Moment Generating Function of Age of Information in Multisource M/G/1/1 Queueing Systems

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Abstract— We consider a multi-source status update system, where each source generates status update packets according to a Poisson process which are then served according to a generally distributed service time. For this multi-source M/G/1/1 queueing model, we consider a self-preemptive packet management policy and derive the moment generating functions (MGFs) of the age of information (AoI) and peak AoI of each source. According to the policy, an arriving fresh packet preempts the possible packet of the same source in the system. Furthermore, we derive the MGFs of the AoI and peak AoI for the globally preemptive and non-preemptive policies, for which only the average AoI and peak AoI have been derived earlier. Finally, we use the MGFs to derive the average AoI and peak AoI in a two-source M/G/1/1 queueing model under each policy. Numerical results show the effect of the service time distribution parameters on the average AoI. The results also highlight the importance of higher moments of the AoI.

Index Terms— AoI, packet management, moment generating function (MGF), multi-source queueing model, M/G/1/1.

I. INTRODUCTION

TIMELY delivery of the status updates of various real-world physical processes plays a critical role in enabling the time-critical Internet of Things (IoT) applications. The age of information (AoI) was first introduced in the seminal work [2] as a destination-centric metric to measure the information freshness in status update systems. A status update packet contains the measured value of a monitored process and a time stamp representing the time at which the sample was generated. Due to wireless channel access, channel errors, fading, etc. communicating a status update packet through the network experiences a random delay. If at a time instant t, the most recently received status update packet contains the time stamp $U(t)$, AoI is defined as the random process

$\Delta(t) = t - U(t)$. Thus, the AoI measures for each source node the time elapsed since the last received status update packet was generated at the source node.

The first queueing theoretic work on AoI is [3] where the authors derived the average AoI for M/M/1, D/M/1, and M/D/1 first-come first-served (FCFS) queueing models. In [4], the authors proposed peak AoI as an alternative metric to evaluate the information freshness. The work in [5] was the first to investigate the AoI in a multi-source setup in which the authors derived an approximate expression for the average AoI in a multi-source M/M/1 FCFS queueing model.

It has been shown that an appropriate packet management policy – in the waiting queue or/and server – has a great potential to improve the information freshness in status update systems [6], [7]. The average AoI for an M/M/1 last-come first-served (LCFS) queueing model with preemption was analyzed in [6]. The average AoI and average peak AoI for three packet management policies named M/M/1/1, M/M/1/2, and M/M/1/2* were derived in [7]. The seminal work [8] introduced the stochastic hybrid systems (SHS) technique to calculate the average AoI. In [9], the authors extended the SHS analysis to calculate the moment generating function (MGF) of the AoI. The SHS technique has been used to analyze the AoI in various queueing models [10], [11], [12], [13], [14], [15], [16], [17], [18], [19] and in gossip networks [20], [21]. The authors of [10] considered a multi-source queueing model in which sources have different priorities and derived the average AoI for two priority based packet management policies. In [11], the author derived the average AoI for a single-source status update system in which the updates follow a route through a series of network nodes where each node has an LCFS queue that supports preemption in service. The work [12] derived the average AoI in a single-source queueing model with multiple servers with preemption in service. In [13], the authors derived the average AoI in a multi-source LCFS queueing model with multiple servers that employ global preemption in service. According to the *globally preemptive* policy, the packets of different sources can preempt each other. The work in [14] derived the average AoI in a multi-source system under a *self-preemptive* packet management policy and packet delivery errors. According to the self-preemptive policy, when a packet arrives, the possible packet of the *same source* in the system is replaced by the fresh packet. The authors of [15], [16] derived the average AoI for a multi-source M/M/1 queueing model under various preemptive and non-preemptive packet management policies. The work in [22] considered a multi-source M/M/1/1 queueing model and studied the optimal generation rate of different

sources to minimize the weighted average AoI under the globally preemptive and self-preemptive policies. In [17], the authors derived the MGF of the AoI for a multi-source M/M/1 queueing model under various packet management policies. The authors of [18] assumed that the status update packets received at the sink need further processing before being used and derived the MGF of the AoI for such a two-server tandem queueing system.

Besides exponentially distributed service time and Poisson process arrivals, AoI has also been studied under various arrival processes and service time distributions in both single-source and multi-source systems. In [23], the authors derived various approximations for the average AoI in a multi-source M/G/1 FCFS queueing model. The work in [24] derived the distribution of the AoI and peak AoI for the single-source PH/PH/1/1 and M/PH/1/2 queueing models. The authors of [25] analyzed the AoI in a single-source D/G/1 FCFS queueing model. The authors of [26] derived a closed-form expression for the average AoI of a single-source M/G/1/1 preemptive queueing model with hybrid automatic repeat request. The stationary distributions of the AoI and peak AoI of single-source M/G/1/1 and G/M/1/1 queueing models were derived in [27]. In [28], the authors derived a general formula for the stationary distribution of the AoI in single-source single-server queueing systems. The work in [29] considered a single-source LCFS queueing model where the packets arrive according to a Poisson process and the service time follows a gamma distribution. They derived the average AoI and average peak AoI for two packet management policies: LCFS with the globally preemptive and non-preemptive. According to the non-preemptive policy, when the server is busy, any arriving packet is blocked and cleared. The work in [30] and [31] derived the average AoI expression for a single-source G/G/1/1 queueing model under two packet management policies. The authors of [32] considered a multi-source M/G/1 queueing system and optimized the arrival rates of each source to minimize the peak AoI. The work in [33] studied the age-upon-decisions in a single-source M/G/1/1 queueing system under the non-preemptive policy. The average AoI and average peak AoI for a multi-source M/G/1/1 queueing model under the globally preemptive policy were derived in [34]. In [35], the authors derived the average AoI for a queueing system with two classes of Poisson arrivals with different priorities under a general service time distribution. They assumed that the system can contain at most one packet and a newly arriving packet replaces the possible currently-in-service packet with the same or lower priority. The average AoI and average peak AoI for a multi-source M/G/1/1 queueing model under the non-preemptive policy were derived in [36].

Besides the age analysis, numerous works have focused on devising AoI-optimal status updating control procedures. Indeed, the optimization of sampling times of each sensor, scheduling, resource management etc. plays a critical role in the performance of status update systems. The works [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49] investigated the optimal sampling problem in status update systems, whereas the works [50], [51], [52], [53], [54] studied the optimal sampling problem with energy harvesting sources. A comprehensive literature review of recent works in AoI can be found in [55].

In this work, we consider a multi-source M/G/1/1 queueing system and derive the MGFs of the AoI and peak AoI under three packet management policies, namely, i) self-preemptive policy [14], ii) globally preemptive policy [34], and iii) non-preemptive policy [36]. The capacity of the system is one packet (i.e., there is no waiting buffer). According to the self-preemptive policy, when a packet arrives, the possible packet of *the same source* in the system is replaced by the fresh packet. According to the globally preemptive policy, a new arriving packet preempts the possible packet in the system regardless of its source index. According to the non-preemptive policy, when the server is busy, any arriving packet is blocked and cleared. By using the MGFs of the AoI and peak AoI, the average AoI and average peak AoI in a two-source M/G/1/1 queueing system under the three policies are derived. The numerical results show that, depending on the system parameters, the considered self-preemptive packet management policy can outperform the globally preemptive and non-preemptive policy proposed in [34] and [36], respectively, from the perspective of average AoI. In addition, they show the importance of higher moments of the AoI by investigating the standard deviation of the AoI under each policy.

The most related work to our paper is [56]. The authors considered a multi-source status update system where packets of each source are generated according to a Poisson process and served according to a distinct phase-type distribution, i.e., the service time distributions are different among the sources. Packet errors are considered so that upon an unsuccessful reception, the packet is re-transmitted with a certain probability. Using the theory of Markov fluid queues, they propose a method to numerically obtain the exact distributions of the AoI and peak AoI for each of the sources under a probabilistically preemptive policy in the considered M/PH/1/1 queue. According to the policy, a new packet arriving from source c is allowed to preempt a packet from source c' in service with a probability depending on c and c'. It is worth noting that the self-preemptive, globally preemptive, and non-preemptive policies are special cases of the probabilistically preemptive policy.

The main differences between [56] and our paper are as follows. On certain aspects, the framework in [56] is more general than ours, which are 1) source-dependent service times, 2) packet errors, 3) and re-transmissions. Namely, the work [56] considers distinct (phase-type) service time distributions among the sources, whereas in our model, the same (general) service time distribution is considered for all sources. Also, [56] incorporates packet errors and re-transmissions, which are not studied in our paper. However, the main distinctive results of our paper are the following. While [56] obtains the distributions of the AoI and peak AoI numerically, we provide the closed-form expressions of the MGFs of the AoI and peak AoI. Furthermore, [56] considers a phase-type distribution for serving the packets; we do not restrict to any specific service time distribution, i.e., our results are valid for any distribution. All these differences between the works also lead to the use of different analytical tools.
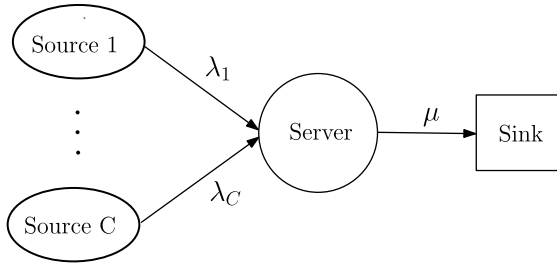
Fig. 1.   Considered multi-source M/G/1/1 queueing system.

