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Metro Scheduling to Minimize Travel Time and Operating Cost Considering Spatial and Temporal Constraints on Passenger Boarding

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ABSTRACT Passengers on metro platforms can board a train only when the train has surplus capacity and the dwell time is sufficient, while the latter condition is omitted in previous studies. Taking into account the impacts of train capacity and dwell time on passengers boarding, this study develops a model on optimizing metro timetable to reduce passenger travel time and metro operating cost, through regulating trains' inter-station run-time, dwell time and headway. The NSGA-II algorithm is employed to obtain the near-optimal Pareto Frontier of the proposed model. To address insufficient dwell time scheduled in the timetable, three operating strategies are proposed and compared: *a*. sticking to nominal timetable; *b*. extending dwell time only; *c*. extending dwell time and recovering delay as soon as possible by compressing train inter-station run-time. Case studies on real-life metro line prove that some passengers cannot board the train during peak hours due to insufficient dwell time. In this context, strategy *a* brings low-quality service because passengers are stranded at platform even though the train has surplus capacity. In contrast, more passengers can board the train with strategies *b* and *c* because dwell time is extended for passengers' boarding when train has surplus capacity. Compared to strategy *b*, strategy *c* reduces the average in-vehicle time of passengers by 2.5% through compressing inter-station run-time to recover the delay. The timetable optimized based on strategy *c* saves total travel time of passengers by 3.1% without increasing operating cost when compared to the practical timetable.

INDEX TERMS Public transportation, urban railway, train scheduling, heuristic algorithms, operating cost, passenger travel time.

I. INTRODUCTION

Metro is a key component of public transit systems, where passengers mainly concern their travel times and expect to arrive at their destinations as soon as possible [1]. Timetable of metro services, specifying the departure and arrival times of each train at each station, has great impacts on passenger travel time, including the waiting time on the platform and the passenger in-vehicle time. For example, the passenger in-vehicle time can be reduced by compressing inter-station run-time. However, once the inter-station run-time is compressed, the operating cost especially the energy consumption of train movements may increase. Because of the conflicting interests of different stakeholders, train timetabling is an inherently multi-objective problem. In this paper, we mainly focus on passenger waiting time, passenger in-vehicle time and operating cost of metro trains.

From the passenger's point of view, the waiting time at station platform can be reduced by shortening service intervals between successive trains at the station. Nevertheless, the minimal service headway should be respected to avoid possible collision between successive trains. Once the number of accumulated passengers in a service interval exceeds the surplus capacity of the next train, some passengers have

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to wait for the following trains. Based on the spatial limitation of train capacity, Niu and Zhou [2] put forward a model to calculate the number of boarding passengers, where the concept of effective passenger loading period has been introduced. However, not only the surplus capacity of trains, but the scheduled dwell time may also restrict passengers to board. When boarding time that required by passenger is longer than the scheduled dwell time, a portion of passengers are unable to board the first arrival train owing to insufficient dwell time. This temporal constraint of dwell time should be considered together with the spatial constraint of train capacity, to calculate the number of in-vehicle passengers and passenger travel time accurately.

In this paper, both temporal and spatial constraints on passenger boarding are taken into account in timetable formulation. Passengers boarding and alighting may affect schedule dwell time due to insufficient get-on or get-off time for passengers. To address the problem of insufficient dwell time, three operating strategies are proposed and compared. Meantime the upper boundary of dwell time in these strategies are also restricted to ensure operation safety and efficiency. The principles of these three strategies are: 1) sticking to the nominal timetable although a part of passengers cannot board the train; 2) extending dwell time to enable the boarding of passengers as much as possible, without changing train inter-station run-time; 3) extending dwell time to enable the boarding of passengers when train capacity is enough, and recover to the nominal timetable by compressing the run-times in the following inter-stations.

Optimizing timetable from passenger's perspective may lead to the increment of operating cost on train movements. Therefore, a multi-objective optimization model on train timetable is proposed to take into account the interests of both passengers and the operator. A NSGA-II algorithm is designed to solve the optimization model and the Pareto front is obtained to depict the interaction between passenger travel time and operating cost. In comparison to the previous studies, this paper has following contributions:

- We put forward a timetable model considering both spatial and temporal constraints on passenger boarding as well as the boundary of dwell time, while previous studies considered spatial constraint on passenger boarding only or ignored the upper boundary of dwell time which could cause deviations in evaluating timetable performance.
- To deal with the potentially insufficient dwell time, we propose three operating strategies which allow the rescheduling of train movements. The performance of these strategies is compared from the aspects of both passenger travel time and operating cost, then the optimal strategy is incorporated into the timetable model.
- As single objective optimization or converting different criterions into one objective cannot reflect the interests of stakeholders, we explore the interaction between different objectives through attaining the near-optimal Pareto front of passenger travel time and operating cost.

The remainder of the paper is organized as follows. The related work is introduced in Section II. Problem description on timetable optimization as well as the interaction between passenger boarding and timetable parameters are presented in Section III. A train scheduling model and operating strategies in the case of insufficient dwell time are proposed to minimize passenger travel time and metro operation cost in Section IV. In Section V, a NSGA-II algorithm is designed to solve the problem and find the near-optimal Pareto solution. Case studies on Beijing Yizhuang line are conducted in Section VI, to demonstrate the effectiveness of the proposed model. Finally, conclusions are given in Section VII.

II. RELATED WORK

A. PASSENGER-ORIENTED TIMETABLE OPTIMIZATION

In order to provide more efficient rail services for passengers, a number of researches studied passenger-oriented timetable design for metro lines, aiming at satisfying passenger demand and improve service quality. Newell [3] discussed the dispatch time of trains to minimize the total waiting time of passengers. Ceder [4], [5] proposed an automatic method to optimize headway or service frequency to reduce passenger waiting time and avoid overcrowding. However, arrival rate of passengers in earlier researches is usually simplified due to the lack of detailed real-life passenger travel data.

As the spatial-temporal dynamics of the passenger demand were noticed by researchers, some studies began to focus on a peak/off-peak timetable to make a better depiction of passenger arrival and departure [6], [7]. For these timetables, a day is divided into several periods (i.e. peak hours and off-peak hours) and the headways within each specific period are fixed. However, the passenger demand tends to vary significantly over time in real life, which is in contrast to the pre-determined fixed headways during peak/off-peak periods. As a result, the peak/off-peak based timetable has a great chance to cause increased passenger waiting time under the time-varying passenger demand.

With the emergence of automated fare collection (AFC) systems, more detailed passenger demand data can be obtained. And most of the recent studies have turned to the optimization of metro timetables under the time-varying origin-destination (OD) passenger demand, which conforms to the reality better. Niu and Zhou [2] proposed a demand-dependent scheduling approach and formulated a nonlinear 0-1 integer model under oversaturated conditions to minimize the passenger waiting time at stations. Sun *et al.* [8] introduced the concept of equivalent time to demonstrate train operations and arrivals of passengers, and developed a mixed integer programming model to optimize departure times of trains at the first station. Niu *et al.* [9] considered further the skip-stop patterns for metro scheduling under time-dependent OD demands and introduced a timetable optimization model aiming at reducing the weighted passenger waiting time. Barrena *et al.* [10] formulated two non-linear mathematical models to reduce average passenger waiting time, where the

departure times of trains at the first station, the number of trains, and running times were all able to be changed.

As well as the waiting time, passenger travel time composed of both passenger waiting time and in-vehicle time is another factor which is usually applied to evaluate metro services. Wang *et al.* [11] developed a model, where the operation of trains and passenger demand characteristics were taken into account, and thus the total passenger travel time was expected to be minimized. Shang *et al.* [12] introduced an S-pattern function to describe the cumulative demand of passengers, and then presented a timetable optimization model to reduce passenger travel time. Zhang *et al.* [13] investigated the timetable optimization problem under congested conditions, and two non-linear models were formulated to design timetables with the objective of minimizing passenger travel time under the constraints of train operations, passenger boarding and alighting processes. Shen *et al.* [14] proposed a timetable optimization model to mitigate the congestion at platforms, and reduce the passenger travel time under a dynamic passenger demand.

It is worth noting that the above researchers all took train dwell times adjustment as a mean of reducing passenger travel time. However, they did not consider the influence of dwell time on passenger boarding, that is the scheduled dwell time may be insufficient for passenger boarding and some passengers may be left behind. In addition, the timetable optimization from the perspective of passengers only is very likely to result in increased operational cost. Therefore, it is necessary to carry out multi-objective optimization on timetable.

B. MULTI-OBJECTIVE OPTIMIZATION ON TIMETABLE

Different stakeholders with different interests are involved in timetable optimization problem. For instance, passengers and operator concern about travel time and operating cost, respectively. Thus, train timetabling should be treated as a multi-objective decision problem. Claessens *et al*. [15] divided the cost of rail operation into three categories, i.e. fixed costs per car per hour, variable costs per car per kilometer and variable costs per train per kilometer. Subsequently, Lindner and Zimmermann [16] adopted this classification and deemed that the operational cost of train movements includes fixed cost and cost per distance. Recently, Tirachini *et al*. [17] considered a more detailed operator cost which includes daily cost per line, vehicle cost per day, vehicle cost per hour and operator cost per vehicle-kilometer. Wang *et al*. [18] and Laporte *et al*. [19] considered the number of required vehicles as operator's concerns in multi-objective timetable optimization problems whereas energy consumption was omitted. In fact, the adjustment of train timetable also has significant effects on energy consumption which distinctly influences operating cost [20]–[22]. Many scholars considered the proper utilization of regenerated energy in timetabling [23]–[25] to relieve environmental concerns and reduce operating cost.

Although many researchers have looked into timetable optimization to improve passenger service quality or reduce operator's cost, the comprehensive evaluation from both passengers' and operator's perspectives has not been fully studied. For example, Yin *et al.* [26] and Huang *et al*. [27] took passenger travel time and energy consumption into consideration in train timetable optimization. However, metro maintenance cost, which depends on the running kilometers of vehicles, was out of consideration in these studies. Moreover, the weighted aggregation method was applied to convert the multi-objective optimization problem into a single-objective optimization problem. This technique requires a good knowledge of the system to appropriately determine the weights of different objectives [28].

C. PASSENGER BOARDING CONDITIONS

Passengers are able to board the train only if two boarding conditions are satisfied simultaneously, i.e. the train has surplus capacity and the boarding time is scheduled long enough. Earlier studies on timetable optimization considered the condition of train capacity, however, the other condition of boarding time is generally not taken into account. In practice, a part of passengers is unable to board the train if the scheduled dwell time is insufficient. This situation is very common in crowded metro lines. For example, observed data of Shanghai metro Line 8 indicates that the actual dwell time reaches three times of the scheduled dwell time in rush hours, and the trains usually attempt to carry passengers as many as they can [29]. A study based on a busy corridor in Netherlands also indicated that delays may arise because of insufficient scheduled dwell time for passenger boarding [30]. These researches reveal that passenger boarding have noticeable impacts on timetabling.

