Developments in General Relativity: Black Hole Singularity and Beyond

I.D.Novikov

Theoretical Astrophysics Center Juliane Maries Vej 30, DK-2100 Copenhagen, Denmark; University Observatory Juliane Maries Vej 30, DK-2100 Copenhagen, Denmark; Astro-Space Center of Lebedev Physical Institute Profsoyuznaya 84/32, Moscow 117997, Russia; NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

October 25, 2018

Abstract

At the 20-th Texas Symposium on Relativistic Astrophysics there was a plenary talk devoted to the recent developments in classical Relativity. In that talk the problems of gravitational collapse, collisions of black holes, and of black holes as celestial bodies were discussed. But probably the problems of the internal structure of black holes are a real great challenge. In my talk I want to outline the recent achievements in our understanding of the nature of the singularity (and beyond!) inside a realistic rotating black hole. This presentation also addresses the following questions:

Can we see what happens inside a black hole?

Can a falling observer cross the singularity without being crushed?

An answer to these questions is probably "yes".

1 Introduction

The problems of the internal structure of black holes are a real great challenge. Inside a black hole the main sights are the singularity.

In this paper I will give a retrospective review and outline the recent achievements in our understanding of the nature of the singularity (and beyond) inside a realistic, rotating black hole. I want to note that there is a strong believe that the interior of black holes is not visible for an external observer, and who cares for invisible things?

Natural question arises: Can one see from outside what happens inside a black hole? My presentation also addressed this question. An answer to this question is probably affirmative.

For systematic discussion of the problems of the internal structure of black holes see [1-8].

The problem of black holes interior was the subject of a very active investigation last decades. There is a great progress in these researches. We know some important properties of the realistic black hole's interior, but some details and crucial problems are still the subject of much debate.

A very important point for understanding the problem of black hole's interior is the fact that the path into the gravitational abyss of the interior of a black hole is a progression in time. We recall that inside a spherical black hole, for example, the radial coordinate is timelike. It means that the problem of the black hole interior is an *evolutionary problem*. In this sense it is completely different from a problem of an internal structure of other celestial bodies, stars for example, or planets.

In principle, if we know the conditions on the border of a black hole (on the event horizon), we can integrate the Einstein equations in time and learn the structure of the progressively deeper layers inside the black hole. Conceptually it looks simple, but there are two types of principal difficulties which prevent realizing this idea consistently.

The first difficulty is the following. Formally the internal structure of a generic rotating black hole even soon after its formation depends crucially on the conditions on the event horizon at very distant future of the external observer (formally at the infinite future). This happens because the light-like signal can propagate from the very distant future to those regions inside a black hole which are deep enough in the hole. The limiting light-like signals which propagate from (formally) infinite future of the external observer form a border inside a black hole which is called a *Cauchy horizon*.

Thus, the structure of the regions inside a black hole depends crucially on the fate of the black hole at infinite future of an external observer. For example, it depends on the final state of the black hole quantum evaporation (because of the Hawking radiation), on possible collisions of the black hole with other black holes, or another bodies, and it depends on the fate of the Universe itself. It is clear that theoreticians feel themselves uncomfortable under such circumstances.

However we will see that all these uncertainties are related with the structure of spacetime of a realistic black hole very close to the singularity inside it. These parts of spacetime are in the region with the curvature greater than the Planck value. Probably the Classical General Relativity is not applicable here (see below, section 6).

The second serious problem is related to the existence of a singularity inside a

black hole. A number of rigorous theorems (see references in [2]) imply that singularities in the structure of spacetime develop inside black holes. Unfortunately these theorems tell us practically nothing about the locations and the nature of the singularities. It is widely believed today that in the singularity inside a realistic black hole the characteristics of the curvature of the spacetime tends to infinity. Close to the singularity, where the curvature of the spacetime approaches the Plank value, the Classical General Relativity is not applicable. We have no a final version of the quantum theory of gravity yet, thus any extension of the discussion of physics in this region would be highly speculative. Fortunately, as we shall see, these singular regions are deep enough in the black hole interior and they are *in the future* with respect to overlying and *preceding* layers of the black hole where curvatures are not so high and which can be described by well-established theory.