A. Contributions

The main contributions of this paper are summarized as follows:

- We consider a self-preemptive packet management policy for a multi-source M/G/1/1 queueing system and derive the MGFs of the AoI and peak AoI under the policy.
- As an extension of [34] and [36], where only the average AoI and peak AoI were derived, we derive the MGFs of the AoI and peak AoI under the globally preemptive and non-preemptive packet management policies.
- By using the MGFs of the AoI and peak AoI, we derive the average AoI and average peak AoI in a two-source M/G/1/1 queueing system under the self-preemptive, globally preemptive, and non-preemptive policies.
- We numerically investigate the standard deviation of the AoI under the policies and show that the average AoI is not sufficient to rigorously evaluate the information freshness of a status update system for a given packet management policy.

B. Organization

The paper is organized as follows. The system model and summary of the main results are presented in Section II. Calculation of the MGFs of the AoI and peak AoI is presented in Section III. Numerical results are presented in Section IV. Finally, concluding remarks are made in Section V.

II. System Model and Main Results

We consider a status update system consisting of a set of independent sources denoted by $\mathcal{C} = \{1, \ldots, C\}$, one server, and one sink, as depicted in Fig. 1. Each source is assigned to send status information about a random process to the sink. Status updates are transmitted as packets, containing the measured value of the monitored process and a time stamp representing the time when the sample was generated. We assume that the packets of source $c \in \mathcal{C}$ are generated according to the Poisson process with the rate λ_c. Since packets of each source are generated according to a Poisson process and the sources are independent, the packet generation in the system follows the Poisson process with rate $\lambda = \sum_{c' \in \mathcal{C}} \lambda_{c'}$. The server serves the packets according to a generally distributed service time with rate μ. We assume that the service times of packets are independent and identically distributed (i.i.d.) random variables following a general distribution. Finally, we consider that the capacity of the system is one (i.e., there is no waiting buffer) and thus, the considered setup is referred to as a multi-source M/G/1/1 queueing system.

A. Packet Management Policies

In this paper, we study the following three packet management policies:

1) Self-Preemptive Policy [14]: According to this policy, a new arriving packet preempts the possible packet of the same source in the system. Whenever the new arriving packet finds a packet of another source under service, the arriving packet is blocked and cleared.

2) Globally Preemptive Policy [34]: According to this policy, a new arriving packet preempts the possible packet in the system regardless of its source index.

3) Non-Preemptive Policy [36]: According to this policy, when the server is busy at the arrival instant of a packet, the arriving packet is blocked and cleared.

B. AoI Definition

For each source, the AoI at the sink is defined as the time elapsed since the last successfully received packet was generated. Formally, let $t_{c,i}$ denote the time instant at which the ith delivered status update packet of source c was generated, and let $t'_{c,i}$ denote the time instant at which this packet arrives at the sink. Let $\bar{t}_{c,i}$ denote the generation time of the ith packet of source c that does not complete service because of the packet management policy (i.e., the packet is either preempted by another packet or it is blocked and cleared). Evolution of the AoI in a two-source system under the self-preemptive packet management policy is illustrated in Fig. 2.

At a time instant τ, the index of the most recently received packet of source c is given by

$$N_c(\tau) = \max\{i' \mid t'_{c,i'} \leq \tau\}, \tag{1}$$

and the time stamp of the most recently received packet of source c is $U_c(\tau) = t_{c,N_c(\tau)}$. The AoI of source c at the sink is defined as the random process $\delta_c(t) = t - U_c(t)$.

Let the random variable

$$Y_{c,i} = t'_{c,i+1} - t'_{c,i} \tag{2}$$

represent the ith interdeparture time of source c, i.e., the time elapsed between the departures of ith and $i + 1$th (delivered) packets from source c. From here onwards, we refer to the ith delivered packet from source c simply as "packet c, i". Moreover, let the random variable

$$T_{c,i} = t'_{c,i} - t_{c,i} \tag{3}$$

represent the system time of packet c, i, i.e., the duration this (delivered) packet spends in the system.

One of the most commonly used metrics for evaluating the AoI of a source is the peak AoI [4]. The peak AoI of source c at the sink is defined as the value of the AoI immediately before receiving an update packet. Accordingly, the peak AoI concerning the ith successfully received packet of source c, denoted by $A_{c,i}$ (see Fig. 2 for the self-preemptive policy), is given by

$$A_{c,i} = Y_{c,i-1} + T_{c,i-1}. \tag{4}$$

We assume that the considered status update system is stationary so that $T_{c,i} =^{\text{st}} T_c$, $Y_{c,i} =^{\text{st}} Y_c$, and $A_{c,i} =^{\text{st}} A_c$, $\forall i$,
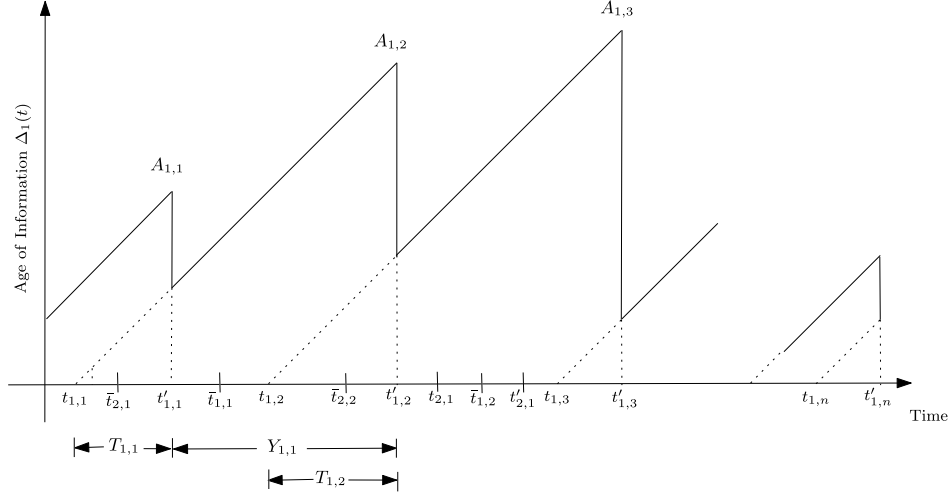
Fig. 2. Example of the evolution of the AoI of source 1 in a two-source system under the self-preemptive packet management policy. The first packet of source 1 is generated at time instant $t_{1,1}$ and this packet is delivered to the sink at time instant $t'_{1,1}$. The first packet of source 2 arrives at the system at time instant $\bar{t}_{2,1}$; however, because the server is serving a source 1 packet, the arrived source 2 packet is blocked and cleared. At time instant $\bar{t}_{1,1}$, a source 1 packet arrives at the empty system and starts the service; however, this packet is replaced by the new packet of source 1 arriving at time instant $t_{1,2}$.

where $=^{\text{st}}$ means stochastically identical (i.e., they have an identical marginal distribution).

Next, the main results of the paper are presented. The results are valid for any service time distribution under the three packet management policies.

C. Summary of the Main Results

The MGFs of the AoI and peak AoI of source c in a multi-source M/G/1/1 queueing system under each of the three packet management policies are given by the following three theorems; the proofs of Theorems 1, 2, and 3 are presented in Section III.

Let S be the random variable representing the service time of any packet in the system.

Theorem 1: The MGFs of the AoI and peak AoI of source c under the self-preemptive packet management policy, denoted by $\bar{M}_{\delta_c}(s)$ and $\bar{M}_{A_c}(s)$, respectively, are given as

$$\bar{M}_{\delta_c}(s) = \frac{M_S(s - \lambda_c)(\bar{M}_{Y_c}(s) - 1)}{s L_{\lambda_c} \bar{M}'_{Y_c}(0)}, \qquad (5)$$

$$\bar{M}_{A_c}(s) = \frac{M_S(s - \lambda_c)\bar{M}_{Y_c}(s)}{L_{\lambda_c}},$$

where $L_{\lambda_c} = \mathbb{E}[e^{-\lambda_c S}]$, $M_S(s - \lambda_c) = \mathbb{E}[e^{(s-\lambda_c)S}]$ is the MGF of the service time S at $s - \lambda_c$, $\bar{M}_{Y_c}(s)$ is the MGF of the interdeparture time Y_c under the policy, which is given as

$$\bar{M}_{Y_c}(s)$$
$$= \frac{a_c(s)M_S(s - \lambda_c)}{(1 - \bar{a}_c(s))\left(1 - \sum_{c' \in \mathcal{C}\backslash\{c\}} \frac{a_{c'}(s)M_S(s - \lambda_{c'})}{1 - \bar{a}_{c'}(s)}\right)}, \quad (6)$$

where

$$a_c(s) = \frac{\lambda_c}{\lambda - s}, \quad \bar{a}_c(s) = \frac{\lambda_c(1 - M_S(s - \lambda_c))}{\lambda_c - s}, \qquad (7)$$

and $\bar{M}'_{Y_c}(0)$ is the first derivative of the MGF of Y_c under the policy, evaluated at $s = 0$, i.e.,

$$\bar{M}'_{Y_c}(0) = \frac{\mathrm{d}(\bar{M}_{Y_c}(s))}{\mathrm{d}s}\bigg|_{s=0}.$$

Theorem 2: The MGFs of the AoI and peak AoI of source c under the globally preemptive packet management policy, denoted by $\hat{M}_{\delta_c}(s)$ and $\hat{M}_{A_c}(s)$, respectively, are given as

$$\hat{M}_{\delta_c}(s) = \frac{\lambda_c M_S(s - \lambda)}{\lambda_c M_S(s - \lambda) - s}, \qquad (8)$$

$$\hat{M}_{A_c}(s) = \frac{\lambda_c M_S^2(s - \lambda)}{L_\lambda(\lambda_c M_S(s - \lambda) - s)}.$$

Theorem 3: The MGFs of the AoI and peak AoI of source c under the non-preemptive packet management policy, denoted by $\tilde{M}_{\delta_c}(s)$ and $\tilde{M}_{A_c}(s)$, respectively, are given as

$$\tilde{M}_{\delta_c}(s) = \frac{\lambda_c M_S(s)(s + \lambda(M_S(s) - 1))}{s(\lambda/\mu + 1)(\lambda - s - (\lambda - \lambda_c)M_S(s))}, \quad (9)$$

$$\tilde{M}_{A_c}(s) = \frac{\lambda_c M_S^2(s)}{\lambda - s - (\lambda - \lambda_c)M_S(s)}.$$

Remark 1: The mth moment of the AoI (peak AoI) is derived by calculating the mth derivative of the MGF of the AoI (peak AoI) when $s \to 0$. For instance, considering the self-preemptive packet management policy, the mth moment of the AoI and peak AoI are given as

$$\bar{\Delta}_c^m = \frac{\mathrm{d}^m(\bar{M}_{\delta_c}(s))}{\mathrm{d}s^m}\bigg|_{s=0}, \qquad (10)$$

$$\bar{A}_c^m = \frac{\mathrm{d}^m(\bar{M}_{A_c}(s))}{\mathrm{d}s^m}\bigg|_{s=0}.$$

In the next three corollaries, by using Theorems 1, 2, and 3 and Remark 1, we derive the average AoI and average peak AoI of source 1 in a *two-source* M/G/1/1 queueing model under each of the three packet management polices.