There are few researches paid attention to passenger boarding time in timetable formulation. Wang *et al*. [31] considered the minimum dwell time which depends on the number of boarding and alighting passengers, and deemed that the practical dwell time should be longer than the minimum dwell time in order to ensure passengers' boarding. However, prolonging dwell time without compressing inter-station running time may influence the predetermined rolling stock circulation. This problem was addressed by Li *et al*. [32], they adjusted dwell time according to the number of passengers at the station and train regulation strategy was also applied to recover to the nominal timetable when delays arise. The objective of reference [41] is to enhance headway regularity and commercial speed of trains, without considering operation cost. In these two studies, the congestion level of passengers within trains or at platforms, which obviously impacts on passenger boarding time, has not been considered.

Recently, taking into account passenger congestion, Li *et al*. [33] calculated the required dwell time according to the demand and speed of passenger boarding/alighting. However, the dwell time was regarded as a parameter rather than a decision variable in timetable optimization. More importantly, all these existing studies ignored the upper boundary

of dwell time, which must be respected to maintain the safe headway. For instance, the dwell time in Wang's research reached 150 s, which is impractical during peak hours on crowded metro lines where the service headway is only a few minutes. Therefore, the boundary of dwell time must be considered in timetable formulation, and the integrated optimization on timetable and operating strategy to deal with insufficient dwell time is called for as the nominal timetable might be disturbed by passengers' boarding and alighting.

III. PROBLEM DESCRIPTION

Generally, passengers at the platform are eager to board the arriving train as soon as possible. However, in practice, some of them cannot board the train due to the limitation of train capacity or insufficient dwell time. Therefore, the impacts of train capacity and dwell time on passenger boarding should be carefully considered in train timetabling. In this section, the interaction between passenger boarding and timetable parameters is analysed, and then the problem of train timetable formulation is presented with the objective of reducing passenger travel time as well as operating cost.

A. TIMETABLE OPTIMIZATION PROBLEM

This paper focuses on train scheduling on a double-track metro line with $2N$ stations. As shown in Fig. 1, the stations are numbered as $i \in [1, 2, \ldots, N, N+1, \ldots, 2N]$, where stations 1 and $2N$, N and $N+1$ denote the start terminal and the return terminal, respectively. The set of service trains are denoted as $j \in [1, 2, 3, \ldots, N_j]$. Each train firstly departs from station 1 in the up-direction and getting to station N, then turnaround to the down direction and runs back to the start terminal $2N$.

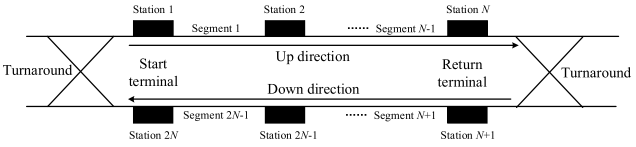
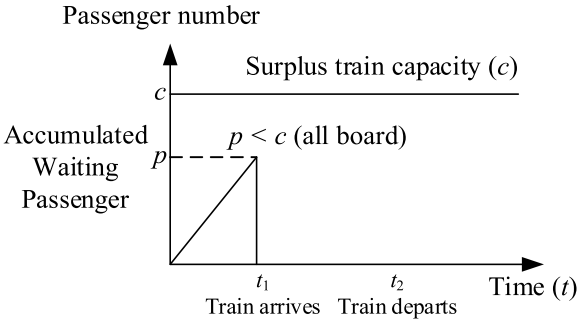


FIGURE 1. Bi-direction metro line with 2N stations.

On the formulation of train timetable, the planning horizon T_p is discretized into a series of time intervals with length Δt, which is set as 1 second in this paper. The cycle time for each train travelling a round-trip is a constant T_c. All trains are assigned with the same running/dwell time in the same segment/station, whereas these assigned times might vary across different segments or stations. The key to timetable formulation is to determine train inter-station runtimes, dwell times and service headways, to minimize operating cost and passenger travel time in the planning horizon T_p.

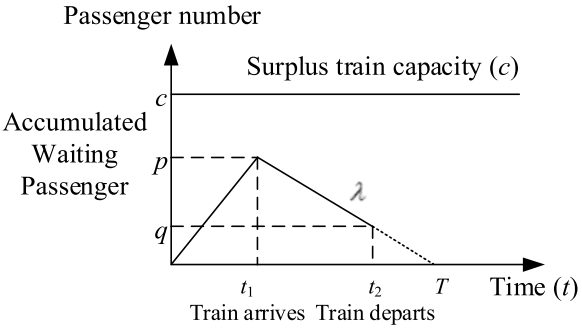
B. INTERACTION BETWEEN PASSENGER BOARDING AND TIMETABLE PARAMETERS

The relationship between timetable and passenger boarding is illustrated in Fig. 2. When the train stops at each station, accumulated waiting passengers at the platform are expected



$c > p \rightarrow$ boarding successful

(a)only considers spatial constraint



$\{c > p\} \wedge \{t_2 \geq T\} \rightarrow$ boarding successful

(b) considers both temporal and spatial constraints

FIGURE 2. Spatial and temporal constraints on passenger boarding.

to be able to board the train, as shown in the Fig. 2. Available train capacity, which is widely considered in previous studies, must be calculated to determine the number of passengers that can practically board the train. For instance, in Fig. 2(a) below, the total number of passenger p is less than surplus train capacity c. Therefore, all waiting passengers are considered to be able to board the train before the train departs.

In addition to the surplus train capacity, the sufficiency of train dwell time also affects the number of boarding passengers, which is however omitted in previous studies. Accumulated waiting passengers need a certain period of time to finish their boarding, and the length of this period is related to passengers' boarding rate λ (passenger per second). As shown in figure (b), the period of time T-t_1 is required for all passengers' boarding, i.e. $(T\text{-}t_1) * \lambda = p$. If train departs the station at time point t_2 which is earlier than T, some passengers (the number equals to q) cannot board the train actually because the dwell time is insufficient. Therefore, the sufficiency of dwell time must be considered together with train capacity constraint to ensure accurate calculation on the number of passengers at platform or in vehicle.

Due to the aforementioned train capacity (spatial) and dwell time (temporal) constraints, some passengers may not board the train within the scheduled dwell time. Different from previous studies that neglected the temporal constraint

on passenger boarding, this paper tackles the impacts of headway and dwell time on passenger boarding. In addition to sticking to the nominal timetable, the other two operating strategies are proposed to deal with the insufficient dwell time. One of the strategies prolongs the scheduled dwell time in busy stations and attempts to recover to the nominal timetable as soon as possible by compressing inter-station run-time, whereas in the other strategy, train's delay time after prolonging dwell time will not be recovered. The implementation effect of different operating strategies will be compared in this paper.

Considering the most common operation of metro system, this research is conducted based on the following assumptions.

1) All passengers at the platform obey First Come First Served (FCFS) rule, and boarding process begins only if the alighting process has finished.

Each station has enough space to accommodate all waiting passengers. Otherwise, other strategies, e.g. passenger control, are implemented to cope with the overcrowded situation.

3) Platform and carriage are very crowded in peak hours and most of passengers are willing, or being dispersed by station attendants, to stay at a relatively spacious area to get a higher possibility to get-on or get-off the train. Therefore, it is assumed that passengers are evenly distributed on the platform, and each carriage of the train has the same congestion degree.

4) All trains stop at every station and stop-skip pattern is not considered.

IV. MODEL FORMULATION

A. NOTATIONS

The parameters and variables employed in the model formulation are given in Table 1.

TABLE 1. Parameters and variables representation.

Parameter	Meaning of representation
i	index of stations / segments, i=1,2,…,2N
j	index of trains, $j\geq 1$
N_j	the number of service trains during the operating period
N_{p}	total number of the power supply interval
$\mathrm{X_i}$	the location of the station i
Δt	the length of time interval
Δx	the length of distance interval
$\eta_{\text{ec-ke}}$	conversion efficiency from electricity to kinetic energy
$\eta_{\text{ke-re}}$	conversion efficiency from kinetic energy to regenerative energy
M	the mass of rolling stock
PSI_{n}	power supply interval n
$E_{trac}^{i,j}$	traction energy consumption of train j in segment i
$E_{re\_all}^{t,PSI_n}$	total amount of the regenerative energy in PSI_{n} at time t
$E_{aux\_self}^{t,PSI_n}$	auxiliary energy consumed by braking trains in PSI_{n} at time t
$E_{non\_brak}^{t,PSI_n}$	total energy consumed by non-braking trains in PSI_n at time t
$E_{aux\_other}^{t,PSI_n}$	auxiliary energy consumed by non-braking trains in PSI_n at time t
E_{re}	utilized regenerative energy
$P^{i,v}(t)$	the number of passengers who arrive station i at time t heading to station v
C	train loading capacity
PA_j^i	the number of alighting passengers per door at station i from train j
PB_j^i	the number of boarding passengers per door at station i to train j
SD_j^i	the number of standee per door at station i in train j
PV_j^i	the number of in-vehicle passengers in train j in the i-th segment
R_j^i	the number of passengers waiting at station i after the departure of train j due to limited boarding time
TC_j^i	the time point that train j is full loaded at station i or reaches scheduled departure time
TD_j^i	the time point that passenger's boarding process can be finished within dwell time
DM_j^i	the time point that passenger's boarding and alighting process is finished
DT_{loss}	the door opening, door closing and safety check time
$DL_{\max}$	the boundary in prolonging dwell time
DS_j^i	the scheduled departure of train j from station i
DP_j^i	the practical departure of train j from station i
AP_j^i	the practical arrival time of train j at station i
Variable	**Meaning of representation**
HD_j	the arrival interval between train j and train j+1
RT_i	the scheduled running time in segment i
DT_i	the scheduled dwell time at station i

B. OPERATING COST

The operating cost consists of energy cost and maintenance cost depending on vehicle-kilometer. The energy cost is related to the energy consumed by traction system and on-board auxiliary equipment, while the regenerative energy during braking is also taken into account. Maintenance cost depends on service frequency which is generally determined according to passenger demand.