The first attempts to investigate the interior of a Schwarzschild black hole have been made in the late 70's [9,10]. It has been demonstrated that in the absence of external perturbations at late times, those regions of the black hole interior which are located long after the black hole formation are virtually free of perturbations, and therefore it can be described by the Schwarzschild geometry for the region with radius less than the gravitational radius. This happens because the gravitational radiation from aspherical initial excitations becomes infinitely diluted as it reaches these regions. But this result is not valid in general case when the angular momentum or the electric charge does not vanish. The reason for that is related to the fact that the topology of the interior of a rotating or/and charged black hole differs drastically from the Schwarzschild one. The key point is that the interior of this black hole possesses a *Cauchy horizon*. This is a surface of infinite blueshift. Infalling gravitational radiation propagates inside the black hole along paths approaching the generators of the Cauchy horizon, and the energy density of this radiation will suffer an infinite blueshift as it approaches the Cauchy horizon.

This infinitely blueshifted radiation together with the radiation scattered on the curvature of spacetime inside a black hole leads to the formation of curvature singularity instead of the regular Cauchy horizon.

We will call this singularity Cauchy horizon singularity. A lot of papers were devoted to investigation of the nature of this singularity. In addition to the papers mentioned above see also [11-26]. Below we consider main processes which are responsible for the formation of the singularity.

2 Processes which are responsible for the formation of the Cauchy horizon singularity

This section discusses the nonlinear effects which trigger the formation of a singularity at the Cauchy horizon inside a black hole. In the Introduction we emphasized that the problem of the black hole interior is an evolutionary problem, and it depends on the initial conditions at the surface of the black hole for all momenta of time up to infinity. To specify the problem, we will consider an isolated black hole (in asymptotically flat spacetime) which was created as a result of a realistic collapse of a star without assumptions about special symmetries.

The initial data at the event horizon of an isolated black hole, which determine the internal evolution at fairly late periods of time, are known with precision because of the no hair property. Near the event horizon we have a Kerr-Newman geometry perturbed by a dying tail of gravitational waves. The fallout from this tail produces an inward energy flux decaying as an inverse power v^{-2p} of advanced time v, where p = 2l + 3 for multipole of order l see [27-30]. See details in Section 3.

Now we should integrate the Einstein equations with the known boundary conditions to obtain the internal structure of the black hole. In general, the evolution with time into the black hole depths looks as follows. The gravitational radiation penetrating the black hole and partly backscattered by the spacetime curvature can be considered, roughly speaking, as two intersecting radial streams of infalling and outgoing gravitational radiation fluxes, the nonlinear interaction of which leads to the formation a non-trivial structure of the black hole interior. However in such a formulation it is a very difficult and still not solved completely mathematical problem. We will consider main achievements in solving it.

What are the processes responsible for formation of the Cauchy horizon singularity? The key factor producing its formation is the infinite concentration of energy density close to the Cauchy horizon as seen by a free falling observer. This infinite energy density is produced by the ingoing radiative "tail".

The second important factor here is a tremendous growth of the black hole internal mass parameter, which was dubbed *mass inflation* [31].

We start by explaining the mechanism responsible for the mass inflation [32-34]. Consider a concentric pair of thin spherical shells in an empty spacetime without a black hole [35]. One shell of mass m_{con} contracts, while the other one of mass m_{exp} expands. We assume that both shells are moving with the speed of light (for example, "are made of photons"). The contracting shell, which initially has a radius greater than the expanding one, does not create any gravitational effects inside it, so that the expanding shell does not feel the existence of the external shell. On the other hand, the contacting shell moves in the gravitational field on the expanding one. The mutual potential of the gravitational energy of the shells acts as a debit (binding energy) on the gravitational mass energy of the external contracting shell. Before the crossing of the shells, the total mass of both of them, measured by an observer outside both shells, is equal to $m_{con} + m_{exp}$ and is constant because the debit of the numerical increase of the negative potential energy is exactly balanced by the increase of the positive energies of photons blueshifted in the gravitational field of the internal sphere.

