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ETERNAL CHAOTIC INFLATION

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ETERNAL CHAOTIC INFLATION *

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ABSTRACT

It is shown that the universe evolution in the chaotic inflation scenario has no end and may have no beginning. According to this scenario, the universe consists of exponentially large number of different mini-universes inside which all possible metastable vacuum states and all possible types of compactification are realized.

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1. There exist two very important problems related to cosmology and to the elementary particle physics. The first one is the cosmological singularity problem. The main part of this problem is not the existence of singularities in the universe, but the statement that the universe does not exist eternally and that there exists "some time at which there is no space-time at all". The second problem is related to the uniqueness of the universe. The essence of this problem was formulated by Einstein in his talk with E.Strauss : "What I am really interested in is whether God could create the world differently". This problem now becomes especially important since the effective potentials $V(\varphi)$ describing symmetry breaking in unified theories of elementary particles often have many different deep minima, and in Kaluza-Klein and superstring theories there is an enormously large number of possible types of compactification of extra dimensions. This makes it extremely difficult to understand why just the world of our type has appeared after compactification and spontaneous symmetry breaking.

As it will be argued below, it is possible to relax or even to resolve these two problems in the context of the chaotic inflation scenario (1-4).

2. In this paper we will consider a simplest model of inflation based on the theory of a massive noninteracting scalar field φ with the Lagrangian

$$L = -\frac{M_P^2}{16\pi} R + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{m^2}{2}\varphi^2 . \qquad (1)$$

Here $M_p \sim 10^{19}$ GeV is the Planck mass, R is the curvature scalar, $M_p^{-2} = G$ is the gravitational constant. A discussion of a theory of an interacting field φ will be contained in a separate publication [5].

Evolution of a sufficiently homogeneous field φ in a locally Friedmann universe with a scale factor $a(t)$ is governed by equations

$$\ddot{\varphi} + 3H\dot{\varphi} = -m^2\varphi \quad , \tag{2}$$

where $H = \frac{\dot{a}}{a}$, and

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{m^2}{2}\varphi^2\right) . \tag{3}$$

Here $k = \pm 1$, 0 for a closed, open or flat universe respectively. At $\varphi > M_p$ solutions of eqs. (2),(3) rapidly approach the asymptotic regime

$$\varphi(t) = \varphi_0 - \frac{mM_p}{2\sqrt{3\pi}}t \quad , \tag{4}$$

$$a(t) = a_0 \exp\left(\frac{2\pi}{M_p^2}\left(\varphi_0^2 - \varphi^2(t)\right)\right) . \tag{5}$$

According to (4),(5), during a time $\tau < \frac{2\sqrt{3\pi}\varphi}{mM_p}$ the value of the field φ remains almost unchanged, and the universe expands quasiexponentially, $a(t+\tau) \approx a(t)\exp(H\tau)$, where

$$H = \frac{2\sqrt{\pi}\, m\varphi}{\sqrt{3}\, M_p} \quad , \tag{6}$$

$H \gg \tau^{-1}$ for $\varphi \gg M_p$. The only possible constraint on the initial value of the field φ at the beginning of the universe evolution is $\frac{m^2\varphi^2}{2} \lesssim M_p^4$. Therefore a typical initial value of the field φ is $\varphi_0 \sim \frac{M_p^2}{m}$ [1-4].

3. **Quasiexponential expansion** (inflation) (4),(5) makes the universe locally homogeneous and isotropic[3]. However, inflation also leads to creation of long-wave fluctuations $\delta\varphi(x)$ of the classical field φ [6-8]. Fluctuations generated during a time $\Delta t = H^{-1}$ in our theory look as a distribution of the classical field $\delta\varphi(x)$ with a time-independent amplitude

$$|\delta\varphi(x)| \sim \frac{H(\varphi)}{\sqrt{2}\,\pi} \sim \sqrt{\frac{2}{3}}\,\frac{m\varphi}{\pi M_p} \tag{7}$$