Corollary 1: The average AoI and average peak AoI of source 1 in a two-source M/G/1/1 queueing model under the

self-preemptive packet management policy are given as

$$\bar{\Delta}_1$$
$$= \frac{L_{\lambda_1}^2(\lambda(1 - L_{\lambda_2}) - \lambda_1\lambda_2 L'_{\lambda_2}) + L_{\lambda_2}^2(\lambda_2(1 - L_{\lambda_1})) - \Psi}{\lambda_1\lambda_2 L_{\lambda_1} L_{\lambda_2}(L_{\lambda_1} + L_{\lambda_2} - L_{\lambda_1}L_{\lambda_2})},$$

$$\bar{A}_1 = \frac{L_{\lambda_1} + L_{\lambda_2} - L_{\lambda_1}L_{\lambda_2} + \lambda_1 L_{\lambda_2} L'_{\lambda_1}}{\lambda_1 L_{\lambda_1} L_{\lambda_2}},$$

where $\Psi = \lambda_1\lambda_2 L_{\lambda_1} L'_{\lambda_1} + L_{\lambda_1} L_{\lambda_2} \lambda_2(1 + \lambda_1 L'_{\lambda_1})$ *and* $L'_{\lambda_c} = \mathbb{E}[Se^{-\lambda_c S}].$

Remark 2: The average AoI under the self-preemptive policy, presented in Corollary 1, generalizes the existing results in [26] and [14]. Specifically, when confining to a single-source case by letting $\lambda_2 \to 0$ *, the average AoI becomes equal to that of the single-source M/G/1/1 queueing model with preemption derived in [26]. Moreover, when we consider an exponentially distributed service time, the average AoI expression coincides with that of the multi-source M/M/1/1 queueing model with preemption derived in [14].*

Corollary 2: The average AoI and average peak AoI of source 1 in a two-source M/G/1/1 queueing model under the globally preemptive packet management policy are given as

$$\hat{\Delta}_1 = \frac{1}{\lambda_1 L_\lambda},$$
$$\hat{A}_1(s) = \frac{1 + \lambda_1 L'_\lambda}{\lambda_1 L_\lambda}.$$

Corollary 3: The average AoI and average peak AoI of source 1 in a two-source M/G/1/1 queueing model under the non-preemptive packet management policy are given as

$$\tilde{\Delta}_1 = \frac{\lambda + \mu}{\lambda_1 \mu} + \frac{\lambda\mu\mathbb{E}[S^2]}{2(\lambda + \mu)},$$
$$\tilde{A}_1(s) = \frac{2\lambda_1 + \lambda_2 + \mu}{\lambda_1 \mu}.$$

It is worth noting that the results in Corollaries 2 and 3 were previously derived in [34] and [36], respectively (without deriving the MGFs). Thus, our derived MGF expressions generalize the results in [34] and [36], i.e., besides the first moment, they can readily be used to derive higher moments of the AoI and peak AoI.

III. DERIVATION OF THE MGFs OF THE AoI AND PEAK AoI

In this section, we prove Theorems 1, 2, and 3. To prove the theorems, we first provide Lemma 1 which presents the MGF of the AoI of source c in the considered multi-source M/G/1/1 queueing model as a function of the MGFs of the system time of source c, T_c, and interdeparture time of source c, Y_c. It is worth noting that the presented MGF expression is valid for the self-preemptive, globally preemptive, and non-preemptive packet management policies.

Lemma 1: The MGFs of the AoI and peak AoI of source c in a multi-source M/G/1/1 queueing model under the self-preemptive, globally preemptive, and non-preemptive packet management policies, denoted by $M_{\delta_c}(s)$ *and* $M_{A_c}(s)$, *respectively, can be expressed as*

$$M_{\delta_c}(s) = \frac{M_{T_c}(s)(M_{Y_c}(s) - 1)}{s\mathbb{E}[Y_c]}, \quad (11)$$

$$M_{A_c}(s) = M_{T_c}(s)M_{Y_c}(s), \quad (12)$$

where $M_{T_c}(s)$ *is the MGF of the system time of a delivered packet of source c and* $M_{Y_c}(s)$ *is the MGF of the interdeparture time of source c; these MGFs need to be determined specifically for each packet management policy.*

Proof: Let an *informative packet* refer to a successfully delivered packet from source c; otherwise, the packet is termed *non-informative*. By invoking the result in [28, Theorem 10] and applying it in our considered multi-source M/G/1/1 queueing system, if the following three conditions are satisfied

1) The arrival rate of the informative packets is positive and finite;
2) The system is stable;
3) The marked point process $\{(t'_{c,i}, T_{c,i})\}_{i=1,2,...}$ is ergodic;

then, the Laplace transform of the AoI of source c, $L_{\delta_c}(s)$, is given as

$$L_{\delta_c}(s) = \bar{\lambda}_c \frac{L_{T_c}(s) - L_{A_c}(s)}{s}, \quad (13)$$

where $\bar{\lambda}_c$ is the arrival rate of informative packets, $L_{T_c}(s)$ is the Laplace transform of the system time of any delivered packet from source c, and $L_{A_c}(s)$ is the Laplace transform of the peak AoI of source c. Next, we verify the conditions for the multi-source M/G/1/1 queueing model under the three packet management policies.

Condition 1: Since the packets of source c, both informative and non-informative, arrive according to the Poisson process with rate λ_c, the mean arrival rate of informative packets is finite. The assumption that the arrival rate of informative packets is positive, i.e., $\bar{\lambda}_c \neq 0$, is a reasonable assumption for any well-behaving status update system, since otherwise the AoI would go to infinity.

Condition 2: Since the capacity of the considered system is one packet, i.e., there are no waiting rooms in the system, the system is stable under the three packet management policies. Moreover, since an informative packet refers to a successfully delivered packet from source c and the system is stable, the mean arrival rate of informative packets of source c, $\bar{\lambda}_c$, is equal to the mean departure rate of the packets which is calculated by $\lim_{\tau\to\infty} \frac{N_c(\tau)}{\tau}$, where $N_c(\tau)$ is the number of delivered packets until time τ.

Condition 3: If we ignore the non-informative packets and just observe the informative packets, the system can be considered as an FCFS queueing model serving (only) the informative packets. In addition, since the system is stable under the three policies, according to [57, Sect. X, Proposition 1.3], the system times of informative packets, $\{T_{c,i}\}_{i=1,2,...}$, form a regenerative process with a finite mean regeneration time. Therefore, it can be verified that $\{(t'_{c,i}, T_{c,i})\}_{i=1,2,...}$ is mixing [58, Page 49], and consequently, it is ergodic.

The Laplace transform and the MGF of the AoI are interrelated as

$$M_{\delta_c}(s) = \mathbb{E}[e^{s\delta_c}] = L_{\delta_c}(-s)$$
$$\overset{(a)}{=} \bar{\lambda}_c \frac{L_{A_c}(-s) - L_{T_c}(-s)}{s}, \quad (14)$$

where (a) follows from (13). Similarly, for the MGF of the peak AoI of source c, $M_{A_c}(s)$, we have $M_{A_c}(s) = L_{A_c}(-s)$; and for the MGF of the system time of a delivered packet of source c, we have $M_{T_c}(s) = L_{T_c}(-s)$. Accordingly, (14) can be written as

$$M_{\delta_c}(s) = \bar{\lambda}_c \frac{M_{A_c}(s) - M_{T_c}(s)}{s}. \tag{15}$$

As shown in (4), the peak AoI of source c can be presented as a summation of two independent random variables, $Y_{c,i-1}$ and $T_{c,i-1}$. Applying basic features of an MGF for the independent random variables $Y_{c,i-1}$ and $T_{c,i-1}$, the MGF of the peak AoI, $M_{A_c}(s)$, is given as the product of the individual MGFs, i.e.,

$$M_{A_c}(s) = M_{T_c}(s)M_{Y_c}(s). \tag{16}$$

Since interdeparture times between consecutive packets of source c under each of the three policies are i.i.d., the number of delivered packets until time τ, $N_c(\tau)$, forms a renewal process. Thus, we have

$$\bar{\lambda}_c = \lim_{\tau \to \infty} \frac{N_c(\tau)}{\tau} = \frac{1}{\mathbb{E}[Y_c]}. \tag{17}$$

Substituting (12) and (17) into (15) completes the proof of Lemma 1. □

According to Lemma 1, the main challenge in calculating the MGFs of the AoI (see (11)) and peak AoI (see (12)) under each packet management policy reduces to deriving the MGF of the system time of source c, $M_{T_c}(s)$, and the MGF of the interdeparture time of source c, $M_{Y_c}(s)$. Note that when we have $M_{Y_c}(s)$, we can easily derive $\mathbb{E}[Y_c]$ (as will be shown in Remark 1).