When shells cross one another, at radius r_0 , the debit is transferred from the contracting shell to the expanding one, but the blueshift of the photons in the con-

traction shell survives. As a result, the masses of both spheres change. The increase of mass m_{con} is called mass inflation. The exact calculation shows that the new mass m'_{con} and m'_{exp} are

$$m'_{con} = m_{con} + \frac{2m_{con}m_{exp}}{\varepsilon}, \qquad \qquad m'_{exp} = m_{exp} - \frac{2m_{con}m_{exp}}{\varepsilon} \tag{1}$$

where $\varepsilon \equiv (r_0 - 2m_{exp})$, (in the paper we use the units G = 1, c = 1). The total massenergy is, of course, conserved $m'_{con} + m'_{exp} = m_{con} + m_{exp}$. If ε is small (the encounter is just outside the horizon of m_{exp}), the inflation of mass of m_{con} can become arbitrary large.

It is not difficult to extend this result to the shells crossing inside a black hole. For simplicity consider at the beginning a spherical charged black hole.

Ori [18] considered a continuous influx (imitating the "tail" of ingoing gravitational radiation) and the outflux as a thin shell (a very rough imitation of the outgoing gravitational radiation scattered by the spacetime curvature inside a black hole). He specified the mass $m_{in}(v)$ to imitate the Price power-law tail (see section 3) and found that the mass function diverges exponentially near the Cauchy horizon as a result of the ingoing flux with the outgoing crossing of the ingoing shell:

$$m \sim e^{k_0 v_-} (k_0 v_-)^{-2p}, \qquad v_- \to \infty,$$
 (2)

where v_{-} is the advanced time in the region lying to the past of the shell, k_0 is constant, the positive constant p depends on perturbations under discussions. Expression (2) describes mass inflation. In this model, we have a scalar curvature singularity since the Weyl curvature invariant Ψ_2 . (Coulomb component) diverges at the Cauchy horizon. Ori [18] emphasizes that in spite of this singularity, there are coordinates in which the metric is finite at the Cauchy horizon. He also demonstrated that though the tidal force in the reference frame of a freely falling observer grows infinitely its action on the free falling observer is rather modest. According to [18] the rate of growth of the curvature is proportional to

$$\sim \tau^{-2} \left| \ln |\tau| \right|^{-2},$$
 (3)

where τ is the observer's proper time, $\tau = 0$ corresponds to the singularity. Tidal forces are proportional to the second time derivatives of the distances between various points of the object. By integrating the corresponding expression twice, one finds that as the singularity is approached ($\tau = 0$), the distortion remains finite.

There is one more effect caused by the outgoing flux. This is the contraction of the Cauchy horizon (which is singular now) with retarded time due to the focusing effect of the outgoing shell-like flux. This contraction continuous until the Cauchy horizon shrinks to r = 0, and a stronger singularity occurs. Ori [18] has estimated the rate of approach to this strong singularity r = 0.

In the case of realistic rotating black hole both processes the infinite concentration of the energy density and mass inflation near the Cauchy horizon also play the key role for formation of the singularity.

3 Decay of physical fields along the event horizon of isolated black holes

Behavior of physical fields along the event horizon determines dynamics of these fields inside a black hole and has an impact on the nature of the singularity inside the black hole. We will consider here the behavior of perturbations in the gravitational field. The first guess about the decay of gravitational perturbations outside Schwarzschild black holes was given in [36], a detailed description was given by Price [27,37]. References for subsequent work see in [2] and in the important work [38]. According to [27], any radiative multipole mode l, m of any initially compact linear perturbation dies off outside a black hole at late time as t^{-2l-3} . The mechanism which is responsible for this behavior is the scattering of the field off the curvature of spacetime asymptotically far from the black hole.

In the case of a rotating black hole the problem is more complicated due to the lack of spherical symmetry. This problem was investigated in many works. See analytical analyses, references and criticism in [38], numerical approach in [39]. For individual harmonics there is a power-law decay which is similar to the Schwarzschild case except that at the event horizon the perturbation also oscillates in the Eddington coordinate v along the horizon's null generators proportional to

$$\sim e^{im\Omega_+ v},$$
(4)

where Ω_+ is the angular velocity of the black hole rotation, $\Omega_+ = a/2Mr_+$, Mand a are mass and specific angular momentum of a black hole correspondingly, $r_+ = M + \sqrt{M^2 - a^2}$. Another important difference from the Schwarzschild case is the following. In the case of the rotating black holes spherical-harmonic modes do not evolve independently. In the linearized theory there is a coupling between spherical harmonics multipole of different l, but with the same m. In the case fully nonlinear perturbations there is the guess that m will not be conserved also. So in the case of arbitrary perturbation the modes with all l which are consistent with the spin weight s of the field will be exited. For the field with spin weight s all modes with $l \ge |s|$ will be exited. Accordingly, the late-time dynamics will be dominated by the mode with l = |s|. The falloff rate is then $t^{-(2|s|+3)}$. In the case of the gravitational field it corresponds to |s| = 2 and t^{-7} .