1) TRACTION ENERGY CONSUMPTION

Traction energy is consumed by rolling stocks to operate on the track, which comprises the traction energy for train motion and the energy used by on-board auxiliary equipment. According to the optimal train control theory [34], the energy-efficient train control strategy includes four operating phases: the maximum acceleration, cruising at a constant speed, coasting and the maximum braking. Train motion is denoted by formula (1)

$$\frac{d^2s}{dt^2}\cdot M(1+\lambda_w)=F_{out}(v,x)-F_{res}(v,x) \tag{1}$$

where F_{out} denotes the traction or braking force when train operates with speed v at location x. A positive value of F_{out} means that the train is motoring. Zero indicates that the train

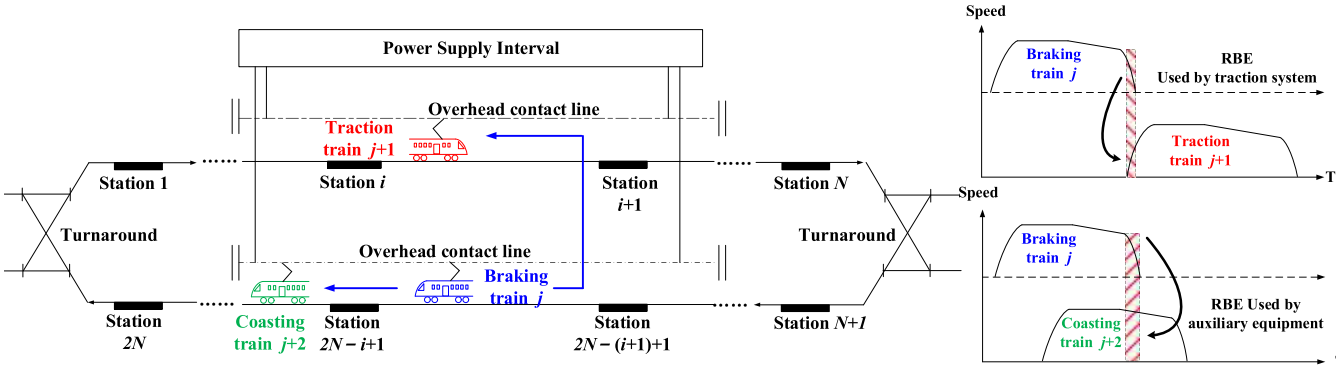


FIGURE 3. The production and utilization of regenerative braking energy.

is coasting and a negative value represents braking; λ_w is the rotary allowance, which is a constant; F_{res} is the total resistance force which is calculated by

$$\begin{aligned} F_{res}(v,x) &= F_{basic}(v) + F_g(x) + F_c(x) \\ &= a + bv + cv^2 + mg\sin(\theta(x)) + \frac{600}{R_c(x)} \end{aligned} \quad (2)$$

where F_{basic} is the well-known Davis resistance formula, coefficients a, b and c depend on train's characteristics; $F_g(x)$ is gradient resistance and $\theta(x)$ represents the gradient at position x. A positive value of $\theta(x)$ indicates uphill and a negative value means downhill; $F_c(x)$ is the curve resistance that relies on the track curve radius at position x.

The value of force F_{out} depends on train's operating phases, and φ is the indicator of different phases. For example, the maximum traction force is applied in full acceleration phase ($\varphi = 1$), the maximum braking force is employed in full braking phase ($\varphi = -1$), and the required force in cruising phases ($\varphi = 0$) is related to the total resistance force. The calculation of the force F_{out} is given in formula (3).

$$F_{out}(v,x) = \begin{cases} F_{trac}(v) & \varphi = 1 \\ -(F_{rb}(v) + F_{mb}(v)) & \varphi = -1 \\ F_{res}(v,x) & \varphi = 0 \\ 0 & otherwise \end{cases} \quad (3)$$

where $F_{trac}(v)$ is the maximum available traction force at the speed of v; $F_{rb}(v)$ is the maximum available regenerative braking force. With modern trains, the total braking force is usually taken as a constant which is the sum of the forces produced by mechanical braking and regenerative braking. As regenerative braking force declines in high-speed conditions, the mechanical braking supplies the required braking force $F_{mb}(v)$ to keep the total braking force a constant [35].

In general, no traction energy will be consumed at the coasting and braking stages. Therefore, the energy consumed in traction system of train j in segment i is equal to the energy that consumed in full accelerating and cruising phases, which can be calculated as:

$$E^{i,j}_{trac}(RT_i) = \sum_{x=X_i}^{X_{i+1}} \frac{\max\left\{F^j_{out}(v,x), 0\right\} \cdot \Delta x}{\eta_{ee-ke}} \quad (4)$$

Besides the traction system, the energy consumed by ion-board auxiliary equipment should also be taken into account. This part of energy depends on the auxiliary power (P^j_{aux}) and the length of operating period. The auxiliary energy consumption of train j in station i and segment i can be calculated as follow.

$$E^{i,j}_{aux}(RT_i, DT_i) = P^j_{aux} \cdot (RT_i + DT_i) \quad (5)$$

2) REGENERATIVE BRAKING ENERGY

Regenerative braking produces electrical energy which can be utilized by other trains. In this paper, calculations of regenerative braking energy are based on our previous study on energy-efficient operation of metro system [36]. Synchronization of traction trains and braking trains in the same Power Supply Interval (PSI) is very important in taking advantage of regenerative braking energy, as most metro lines have not equipped with energy storage device. The regenerative energy first feeds the auxiliary equipment onboard the braking train, and the remaining energy is then injected into the catenary which can be consumed by other trains in the same PSI. For example, three trains are operating in the same PSI as shown in the Fig. 3, the regenerative energy generated by braking train j is firstly consumed by auxiliary equipment on train j. Then, the rest of the regenerative energy is fed back to the catenary and it can be used by traction train $j+1$ and the auxiliary equipment on train $j+2$.

It should be noted that not only the trains in the maximum braking phase produce regenerative energy, but also trains in the cruising phase could apply braking force to keep the speed as a constant in downhill slopes. Once braking is required in cruising phase, the regenerative braking is prior to the mechanical braking. A set γ which includes all the decision variables is used to simplify equations. The total amount of the regenerated energy produced by braking trains in the same PSI at time step t can be calculated in

formula (6). In this formula, train's regenerative braking force is firstly calculated by subtracting mechanical braking force (F_{mb} is a negative value). Then, total regenerative braking energy is attained by multiplying train's speed and unit time of 1 second.

$$E_{re\_all}^{t,PSI_n}(\gamma) = \sum_{j \in PSI_n} \eta_{ke-re} \cdot \left| \min \left\{ \min \left\{ F_{out}^{j}(t), 0 \right\} + F_{mb}(v_j(t)), 0 \right\} \right| \cdot v_j(t) \Delta t \quad (6)$$

The energy consumed by auxiliary equipment on braking train(s) at the same time step is calculated by formula (7).

$$E_{aux\_self}^{t,PSI_n}(\gamma) = \sum_{j} P_{aux}^{j} \cdot \Delta t, \quad \forall j \in PSI_n \wedge F_{out}^{j}(t) < 0 \quad (7)$$

The amount of energy required by other non-braking trains in the same PSI consists of two parts: traction energy and auxiliary equipment, as shown in formula (8), as shown at the bottom of the next page. The actually utilized regenerative energy can be calculated by formula (9), as shown at the bottom of the next page. The regenerative energy is firstly used to support auxiliary equipment on braking trains. If regenerated energy is greater than $E_{aux\_self}^{t,PSI_n}$, non-braking trains can utilize the surplus regenerative braking energy.

Finally, the net energy cost is the sum of traction energy consumption and auxiliary energy, then subtract utilized regenerated energy. Energy consumption cost can be calculated by multiplying the unit price of energy(ω_e), as shown in formula (10).

$$C_{energy} = \omega_e \cdot \left(\sum_{j=1}^{N_j} \sum_{i=1}^{2N} E_{trac}^{i,j} + \sum_{j=1}^{N_j} \sum_{i=1}^{2N} E_{aux}^{i,j} - E_{re} \right) \quad (10)$$

3) METRO MAINTENANCE COST

Metro maintenance cost depends on the vehicle-kilometer, which is determined by service frequency, fleet size and length of metro lines. For a given metro line, fleet size and line length are constant at operational level. Therefore, higher service frequency leads to more maintenance cost. In this paper, service frequency is a predetermined value and can be adjusted before optimizing the timetable. Then, service frequency is set as a constant during optimizing timetable parameters.

The relationship between maintenance cost and service frequency is expressed in formula (11).

$$C_{maint} = 2L \cdot m \cdot N_j \cdot \omega_{maint} \quad (11)$$

where L is the length of track in one direction; m is the fleet size; ω_{maint} is maintenance cost per car kilometer and N_j is the number of services within the planning horizon.

C. PASSENGER TRAVEL COST

1) PASSENGER BOARDING STRATEGIES

There are two conditions should be satisfied if passengers want to board a train even when they arrived at the platform before the departure of the train. First, the train remains enough capacity to carry passengers. Second, the scheduled dwell time is sufficient for passengers to finish the alighting and boarding process.

Due to the first condition, passengers might be unable to board the train j which does not have enough capacity, even if they arrive at the platform before the departure of train j. Thus, the latest arrival time for passengers that can board train j at station i is defined as TC_j^i, which is also called effective loading time and can be calculated by

$$TC_j^i = \min \left\{ DS_j^i, \max \left\{ \tau \mid \sum_{t=TC_{j-1}^i}^{\tau} \sum_{v=i+1}^{2N} P^{i,v}(t) \le C - PV_j^{i-1} + PA_j^i \right\} \right\},$$
$$j = 2, 3, \ldots N_j, \quad i = 1, 2, \ldots 2N-1 \quad (12)$$

Considering the fair of evaluations on alternative solutions, a dummy train is assumed to take away all of the passengers waiting at platforms before the study period. In this condition, passengers arrive at platforms after the time point of DS_1^i, and $TC_1^i = DS_1^i$. The scheduled departure time DS and arrival time AR of train j at station i can be calculated by formulas (13) and (14).

$$DS_j^i = \sum_{1}^{i-1} RT_i + \sum_{1}^{i} DT_i + \sum_{1}^{j-1} HD_j \quad (13)$$

$$AR_j^i = DS_j^i - DT_i \quad (14)$$

The second condition indicates that passenger's alighting and boarding process takes a period of time. The minimum dwell time to complete the alighting and boarding can be estimated by regression models. Lam *et al*. [37] put forward an equation to calculate the minimum time for the whole alighting and boarding process according to the number of passengers, whereas the crowdedness in the carriage was not considered. Afterward, Puong [38] took the crowdedness into account and proposed a new model to estimate dwell time in high-frequency metro lines. Recent research also shows that the crowd level is an important factor in determining the dwell time besides the number of boarding and alighting passengers [39].

In this study, Puong's model is adopted to calculate the minimum dwell time (in the form of required departure time DM) to complete passenger's alighting and boarding process, as shown in formula (15). The dwell time composes four parts: loss time for train operation, boarding time, alighting time and additional time associated with congestion level. The congestion level is indicated by the number of standee per door (SD), which can be calculated by the number of in-vehicle passengers, in-vehicle area, quantity of door and seat. All waiting Passengers are able to board the train only if the

scheduled departure time *DS* is later than *DM*.

$$DM_j^i = 12.22 + 2.27PB_j^i + 1.82PA_j^i + 6.2 \cdot 10^{-4}(SD_j^i)^3 PB_j^i + AR_j^i \quad (15)$$

Table 2 gives the potential relationships among effective loading time *TC*, scheduled departure time *DS* and required departure time *DM*, as well as the practical implications. The relationship between *TC* and *DS* embodies the constraint on train capacity, which indicates that only passengers who arrived before *TC* are able to board the train. According to the aforementioned second boarding condition, passengers who arrived before *TC* require a minimum departure time of *DM* for boarding and alighting, and the scheduled departure time *DS* should be greater than *DM*. Otherwise, a part of waiting passengers may be unable to board the train.

TABLE 2. The relationship between three types of time indices.

