Received June 26, 2020, accepted July 9, 2020, date of publication July 14, 2020, date of current version July 29, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3009102

COMMENTS AND CORRECTIONS

Comments and Corrections to "Design and Implementation of Novel Fractional-Order Controllers for Stabilized Platforms"

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This work was supported in part by the Slovak Grant Agency for Science under Grant VEGA 1/0365/19, and in part by the Slovak Research and Development Agency under Contract APVV-14-0892 and Contract APVV-18-0526.

In the above article [1], the author found some mistakes in equations as well as the proposed algorithm.

In the article [1], an incorrect Grünwald–Letnikov definition (GLD) of fractional derivative was given as

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{\Gamma(\alpha)h^{\alpha}} \sum_{k=0}^{(t-a)/h} \frac{\Gamma(\alpha+k)}{(k+1)} f(t-kh), \qquad (1)$$

where

$$\binom{n}{r} = \frac{\Gamma(\alpha+k)}{\Gamma(k+1)}. \qquad (2)$$

Both equations (1) and (2) of the article [1] are incorrect. Even the symbols in (2) do not correspondence on the left and right sides of the equation, nevertheless, such expression of the binomial coefficients (2) is not used in (1).

The corrected GLD is given as follows [2]–[4]:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\left\lfloor \frac{t-a}{h} \right\rfloor} (-1)^k \binom{\alpha}{k} f(t-kh), \qquad (3)$$

where $\lfloor z \rfloor$ is the floor function, i.e., the greatest integer smaller than z, and where

$$\binom{\alpha}{k} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\,\Gamma(\alpha-k+1)} \qquad (4)$$

are the binomial coefficients for $\binom{\alpha}{0} = 1$.

In addition, the equation for the definition of the integer-order controller [1, eq. (5)] is incorrect too. There is

$$G_i(s) = K_p + K_i s + K_d. \qquad (5)$$

The correct expression for the integer-order PID controller is

$$G_i(s) = K_p + K_i s^{-1} + K_d s \qquad (6)$$

when the authors considered the fractional-order controller in the corresponding form

$$G_f(s) = K_p' + K_i' s^{-\lambda} + K_d' s^{\mu} \qquad (7)$$

with λ as an integral order and μ as a derivative order.

Moreover, such a fractional-order PID controller is not so novel as the authors declared in the article title. The first references of the fractional-order PID controller in this form were presented in 1994 in [5] and [6]. For the design of the fractional-order controller parameters (7) various exact tuning techniques, even for a position servo system, can be used [7], [8].

In the article [1], the authors mentioned that fractional calculus operators have some interesting properties, but they did not specify them, except one of them, i.e., it can reduce the number of calculation data sets by reducing the memory duration. This is not the property of fractional calculus but of the method called "short memory principle" proposed by Podlubny for numerical calculation of the fractional derivatives and integrals [4]. However, the relation described in the article [1, eq. (11)] is not complete and it has the form:

$$ {}_aD_t^{\alpha} f(t) \approx \frac{1}{h^{\alpha}} \sum_{k=0}^{N(t)} \omega_k f(t-kh), \qquad (8)$$

where

$$N(t) = \min\left\{ \left[\frac{t-t_0}{h}\right], \frac{L}{h} \right\}. \qquad (9)$$

In (8) there is the parameter ω_k, which is not defined. According to the GLD, it is the binomial coefficient which can be defined as follows [4], [9]:

$$\omega_0 = 1, \qquad \omega_k = \left(1 - \frac{1+\alpha}{k}\right)\omega_{k-1}. \qquad (10)$$

In the relation (9) the parameter t_0 is not defined as well. Moreover, for the lower moving limit $a = t - L$ in expression (8), the relation (9) should be in the form [4], [9]:

$$N(t) = \min\left\{ \left[\frac{t}{h}\right], \left[\frac{L}{h}\right] \right\}, \qquad (11)$$

where operation [.] means an integer part.

Authors declared in the article [1] that based on the above the consideration we can see that "*Using this feature to reasonably reduce the calculation step size can effectively reduce the amount of calculation and storage space*" and stated that memory length $L = 100$ is an appropriate choice.

Regarding the memory length, we have to take into account that for determining the L for required accuracy ϵ the following inequality can be used:

$$L \geq \left(\frac{M}{\epsilon |\Gamma(1-\alpha)|} \right)^{1/\alpha} \tag{12}$$

where

$$M = \max_{[0,\infty]} |f(t)|. \tag{13}$$

The evaluation of the short memory effect and convergence relation of the error between short and long memory was clearly described and also proved in [4]. Obviously, for this simplification, we pay a penalty in the form of some inaccuracy. It is necessary to notice that time step h versus memory length L must be considered as well. Since simulation time and memory length were mentioned in the article, the step of calculation h or sampling period was not in spite of the fact that computer simulation and HW Arduino-based experiments were done. It is extremely important information which is missing in the article.

The authors in the article [1] described a numerical scheme for calculation of the fractional derivative in the time domain but, in Fig. 8, we can see that they deal with approximation in Laplace s-domain. Such kind of approximation is different from the time domain approximation (8) and brings different results. Some comparisons of the results can be found, for instance, in [7], [10], and [11].

For the implementation of the fractional-order controller in discrete (digital) form the following [11]–[13] should be addressed. There are described simple methods and algorithms for such implementation in processor-based devices, as well as the MATLAB simulations.

Based on the above considerations, comments, and corrections an important question appears. Are the results presented in the article [1] correct or not?

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