Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2009, Article ID 164303, 9 pages
doi:10.1155/2009/164303

Research Article

Sharp Condition for Global Existence and Blow-Up on Klein-Gordon Equation

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Received 26 November 2008; Revised 17 March 2009; Accepted 17 April 2009

Recommended by Ben T. Nohara

We study the initial boundary value problem of the nonlinear Klein-Gordon equation. First we introduce a family of potential wells. By using them, we obtain a new existence theorem of global solutions and show the blow-up in finite time of solutions. Especially the relation between the above two phenomena is derived as a sharp condition.

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1. Introduction

Klein-Gordon equation is one of the famous evolution equations arising in relativistic quantum mechanics. There are a lot of literature giving the outline of its study trace. For the following type nonlinear Klein-Gordon (NLKG) equation:

$$\varphi_{tt} - \Delta\varphi + \varphi = |\varphi|^{p-1}\varphi, \tag{1.1}$$

a lot of papers show the global and local well-posedness and blow-up properties for the Cauchy problem of the above NLKG equation, which can be found in [1–5]. Especially Zhang derived a sharp condition for the global existence of the Cauchy problem of the above NLKG equation in [6]. By introducing a so-called ground state solution, which is the positive solution of the nonlinear Euclidean scalar field equation $\Delta u - u + u^p = 0$, he applied a host of very useful properties of the ground state solution to show the sharp condition for this Cauchy problem. In the present paper, we try to make use of the classical potential wells argument [7], which is different from that in [6], to clarify the sharp condition for initial boundary value problem (IBVP) of the same NLKG equation.

2. Potential Wells and Their Properties

In this paper, we study the initial boundary value problem of nonlinear Klein-Gordon equation

$$\begin{gathered}\varphi_{tt} - \triangle\varphi + \varphi = |\varphi|^{p-1}\varphi, \quad x \in \Omega, \ t > 0, \\ \varphi(x,0) = \varphi_0(x), \quad \varphi_t(x,0) = \varphi_1(x), \quad x \in \Omega, \\ \varphi(x,t) = 0, \quad x \in \partial\Omega, \ t \geqslant 0,\end{gathered} \tag{2.1}$$

where $1 < p < \infty$ for $n \leqslant 2$; $1 < p < (n+2)/(n-2)$ for $n \geqslant 3$.

For problem (2.1), we define the energy function and some functionals as follows:

$$\begin{aligned}E(t) &= \frac{1}{2}\|\varphi_t\|^2 + \frac{1}{2}\|\varphi\|^2 + \frac{1}{2}\|\nabla\varphi\|^2 - \frac{1}{p+1}\|\varphi\|_{p+1}^{p+1}, \\ J(\varphi) &= \frac{1}{2}\|\varphi\|^2 + \frac{1}{2}\|\nabla\varphi\|^2 - \frac{1}{p+1}\|\varphi\|_{p+1}^{p+1}, \\ I(\varphi) &= \|\varphi\|^2 + \|\nabla\varphi\|^2 - \|\varphi\|_{p+1}^{p+1}, \\ I_\delta(\varphi) &= \delta\|\nabla\varphi\|^2 + \|\varphi\|^2 - \|\varphi\|_{p+1}^{p+1}.\end{aligned} \tag{2.2}$$

In aid of the above functionals, we define the potential well as follows:

$$W = \left\{\varphi \in H_0^1(\Omega) \mid I(\varphi) > 0, J(\varphi) < d\right\} \cup \{0\}, \tag{2.3}$$

where

$$\begin{gathered}d = \inf_{\varphi \in \mathcal{N}} J(\varphi), \\ \mathcal{N} := \left\{\varphi \in H_0^1(\Omega) \mid I(\varphi), \|\nabla\varphi\| \neq 0\right\}.\end{gathered} \tag{2.4}$$

Then we further give the following definitions

$$\begin{gathered}d(\delta) = \inf_{\varphi \in \mathcal{N}_\delta} J(\varphi), \\ \mathcal{N}_\delta := \left\{\varphi \in H_0^1(\Omega) \mid I_\delta(\varphi) = 0, \|\nabla\varphi\| \neq 0\right\}.\end{gathered} \tag{2.5}$$