Situations	Implications	
	Enough train capacity	Enough dwell time
TC < *DS* <*DM*	×	×
DM < *DS* <*TC*	√	√
DS < *TC* <*DM*	√	×
DM < *TC* <*DS*	×	√
DS < *DM* <*TC*	√	×
TC < *DM* <*DS*	×	√

In case of insufficient dwell time for passengers boarding, three operating strategies are proposed which take temporal constraint on passenger boarding into account. The impractical operating strategy, which ignores temporal constraint on passenger boarding, is also introduced and compared with three strategies proposed in this paper. It should be noted that these strategies are used respectively, and only one strategy is applied during each optimization. All these four strategies consider the spatial constraint on train capacity.

a) OTAB: *sticking to the Original Timetable, and All waiting passengers are assumed to Board the train (an impractical strategy but widely applied in previous studies)*

The temporal boarding condition is ignored in this strategy. All passengers are assumed to board the train within the schedule dwell time as long as the train has enough capacity. The number of alighting and boarding passengers of train *j* at station *i* are determined by the effective loading period $(TC_{j-1}^i, TC_j^i]$, which can be calculated:

$$PA_j^i = \sum_{u=1}^{i-1} \sum_{t=TC_{j-1}^i}^{TC_j^i} P^{u,i}(t), \quad j = 2, 3, \ldots, N_j, \quad i = 2, 3, \ldots, 2N \quad (16)$$

$$PB_j^i = \sum_{t=TC_{j-1}^i}^{TC_j^i} \sum_{v=i+1}^{2N} P^{i,v}(t), \quad j = 2, 3, \ldots, N_j, \quad i = 1, 2, \ldots, 2N-1 \quad (17)$$

It should be noted that $P^{i,v}(t) = 0$ if *i* and *v* are in different operating directions. After the departure of train *j* from station *i*, the number of passengers in train *j* is

$$PV_j^i = PV_j^{i-1} + PB_j^i - PA_j^i, \quad j = 2, 3, \ldots, N_j, \ i = 1, 2, \ldots 2N-1 \quad (18)$$

This operating strategy cannot exist when the scheduled departure time is earlier than required departure time, i.e. $DS < DM$. In this case, a part of passengers actually cannot board the train due to insufficient dwell time. This strategy is widely applied in previous studies that only considered the constraint of train capacity, whereas the results are inaccurate when the dwell time is insufficient for passengers' boarding and alighting.

b) OTWT: *sticking to the Original Timetable, a part of passengers Wait for the next Train*

Different from the OTAB strategy, the temporal boarding condition is considered in this strategy. A part of passengers has to wait for the next train if the scheduled dwell time is insufficient, i.e. $DS < DM$. It should be noted that formula (12) has to be amended to formula (19) due to the number of waiting passenger contains newly arrived passenger and left-behind passenger. R_{j-1}^i denotes the number of passengers who arrived before TC_{j-1}^i but have to wait for the next train due to insufficient dwell time. For the first dummy train, $R_1^i = 0$.

$$TC_j^i = \min\left\{DS_j^i, \max\left\{\tau \,|\, \sum_{t=TC_{j-1}^i}^{\tau} \sum_{v=i+1}^{2N} P^{i,v}(t) + R_{j-1}^i \le C - PV_j^{i-1} + PA_j^i\right\}\right\}, \quad j = 2, 3, \ldots N_j, \quad i = 1, 2, \ldots 2N-1 \quad (19)$$

The effective loading time *TC* determines the number of passengers that can be accommodated in the train. However, the boarding process takes a period of time after the alighting of passengers. Only a part of passengers can board the train in time if the scheduled dwell time is not long enough. The number of passengers who can actually board the train is

$$E_{non\_brak}^{t,PSI_n}(\gamma) = E_{trac}^{t,PSI_n} + E_{aux\_other}^{t,PSI_n} = \sum_j \left(\frac{\max\left\{F_{out}^j(t), 0\right\} \cdot v_j(t)\Delta t}{\eta_{ee-ke}} + P_{aux}^j \cdot \Delta t\right), \quad \forall j \in PSI_n \wedge F_{out}^j(t) \ge 0 \quad (8)$$

$$E_{re}(\gamma) = \sum_{t \in T_p} \sum_{PSI_n \in N_p} \min\left\{E_{re\_all}^{t,PSI_n}, E_{aux\_self}^{t,PSI_n}\right\} + \min\left\{\max\left\{E_{re\_all}^{t,PSI_n} - E_{aux\_self}^{t,PSI_n}, 0\right\}, E_{non\_brak}^{t,PSI_n}\right\} \quad (9)$$

calculated as:

$$PB_j^i = \min\left\{1, \frac{DT_i - 12.22 - 1.82PA_j^i}{2.27PB_j^i + 6.2\cdot 10^{-4}(SD_j^i)^3 PB_j^i}\right\} \times \left(\sum_{t=TC_{j-1}^i}^{TC_j^i}\sum_{v=i+1}^{2N} P^{i,v}(t) + R_{j-1}^i\right), \quad j = 2,3,\ldots,N_j, \quad i = 1,2,\ldots,2N-1 \tag{20}$$

In formula (20), the ratio between surplus dwell time for passenger boarding and required boarding time for all passengers is firstly calculated. In extreme cases, passenger alighting could consume more time than the dwell time, i.e. $DT_i - 12.22 < 1.82PA_j^i$, and no time is left for passengers' boarding. However, this situation will not be discussed in this paper. The obtained proportion is multiplied by the number of waiting passengers to estimating the number of passengers who can actually board the train within the dwell time. Then, time point *TD*, which means passengers arriving before *TD* can board the train within the scheduled dwell time, can be obtained by formula (21). In contrast, passengers arriving after *TD* have to wait for the next train even if train j has surplus capacity.

$$TD_j^i = \begin{cases} TC_j^i, & DM_j^i < DS_j^i \\ \max\left\{\gamma \mid \sum_{t=TC_{j-1}^i}^{\gamma}\sum_{v=i+1}^{2N} P^{i,v}(t) + R_{j-1}^i \le PB_j^i\right\}, & DS_j^i \le DM_j^i \end{cases} \tag{21}$$

If the boarding process can be finished within dwell time, *TD* is equal to *TC*, which means all passengers can board the train. Otherwise, some passengers arriving at the platform during [*TD*, *TC*] may not board the train. Therefore, formulas (16) and (17) which represent the number of actual alighting and boarding passengers are revised as formula (22) and (23). The number of left-behind passengers due to insufficient dwell time is calculated by formula (24).

$$PA_j^i = \sum_{u=1}^{i-1}\sum_{t=TC_{j-1}^i}^{TD_j^i} P^{u,i}(t), \quad j = 2,3,\ldots,N_j, \quad i = 2,3,\ldots,2N \tag{22}$$

$$PB_j^i = \sum_{t=TC_{j-1}^i}^{TD_j^i}\sum_{v=i+1}^{2N} P^{i,v}(t), \quad j = 2,3,\ldots,N_j, \quad i = 1,2,\ldots,2N-1 \tag{23}$$

$$R_j^i = \sum_{TD_j^i}^{TC_j^i}\sum_{v=i+1}^{2N} P^{i,v}(t) \tag{24}$$

c) EXDL: *EXtending dwell time and DeLay occurs*

Different from the OTWT strategy, dwell time can be extended in a reasonable range to accommodate more waiting passengers in this strategy. Firstly, the number of passengers who can be accommodated in the train is obtained according to effective loading time. Then, the required departure time for these passengers completing boarding is calculated. After that, three scenarios are defined according to the relationship among scheduled departure time *DS*, required departure time *DM* and upper boundary of extended departure time $DL_{\max}$:

- $DS > DM$, dwell time is sufficient for passenger alighting and boarding, and scheduled timetable will not be unchanged.
- $DS < DM \wedge (DM\text{-}DS) \le DL_{\max}$, dwell time is insufficient and scheduled departure time is delayed to match passenger demand.
- $DS < DM \wedge (DM\text{-}DS) > DL_{\max}$, the required extension of dwell time exceeds the upper boundary, and the practical departure time should be equal to $DS + DL_{\max}$.

In the first two scenarios, formulas (16-18) that applied in OTAB strategy are implemented to calculate effective loading time and the number of boarded passengers. This is because passenger's boarding process can be finished within the scheduled or prolonged dwell time. In the third situation, a part of passengers cannot board the train even if the dwell time has been prolonged. In this case, formulas (19-24) in OTWT strategy are used to describe the passenger boarding process. The delay caused by extending dwell time will not be recovered in EXDL strategy. Therefore, practical departure and arrival times of all the stations after station *ir*, where delay occurs, will be postponed, as shown in formulas (25) and (26). Also, train's cycle time is extended due to the delay. It should be noted that $DL_{\max}$ is used for limiting the upper boundary of practical dwell time, in order to ensure operating safety and a certain extent of fairness, as well as reserve some capacity for downstream stations.

$$DS_j^i < DP_j^i < DS_j^i + DL_{\max} \tag{25}$$

$$AP_j^{i+1} = DS_j^i + RT_i, \quad \forall i > ir \tag{26}$$

d) EXRE: *EXtending dwell time and REcover delay time*

In accordance with EXDL strategy, scheduled departure time can be extended in the EXRE strategy to enable more passengers to board the train. If the required extension of dwell time is less than the acceptable range, formulas (16-18) in OTAB strategy can be used for calculating the number of in-vehicle passengers. Otherwise, formulas (19-24) in OTWT strategy will be implemented owing to some passengers cannot board the train.

Differently, the deviation from the nominal timetable is expected to be eliminated as soon as possible via compressing inter-station running times in EXRE strategy. Even though the extent of compressing running time is rather limited in one inter-station, the delay can be gradually recovered in the next few inter-stations. The rescheduled timetable configurations are attained by formula (27) -(30).

First, the station where delay occurs is marked with *id*. The sum of available inter-station run-time is used for calculating

the station where delay can be fully recovered, and this station is marked as *idr*.

$$idr = \min\left\{ i | \sum_{i=id}^{2N} RT_i - RT_i^{\min} > \min\left\{ \max\left\{ DM_j^i - DS_j^i, 0 \right\}, DL_{\max} \right\} \right\} \quad (27)$$

Then, minimum inter-station run-time is applied in segments *id* to *idr*, to recover the scheduled timetable. Rest of the delay time is recovered in segment *idr*.

$$RT_i = \begin{cases} RT_i^{\min}, & i = id, id+1, \ldots, idr-1 \\ RT_i - (\min\left\{ \max\left\{ DM_j^i - DS_j^i, 0 \right\}, DL_{\max} \right\} \\ \quad - (\sum_{i=id}^{idr-1} RT_i - RT_i^{\min})), & i = idr \end{cases} \quad (28)$$

After than, the practical train departure and arrival times in the following stations are also changed.

$$DP_j^i = \begin{cases} DS_j^i + \min\left\{ \max\left\{ DM_j^i - DS_j^i, 0 \right\}, DL_{\max} \right\}, \\ \quad i = id \\ DS_j^i + \min\{\max\{DM_j^{id} - DS_j^{id}, 0\}, DL_{\max}\} \\ \quad - (\sum_{i=id}^{idr-1} RT_i - RT_i^{\min}), \quad i = id+1, \ldots, idr \end{cases} \quad (29)$$

$$AP_j^i = DS_j^i + RT_i, \quad i = id, id+1, \ldots, idr \quad (30)$$

Fig. 4 summarized the possible scenarios for passenger boarding and the corresponding operating strategies. Firstly, TC_j^i is calculated according to train capacity and scheduled departure time. Based on TC_j^i, the numbers of boarding and alighting passengers are obtained. This part of passengers and congestion level are used to calculate the required departure time *DM*. The comparison of DM and DS is the starting condition for prolonging the dwell time. If $DM \leq DS$, trains will depart from the station on time and four different strategies have the same result. In contrast, when $DM > DS$, a part of passengers is unable to board the train if the train strictly complies with the scheduled departure time (OTAB and OTWT strategies). Alternatively, dwell time will be prolonged in EXRE and EXDL strategies to enable more passengers to board the train.