4 Nature of the singularity of a rotating black hole

As we mentioned in Section 2, we can use the initial data at the event horizon which we discussed in the previous Section 3 to determine the nature of the singularity inside a black hole. Main processes which are responsible for formation of the singularity were discussed in Section2. We start from the discussion of the singularity which arises at late time, long after the formation of an isolated black hole.

In general the evolution with time into the black hole deeps looks like the following. There is a weak flux of gravitational radiation into a black hole through the horizon because of small perturbations outside of it. When this radiation approaches the Cauchy horizon it suffers an infinite blueshift. The infinitely blueshift radiation together with the radiation scattered by the curvature of spacetime inside the black hole results in a tremendous growth of the black hole internal mass parameter ("mass inflation", see Section 2) and finally leads to formation of the curvature singularity of the spacetime along the Cauchy horizon. The infinite tidal gravitational forces arise here. This result was confirmed by considering different models of the ingoing and outgoing fluxes in the interior of charged and rotating black holes([6],[4]).

In the case of a rotating black hole the growth of the curvature (and mass function) when we coming to the singularity is modulated by the infinite number of oscillations. This oscillatory behavior of the singularity is related to the dragging of the inertial frame due to rotation of a black hole. Ori [4] calculated the asymptotic of the curvature scalar $K \equiv R_{iklm}R^{iklm}$ near the Cauchy horizon singularity:

$$K \approx A|\tau|^{-2} \left[\ln(-\tau/M)\right]^{-7} \times \sum_{m=1,2} C_m \exp\left\{-im\left[a(M^2 - a^2)^{-1/2}\ln(-\tau/M)\right]\right\},(5)$$

where τ is a proper time of a free falling observer, $\tau = 0$ at the singularity, A is a constant that depends on the geodesic's constants of motion, C_m are coefficients that depend on point at the singularity which is hit by the observer.

It was shown [18] that the singularity at the Cauchy horizon is quite weak. In particular, the integral of the tidal force in the freely falling reference frame over the proper time remains finite. It means that the infalling object would then experience the finite tidal deformations which (for typical parameters) are even negligible. While an infinite force is extended, it acts only for a very short time. This singularity exists in a black hole at late times from the point of view of an external observer, but the singularity which arises just after the gravitational collapse of a star is much stronger. According to the Tipler's terminology [41] (see also generalization of the classification in [42]) this is a weak singularity. It seems likely that an observer falling into a black hole with the collapsing star encounters a crushing singularity (strong singularity in the Tipler's classification). This is so called Belinsky-Khalatnikov-Lifshitz (BKL) space-like singularity [40]. On the other hand an observer falling into an isolated black hole in a late times generally reaches a weak singularity described above.

The weak Cauchy horizon singularity arises first at very late time (formally infinite time) of the external observer and its null generators propagate deeper into black hole and closer to the event of the gravitational collapse. They are subject of the focusing effect under the action of the gravity of the outgoing scattered radiation (see Section 2). Eventually the weak null singularity shrinks to r = 0, and strong BKL singularity occurs. This picture was considered in details in the case of a charged spherical black hole but I do not know the strict proof of it in the case of a rotating black hole [4,25]

There is one more important question: how generic are considering singularities? The solution which describes a singularity is called general if it depends on the total number of arbitrary functions of three independent variables which corresponds to the inherent degrees of freedom (plus the number of unfixed gauge degrees of freedom). In the case of gravitational field in vacuum this inherent degrees of freedom are equal 4.

The authors of [40] have had demonstrated that BKL singularity is general. Ori and Flanagan [21] have demonstrated that weak Cauchy horizon singularity is general also. So on principle both of these types of singularity are stable and can arise inside black holes. But of course which singularity (or both) arises depends on concrete situation, and this problem should be analyzed.