Now, it is ready for us to define a family of potential wells and the outside sets of the corresponding potential wells sets as follows:

$$W_\delta = \left\{\varphi \in H_0^1(\Omega) \mid I_\delta(\varphi) > 0, J(\varphi) < d(\delta)\right\} \cup \{0\}, \quad 0 < \delta < 1,$$
$$V = \left\{\varphi \in H_0^1(\Omega) \mid I(\varphi) < 0, J(\varphi) < d\right\}, \tag{2.6}$$
$$V_\delta = \left\{\varphi \in H_0^1(\Omega) \mid I_\delta(\varphi) < 0, J(\varphi) < d(\delta)\right\}.$$

The following lemmas are given to show the relations between the functional $I_\delta(\varphi)$ and $\|\nabla\varphi\|$.

Lemma 2.1. *If* $0 < \|\nabla\varphi\| < r(\delta)$ *then* $I_\delta(\varphi) > 0$, *where* $r(\delta) = (\delta/C_*^{p+1})^{1/(p-1)}$ *and* $C_* = \sup(\|\varphi\|_{p+1}/\|\nabla\varphi\|)$.

Proof. If $0 < \|\nabla\varphi\| < r(\delta)$, then

$$I_\delta(\varphi) = \delta\|\nabla\varphi\|^2 - \left(\|\varphi\|_{p+1}^{p+1} - \|\varphi\|^2\right),$$
$$\|\varphi\|_{p+1}^{p+1} - \|\varphi\|^2 \leqslant C_*^{p+1}\|\nabla\varphi\|^{p+1} \leqslant \delta\|\nabla\varphi\|^2. \tag{2.7}$$

Hence $I_\delta(\varphi) > 0$. □

Lemma 2.2. *If* $I_\delta(\varphi) < 0$ *then* $\|\nabla\varphi\| > r(\delta)$.

Proof. Note that $I_\delta(\varphi) < 0$ gives

$$\delta\|\nabla\varphi\|^2 < \|\varphi\|_{p+1}^{p+1} - \|\varphi\|^2 < \|\varphi\|_{p+1}^{p+1} \leqslant C_*^{p+1}\|\nabla\varphi\|^{p-1}\|\nabla\varphi\|^2, \tag{2.8}$$

which implies

$$\|\nabla\varphi\|^{p-1} > \frac{\delta}{C_*^{p+1}} = r^{p-1}(\delta), \tag{2.9}$$

that can be deduced from (2.8). □

Lemma 2.3. *As the function of* δ, $d(\delta)$ *is increasing on* $0 < \delta \leq 1$, *decreasing on* $1 \leq \delta \leq (p+1)/2$, *and takes the maximum* $d = d(1)$ *at* $\delta = 1$.

Proof. Now we prove that $d(\delta^1) < d(\delta^2)$ for any $0 < \delta^1 < \delta^2 < 1$ or $1 < \delta^2 < \delta^1 < (p+1)/2$. Clearly, it is enough to prove that for any $0 < \delta^1 < \delta^2 < 1$ or $1 < \delta^2 < \delta^1 < (p+1)/2$ and for any $\varphi \in \mathcal{N}_{\delta^2}$, there exist a $v \in \mathcal{N}_{\delta^1}$ and a constant $\varepsilon(\delta^1, \delta^2) \geq \mathcal{N}_{\delta^1}$ such that $J(v) <$

$J(\varphi)-\varepsilon(\delta^1,\delta^2)$. In fact, for the previous φ we also define $\lambda(\delta)$ by $\delta(\|\lambda\varphi\|^2+\|\lambda\nabla\varphi\|^2) = \|\lambda\varphi\|_{p+1}^{p+1}$, then $I_\delta(\lambda(\delta)\varphi) = 0$, $\lambda(\delta^2) = 1$. Let $g(\lambda) = J(\lambda\varphi)$, then

$$\begin{aligned}\frac{d}{d\lambda}g(\lambda) &= \frac{1}{\lambda}\left(\|\lambda\varphi\|^2 + \|\lambda\nabla\varphi\|^2 - \|\lambda\varphi\|_{p+1}^{p+1}\right)\\ &= \frac{1}{\lambda}\left((1-\delta)\|\lambda\nabla\varphi\|^2 + I_\delta(\lambda\varphi)\right)\\ &= (1-\delta)\|\lambda\nabla\varphi\|^2.\end{aligned} \tag{2.10}$$

Take $v = \lambda(\delta^1)\varphi$, then $v \in \mathcal{N}_{\delta^1}$.