Next, we will derive the MGFs of the AoI and peak AoI under the self-preemptive, globally preemptive, and non-preemptive packet management policies.

A. MGFs of AoI and Peak AoI Under the Self-Preemptive Packet Management Policy

To derive the MGF of the system time of source c, we first derive the probability density function (PDF) of the system time, $f_{T_c}(t)$, which is given by the following lemma.

Lemma 2: The PDF of the system time of source c, $f_{T_c}(t)$, is given by

$$f_{T_c}(t) = \frac{f_S(t)e^{-\lambda_c t}}{L_{\lambda_c}}. \tag{18}$$

Proof: The system time of a delivered packet from source c is equal to the service time of the packet. Let X_c be a random variable representing the interarrival time between two consecutive packets of source c. Thus, the distribution of T_c is given by $\Pr(T_c > t) = \Pr(S > t \mid S < X_c)$. Hence, $f_{T_c}(t)$ is calculated as

$$f_{T_c}(t) = \lim_{\epsilon \to 0} \frac{\Pr(t < T_c < t + \epsilon)}{\epsilon}$$
$$= \lim_{\epsilon \to 0} \frac{\Pr(t < S < t + \epsilon \mid S < X_c)}{\epsilon}$$
$$= \lim_{\epsilon \to 0} \frac{\Pr(t < S < t + \epsilon)\Pr(S < X_c \mid t < S < t + \epsilon)}{\epsilon \Pr(S < X_c)}$$
$$= \frac{f_S(t)\Pr(X_c > t)}{\Pr(S < X_c)} \stackrel{(a)}{=} \frac{f_S(t)e^{-\lambda_c t}}{L_{\lambda_c}}, \tag{19}$$

where (a) follows from the fact that i) the interarrival times of the source c packets follow the exponential distribution with parameter λ_c and thus, $\Pr(X_c > t) = 1 - F_{X_c}(t) = e^{-\lambda_c t}$, where $F_{X_c}(t)$ is the cumulative distribution function (CDF) of the interarrival time X_c and ii) $\Pr(S < X_c)$ is calculated as

$$\Pr(S < X_c) = \int_0^\infty \Pr(S < X_c \mid X_c = t)f_{X_c}(t)\mathrm{d}t$$
$$= \int_0^\infty F_s(t)\lambda_c e^{-\lambda_c t}\mathrm{d}t \stackrel{(b)}{=} L_{\lambda_c}, \tag{20}$$

where $F_s(t)$ is the CDF of the service time S, and (b) follows from the fact that according to the feature of the Laplace transform, for any function $f(y), y \geq 0$, we have [59, Sec. 13.5]:

$$L_{\int_0^y f(b)\mathrm{d}b}(s) = \frac{L_{f(y)}(s)}{s}, \tag{21}$$

where $L_{f(y)}(s)$ is the Laplace transform of $f(y)$. □

Using Lemma 2, the MGF of the system time of source c, $\bar{M}_{T_c}(s) = \int_0^\infty e^{st} f_{T_c}(t)\mathrm{d}t$, is given as

$$\bar{M}_{T_c}(s) = \frac{1}{L_{\lambda_c}} \int_0^\infty e^{(s-\lambda_c)t} f_S(t)\mathrm{d}t$$
$$= \frac{M_S(s - \lambda_c)}{L_{\lambda_c}}. \tag{22}$$

The next step is to derive the MGF of the interdeparture time Y_c, $\bar{M}_{Y_c}(s)$, which is given by the following proposition.

Proposition 1: The MGF of the interdeparture time of source c, $\bar{M}_{Y_c}(s)$, is given by

$$\bar{M}_{Y_c}(s)$$
$$= \frac{a_c(s)M_S(s - \lambda_c)}{(1 - \bar{a}_c(s))\left(1 - \sum_{c' \in \mathcal{C}\setminus\{c\}} \frac{a_{c'}(s)M_S(s - \lambda_{c'})}{1 - \bar{a}_{c'}(s)}\right)}, \tag{23}$$

where $\bar{a}_c(s)$ and $a_c(s)$ were defined in (7).

Proof: The MGF of the interdeparture time of source c packets is defined as $\bar{M}_{Y_c}(s) = \mathbb{E}[e^{sY_c}]$. To derive $\bar{M}_{Y_c}(s)$, we need to first characterize Y_c. To this end, Fig. 3 depicts a semi-Markov chain that represents the different system occupancy states (indicated by q's) and their transition probabilities (indicated by p's) in relation to Y_c, i.e., the dynamics of the system occupancy of the C different sources' packets in relation to Y_c. Thus, the graph captures all the probabilistic queueing-related events that constitute the interdeparture time Y_c, allowing us to derive Y_c.

For the graph in Fig. 3, the $C + 2$ states $\{q_0, q_1, q_2, \ldots, q_C, q_0'\}$ are explained as follows. When a source c packet is successfully delivered to the sink, the system goes to idle state q_0, where it waits for a new arrival from any source. State $q_{c'}$, $c' \in \mathcal{C}$, indicates that a source c' packet is under service. State q_0' indicates that a packet of source $c' \in \mathcal{C}_{-c}$ is successfully delivered to the sink and the system becomes empty, where $\mathcal{C}_{-c} = \mathcal{C}\setminus\{c\}$. From the graph, the interdeparture time Y_c is calculated by characterizing the required time to start from state q_0 and return to q_0. Let $\bar{X}_c = \min_{c' \in \mathcal{C}_{-c}} X_{c'}$; then, the transitions between the states are explained in the following:
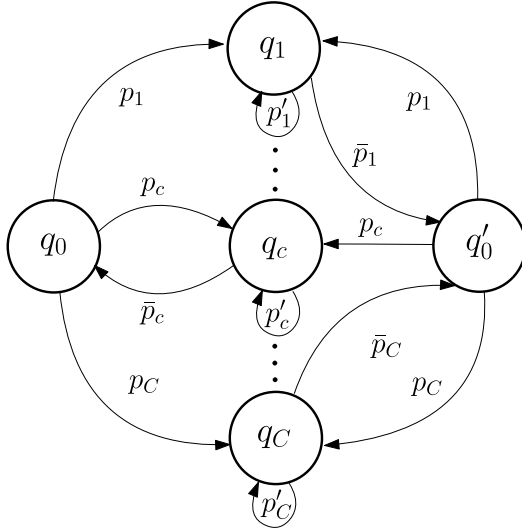
Fig. 3. Semi-Markov chain corresponding to the interdeparture time of two consecutive packets of source c under the self-preemptive policy, Y_c.

1) $q_0 \to q_{c'}, \forall c' \in \mathcal{C}$: The system is in the idle state q_0 and a source c' packet arrives. This transition happens if the interarrival time of source c' packet, $X_{c'}$, is shorter than the minimum interarrival time among all the other sources, $\bar{X}_{c'}$. Thus, the transition occurs with probability $p_{c'} = \Pr(X_{c'} < \bar{X}_{c'})$. The sojourn time of the system in state q_0 before this transition, denoted by $\eta_{c'}$, has the distribution $\Pr(\eta_{c'} > t) = \Pr(X_{c'} > t \mid X_{c'} < \bar{X}_{c'})$.

2) $q_{c'} \to q_{c'}, \forall c' \in \mathcal{C}$: The system is in state $q_{c'}$, i.e., serving a source c' packet, while a new source c' packet arrives and enters the system due to the self-preemptive packet management policy. This transition happens with probability $p'_{c'} = \Pr(X_{c'} < S)$. The sojourn time of the system in state $q_{c'}$ before this transition, denoted by $\eta'_{c'}$, has the distribution $\Pr(\eta'_{c'} > t) = \Pr(X_{c'} > t \mid X_{c'} < S)$.

3) $q_c \to q_0$: The system is in state q_c and the source c packet completes service and is delivered to the sink. This transition happens with probability $\bar{p}_c = \Pr(S < X_c)$. The sojourn time of the system in state q_c before this transition, denoted by $\bar{\eta}_c$, has the distribution $\Pr(\bar{\eta}_c > t) = \Pr(S > t \mid S < X_c)$.

4) $q_{c'} \to q'_0, \forall c' \in \mathcal{C}_{-c}$: The system is in state $q_{c'}, \forall c' \in \mathcal{C}_{-c}$, and the source c' packet completes service and is delivered to the sink. This transition happens with probability $\bar{p}_{c'} = \Pr(S < X_{c'})$. The sojourn time of the system in state $q_{c'}$ before this transition has the distribution $\Pr(\bar{\eta}_{c'} > t) = \Pr(S > t \mid S < X_{c'})$.

5) $q'_0 \to q_{c'}, \forall c' \in \mathcal{C}$: This transition is the same as transition $q_0 \to q_{c'}$.

Next, we derive the transition probabilities and the sojourn time distributions.