5 Quantum effects

As we mentioned in Introduction quantum effects play crucial role in the very vicinity of the singularity. In addition to that the quantum processes probably are important also for the whole structure of a black hole. Indeed, in the previous discussion we emphasized that the internal structure of black holes is a problem of evolution in time starting from boundary conditions on the event horizon for all moments of time up to the infinite future of the external observer.

It is very important to know the boundary conditions up to infinity because we observed that the essential events - mass inflation and singularity formation - happened along the Cauchy horizon which brought information from the infinite future of the external spacetime. However, even an isolated black hole in an asymptotically flat spacetime cannot exist forever. It will evaporate by emitting Hawking quantum radiation. So far we discussed the problem without taking into account this ultimate fate of black holes. Even without going into details it is clear that quantum evaporation of the black holes is crucial for the whole problem.

What can we say about the general picture of the black hole's interior accounting for quantum evaporation? To account for the latter process we have to change the boundary conditions on the event horizon as compared to the boundary conditions discussed above. Now they should include the flux of negative energy across the horizon, which is related to the quantum evaporation. The last stage of quantum evaporation, when the mass of the black hole becomes comparable to the Plank mass $m_{Pl} = (\hbar c/G)^{1/2} \approx 2.2 \times 10^{-5}g$, is unknown. At this stage the spacetime curvature near the horizon reaches l_{Pl}^{-2} , where l_{Pl} is the Plank length:

$$l_{Pl} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \approx 1.6 \times 10^{-33} \text{cm.}$$
(6)

This means that from the point of view of semiclassical physics a singularity arises here. Probably at this stage the black hole has the characteristics of an extreme black hole, when the external event horizon and internal Cauchy horizon coincide. As for the processes inside a true singularity in the black hole's interior, they can be treated only in the framework of an unified quantum theory incorporating gravitation, which is unknown. Thus when we discuss any singularity inside a black hole, we should consider the regions with the spacetime curvature bigger than l_{pl}^{-2} as physical singularity from the point of view of semiclassical physics¹. About different aspects of quantum effects in black holes see also [45,46].

6 Truly realistic black holes

So far we discussed the isolated black holes which were formed as the result of a realistic gravitational collapse without any assumptions about symmetry. Still they are not truly realistic black holes. For the truly realistic black holes we should account matter and radiation falling down through the event horizon at all times up to infinity (or up to the evaporation of a black hole).

The perturbations at the event horizon which arise as a result of the collapse of the non-symmetrical body have a compact support at some initial time. Subsequent perturbations, for example the perturbations which arise from the capture of photons which originate from the relic cosmic background radiation, have non-compact support.

We should account also the difference of the curvature of spacetime in the real Universe from the curvature in the ideal model asymptotically far from the black hole (the scattering of the field in these regions is responsible for the formation of the late-time power-law radiative tails). First steps in the investigation of the truly realistic black holes were done recently.

Burko [25] studied numerically the origin of the singularity in a simple toy model of a spherical charged black hole which was perturbed nonlinearly by a self-gravitating spherical scalar field. This field was specified in such a way that it had a noncompact support. Namely, it grows logarithmically with advanced time along an outgoing characteristic hypersurface. It was demonstrated that in this case the weak null Cauchy horizon singularity was formed. The null generators of the singularity contract with retarded time, and eventually the central spacelike strong singularity forms. Thus in this case the casual structure of the singularity is the same as in the case of the perturbations with a compact support at some initial time. Of course, this example is very far to be a realistic one.

In another work Burko [26] demonstrated numerically that the scalar field can be chosen along an outgoing characteristic hypersurface in such a way that only spacelike strong singularity forms. The scalar field has a non-compact support in this case.

It is an open question whether these results hold also for rotating black holes, and what would be a result in the case of a realistic source of perturbations for realistic

¹Quantum effects may manifest themselves in the region with the spacetime curvature smaller than l_{pl}^{-2} , see [44,45,2].

black holes.