2) PASSENGER TOTAL TRAVEL TIME

In general, passengers expect to arrive at their destinations as soon as possible. To this end, the timetable formulation aims to reduce the total travel time of passengers, including waiting time and in-vehicle time. Passenger in-vehicle time comprises inter-station running time and dwell time. Consequently, the

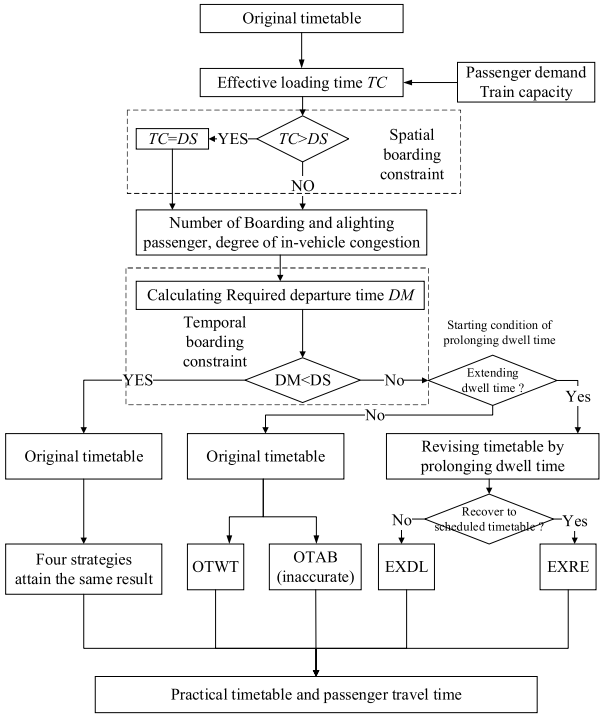


FIGURE 4. Passenger boarding scenarios and different operating strategies with the consideration of temporal and spatial boarding constraints.

total travel time can be calculated by formula (31).

$$C_{time}(\gamma) = \sum_{j=1}^{J} \sum_{u=1}^{2N-1} \sum_{v=u+1}^{2N} \sum_{t=TD_j^{u-1}}^{TD_j^u} \left[\left((DP_j^i - t) + \sum_{i=u}^{v-1} RT^i + \sum_{i=u+1}^{v-1} DT^i \right) \cdot P^{u,v}(t) \right] \quad (31)$$

It should be noticed that the temporal boarding constraint is not considered in OTAB strategy, and *TD* is set the same value as *TC* in OTAB strategy because all passengers are assumed to be able to board the train if they arrived before *TC*. Besides, the practical arrival/departure time (*AP/DP*) in OTAB and OTWT strategies equal to the scheduled time (*DS/AR*). In EXDL and EXRE strategies, however, practical arrival/departure time may be revised according to passenger alighting and boarding situations

D. OBJECTIVE FUNCTION

The objective of timetabling problem in this research is to minimize metro operating cost and passenger travel time. It is impossible to find a solution that can simultaneously minimize these two contradictory objectives. Therefore, a multi-objective optimization model is developed to explore the relationship between each objective. Decision maker could select the most suitable solution according to practical operation requirements. The objective function is formulated

as:

$$min\, C_{total}(\gamma) = \{C_{energy} + C_{maint}, C_{time}\} \tag{32}$$

E. CONSTRAINTS

To ensure operational safety and service efficiency, the following constraints should be satisfied.

1) RUNNING TIME CONSTRAINT

the scheduled running time in the inter-station i should be bounded with the consideration of service quality and the maximum operating speed. The improvement on RT has impacts on both energy consumption and passenger travel time. For example, more traction energy will be consumed if RT is lessened. Meanwhile, passenger travel time is likely to be reduced. Especially, we treat the turnaround (i.e., the segment N and $2N$) time as a special segment running time.

$$T_i^{\min} \le T_i \le T_i^{\max}, \quad i = 1, 2, 3, \ldots, N, \ldots, 2N-1, 2N \tag{33}$$

2) CYCLE TIME CONSTRAINT

the cycle time, defined as the sum of DT and RT of all the stations, is predetermined. However, the cycle time may be prolonged in EXDL strategy when delays occur.

$$T_c = \sum_{1 \le i \le 2N} DT_i + RT_i \tag{34}$$

3) HEADWAY CONSTRAINT

the service headway between two successive trains must be no less than the minimum headway for operation safety. Meanwhile, the deviation of service headway from the average headway (H_{ave}) should be limited within an appropriate range ρ, for the reason that headway regularity provides better service fairness for passengers.

$$\left|H_j - H_{ave}\right| < \rho \tag{35}$$

4) POWER PEAK CONSTRAINT

the instantaneous traction power in the same PSI should not exceed the maximum capacity of the power supply system.

$$\sum_{t \in T_p} \sum_{j \in PSI_n} F_{out}^j(t) \cdot v_j(t) \le P_{\max}^n, \quad \forall j \in PSI_n \wedge F_{out}^j(t) > 0 \tag{36}$$

5) MAXIMUM DWELL TIME CONSTRAINT

The scheduled departure time of train j from station i can be postponed to satisfy passengers' boarding and alighting. From aspects of operating safety and fairness, practical departure time's (DP) deviation from the scheduled departure time (DS) should be restricted in a reasonable range ($DL_{\max}$).

$$DP_j^i = \max\{DS_j^i, \min\{DM_j^i, DS_j^i + DL_{\max}\} \tag{37}$$

6) TRAIN CAPACITY CONSTRAINT

Train capacity is always a practical and necessary in this model. Passenger can never board train j which has no surplus capacity left at station n.

$$\sum_{i=1}^{n} PB_j^i - \sum_{i=1}^{n} PA_j^i \le C \tag{38}$$

V. SOLUTION ALGORITHM

In multi-objective optimization problem, there is a set of acceptable trade-off optimal solutions rather than a unique solution. These acceptable solutions compose the Pareto optimal set which means any solutions in the Pareto set cannot improve at least one of the objectives without degradation any other objectives. To find the Pareto optimal set of the proposed model, a Non-dominated Sorting Genetic Algorithm II (NSGA-II) algorithm is developed in this study. The NSGA-II is a powerful multi-objective evolutionary algorithm which is competent to find a much better spread of solutions and better convergence near the true Pareto optimal set [40]. Decision maker can flexibly select a passenger-oriented or economy-efficient timetable from the Pareto front, according to practical operating conditions. Fig. 5 gives the flowchart of the developed NSGA-II.

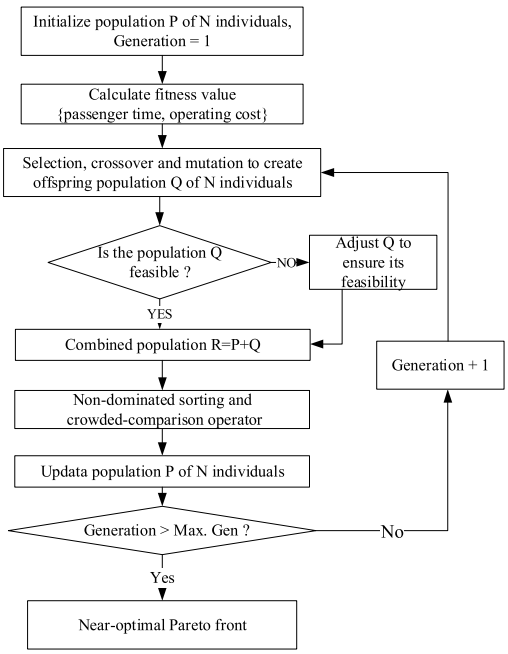


FIGURE 5. The flowchart of NSGA-II algorithm.

A. CHROMOSOME CODING AND INITIALIZING

An initial population P with N individuals is generated based on the practical timetable configurations, using integer coding. There are n stations in one direction, thus the length of each individual chromosome is $4n + N_{\text{car}} - 1$, which can be divided into three parts: dwell times (Gene position 1 to $2n$), running times (Gene position $2n + 1$ to $4n$) and headways

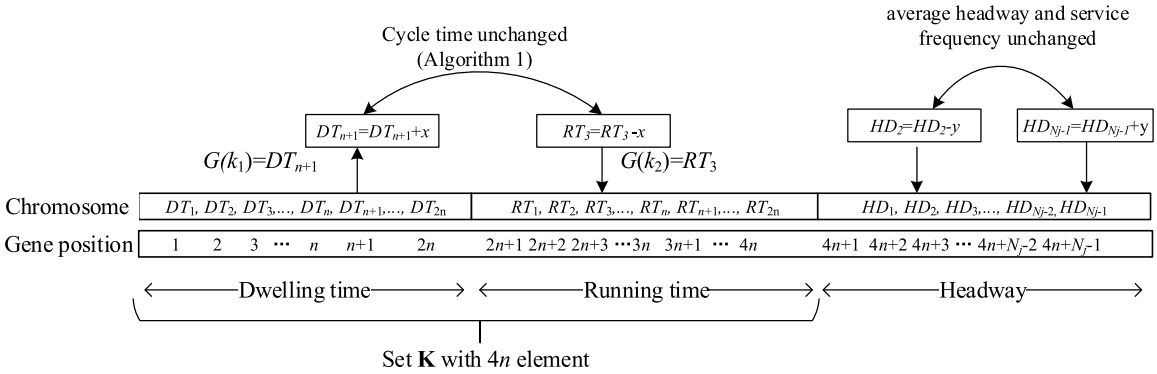


FIGURE 6. Generating chromosome based on practical timetable configurations with constant cycle time.

Algorithm 1 Generating Feasible Solutions

Input: Practical DT and RT, variation range δ
Output: Feasible solutions
While K $\neq \varnothing$
  $x = [\delta - 2 \times rand(\delta)]$
  randomly pick $k_1 \in$ K
  $G(k_1) = G(k_1) + x$
  randomly pick $k_2 \in$ K
  $G(k_2) = G(k_2) - x$
end while
Output [DT, RT]

(Gene position $4n + 1$ to $4n + N_{\text{car}} - 1$), as shown in Fig.6. The k-th gene position in the chromosome is denoted as $G(k)$. It should be noted that the cycle time is considered as a fixed value. The computational procedure of generating a chromosome with a fixed cycle time is shown as the pseudo-code of Algorithm 1.

To keep the cycle time as a constant, new populations are generated by increasing or reducing inter-station run-times and dwell times in a practical timetable. Firstly, all dwell times and running times in the chromosome compose the set K with $4n$ elements, which are indicated as $G(1), G(2) \ldots, G(4n)$. Secondly, a gene position $G(k_1)$ is randomly selected in the set K and a random variable x is added to $G(k_1)$. The value of random variable x is determined within a variation range δ, i.e. $-\delta \leq x \leq \delta$. Thirdly, another gene position $G(k_2)$ is randomly selected in K, and the previously used random variable x is now subtracted from $G(k_2)$. The above process is repeated until all elements in K are revised and the variables k_1, k_2 and x are generated randomly in each revision. As such, cycle time remains unchanged because the sum of inter-station run-times and dwell times is a constant.