For $0 < \delta^1 < \delta^2 < 1$, we have

$$J(\varphi) - J(v) = g(1) - g\left(\lambda\left(\delta^1\right)\right) > \left(1-\delta^2\right)r^2\left(\delta^2\right)\lambda\left(\delta^1\right)\left(1-\lambda\left(\delta^1\right)\right) \equiv \varepsilon\left(\delta^1,\delta^2\right). \tag{2.11}$$

For $1 < \delta^2 < \delta^1 < (p+1)/2$, we have

$$J(\varphi) - J(v) = g(1) - g\left(\lambda\left(\delta^1\right)\right) > \left(\delta^2-1\right)r^2\left(\delta^2\right)\lambda\left(\delta^2\right)\left(\lambda\left(\delta^1\right)-1\right) \equiv \varepsilon\left(\delta^1,\delta^2\right). \tag{2.12}$$

These give the conclusion. □

3. Sharp Condition for Global Existence and Blow-Up

Definition 3.1 (weak solution). The function $\varphi(x,t) \in L^\infty(0,T;H_0^1(\Omega))$ with $\varphi_t(t,x) \in L^\infty(0,T;L^2(\Omega))$ is called a weak solution of problem (2.1) for $t \in [0,T)$ if the following conditions are satisfied:

(1) $(\varphi_t,v) + \int_0^T(\nabla\varphi,\nabla v)d\tau + \int_0^T(\varphi,v)d\tau = \int_0^T(\varphi|\varphi|^{p-1},v)d\tau + (\varphi_1(x),v)$,

(2) $\varphi(x,0) = \varphi_0(x)$ in $H_0^1(\Omega)$, $\varphi_t(x,0) = \varphi_1(x)$ in $L^2(\Omega)$,

for all $v \in H_0^1(\Omega)$.

Theorem 3.2 (global existence). *Let p satisfy*

(H) $1 < p < \infty$ *for* $n \le 2$; $1 < p < (n+2)/(n-2)$ *for* $n \ge 3$.

Let $\varphi_0(x) \in H_0^1(\Omega)$, $\varphi_1(x) \in L^2(\Omega)$. Suppose that $0 < E(0) < d$, $I(\varphi_0) > 0$, or $\|\nabla\varphi_0\| = 0$. Then problem (2.1) admits a global weak solution $\varphi(x,t) \in L^\infty(0,\infty;H_0^1(\Omega))$, $\varphi_t(x,t) \in L^\infty(0,\infty;L^2(\Omega))$ with $\varphi(t) \in W$.

Proof. Let $\{w_j(x)\}$ be a system of base functions in $H_0^1(\Omega)$. Construct the approximate solutions $\varphi_m(x,t)$ of problem (2.1) as done in [7]

$$\varphi_m(x,t) = \sum_{j=1}^m g_{jm}(t)w_j(x), \quad m = 1,2,\ldots, \tag{3.1}$$

satisfying

$$
\begin{gathered}
(\varphi_{mtt}, w_s) + (\nabla\varphi_m, \nabla w_s) + (\varphi_m, w_s) = \left(\varphi_m |\varphi_m|^{p-1}, w_s\right), \qquad s = 1,2,\ldots, \\
\varphi_m(x,0) = \sum_{j=1}^{m} a_{jm} w_j(x) \longrightarrow \varphi_0(x) \quad \text{in } H_0^1(\Omega), \\
\varphi_{mt}(x,0) = \sum_{j=1}^{m} b_{jm} w_j(x) \longrightarrow \varphi_1(x) \quad \text{in } L^2(\Omega).
\end{gathered} \tag{3.2}
$$