and with initial wavelength $\Delta\ell \sim H^{-1}$. Later their wavelength exponentially grows as $a(t)$ (5), but at the same time new perturbations with the wavelength $\Delta\ell \sim H^{-1}$ are generated, etc. Inhomogeneities of the field φ give rise to density perturbations $\delta\rho(x)$, which on a galaxy scale have a relative amplitude $\frac{\delta\rho}{\rho} \sim 10\,\frac{m}{M_p}$. This gives a desirable value $\frac{\delta\rho}{\rho} \sim 10^{-4}$ for $m \sim 10^{-5}M_p \sim 10^{14}$ GeV (8,9,3). The value of $\frac{\delta\rho(x)}{\rho}$ logarithmically grows at large scales and becomes O(1) at a scale $\ell \sim M_p^{-1}\exp(2\pi\frac{M_p}{m}) \sim 10^{10^5}$ cm [3]. Perturbations of the scalar field on such a scale are generated when the average field φ (4) is of the order $\frac{M_p}{3}\sqrt{\frac{M_p}{m}} \sim 100\, M_p$. The physical meaning of this result is very interesting.

4. During a typical time $\Delta t = H^{-1}$ the average field φ (4) decreases by

$$\Delta\varphi = \frac{mM_p}{2\sqrt{3\pi}H} = \frac{M_p^2}{4\pi\varphi} . \tag{8}$$

From (7),(8) it follows that $|\delta\varphi| \lesssim \Delta\varphi$ for $\varphi \lesssim \frac{M_p}{3}\sqrt{\frac{M_p}{m}}$ and $|\delta\varphi| \gtrsim \Delta\varphi$ for $\varphi \gtrsim \frac{M_p}{3}\sqrt{\frac{M_p}{m}}$. This means that the region $M_p \lesssim \varphi \lesssim M_p \cdot \frac{M_p}{m}$ ($M_p \lesssim \varphi \lesssim 10^5 M_p$ for $m \sim 10^{-5} M_p$), at which inflation occurs and the inequality $\frac{m^2\varphi^2}{2} \lesssim M_p^4$ is satisfied, is divided into two regions. At $\varphi \lesssim \frac{M_p}{3}\sqrt{\frac{M_p}{m}} \sim 100\, M_p$ evolution of the scalar field φ everywhere with a good accuracy is described by eq. (4). However, at $\varphi \gtrsim \frac{M_p}{3}\sqrt{\frac{M_p}{m}} \sim 100\, M_p$ (i.e. in the main part of the region $M_p \lesssim \varphi \lesssim 10^5 M_p$) only the average field φ (averaged over the initial coordinate volume of the universe [5]) obeys eq. (4), and the role of fluctuations is very important.

Let us consider a domain of the universe of a size $\Delta\ell \gtrsim O(H^{-1}(\varphi))$ containing a sufficiently homogeneous field $\varphi \gg \frac{M_p}{3}\sqrt{\frac{M_p}{m}} \sim 100\, M_p$. According to the "no hair" theorem for de Sitter space, inflation in such a domain proceeds independently of what occurs outside it [10], and in this sense such a domain can be considered as a separate inflationary mini-universe. After the typical time $\Delta t = H^{-1}$ the size of this domain grows e times, its volume grows e^3 times, and it becomes effectively divided into $O(e^3)$ mini-universes of a size $O(H^{-1})$ containing the field $\varphi - \Delta\varphi + \delta\varphi(x) \approx \varphi + \delta\varphi(x)$. Since a typical wavelength of the field $\delta\varphi(x)$ generated during the time $\Delta t = H^{-1}$ is $O(H^{-1})$, and the amplitude of $\delta\varphi(x)$ is $O(H)$, the value of the field φ approximately in a half of the mini-universes of a size $O(H^{-1})$ decreases by $O(H)$, whereas in another half of these domains the field φ grows, $\varphi + \delta\varphi(x) - \Delta\varphi \approx \varphi + O(H)$. Thus, the physical volume of the universe occupied by the growing field φ during each time interval $\Delta t = H^{-1}$ grows $O(\frac{e^3}{2})$ times. Therefore, though the average field φ decreases due to the slow rolling to the minimum of its potential energy (4), the total physical volume of the universe filled with a permanently growing field φ increases as $\exp((3-\ln 2)Ht) \gtrsim \exp\frac{4m\varphi}{M_p}t$, and the physical volume of domains, in which the field φ does not decrease, grows almost as fast as $\frac{1}{2}\exp(3Ht)$. This leads to two important consequences.