I want to do the following remark. As I mentioned in Section 5, when inside a black hole we come to the singularity close enough, where the spacetime curvature reaches l_{pl}^{-2} , we should consider this region as a singular one from the point of view of semiclassical physics. This means that any details of the classical spacetime structure in the singular quantum region make no sense. This means that if we are interested in the spacetime structure only outside the singular quantum region and want to investigate this structure in some definite region at the singularity we should take into account radiation coming to the event horizon during the restrict period of time t_0 only. All radiation which comes to the border of a black hole later will come to the region under consideration inside the quantum singular region and does not influence on the structure of the spacetime outside it at this place. It is rather easy to estimate this period t_0 . It is

$$t_0 \approx 3 \cdot 10^6 \mathrm{sec} \left(\frac{M}{10^9 M_{\odot}}\right). \tag{7}$$

7 Can one see what happens inside a Black Hole?

Is it possible for a distant observer to receive information about the interior of a black hole? Strictly speaking, this is forbidden by the very definition of a black hole. What we have in mind in asking this question is the following. Suppose there exists a stationary or static black hole. Can we, by using some device, get information about the region lying inside the apparent horizon?

Certainly it is possible if one is allowed to violate the weak energy condition. For example, if one sends into a black hole some amount of "matter" of negative mass, the surface of black hole shrinks, and some of the rays which previously were trapped inside the black hole would be able to leave it. If the decrease of the black hole mass during this process is small, then only a very narrow region lying directly inside the horizon of the former black hole becomes visible.

In order to be able to get information from regions not close the apparent horizon but deep inside an original black hole, one needs to change drastically the parameters of the black hole or even completely destroy it. A formal solution corresponding to such a destruction can be obtained if one considers a spherically symmetric collapse of negative mass into a black hole. The black hole destruction occurs when the negative mass of the collapsing matter becomes equal to the original mass of the black hole. In such a case an external observer can see some region close to the singularity. But even in this case the four-dimensional region of the black hole interior which becomes visible has a four-dimensional spacetime volume of order M^4 . It is much smaller than four-volume of the black hole interior, which remains invisible and which is of order M^3T , where T is the time interval between the black hole formation and its destruction (we assume $T \gg M$). The price paid for the possibility of seeing even this small part of the depths of the black hole is its complete destruction. Does this mean that it is impossible to see what happens inside the apparent horizon without a destructive intervention? We show that such a possibility exists (Frolov and Novikov [47,48]). In particular, in these works we discuss a gedanken experiment which demonstrates that traversable wormholes (if only they exist) can be used to get information from the interior of a black hole practically without changing its gravitational field. Namely, we assume that there exist a traversable wormhole, and its mouths are freely falling into a black hole. If one of the mouths crosses the gravitational radius earlier than the other, then rays passing through the first mouth can escape from the region lying inside the gravitational radius. Such rays would go through the wormhole and enter the outside region through the second mouth, see details in [2,49].

Acknowledgments

This paper was supported in part by the Danish natural Science Research Council through grant No. 9701841 and also in part by Danmarks Grundforskningsfond through its support for establishment of the TAC.

References

- Burko, L.M. and A. Ori; editors. Internal Structure of Black Holes and Spacetime Singularity, Institute of Physics Publishing, Bristol and Philadelphia, and Izrael Physical Society, Jerusalem, (1997).
- [2] Frolov, V. P. I.D. Novikov, Black Hole Physics: Basic Concepts and New Developments, Kluwer Academic Publisher, Dordrecht (1998)
- [3] Brady, P.R., Droz, S. and Morsink, S.M. gr-qc/9805008 (1998).
- [4] Ori, A. Phys. Rev. Letters. 83, 5423 (1999).
- [5] Ori, A. Phys. Rev. D, 58, 084016, (1998).
- [6] Ori, A. Phys. Rev. D, **61**, 024001, (1999).
- [7] Berger B.K., gr-qc/0201056 (2002)
- [8] Ori, A., gr-qc/0103012 (2001)
- [9] Doroshkevich, A.G. and Novikov, I.D., Zh. Eksp. Teor Fiz., 74, 3 (1978).
- [10] Poisson, E. and W. Israel, Class. Quantum Grav. 5, L201, (1998).
- [11] Penrose, R., Structure of Space-Time in C,M,DeWitt and J.A.Wheeler (eds) Battelle Recontres, Benjamin, NY (1968).