Multiplying (3.2) by $g'_{sm}(t)$ and summing for s we can obtain

$$
\frac{1}{2}\frac{d}{dt}\|\varphi_{mt}\|^2 + \frac{1}{2}\frac{d}{dt}\|\nabla\varphi_m\|^2 + \frac{1}{2}\frac{d}{dt}\|\varphi_m\|^2 - \frac{1}{p+1}\frac{d}{dt}\|\varphi_m\|_{p+1}^{p+1} = 0. \tag{3.3}
$$

Integrating with respect to t we obtain

$$
\begin{aligned}
E_{mt}(t) &= \frac{1}{2}\|\varphi_{mt}\|^2 + \frac{1}{2}\|\nabla\varphi_m\|^2 + \frac{1}{2}\|\varphi_m\|^2 - \frac{1}{p+1}\|\varphi_m\|_{p+1}^{p+1} \\
&= \frac{1}{2}\|\varphi_{mt}(0)\|^2 + \frac{1}{2}\|\nabla\varphi_m(0)\|^2 + \frac{1}{2}\|\varphi_m(0)\|^2 - \frac{1}{p+1}\|\varphi_m(0)\|_{p+1}^{p+1} \\
&= E_m(0).
\end{aligned} \tag{3.4}
$$

For the cases $E(0) < d$ and $I(\varphi_0) > 0$ or $\|\nabla\varphi_0\| = 0$, we have

$$
\begin{aligned}
&\frac{1}{2}\|\varphi_{mt}\|^2 + J(\varphi_m) = E_m(0) < d, \quad 0 \leqslant t < \infty, \\
&J(\varphi_m) = \frac{1}{2}\|\nabla\varphi_m\|^2 + \frac{1}{2}\|\varphi_m\|^2 - \frac{1}{p+1}\|\varphi_m\|_{p+1}^{p+1} \\
&\qquad = \left(\frac{1}{2} - \frac{1}{p+1}\right)\left(\|\nabla\varphi_m\|^2 + \|\varphi_m\|^2\right) + \frac{1}{p+1} I(\varphi_m) \\
&\qquad \geqslant \frac{p-1}{2(p+1)}\left(\|\nabla\varphi_m\|^2 + \|\varphi_m\|^2\right).
\end{aligned} \tag{3.5}
$$

Hence we arrive at

$$
\frac{1}{2}\|\varphi_{mt}\|^2 + \frac{p-1}{2(p+1)}\left(\|\nabla\varphi_m\|^2 + \|\varphi_m\|^2\right) < d, \tag{3.6}
$$

then

$$\begin{aligned} \|\varphi_{mt}\|^2 &\leqslant 2d, \\ \|\nabla\varphi_m\|^2 &\leqslant \frac{2(p+1)}{p-1}d, \\ \|\varphi_m\|^2 &\leqslant \frac{2(p+1)}{p-1}d. \end{aligned} \tag{3.7}$$

Hence, there exist a φ and a subsequence φ_v such that $\varphi_v \to \varphi$ in $L^\infty(0,\infty;H^1(\Omega))$ weak star and a.e. in $Q = \Omega \times [0,\infty)$, $|\varphi_v|^{p-1}\varphi_v \to |\varphi|^{p-1}\varphi$ in $L^\infty(0,\infty;L^q(\Omega))$ weak star, $\varphi_{vt} \to \varphi_t$ in $L^2(0,\infty;L^2(\Omega))$ weakly.

In (3.2) for fixed s, letting $m = v \to \infty$, we get

$$(\varphi_t, w_s) + (\nabla\varphi, \nabla w_s) + (\varphi, w_s) = \left(|\varphi|^{p-1}\varphi, w_s\right), \quad \forall s. \tag{3.8}$$

Integrating t from 0 to t, we obtain that $\varphi(t,x) \in L^\infty(0,\infty;H_0^2(\Omega))$, $\varphi_t(x,t) \in L^\infty(0,\infty;L^2(\Omega))$ is a global weak solution of problem (2.1).