Lemma 3: The transition probabilities $p_{c'}$, $p'_{c'}$, and $\bar{p}_{c'}$ for all $c' \in \mathcal{C}$ are given as follows:

$$p_{c'} = \frac{\lambda_{c'}}{\lambda}, \qquad \bar{p}_{c'} = L_{\lambda_{c'}}, \qquad p'_{c'} = 1 - L_{\lambda_{c'}}. \quad (24)$$

Proof: Since $\bar{X}_{c'}$ is the minimum of independent exponentially distributed random variables X_j, $j \in \mathcal{C}_{-c'}$, it follows

the exponential distribution with parameter $\bar{\lambda}_{c'} = \sum_{j \in \mathcal{C}_{-c'}} \lambda_j$. Thus,

$$
\begin{aligned}
p_{c'} &= \Pr(X_{c'} < \bar{X}_{c'}) \\
&= \int_0^\infty \Pr(X_{c'} < \bar{X}_{c'} \mid \bar{X}_{c'} = t) f_{\bar{X}_{c'}}(t)\mathrm{d}t \\
&= \int_0^\infty (1 - e^{-\lambda_{c'} t})\bar{\lambda}_{c'} e^{-\bar{\lambda}_{c'} t}\mathrm{d}t = \frac{\lambda_{c'}}{\lambda}. \quad (25)
\end{aligned}
$$

The probability $\bar{p}_{c'} = \Pr(S < X_{c'}) = L_{\lambda_{c'}}$ was derived in (20). In addition, we have $p'_{c'} = \Pr(X_{c'} < S) = 1 - \Pr(X_{c'} > S) = 1 - \bar{p}_{c'} = 1 - L_{\lambda_{c'}}$. $\qquad \square$

Lemma 4: The PDFs of the random variables $\eta_{c'}$, $\bar{\eta}_{c'}$, and $\eta'_{c'}$ for all $c' \in \mathcal{C}$ are given as follows:

$$
\begin{aligned}
f_{\eta_{c'}}(t) &= \lambda e^{-\lambda t}, \\
f_{\bar{\eta}_{c'}}(t) &= \frac{f_s(t)e^{-\lambda_{c'} t}}{L_{\lambda_{c'}}}, \\
f_{\eta'_{c'}}(t) &= \frac{\lambda_{c'} e^{-\lambda_{c'} t}(1 - F_s(t))}{1 - L_{\lambda_{c'}}}. \quad (26)
\end{aligned}
$$

Proof: We only prove the PDF of the random variable $\eta_{c'}$; the other PDFs can be derived using the same approach. The PDF of the random variable $\eta_{c'}$ is given as

$$
\begin{aligned}
&f_{\eta_{c'}}(t) \\
&= \lim_{\epsilon \to 0} \frac{\Pr(t < \eta_{c'} < t + \epsilon)}{\epsilon} \\
&= \lim_{\epsilon \to 0} \frac{\Pr(t < X_{c'} < t + \epsilon \mid X_{c'} < \bar{X}_{c'})}{\epsilon} \\
&= \lim_{\epsilon \to 0} \frac{\Pr(t < X_{c'} < t+\epsilon)\Pr(X_{c'} < \bar{X}_{c'} \mid t < X_{c'} < t+\epsilon)}{\epsilon \Pr(X_{c'} < \bar{X}_{c'})} \\
&= \frac{(1 - F_{\bar{X}_{c'}}(t))f_{X_{c'}}(t)}{\Pr(X_{c'} < \bar{X}_{c'})} = \lambda e^{-\lambda t}. \quad (27)
\end{aligned}
$$

$\qquad \square$

To reiterate, according to Fig. 3, the interdeparture time between two consecutive packets from source c is equal to the total sojourn time experienced by the system between starting from q_0 and returning to q_0. That is, this total sojourn time consists of a summation of the individual sojourn times – which are specific to each state and its related transitions – for all possible paths $\{q_0, \ldots, q_0\}$. Thus, random variable Y_c can be characterized by the sojourn time random variables $\eta_{c'}$, $\bar{\eta}_{c'}$, and $\eta'_{c'}$ for all $c' \in \mathcal{C}$, and their numbers of occurrences, which are denoted by $k_{c'}$, $\bar{k}_{c'}$, and $k'_{c'}$, respectively. Consequently, Y_c can be presented as

$$Y_c = \sum_{c' \in \mathcal{C}} k_{c'}\eta_{c'} + \sum_{c' \in \mathcal{C}} \bar{k}_{c'}\bar{\eta}_{c'} + \sum_{c' \in \mathcal{C}} k'_{c'}\eta'_{c'}. \quad (28)$$

Having defined Y_c in (28), we proceed to derive the MGF $\bar{M}_{Y_c}(s) = \mathbb{E}[e^{sY_c}]$. Let $K_{c'}, \bar{K}_{c'}$, and $K'_{c'}$ denote the random variables representing the numbers of occurrences of random variables $\eta_{c'}$, $\bar{\eta}_{c'}$, and $\eta'_{c'}$, respectively. Then, using (28), the MGF of Y_c is given by (29), as shown at the bottom of the next page, where equality (a) follows because i) random variables $\eta_{c'}$, $\bar{\eta}_{c'}$, and $\eta'_{c'}$ for all $c' \in \mathcal{C}$ are independent, and ii) because of the independence of paths, $\Pr\big((K_1, \cdots, K_C, \bar{K}_1, \cdots, \bar{K}_C, K'_1, \cdots, K'_C) = (k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)\big)$ is equal to the summation of the probabilities of all the possible

paths corresponding to the occurrence combination $(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)$, which is given by the term

$$\prod_{c'=1}^{C} p_{c'}^{k_{c'}} \prod_{c'=1}^{C} \bar{p}_{c'}^{\bar{k}_{c'}} \prod_{c'=1}^{C} p'^{k'_{c'}}_{c'} Q(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C),$$

where $Q(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)$ is the number of paths with the occurrence combination $(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)$.

In the following remark, the values of $\mathbb{E}[e^{s\eta_{c'}}]$, $\mathbb{E}[e^{s\bar{\eta}_{c'}}]$, and $\mathbb{E}[e^{s\eta'_{c'}}]$ for all $c' \in \mathcal{C}$ are given.

Remark 3: By using the PDFs presented in Lemma 4, we have

$$\mathbb{E}[e^{s\eta_{c'}}] = \frac{\lambda}{\lambda - s},$$

$$\mathbb{E}[e^{s\bar{\eta}_{c'}}] = \frac{M_S(s - \lambda_{c'})}{L_{\lambda_{c'}}},$$

$$\mathbb{E}[e^{s\eta'_{c'}}] = \frac{\lambda_{c'}(1 - M_S(s - \lambda_{c'}))}{(\lambda_{c'} - s)(1 - L_{\lambda_{c'}})}. \tag{30}$$

What remains in deriving $\bar{M}_{Y_c}(s)$ given by the right-hand side of equality (a) of (29) are: i) the calculation of $Q(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)$, i.e., the number of paths with the occurrence combination $(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)$, and ii) calculation of the summation over the different occurrence combinations. While a direct analytical solution seems difficult, we cope with this challenge through the following lemma, providing an effective tool for the remaining calculation.

Lemma 5: Consider a directed graph $G = (\mathcal{V}, \mathcal{E})$ consisting of a set $\mathcal{V}$ of V nodes, a set $\mathcal{E}$ of E edges, an algebraic label $e_{v' \to \bar{v}}$ on each edge $e \in \mathcal{E}$ from node v' to $\bar{v}$, and a node $u \in \mathcal{V}$ with no incoming edges. Let the transfer function $H(v)$ denote the weighted sum over all paths from u to v where the weight of each path is the product of its edge labels. Then, the transfer functions $H(v)$, $\forall v \in \mathcal{V}$, are calculated by solving the following system of linear equations:

$$\begin{cases} H(u) = 1 \\ H(v) = \displaystyle\sum_{v' \in \mathcal{E}} e_{v' \to v} H(v'), & u \neq v. \end{cases} \tag{31}$$

Proof: See [60, Sec. 6.4]. □

We adopt Lemma 5 to calculate $\bar{M}_{Y_c}(s)$ as follows. We form the directed graph $G = (\mathcal{V}, \mathcal{E})$ by defining its set of nodes $\mathcal{V}$,

the directed edges $\mathcal{E}$ of weights $e_{v' \to \bar{v}}$, and the transfer functions of each node, $H(v)$, $v \in \mathcal{V}$, so that the right-hand side of equality (a) in (29) becomes equal to the transfer function of a node $\bar{v} \in \mathcal{V}$, $H(\bar{v})$. That is, we seek for the relation $\bar{M}_{Y_c}(s) = H(\bar{v})$. The formation of such graph G can readily be understood by perceiving its high similarity to the structure of the semi-Markov chain – a directed graph – in Fig. 3, which was used to characterize Y_c through paths $\{q_0, \ldots, q_0\}$. In order to define the node $u \in \mathcal{V}$ with no incoming edges, we remove the incoming links of q_0, thus representing the node u, and as a countermeasure, we introduce a virtual node $\bar{q}_0$ to account for the system state after completing the service of a source c packet. Finally, observing the factors that represent the edge weights on the right-hand side of equality (a) in (29), we depict the directed graph G in Fig. 4. According to this graph, $\bar{M}_{Y_c}(s)$ is given by the transfer function from node q_0 to node $\bar{q}_0$, $H(\bar{q}_0)$. In other words, we have $\bar{M}_{Y_c}(s) = H(\bar{q}_0)$, which now leads us to solve for $H(\bar{q}_0)$ based on (31).