Headways in chromosomes are generated similarly. The only difference is that the number of elements may be odd numbers. In this regard, the final selected element remains unchanged. Therefore, different chromosomes share the same average headway and service frequency within the operation period.

B. SELECTION, CROSSOVER AND MUTATION

The individuals in P are selected by a roulette-wheel, in which individuals with higher fitness have a higher possibility to be selected. Crossover and mutation operations are carried out among three different gene positions and different parts of decision variables are exchanged, as shown in Fig. 7. The value of a gene position is regenerated randomly if mutation operation is activated according to mutation possibility. Offspring population Q is formed based on the selected individuals in population P through crossover and mutation. Finally, a new population R with $2N$ individuals is generated by mixing populations P and Q.

The crossover and mutation operations may change the cycle time and make the solution infeasible. If cycle time exceeds the scheduled value, a gene position which represents running time or dwell time is randomly selected and subtracted by 1 second. The selection and subtraction repeat until cycle time equals to the scheduled value. Similarly, the cycle time can be prolonged by randomly adding seconds in some gene positions.

C. NON-DOMINATED SORTING AND CROWDED-COMPARISON

Each individual in population R is compared with every other solution in order to identify the non-dominated level. If two individuals satisfy the rule that $\text{Obj}_1(\text{R}_i) < \text{Obj}_1(\text{R}_j)$ *and* $\text{Obj}_2(\text{R}_i) < \text{Obj}_2(\text{R}_j)$ in a minimization problem with two objectives, R_j is dominated by R_i and R_i is marked as a non-dominated solution.

In non-dominated sorting operation of R individuals, all the non-dominated solution (N_1) is assigned rank 1 and removed from the population. For the remaining R-N_1 dominated solutions, they are sorted again and forming a new set of non-dominated solutions (N_2) which are assigned rank 2. This process continues until all the individuals are ranked. For the individuals with the same rank level, the crowding distance for each individual is calculated by formula (39).

$$d_i = \sum_{obj\_n=1}^{2} \frac{f_{obj\_n}^{i-1} - f_{obj\_n}^{i+1}}{f_{obj\_n}^{\max} - f_{obj\_n}^{\min}} \tag{39}$$

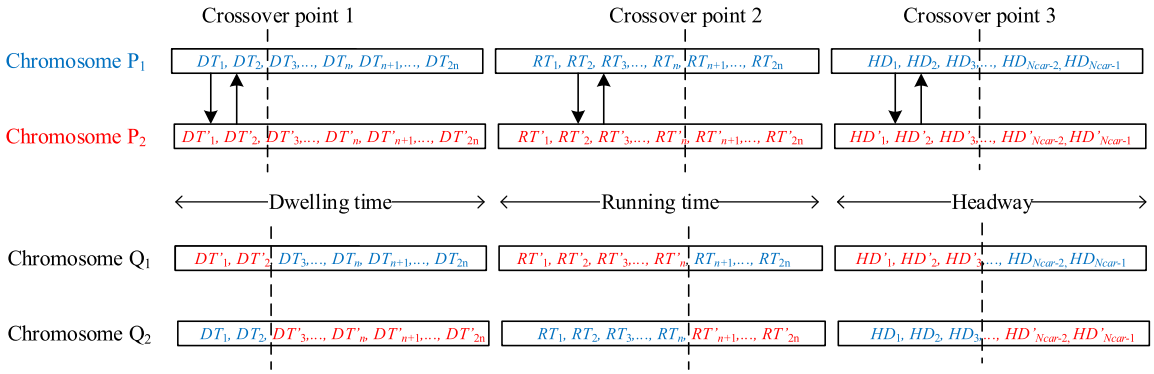


FIGURE 7. Crossover operation in generating new population.

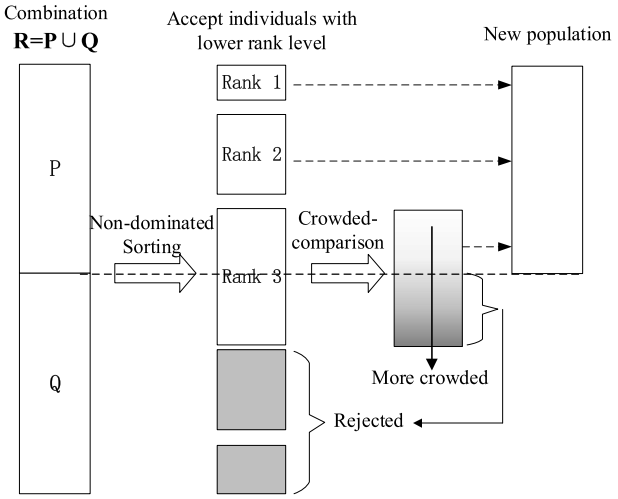


FIGURE 8. Generating new population by Non-dominated sorting and crowded-comparison operations.

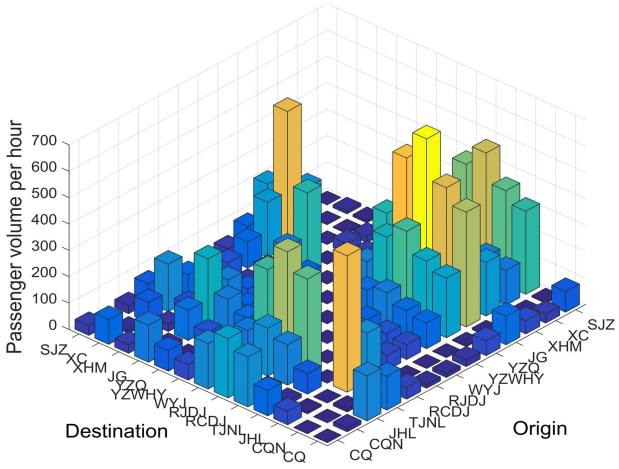


FIGURE 9. Passenger volume in peak hours.

d_i denotes the crowding distance of the individual i in a non-dominated front; $f^i_{obj\_n}$ is the value of n^{th} objective of the individual i; $f^{\max}_{obj\_n}$ and $f^{\min}_{obj\_n}$ are the maximum and minimum values of n^{th} objective, respectively.

The new population of N individuals is picked from population R based on the non-dominated rank and the crowding distance, as shown in Fig. 8. Individuals with the lowest rank level are preferred. Afterwards, solutions belonging to the same rank level are sorted according to the crowding distance, and individuals located in lesser crowded regions (greater d_i) are selected to form the new population. If the evolution reaches the maximum generation, the algorithm will be terminated and export the optimized solutions.

VI. CASE STUDIES

Beijing Metro Yizhuang line, which is 21.5 km with 13 stations, is selected to conduct case studies. The study period is peak hours from 7:00 a.m. to 9:00 a.m. on weekdays. The Origin-Destination matrix is given in Fig. 9, which is obtained from Beijing Metro Operation Ltd. The scheduled inter-station running time and station dwell time in the real-world timetable as well as the distance between two adjacent stations are listed in Table 3. It should be noticed that the up direction is from CQ to SJZ. The running time in segment n denotes the scheduled run-time between station n and station $n+1$. The whole line is divided into 6 power supply intervals which are shared by both directions. Train parameters are listed in Table 4. The mathematical model is solved by Matlab® 2016a on a personal laptop with 4 cores of 2.3 GHz and 8 GB RAM.

TABLE 3. Vehicle performance and parameters settings.

Parameters	Value (unit)	Parameters	Value
M	282,000 kg	C	1400 pax/train
Maximum acceleration	1.0 m/s^2	P_{aux}	30 kWh
Maximum deceleration	1.0 m/s^2	$DL_{\max}$	20 s
Davis resistance equation $[a,b,c]$	[0.8636, 0.0102, 2.38×10^{-4}]	ω_{maint}	¥ 15/car-kilometer
$\eta_{\text{ee-ke}}$	0.7	ω_{e}	¥ 1/kWh
$\eta_{\text{ke-re}}$	0.7	Δt	1 s

TABLE 4. Nominal timetable of Beijing Metro Yizhuang line in real-world.

Stations	Length (m)	Up direction		Down direction		PSI
		Dwell time (s)	Running time (s)	Dwell Time (s)	Running time (s)	
SJZ	2631	45	-	45	190	1
XC	1275	30	190	30	108	
XHM	2366	30	108	30	157	2
JG	1982	30	157	30	135	
YZQ	993	35	135	35	90	3
YZWH	1728	30	90	30	114	
WYJ	1090	30	114	30	103	
RJDJ	1355	30	103	30	104	4
RCDJ	2337	30	104	30	164	
TJNL	2301	30	164	30	150	5
JHL	2055	30	150	30	140	
CQN	1281	35	140	35	102	6
CQ	-	45	102	45	-	

In the case study section, the interaction between service headways and required station dwell times is analyzed firstly. The performance of different operating strategies in response to insufficient dwell time is then compared followed by the sensitivity analysis on the boundary in prolonging station dwell time. Finally, the current timetable of Beijing Metro Yizhuang line is optimized to tackle the concerns of stakeholders, based on the optimal operating strategy.

A. THE INTERACTION BETWEEN SERVICE HEADWAY AND REQUIRED DWELL TIME

Service headways may have a great impact on station dwell time, especially in peak hours on busy metro lines. For example, a larger service headway results in more passengers waiting at the platform, which in turn requires longer dwell time. This inherent relationship between service headway and dwell time is considered and analyzed in this section, assuming that passenger volume is even distributed in study period.

Fig. 10 and Fig.11 depict the required dwell time (*RDT*) for passengers to alight and board under different headways in up and down directions, respectively. It should be noted that the constraint of train's surplus capacity is always considered in case studies. The bold red line in figures represents the scheduled dwell time at each station in the nominal timetable. It is obvious that *RDT* increases with headway, but growth rates vary across stations. In some cases, the *RDT* exceeds the scheduled dwell time significantly (e.g. the JG station in down direction), which means that a great number of waiting passengers cannot board the train within the scheduled dwell time. Therefore, it is necessary to consider the interaction between service headways and station dwell times in metro timetable formulation. Otherwise, dwell time may be

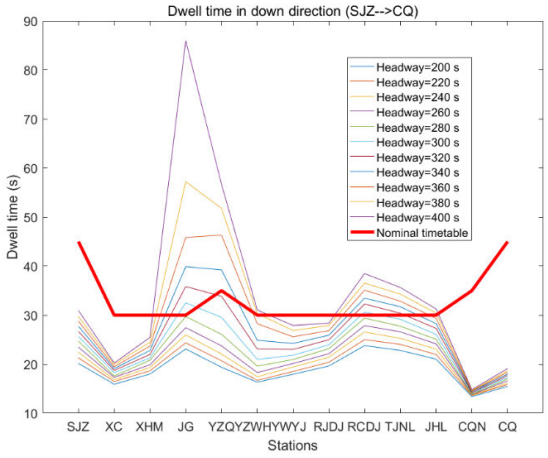


FIGURE 10. Dwell times in down direction with different headways.