- [12] McNamara, J.M. Proc. R. Soc. London A358, 499, (1978).
- [13] McNamara, J.M. Proc. R. Soc. London A364, 121, (1978).
- [14] Gürsel, Y., I.D.Novikov, V. Sandberg, and A.A.Starobinsly Phys. Rev. D, 20, 1260, (1979).
- [15] Gürsel, Y., V. Sandberg, I.D.Novikov, and A.A.Starobinsly *Phys. Rev. D*, **19**, 413, (1979).
- [16] Matzner, R.A., N. Zamorano and V.D Sandberg, *Phys. Rev. D*, **19**, 2821, (1979).
- [17] Chandrasekhar, S, and J.B. Hartle Proc. R. Soc. London A384, 301, (1982).
- [18] Ori, A. Phys. Rev. Letters. 67, 789, (1991).
- [19] Ori, A.) Phys. Rev. Letters. 68, 2117, (1992).
- [20] Brady, P.R., and J.D. Smith *Phys. Rev. Letters.* **75**, 1256, (1995).
- [21] Ori, A., and E.E. Flanagan, *Phys. Rev. D.* 53, R1754, (1996).
- [22] Ori, A. Phys. Rev. D. 55, 4860, (1997).
- [23] Burko, L.M. and A. Ori Phys. Rev. D. 57, R7084, (1998).
- [24] Gurtug, O., M. Halilsoy, gr-qc/0203019 (2002)
- [25] Burko, L.M. gr-qc/0206012 (2002)
- [26] Burko, L.M. gr-qc/0209084 (2002)
- [27] Price, R.H., Phys. Rev. D. 5, 2439, (1972).
- [28] Gundlach, C., R.H. Price, and J. Pullin, *Phys. Rev. D.* 49, 883, (1994).
- [29] Gundlach, C., R.H. Price, and J. Pullin, *Phys. Rev. D.* 49, 890, (1994).
- [30] Ori, A. Gen. Rel. & Grav. 29, 881, (1997).
- [31] Poisson, E. and W. Israel, *Phys. Rev. D.* 41, 1796, (1990).
- [32] Dray, T. and G. 't Hooft, Commun. Math. Phys. 99, 613, (1985).
- [33] Barrabès, C., W. Israel, E. Poisson, *Class. Quantum Grav.* 7, L273, (1990).
- [34] Bonanno, A., S. Droz, W. Israel, and S. Morsink, Proc. R. Soc. London A450, 553, (1995).
- [35] Droz, S., W. Israel, and S. Morsink, *Physics World* 9, 34, (1995).

- [36] Doroshkevich, A.G., Ya. B. Zeldovich an I.D. Novikov, Zh/ Eksp. Teor. Fiz 49, 170 (1965).
- [37] Price, R.H., *Phys. Rev. D.* 5, 2419 (1972).
- [38] Barack, L. and A. Ori, *Phys. Rev. D.* **60**, 124005 (1999).
- [39] Burko, L.M. and G. Klama, gr-qc/0209107 (2002).
- [40] Belinsky, V.A., I.M. Khalatnikov, and E.M. Lifshitz, Usp. Fiz. Nauk 102, 463 (1970), [Adv. Phys. 19, 525 (1970)]
- [41] Tipler, F.J., Phys. Lett 64A, 8 (1977).
- [42] Ori, A. Phys. Rev. D. **61**, 064016, (2000).
- [43] Novikov, I.D. and A.A. Starobinsky, Zh. Eksp. Teor. Fiz 78, 3 (1980).
- [44] Berezin, V.A. *Preprint NRI* p-0183, Moscow (1980).
- [45] Anderson W.G., P.R. Brady, W. Israel, and S.M. Morsink, *Phys. Lett* 70, 1041 (1993).
- [46] Balbinot, R. and E. Poisson, *Phys. Lett* **70**, 13 (1992).
- [47] Frolov, V.P. and I.D. Novikov, *Phys. Rev. D*, **42**, 1057 (1990).
- [48] Frolov, V.P. and I.D. Novikov, *Phys. Rev. D*, 48, p.1607 (1993).
- [49] Artemova, I.V. and I.D. Novikov, in Proceeding of th 9-th Course of Astrofundamental Physics, The Early Universe and Cosmic Microwave Background: Theory and Observations(2001) Palermo- Sicily, (2002) N.Sanchez (eds) in press (2003).