The system of linear equations in (31) corresponding to the graph in Fig. 4 is given as

$$H(q_0) = 1,$$

$$H(q'_0) = \sum_{c' \in \mathcal{C}_{-c}} \bar{p}_{c'} \mathbb{E}[e^{s\bar{\eta}_{c'}}] H(q_{c'}),$$

$$H(\bar{q}_0) = \bar{p}_c \mathbb{E}[e^{s\bar{\eta}_c}] H(q_c)$$

$$H(q_{c'}) = p_{c'} \mathbb{E}[e^{s\eta_{c'}}] H(q_0) + p'_{c'} \mathbb{E}[e^{s\eta'_{c'}}] H(q_{c'})$$
$$\qquad + p_{c'} \mathbb{E}[e^{s\eta_{c'}}] H(q'_0), \forall c' \in \mathcal{C}. \tag{32}$$

By solving the system of linear equations in (32), $H(\bar{q}_0)$ is given as

$$H(\bar{q}_0)$$
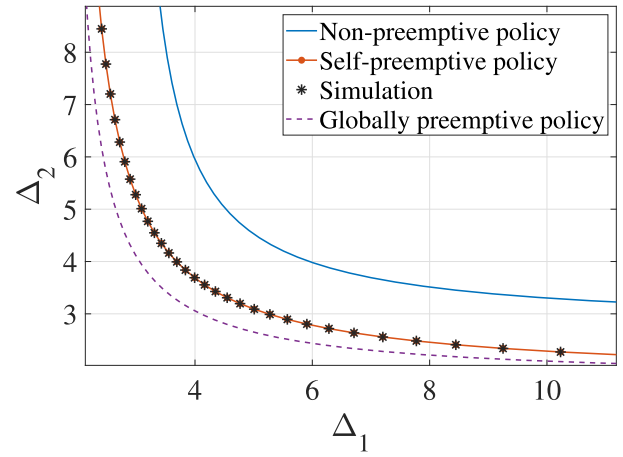
$$= \frac{p_c \mathbb{E}[e^{s\eta_c}] \bar{p}_c \mathbb{E}[e^{s\bar{\eta}_c}]}{\left(1 - p'_c \mathbb{E}[e^{s\eta'_c}]\right)\left(1 - \sum_{c' \in \mathcal{C}_{-c}} \dfrac{p_{c'} \mathbb{E}[e^{s\eta_{c'}}] \bar{p}_{c'} \mathbb{E}[e^{s\bar{\eta}_{c'}}]}{1 - p'_{c'} \mathbb{E}[e^{s\eta'_{c'}}]}\right)}. \tag{33}$$

Finally, substituting the probabilities $p_{c'}$, $p'_{c'}$, and $\bar{p}_{c'}$ given in Lemma 3 and the values of $\mathbb{E}[e^{s\eta_{c'}}]$, $\mathbb{E}[e^{s\eta'_{c'}}]$, and $\mathbb{E}[e^{s\bar{\eta}_{c'}}]$ given in Remark 3 into (33) results in the MGF of the interdeparture time of source c, $\bar{M}_{Y_c}(s)$, as given in Proposition 1. □

Finally, substituting the MGF of the system time of source c derived in (22) and the MGF of the interdeparture time of source c derived in (23) into (11) results in the MGF of the AoI under the self-preemptive policy, $\bar{M}_{\delta_c}(s)$, given in

---

$$\bar{M}_{Y_c}(s) = \mathbb{E}[e^{sY_c}] = \mathbb{E}\big[\mathbb{E}[e^{sY_c} \mid (K_1, \cdots, K_C, \bar{K}_1, \cdots, \bar{K}_C, K'_1, \cdots, K'_C) = (k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)]\big]$$

$$= \sum_{k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C} \mathbb{E}\big[e^{s(\sum_{c' \in \mathcal{C}} k_{c'} \eta_{c'} + \sum_{c' \in \mathcal{C}} \bar{k}_{c'} \bar{\eta}_{c'} + \sum_{c' \in \mathcal{C}} k'_{c'} \eta'_{c'})}\big]$$

$$\Pr\big((K_1, \cdots, K_C, \bar{K}_1, \cdots, \bar{K}_C, K'_1, \cdots, K'_C) = (k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C)\big)$$

$$\stackrel{(a)}{=} \sum_{k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C} \prod_{c'=1}^{C} \mathbb{E}[e^{s\eta_{c'}}]^{k_{c'}} \prod_{c'=1}^{C} \mathbb{E}[e^{s\bar{\eta}_{c'}}]^{\bar{k}_{c'}} \prod_{c'=1}^{C} \mathbb{E}[e^{s\eta'_{c'}}]^{k'_{c'}}$$

$$\prod_{c'=1}^{C} p_{c'}^{k_{c'}} \prod_{c'=1}^{C} \bar{p}_{c'}^{\bar{k}_{c'}} \prod_{c'=1}^{C} p'^{k'_{c'}}_{c'} Q(k_1, \cdots, k_C, \bar{k}_1, \cdots, \bar{k}_C, k'_1, \cdots, k'_C), \tag{29}$$

Fig. 4. Directed graph to calculate the MGF of the interdeparture time under the self-preemptive policy.

Theorem 1. In addition, substituting (22) and (23) into (12) results in the MGF of the peak AoI under the self-preemptive policy, $\bar{M}_{A_c}(s)$, given in Theorem 1.

B. MGFs of AoI and Peak AoI Under the Globally Preemptive and Non-Preemptive Policies

For the globally preemptive policy, Lemmas 2 and 3 in [34] provide the MGFs of the system time of source c, $\hat{M}_{T_c}(s)$, and the interdeparture time of source c, $\hat{M}_{Y_c}$, respectively, which are given as

$$\hat{M}_{T_c}(s) = \frac{M_S(s - \lambda)}{L_\lambda},$$
$$\hat{M}_{Y_c}(s) = \frac{\lambda_c M_S(s - \lambda)}{\lambda_c M_S(s - \lambda) - s}. \tag{34}$$

Substituting $\hat{M}_{T_c}(s)$ and $\hat{M}_{Y_c}(s)$ in (34) into (11) results in the MGF of the AoI under the globally preemptive policy, $\hat{M}_{\delta_c}(s)$, given in Theorem 2. Substituting (34) into (12) results in the MGF of the peak AoI of source c under the globally preemptive policy, $\hat{M}_{A_c}(s)$, given in Theorem 2.

Under the non-preemptive policy, the system time of a delivered packet is equal to the service time of the packet. Thus, the MGF of the system time of source c under the non-preemptive policy is given by $\tilde{M}_{T_c}(s) = M_S(s)$. Equation (13) in [36] provides the MGF of the interdeparture time of source c under the non-preemptive policy, $\tilde{M}_{Y_c}(s)$, which is given as

$$\tilde{M}_{Y_c}(s) = \frac{\lambda_c M_S(s)}{\lambda - s - (\lambda - \lambda_c) M_S(s)}. \tag{35}$$

Substituting $\tilde{M}_{T_c}(s) = M_S(s)$ and $\tilde{M}_{Y_c}(s)$ in (35) into (11) results in the MGF of the AoI under the non-preemptive policy, $\tilde{M}_{\delta_c}(s)$, given in Theorem 3. Substituting $\tilde{M}_{T_c}(s) = M_S(s)$ and $\tilde{M}_{Y_c}(s)$ in (35) into (12) results in the MGF of the peak AoI of source c under the non-preemptive policy, $\tilde{M}_{A_c}(s)$, given in Theorem 3.

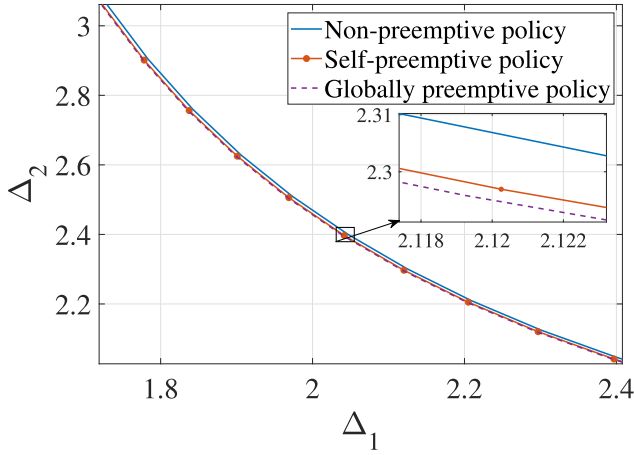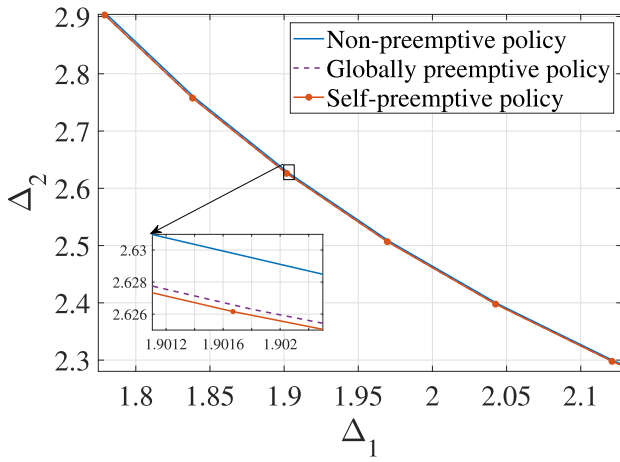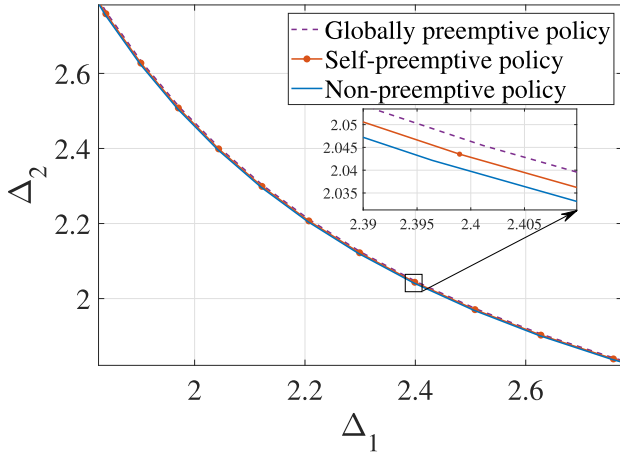(a) $\kappa = \beta = 0.5$

(b) $\kappa = \beta = 1.7$

(c) $\kappa = \beta = 3$

Fig. 5. Contours of achievable average AoI pairs under the gamma distribution for the different sets of parameters with $\mu = 1$.