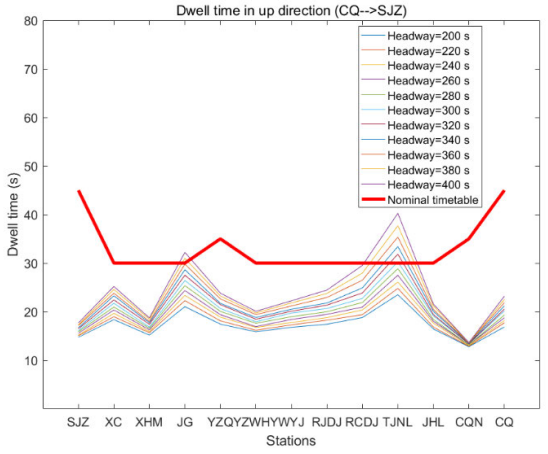


FIGURE 11. Dwell times in up direction with different headways.

TABLE 5. Passenger alighting and boarding with different headways.

HD (s)	Passenger volumes (pax)				Dwell time configurations (s)			
	PA_{all}	*SD*	PB_{all}	PB_{suc}	T_{PA}	T_{PB}	t_{PB}	*RDT*
200	5	1.08	111	111	0.38	10.5	2.27	23
250	6	4.67	144	144	0.46	14.0	2.33	27
300	9	8.46	178	155	0.68	19.6	3.04	33
350	10	12.2	211	120	0.76	29.9	3.73	43

insufficient for passenger boarding in peak hours with large headway.

Table 5 gives a detailed comparison of required dwell times for passengers boarding a train at the JG station (down direction) under different headways, and the scheduled dwell time of the nominal timetable is 30 s. The total number of passengers alighting the train (PA_{all}), total waiting passengers at the platform (PB_{all}) and the standees per door in the train (*SD*) increase with the headway, whereas the number of passengers who board the train (PB_{suc}) successfully declines after headway reaches a certain level. The reason for the reduction of PB_{suc} is that the growing congestion level decreases the speed

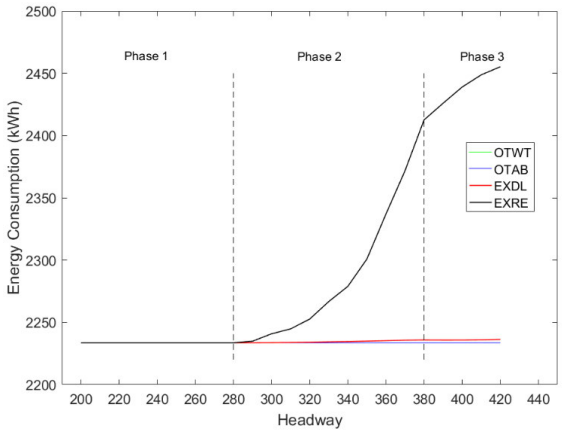


FIGURE 12. Energy consumption of different operating strategies.

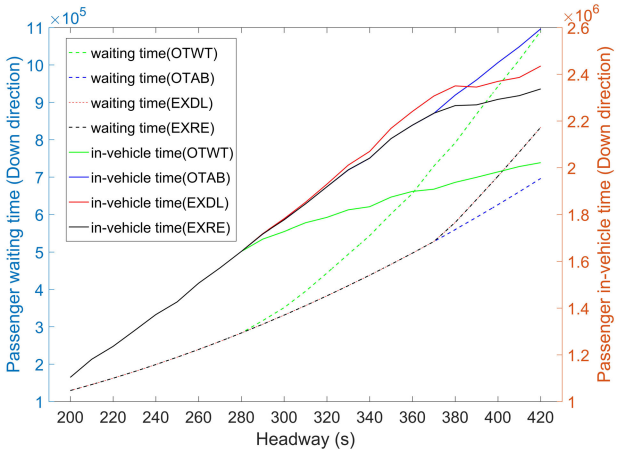


FIGURE 13. Passenger waiting and in-vehicle times in down direction.

of passenger boarding, due to more conflicts among alighting, boarding and standing passengers. Passengers need more time to board (t_{PB}) a more crowded train and the boarding process (T_{PB}) lasts for a longer time. The alighting process (T_{PA}) is finished within a second because very few passengers alight at JG station.

B. THE COMPARISON OF DIFFERENT OPERATING STRATEGIES

The real-world operating data of Yizhuang line involving five trains scheduled with regular headways is employed to explore the performance of the four operating strategies proposed in this study. Figs. 12 - 14 give the performance of different operating strategies under different headways, in terms of train energy consumption and passenger travel time. It should be noted that the length of study period varies due to the changed headway.

In Fig. 12, a notable increment in energy consumption is observed in EXRE strategy when the headway is prolonged. The reason is that train running times in several inter-stations are compressed under EXRE. However, the impacts of

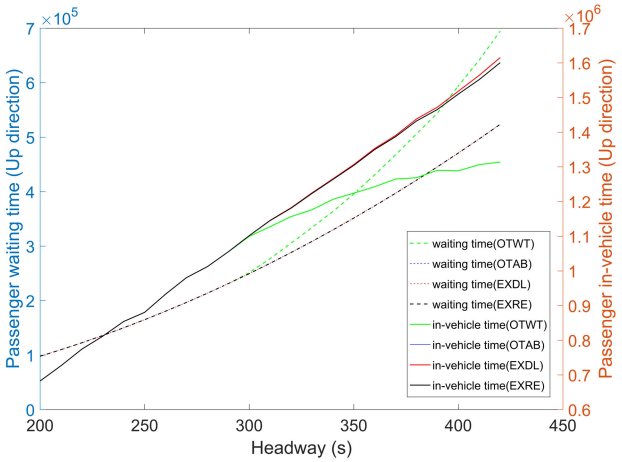


FIGURE 14. Passenger waiting and in-vehicle times in up direction.

service headway on energy consumption are very minor under the other three strategies, as the inter-station run-time remains the same with OTWT, OTAN, and EXDL.

When the headway is shorter than 280 s, all four operating strategies consume the same energy as all passengers can board the train within dwell time.

Once the headway is between 280 s and 380 s, the required time to complete the boarding of all passengers exceeds the scheduled dwell time in some stations. As such, EXDL strategy prolongs the dwell time and the energy increases slightly as the auxiliary equipment consumption grows with time. However, with EXRE strategy, not only the dwell time is prolonged, but also the inter-station running time is compressed. Longer headway induces more extension of dwell time, which also indicates more running time has to be compressed. As a result, the energy consumption of EXRE strategy increases significantly when the headway grows from 280 s to 380 s.

When the headway exceeds 380 s, the rising trend of EXRE strategy is retarded because one of the stations has reached the maximum prolonged dwell time ($DL_{\max}$), and running time in the following inter-stations will not be further compressed. In addition, energy consumption of OTWT and OTAB strategies keep as a constant because these two strategies always follow the original timetable regardless of passenger boarding.

The dotted line in Fig. 13 represents passenger waiting time under different operating strategies and headways. OTAB strategy always leads to the lowest waiting time because the temporal constraint on passenger boarding is ignored, thus all waiting passengers are assumed to board the train as long as the train has enough capacity.

In contrast, OTWT strategy considers the *RDT* may be larger than scheduled dwell time. In this case, some passengers cannot board the train even if its capacity is sufficient and passengers have to wait for the next train. Because of no extension of dwell time, the OTWT results in the largest waiting time.

The passenger waiting time of EXDL and EXRE strategies equals to that of OTAB strategy, when headway is smaller than 380 s. The reason is that the dwell time is prolonged to complete the boarding of passengers.

When the headway is greater than 380 s, *RDT* at JG station exceeds the maximum prolonged dwell time and some passengers have to wait for the next train. Consequently, the waiting time of EXDL and EXRE strategies are higher than that of OTAB strategies when headway is larger than 380 s. It should be noted that EXDL and EXRE strategies share the same waiting time and the red dotted line which represents waiting time in EXDL model is covered by the black dotted line.

When it comes to the in-vehicle time, as shown by the solid line in Fig. 13, the in-vehicle time of OTAB strategy increases steadily with the service headway, because enlarged headway results in more waiting passengers and all of them can board the train.

Whereas in OTWT strategy, some passengers cannot board the train due to temporal boarding constraint. The reduction on in-vehicle passengers results in less passenger in-vehicle time.

EXDL and EXRE strategies involve similar in-vehicle time when headway is smaller than 280 s. When the service headway is between 280 s and 380 s, the in-vehicle time of EXDL strategy is slightly higher because the delay caused by the prolonged dwell time is not recovered, which goes against the benefit of in-vehicle passengers.

When the headway continues increasing, *RDT* at JG station exceeds the maximum prolonging time. Under such circumstances, some passengers cannot board the train as the dwell time reaches the upper boundary. Therefore, the in-vehicle times of EXDL and EXRE strategies are lower than the in-vehicle time of OTAB strategy, because the total numbers of served passengers of EXDL and EXRE are less than that of OTAB strategy.

Fig. 14 depicts the passenger waiting time and in-vehicle time in the up direction, which is quite similar to the tendency of that in the down direction. The main difference is that the results of EXDL and EXRE strategies are alike. The reason is that passenger demand in the up direction is relatively low and the required dwell time does not reach the maximum allowable dwell time.

OTWT strategy leads to the largest waiting time because a part of passengers has to wait for the next train, whereas the other three strategies involve the same waiting time.

In respect of in-vehicle time, OTWT induces the least in-vehicle time due to that less passengers are serviced. EXDL results in the longest in-vehicle time because dwell times at a few stations are prolonged and the inter-station running times remain the same.

A Comprehensive comparison of four operating strategies is listed in Table 6. Compared to OTAB strategy, OTWT strategy induces longer waiting time when the headway exceeds 300 s. It reveals that a part of passengers cannot board the train within the scheduled dwell time, and the average waiting time

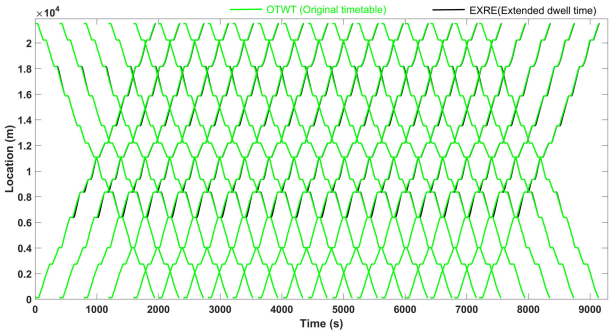


FIGURE 15. The comparison of original timetable to revised timetable in EXRE strategy.

is underestimated in OTAB strategy. Therefore, the OTAB strategy deviates from the practical operating condition and the calculation results may be incorrect, especially in peak hours.

Different from OTWT strategy, scheduled dwell time can be extended within a reasonable range in EXDL and EXRE strategies. Table 6 shows that EXDL and EXRE perform much better than OTWT in terms of waiting time, average in-vehicle time and the total number of served passengers. Particularly, the in-vehicle time of EXRE was significantly reduced in comparison with that of EXDL as inter-station running times might be compressed in EXRE.

To sum up, these strategies are feasible and can achieve the same result when scheduled dwell time can satisfy passenger's alighting and boarding. In crowded conditions, however, OTAB strategy becomes infeasible because the constraint of boarding time is not considered. Case studies proved that dwell time should be appropriately extended to improve passenger service, and EXRE strategy can effectively improve the service quality with a slight increase in energy consumption.