Next we prove the fact that $\varphi(t) \in W$ for $0 \leq t < \infty$. First of all, we will show that $\varphi_0(x) \in W$. Let $\varphi(t)$ be any solution of problem (2.1) with

$$E(0) = \frac{1}{2}\|\varphi_1\|^2 + J(\varphi_0) < d, \tag{3.9}$$

which gives that $J(\varphi_0) < d$. If $I(\varphi_0) > 0$ then from the definition of W we obtain $\varphi_0(x) \in W$. If $\|\nabla\varphi_0\| = 0$ then $\varphi_0(x) \in W$ also. It is easy to see $\varphi_{m0}(x) \in W$ for sufficiently large m.

It is enough for us to prove $\varphi_m(t) \in W$ for sufficiently large m and $t > 0$. If it is false, then there must exist a $t_0 > 0$ for sufficiently large m such that $\varphi_m(t_0) \in \partial W$, that is,

$$I(\varphi_m(t_0)) = 0, \qquad \|\nabla\varphi_m(t_0)\| \neq 0, \quad \text{or} \quad J(\varphi_m(t_0)) = d. \tag{3.10}$$

From the energy inequality $E(0) < d$, we get $E_m(0) < d$ for sufficiently large m, that is,

$$\frac{1}{2}\|\varphi_{mt}\|^2 + J(\varphi_m) = E_m(0) < d. \tag{3.11}$$

Then we can see that $J(\varphi_m(t_0)) = d$ is impossible. On the other hand, if $I(\varphi_m(t_0)) = 0$, $\|\nabla\varphi_m(t_0)\| \neq 0$ we obtain $\varphi_m(t_0) \in \mathcal{N}$. By the definition of $\mathcal{N}$, we get $J(\varphi_m(t_0)) \geqslant d$, which contradicts (3.11). Hence $\varphi_m(t) \in W$ is true. □

Theorem 3.3 (blow-up). *Assume that $\varphi_0(x) \in H_0^1(\Omega)$, $\varphi_1(x) \in L^2(\Omega)$, $E(0) < d$, and $I(\varphi_0) < 0$, then the solution of problem* (2.1) *must blow up in finite time, that is, there exists a $T > 0$ such that $\lim_{t\to T}\|\varphi(t)\| = +\infty$.*

Proof. Let $\varphi(t)$ be any solution of problem (2.1) with $E(0) < d$ and $I(\varphi_0) < 0$. Set $F(t) = \|\varphi\|^2$, then $(F(t))' = 2(\varphi_t, \varphi)$,

$$(F(t))'' = 2\|\varphi_t\|^2 + 2(\varphi_{tt}, \varphi) = 2\|\varphi_t\|^2 - 2I(\varphi), \tag{3.12}$$

$$(F(t))'' \geqslant (p+3)\|\varphi_t\|^2 + (p-1)(\lambda_1 + 1)F(t) - 2(p+1)E(0), \tag{3.13}$$

where $\lambda_1 > 0$ is the first eigenvalue of problem $\Delta\varphi + \lambda\varphi = 0$, $\varphi(x,t)|_{\partial\Omega} = 0$.

Now we will consider the following two cases to finish the proof:

(i) if $E(0) \leqslant 0$, then $(F(t))'' \geqslant (p+3)\|\varphi_t\|^2$;

(ii) if $0 < E(0) < d$, we should discuss this case in aid of set V_δ.

Let $\delta_1 < \delta_2$ be two roots of equation $d(\delta) = E(0)$. For any $\delta \in (\delta_1, \delta_2)$ we will prove $\varphi(t) \in V_\delta$.

First let us prove $\varphi_0 \in V_\delta$. From the energy equality

$$\frac{1}{2}\|\varphi_1\|^2 + J(\varphi_0) = E(0) < d(\delta), \tag{3.14}$$

we get

$$J(\varphi_0) < d(\delta) \quad \text{for } \delta_1 < \delta < \delta_2, \tag{3.15}$$

and $I(\varphi_0) < 0$ gives $I_\delta(\varphi_0) < 0$ for $\delta_1 < \delta < \delta_2$. Thereby we obtain $\varphi_0 \in V_\delta$.