IV. NUMERICAL RESULTS

In this section, we use Corollaries 1, 2, and 3 to validate the derived results for the average AoI under the self-preemptive packet management policy in a two-source system and compare the performance of the three policies in terms of the

(a) $\alpha = 2.4, \omega = 0.0583$

(b) $\alpha = 2.7, \omega = 0.630$

(c) $\alpha = 4, \omega = 0.750$

Fig. 6. Contours of achievable average AoI pairs under the Pareto distribution for the different sets of parameters with $\mu = 1$.

average AoI and sum average AoI. In addition, using the MGFs of the AoI derived in Theorems 1, 2, and 3, we investigate the standard deviation of the AoI to assess the variation of the AoI around the mean.

We investigate two service time distributions: i) gamma distribution and ii) Pareto distribution.
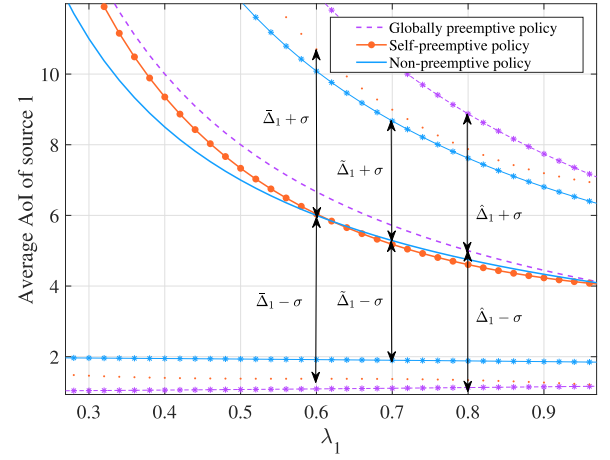
(a) $\kappa = 2$ and $\beta = 1$.
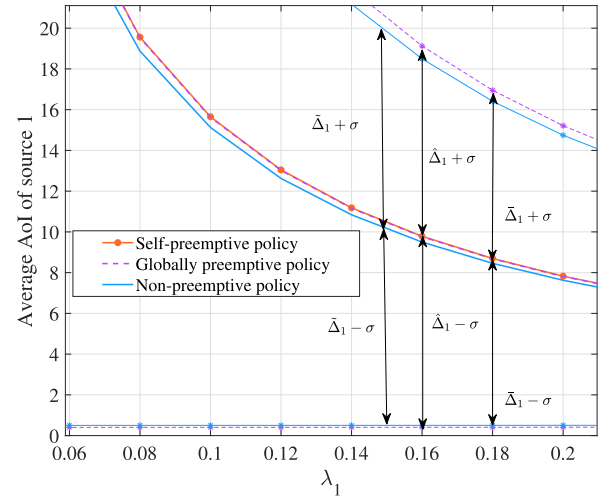
(b) $\kappa = 2$ and $\beta = 4$.

Fig. 7. Average AoI of source 1 and its standard deviation (σ) as a function of λ_1 under the gamma distribution.

- The PDF of a random variable S following a gamma distribution is defined as $f_S(t) = \dfrac{\beta^\kappa t^{\kappa-1} \exp(-\beta t)}{\Gamma(\kappa)}$, $t > 0$, for parameters $\kappa > 0$ and $\beta > 0$, where $\Gamma(\kappa)$ is the gamma function at κ. The service rate is $\mu = 1/\mathbb{E}[S] = \beta/\kappa$.
- The PDF of a random variable S following a Pareto distribution is defined as $f_S(t) = \dfrac{\alpha\omega^\alpha}{t^{\alpha+1}}$, for $t \in [\omega, \infty]$ and parameters $\omega > 0$ and $\alpha > 2$. The service rate is $\mu = \dfrac{\alpha-1}{\alpha\omega}$.

To investigate the effect of distributional properties of the service time on the average AoI under each policy, we use the squared coefficient of variation (SCoV) of the service time S, defined as $\Phi^2 = \sigma^2\mu^2$ [61, Page 39], where σ^2 is the variance of the service time. The SCoV plays a key role in analyzing the benefits of preemption in queueing, as will be detailed in the next section. The SCoV of the gamma distribution is given as $\Phi^2_{\text{gamma}} = \dfrac{1}{\kappa}$ and the SCoV of the Pareto distribution is given as $\Phi^2_{\text{Pareto}} = \dfrac{1}{\alpha(\alpha-2)}$.
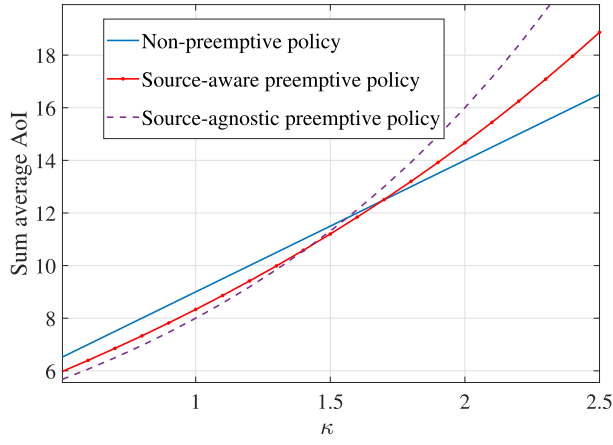
(a) Gamma distribution with $\beta = 1$.
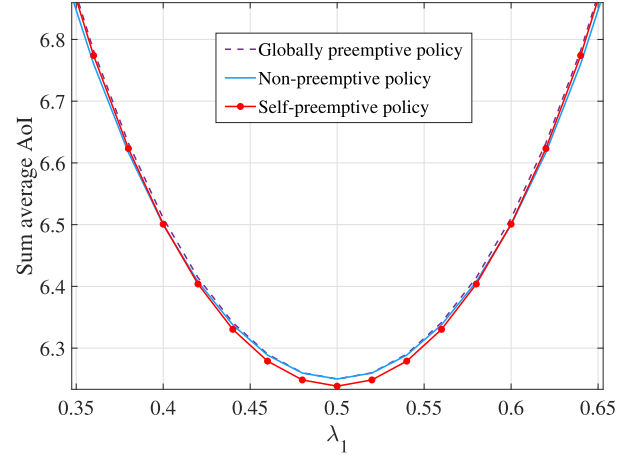
(a) Gamma distribution with $(\kappa = 2, \beta = 4)$.
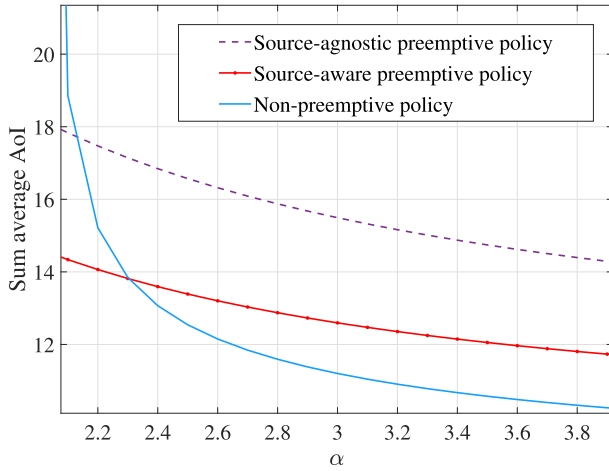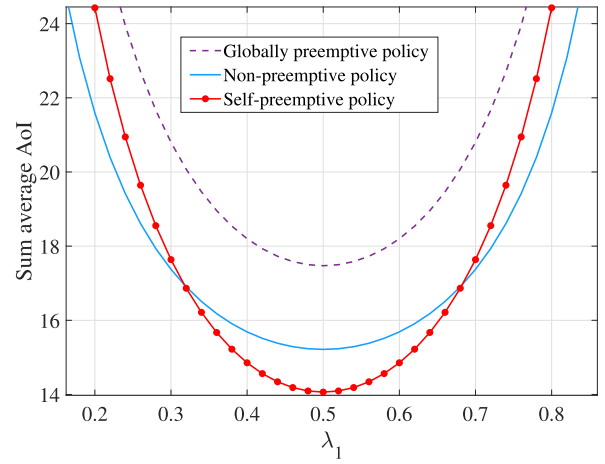
(b) Pareto distribution with $\omega = 1$.

Fig. 8. Sum average AoI under gamma and Pareto distributions with $\lambda = 0.5$.

(b) Pareto distribution with $(\alpha = 2.2, \omega = 1)$.

Fig. 9. Sum average AoI as a function of λ_1.

In all the figures, we have $\lambda = \lambda_1 + \lambda_2 = 1$. Next, we investigate the contours of achievable average AoI pairs, standard deviation of the AoI, the sum average AoI, and the effect of the number of sources on the AoI under each policy.

A. Contours of Achievable Average AoI Pairs

Fig. 5 illustrates the contours of achievable average AoI pairs (Δ_1, Δ_2) for the considered self-preemptive, globally preemptive, and the non-preemptive policies under the gamma distribution with service rate $\mu = 1$ for the parameters $\kappa = \beta = 0.5$, $\kappa = \beta = 1.7$, and $\kappa = \beta = 3$. Recall that increasing κ makes the gamma distribution to have a smaller SCoV, $\Phi^2_{\text{gamma}} = \frac{1}{\kappa}$. For the parameters $\kappa = \beta = 0.5$, the globally preemptive policy outperforms the others and the non-preemptive is the worst policy (Fig. 5a); for the parameters $\kappa = \beta = 1.7$, the self-preemptive policy outperforms the others and the non-preemptive is the worst policy (Fig. 5b); and for the parameters $\kappa = \beta = 3$, the non-preemptive policy outperforms the others and the globally preemptive policy is the worst one (Fig. 5c). In addition, we can see that the simulated curve for the self-preemptive policy matches with the derived expression in Corollary 1 (Fig. 5a).