i) Domains (mini-universes) of initial size $\ell \gtrsim O(H^{-1})$ containing the field $\varphi \gtrsim \frac{M_P}{3}\sqrt{\frac{M_P}{m}}$ infinitely reproduce other mini-universes with $\varphi \gtrsim \frac{M_P}{3}\sqrt{\frac{M_P}{m}}$. This process occurs without end, and therefore there is no end of the universe evolution in the chaotic inflation scenario.

Moreover, there may be no need to assume that the universe was created as a whole at some initial moment $t = 0$: The process of the universe creation and self-reproduction may occur eternally, it may have no beginning and no end. This possibility seems especially interesting and requires a more detailed investigation (5,11).

For completeness one should mention that the process of the universe self-reproduction may occur (12-16) in the old inflationary universe scenario(12) and in the new inflationary universe scenario(17).

Since no realistic versions of the old and of the new inflationary universe scenario have been suggested so far (see (3,4) for a discussion of the present status of the inflationary universe scenario), we will not consider this possibility here. It is worth mentioning, however, that in the context of the chaotic inflation scenario the possibility that the universe has no beginning is more likely (11) than the analogous possibility suggested in the context of the new inflationary universe scenario (15).

ii) The process of formation of mini-universes occurs at $\varphi \gtrsim M_P\sqrt{\frac{M_P}{m}}$, i.e. at $\frac{m^2}{2}\varphi^2 \gtrsim M_p^4 \cdot \frac{m}{M_p} \sim 10^{-5} M_p^4$. Therefore this process may occur at densities which are much smaller than the Planck density $O(M_p^4)$. At small densities quantum fluctuations of metric in the mini-universe at the moment of its formation are small, and the structure of space-time inside the new-born mini-universes remains unchanged. However, at the same time large-scale fluctuations of all scalar fields Φ with masses $m_\Phi \ll H$, which may be present in the theory, are formed (6-8). Since the Hubble parameter $H \sim \frac{2m\varphi}{M_P}$ at that time is greater than $O(10^{-2})M_p$, fluctuations of the scalar fields Φ are strong enough to transfer the classical fields Φ in the new-born mini-universe from one local minimum of the effective potential $V(\Phi, \varphi)$ to another.(3) This changes the low-energy elementary particle physics inside this mini-universe, which after inflation becomes exponentially large.

In the mini-universe of initial size $O(H^{-1}) = O(M_p^{-1})$ formed at $\frac{m^2}{2}\varphi^2 \sim M_p^4$ fluctuations of metric at a scale $O(M_p^{-1})$ are of the order of unity. Therefore, if the theory (1) can be considered as a part of a Kaluza-Klein (or superstring) theory, then at the moment of the mini-universe formation the type of compactification (and the number of compactified dimensions) inside the mini-universe can be changed almost independently of what occurs in causally-disconnected regions outside it. (The only possible constraint on the local change of space-time structure comes from topology and may lead e.g. to creation of a pair of mini-universes with opposite topological numbers.) Note, that this process may occur in the chaotic inflation scenario only, since only in this scenario inflation may occur at densities of the order of M_p^4 (3,II).

As a result, the universe becomes divided into exponentially large number of domains, and all possible types of compactification and all possible types of metastable vacuum states should be realized in different domains of the universe. All those domains (mini-universes), in which inflation remains possible after their formation, later become exponentially large. It seems therefore that God not only could create the universe differently, but in His wisdom He created the universe which has been unceasingly producing different universes of all possible types.

According to this scenario, we live in the mini-universe of our type not for the reason that it is the only possible universe, but for the reason that there exist many different mini-universes, and life of our type cannot exist in domains with a different dimensionality and with different types of symmetry breaking (3). This provides a justification of the anthropic principle in the inflationary cosmology.

A more detailed discussion of the problems touched upon in the present paper will be published elsewhere (5,II).

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