Next let us show that $\varphi(t) \in V_\delta$ for $\delta_1 < \delta < \delta_2$ and $t \geqslant 0$. If it is false, we can find a $t_0 \in (0, +\infty)$ as the first time such that $\varphi(t_0) \in \partial V_\delta$, that is, $J(\varphi(t_0)) = d(\delta)$ or $I_\delta(\varphi(t_0)) = 0$ for some $\delta_1 < \delta < \delta_2$. However from the conservation law we can see that $J(\varphi(t)) = d(\delta)$ is impossible. If $I_\delta(\varphi(t_0)) = 0$ then $I_\delta(\varphi(t)) < 0$ for $0 \leqslant t < t_0$. At the same time, Lemma 2.2 yields that $\|\nabla\varphi(t)\| \geqslant r(\delta) > 0$ and $\|\nabla\varphi(t_0)\| \geqslant r(\delta)$. Hence, by the definition of $d(\delta)$ we get $J(\varphi(t)) \geqslant d(\delta)$, which contradicts $J(\varphi) < d(\delta)$. So we obtain $\varphi(t) \in V_\delta$ for $\delta_1 < \delta < \delta_2$ and $t \geqslant 0$. Hence, $I_\delta(\varphi) < 0$ and $\|\nabla\varphi\| \geqslant r(\delta)$. Let $\delta \to \delta_2$, then $I_{\delta_2}(\varphi) \leqslant 0$ and $\|\nabla\varphi\| \geqslant r(\delta_2)$. By (3.12) we obtain

$$\begin{aligned}(F(t))'' &= 2\|\varphi_t\|^2 - 2I(\varphi) \geqslant -2I(\varphi) \\ &= 2(\delta_2 - 1)\|\nabla\varphi\|^2 - 2I_{\delta_2}(\varphi) \\ &\geqslant 2(\delta_2 - 1)r^2(\delta_2) \equiv a^0, \quad t \geqslant 0.\end{aligned} \tag{3.16}$$

For $a = \min\{(p+3)\|\varphi_t\|^2, a^0\}$ we have

$$(F(t))' \geqslant at + (F(0))', \quad \forall t \geqslant 0. \tag{3.17}$$

Hence there exists a $t_0 \geqslant 0$ such that

$$(F(t))' \geqslant (F(t_0))' > 0, \quad t \geqslant t_0, \tag{3.18}$$

which gives

$$F(t) \geqslant (F(t_0))'(t-t_0) + F(t_0), \quad t \geqslant t_0. \tag{3.19}$$

By (3.13) for sufficiently large t, we obtain

$$(F(t))'' F(t) - \frac{p+3}{4} F(t)' \geqslant (p+3)\left(\|\varphi_t\|^2\|\varphi\|^2 - (\varphi_t,\varphi)^2\right) \geqslant 0. \tag{3.20}$$

By a direct computation we can see that

$$\left(F(t)^{-\alpha}\right)'' = -\alpha F^{-\alpha-2}\left(FF'' + (-\alpha-1)\left(F'\right)^2\right). \tag{3.21}$$

Let $\alpha = (p-1)/4$, then we get

$$FF'' + (-\alpha-1)\left(F'\right)^2 \geqslant 0, \tag{3.22}$$

that is,

$$\left(F(t)^{-\alpha}\right)'' \leqslant 0. \tag{3.23}$$

Applying properties of concave function we can get that there exists a bounded $T > 0$ such that

$$\lim_{t\to T} F(t) = +\infty. \tag{3.24}$$

□

From the above two theorems we can easily get a sharp condition for global existence and blow-up of solutions to problem (2.1) like the following.

Let p satisfy (H). Assume that $\varphi_0(x) \in H_0^1(\Omega)$, $\varphi_1(x) \in L^2(\Omega)$, $0 < E(0) < d$. Then $I(\varphi_0) > 0$ supports problem (2.1) to admit a global weak solution, and $I(\varphi_0) < 0$ leads blow-up of solutions for problem (2.1).

Acknowledgments

This work is supported by Natural Science Foundation of Heilongjiang Province (A200702; A200810); Science and Technology Foundation of Education Office of Heilongjiang Province (11541276).

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