Fig. 6 illustrates the contours of achievable average AoI pairs (Δ_1, Δ_2) for the packet management policies under the Pareto distribution with $\mu = 10$ for the sets of parameters $(\alpha = 2.4, \omega = 0.0583)$, $(\alpha = 2.7, \omega = 0.630)$, and $(\alpha = 4, \omega = 0.750)$. Note that increasing α makes the Pareto distribution to have a smaller SCoV, $\Phi^2_{\text{Pareto}} = \frac{1}{\alpha(\alpha - 2)}$. Similar to the observations made for the gamma distribution, for the parameters $(\alpha = 2.4, \omega = 0.0583)$, the globally preemptive policy outperforms the others and the non-preemptive policy is the worst one (Fig. 6a); for the parameters $(\alpha = 2.7, \omega = 0.630)$, the self-preemptive policy outperforms the others and the non-preemptive policy is the worst one (Fig. 6b); and for the parameters $(\alpha = 4, \omega = 0.750)$, the non-preemptive policy outperforms the others and the globally preemptive policy is the worst one (Fig. 6c).

Figs. 5 and 6 show that by fixing the mean service time and adjusting the set of parameters so that the SCoV becomes greater than 1 ($\Phi^2 > 1$), i.e., $\kappa = \beta = 0.5$ for the gamma distribution and $(\alpha = 2.4, \omega = 0.0583)$ for the Pareto distribution, the globally preemptive policy is the best policy. This is due to the fact that when $\Phi^2 > 1$, the average service time of a new arriving packet is smaller than the average

(a) $\kappa = \beta = 0.5$
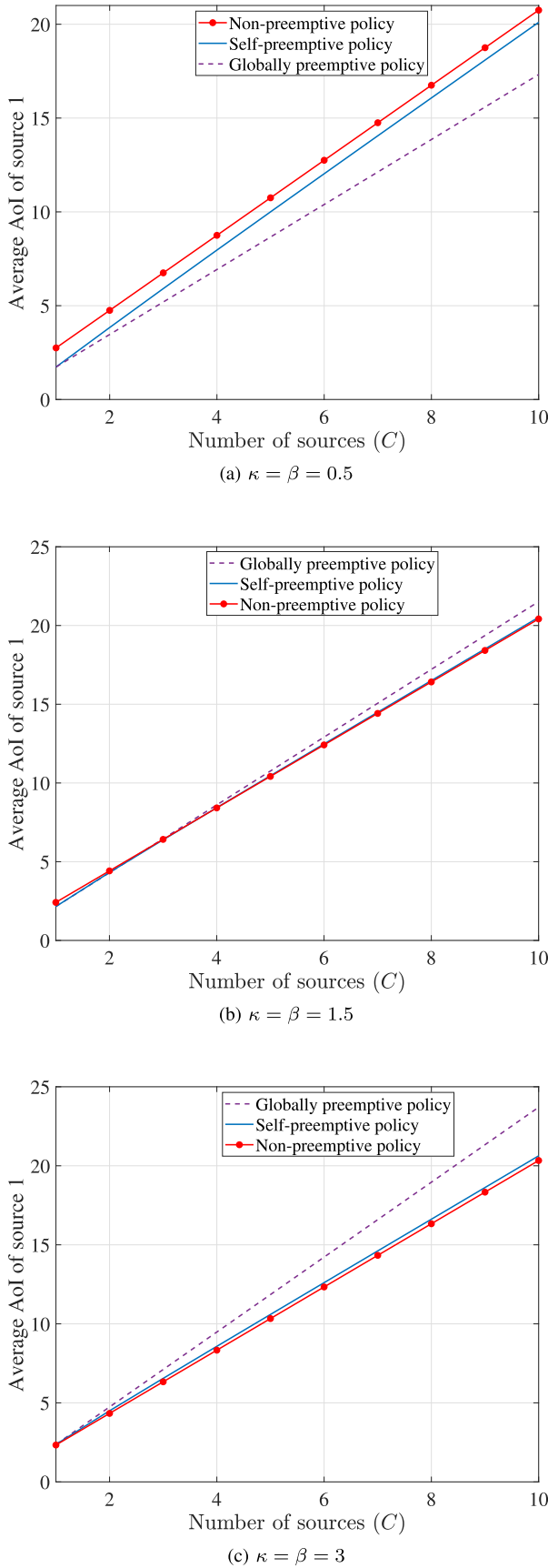

(b) $\kappa = \beta = 1.5$

(c) $\kappa = \beta = 3$

Fig. 10. Average AoI of source 1 under the gamma distribution for the different sets of parameters with $\lambda = 1$ and $\mu = 1$.

residual service time of the packet in service [61, Page 39] and [56, Sec. V.B]. In other words, for $\Phi^2 > 1$, a new arriving packet is not only fresher than the packet in service but also it requires, on average, a shorter time to be served; hence, (global) preemption is indisputably beneficial. However, the case is not this clear for $\Phi^2 < 1$. Namely, while an arriving packet is fresher than the packet in service, it requires, on the average, longer time to be served, and thus, there is a trade-off between the self-preemption and blocking. As it can be seen, when $\Phi^2 < 1$ is sufficiently small (e.g., the cases $\kappa = \beta = 3$ for the gamma distribution and ($\alpha = 4$, $\omega = 0.750$) for the Pareto distribution), the non-preemptive policy is the best one. This is expected because retaining to serve the current, albeit already slightly staled, packet with significantly shorter service time leads to the lowest AoI. Interestingly, when $\Phi^2 < 1$ is only slightly lower than 1 (e.g., the cases $\kappa = \beta = 1.7$ for the gamma distribution and ($\alpha = 2.7$, $\omega = 0.630$) for the Pareto distribution), the self-preemptive policy, which is a partially preemptive policy, is the best one.

B. Standard Deviation of the AoI

Fig. 7 depicts the average AoI of source 1 and its standard deviation (σ) as a function of λ_1 under the gamma distribution[1] with parameters $\kappa = 2$, $\beta = 1$, $\mu = 0.5$ (Fig. 7a) and $\kappa = 2$, $\beta = 4$, $\mu = 2$ (Fig. 7b). The standard deviation measures the dispersion of the values of the AoI relative to its mean; we show this by the curves $\Delta_1 + \sigma$ and $\Delta_1 - \sigma$. The figure exemplifies that the standard deviation of the AoI might have a large value even though the average AoI remains low. For example, while the average AoI performance of the non-preemptive policy is inferior to the other two policies for smaller arrival rates (around $\lambda_1 < 0.62$), the non-preemptive policy results in the least variation of the AoI around its mean for all arrival rates. This demonstrates that the average AoI does not provide complete characterization for the information freshness and thus, higher moments of the AoI need to taken into account when designing and evaluating a reliable status update system. Indeed, besides the requirement of a low average AoI value, maintaining low variation of the AoI values is crucial for time-critical applications.

C. Sum Average AoI

Fig. 8a depicts the sum average AoI, $\Delta_1 + \Delta_2$, under the gamma distribution as a function of parameter κ with $\beta = 1$. Fig. 8b depicts the sum average AoI under the Pareto distribution as a function of parameter α with $\omega = 1$. Fig. 9a illustrates the sum average AoI under the gamma distribution with ($\kappa = 2, \beta = 4$), and Fig. 9b illustrates the sum average AoI under the Pareto distribution with ($\alpha = 2.2, \omega = 1$). Similar to the observations made above, Figs. 8 and 9 exemplify that we can find a parametrization of the gamma and Pareto distributed service times so that each of the three policies, in turn, outperforms the others.

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[1]It is worth noting that since the MGF of the Pareto distribution does not exist, the standard deviation of the AoI under the Pareto distribution cannot be derived.

D. Effect of the Number of Sources

Here, we study the impact of having different numbers of sources $C \in \{1, 2, \ldots, 10\}$ in the system. Fig. 10 depicts the average AoI of source 1 under the gamma distribution for the different sets of parameters κ and β with $\lambda = 1$ and $\mu = 1$. We consider equal arrival rates, i.e., $\lambda_c = \lambda/C$, $\forall c \in \mathcal{C}$, and plot the average AoI of source 1; the average AoI of the other sources is the same. The figure shows a general trend that for all the policies, the average AoI of source 1 increases when the number of sources increases. This is because the server's serving power is shared among a higher number of sources, resulting in the increased average AoI of each individual source. Interestingly, we can see that the relative performance of the policies depends both on the parameters of the gamma distribution and on the number of sources. For example, by observing the case $\kappa = \beta = 1.5$ in Fig. 10b, when the system contains at most three sources (i.e., $C \in \{1, 2, 3\}$), the self-preemptive policy is the best policy, whereas when there are at least four sources (i.e., $C \geq 4$), the non-preemptive policy is the best one.

V. Conclusion

We derived the MGFs of the AoI and peak AoI in a multi-source M/G/1/1 queueing model under the considered self-preemptive packet management policy and the globally preemptive and non-preemptive policies studied earlier. Using the derived MGFs, we derived the average AoI and average peak AoI in a two-source M/G/1/1 queueing system under the three packet management policies. The numerical results showed that, depending on the system parameters, i.e., the packet arrival rates and the distribution of the service time, each policy can outperform the others. In addition, by visualizing the standard deviation of the AoI, the results demonstrated that the average AoI falls short in thoroughly characterizing the information freshness so that higher moments of the AoI need to be taken into account for the design of reliable status update systems.

An interesting future work would be to extend the conducted AoI analysis to the system that has one waiting room for each source and where an arriving packet of a source replaces the possible older packet of the same source waiting in the queue.

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