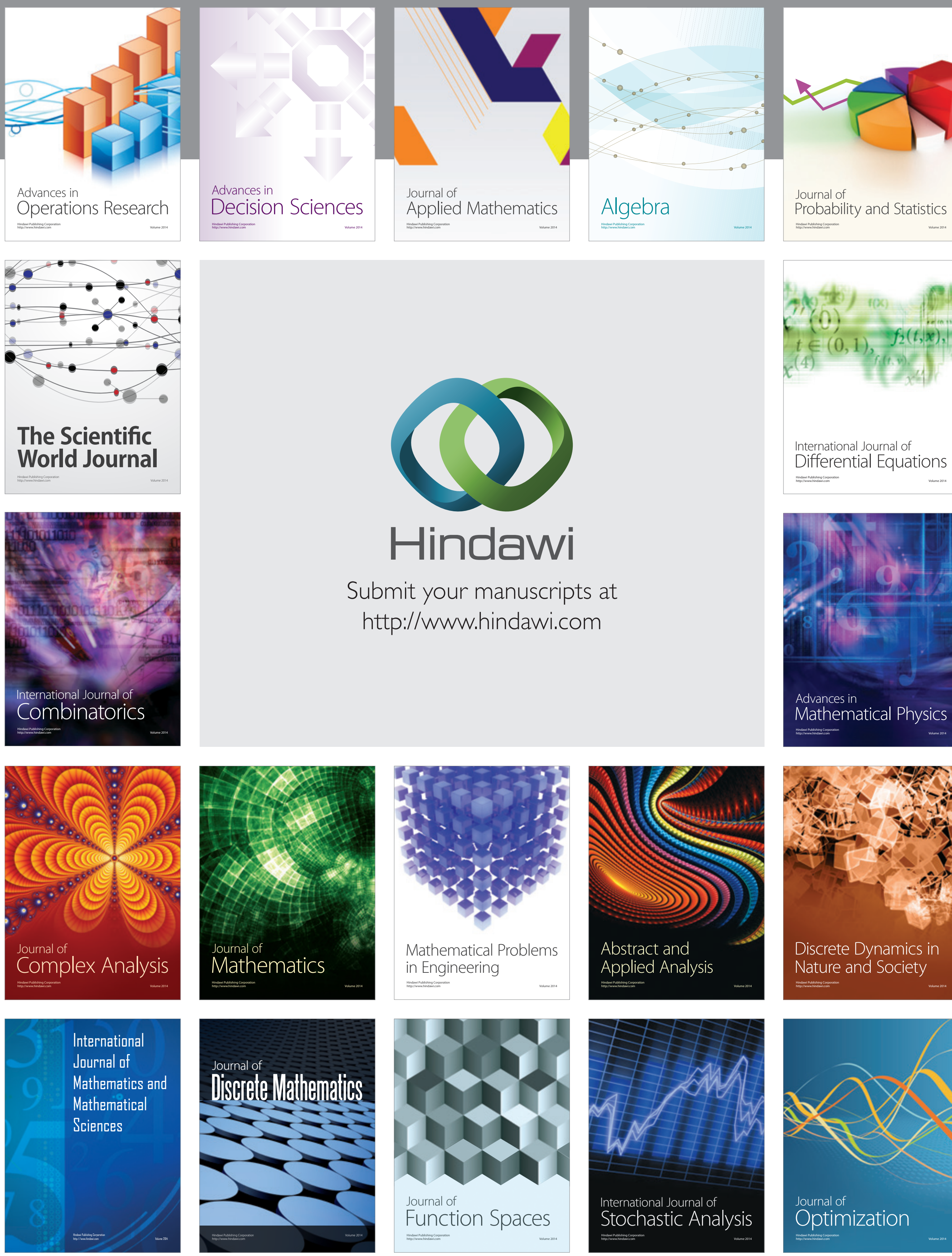

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