C. SENSITVITY ANALYSIS ON THE BOUNDARY IN PROLONGING STATION DWELL TIME

As aforementioned, EXRE strategy is recommended because passenger travel time is significantly saved by extending dwell time and compressing inter-station run-time. The reduction in passenger travel time largely depends on the boundary in prolonging dwell time $DL_{\max}$, which is set as a constant in aforementioned cases.

The time-space diagrams of the train movement in OTWT and EXRE strategies are shown in Fig. 15. It can be found that dwell time is extended in some crowded stations, e.g. JG station (location 6272 m). Then, extended dwell time is recovered in the following stations by compressing inter-station running time.

To investigate the performance of EXRE strategy under different boundaries in prolonging dwell time, extensive analyses on serviced passengers as well as their waiting and in-vehicle times are illustrated in Fig. 16. The headway is set as 400 s and the $DL_{\max}$ is a discrete variable between 0 s and 40 s. The performance of OTWT and OTAB is constant with

TABLE 6. Comparison of four operating strategies in practical timetable configurations.

Headway (s)	Operating strategies	Energy consumption (kWh)	Total waiting time (s)	Total in-vehicle time (s)	Average waiting time (s/pax)	Average in-vehicle time (s/pax)	Total serviced (pax)	Total arrived (pax)
250	OTWT	2234	383618	2306982	109	660	3534	3534
	OTAB	2234	383618	2306982	109	660	3534	3534
	EXDL	2234	383618	2306982	109	660	3534	3534
	EXRE	2234	383618	2306982	109	660	3534	3534
300	OTWT	2234	603903	2825862	139	665	4249	4349
	OTAB	2234	580683	2879262	134	662	4349	4349
	EXDL	2234	580683	2883464	134	663	4349	4349
	EXRE	2241	580683	2878373	134	662	4349	4349
350	OTWT	2234	1008551	3097350	195	688	4499	5163
	OTAB	2234	818471	3428460	159	664	5163	5163
	EXDL	2235	818471	3475056	159	673	5163	5163
	EXRE	2301	818471	3428883	159	664	5163	5163
400	OTWT	2234	1530831	3283544	256	702	4679	5977
	OTAB	2234	1096981	3959409	184	663	5977	5977
	EXDL	2236	1173171	3893813	196	684	5689	5977
	EXRE	2439	1173171	3794587	196	667	5689	5977

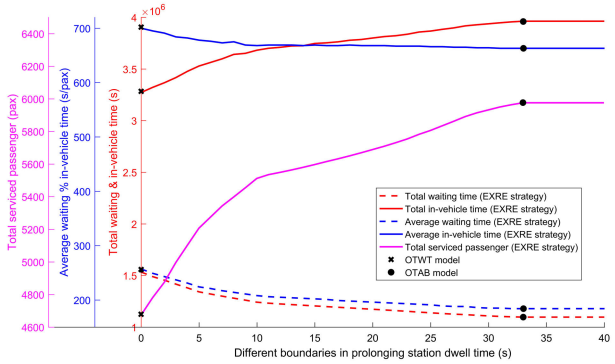


FIGURE 16. The performance of EXRE strategy with different boundaries in prolonging station dwell time.

different values of $DL_{\max}$, since the nominal timetable is not regulated in these two strategies.

When $DL_{\max}$ equals to 0, EXRE and OTWT have the same results because dwell time will not be extended and a part of passengers may be unable to board the train. As $DL_{\max}$ increases, more passengers can board the train within the extended dwell time and the performance of EXRE strategy tends to approach the OTAB strategy. When $DL_{\max}$ is equal to or greater than 33 s, passenger alighting and boarding can be completed within the extended/scheduled dwell time. In other words, a proper extension of dwell time is contributing to improve service quality. The extent in prolonging station dwell time depends on the passenger demand and the schedule of the following train.

D. MULTI-OBJECTIVE OPTIMIZATION OF TRAIN TIMETABLE

To verify the effectiveness of the proposed timetable model, the practical timetable of Beijing Metro Yizhuang line with the planning horizon of 2 hours in the morning peak is optimized without changing the number of train services, which is set as 25 in each direction and the average headway is 300 s. Extensive studies with different numbers of train services are then carried out to explore the relationships among the service frequency, passenger travel time and operating cost. On the algorithm parameters applied in the above cases, the crossover rate is 0.8; the mutation rate is 0.1; the size of population is 60 and the maximum generation is 100 which are selected based on empirical analyses and data experiments. In this case, the computational cost of the proposed approach is about half an hour.

Table 7 lists the performance of four operating strategies in the practical timetable. The different performance of OTAB and OTWT strategies clearly validates the necessity of considering temporal boarding constraint in timetable formulation. With the practical timetable, a part of passengers is unable to board the train within the scheduled dwell time and they have to wait for the next train. EXDL and EXRE strategies allow more passengers to board the currently-arrived train by extending dwell time, at the cost of energy consumption. The results demonstrate that EXDL and EXRE strategies contribute to the improvement on service quality, while EXRE strategies can also reduce in-vehicle time by compressing inter-stations running times at the expense of slight increment in energy consumption. Therefore, the EXRE is implemented in the multi-objective timetable optimization.

Fig. 17 gives the near-optimal Pareto front of the optimized timetables under EXRE strategy. The performance of the practical timetable with EXRE is also marked as red cross. It can be found that the practical timetable can be improved from both perspectives of passenger travel time and operator cost. For passengers, total travel time in optimized timetable can be saved by 3.09%, from 26615342 s to 25792132 s, and the operator's cost remains unchanged. At the same time, the operator's cost can be reduced with the optimized timetable by reallocating inter-station run-times for traction

TABLE 7. The performance of practical timetable with different operating strategies.

Operating strategies	Energy consumption (kWh)			Passenger waiting time (s)	Passenger in-vehicle time (s)	Total serviced passenger (pax)	Average waiting time (s/pax)	Average in-vehicle time (s/pax)
	Traction	Auxiliary Equipment	Utilized regenerative energy					
OTAB	10340	828	714	4210806	21834749	31544	133	692
OTWT	10340	828	714	9052616	21077651	30516	287	691
EXDL	10340	830	769	4210806	21906038	31544	133	694
EXRE	10425	828	735	4210806	21834348	31544	133	692

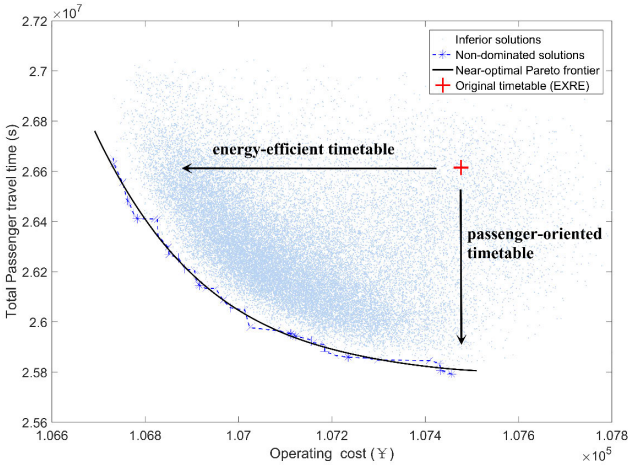


FIGURE 17. The Near-optimal Pareto frontier with EXRE operating strategy.

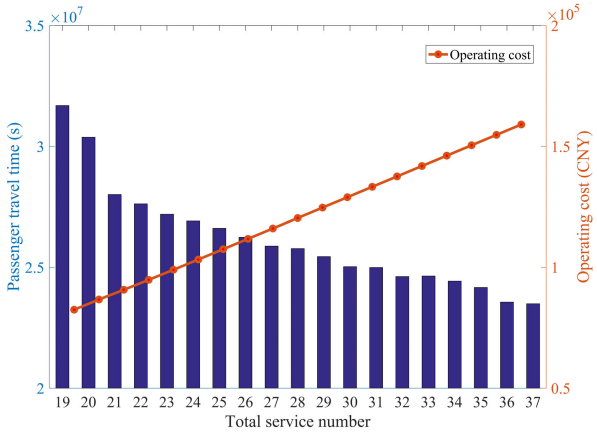


FIGURE 18. Passenger travel time and operating cost with different train services.

energy saving and improving the utilization of regenerative braking energy. Based on practical operating conditions of metro lines, decision-makers can select a particular solution from the obtained near-optimal Pareto frontier in all runs. It should be noted that service frequency is not changed in Fig. 17, therefore, the maintenance cost is a constant.

Fig. 18 shows the impacts of train service frequency on operator cost and passenger travel time. Operating cost, including energy consumption and maintenance investment, almost increases linearly with the number of services. On the contrary, passenger travel time has a notable decline when the number of services increases from 19 to 21, while the rate of decrease is smooth otherwise. In this case, the service number of trains is recommended to be greater than 20 so that passenger travel time can be saved significantly. Another result that can be concluded from these cases is the scalability of the proposed approach. Because various operating strategies and operating conditions can be applied flexibly.

VII. CONCLUSION

This paper has developed a multi-objective timetable optimization model with the consideration of both passenger travel time and operating cost. The inherent relationship between service headway and dwell time is analyzed. To cope with the insufficient dwell time that may arise in practice, different operating strategies are proposed and compared in terms of operating cost and passenger travel time. A NSGA-II algorithm is developed to explore the relationship between the objectives and the near-optimal Pareto front is attained by adjusting headways, inter-station running times as well as dwell times. Case studies are conducted on the real-world data of Beijing Yizhuang line.

Case studies have demonstrated that apart from the well-considered spatial constraint in existing studies, temporal constraint in passenger boarding should also be taken into account. The rationality of dwell time in peaks hours of busy metro lines has notable impacts on passenger service. The OTAB strategy which was widely employed in previous studies has been proved to be impractical and inaccurate when passenger requires more boarding time than scheduled dwell. Results of EXRE strategy proved that it is worthwhile to prolong the scheduled dwell time when passenger boarding and alighting require more time than the scheduled time. Because passenger travel time can be significantly reduced in EXRE strategy at the price of little increment in energy consumption, and we recommend this applicable strategy in busy metro lines. Additionally, all the proposed strategies achieve the same result in unsaturated conditions.

In future studies, an integrated optimization on train timetable and rolling stock scheme will be explored, as the regulation on train timetable might affect the predetermined operation scheme for rolling stocks. Besides, the proposed

algorithm can be further optimized to achieve better computational efficiency and high-quality solutions.

APPENDIX
NOMENCLATURE

Due to a lot of parameters and variables have been used through the paper, a nomenclature is added here to for the reference. All parameters are divided into 5 categories according to different usages and the meanings are listed in the table below.

Category	Parameter	Meaning
Metro lines	i	The number of stations
	N	Total number of stations
	N_p	Total number of the power supply interval
	PSI_{n}	The number of power supply interval n
	$P^n_{\max}$	The maximum capacity of the power supply system
	$\mathrm{X_i}$	The location of the station i
	Δx	The length of train movement within certain time interval
	L	The length of track in one direction
Metro trains	j	The number of trains
	N_j	Total number of trains
	$\eta_{\mathrm{ee-ke}}$	Conversion efficiency from electricity to kinetic energy
	$\eta_{\mathrm{ke-re}}$	Conversion efficiency from kinetic energy to regenerative energy
	M	The mass of rolling stock
	λ_{w}	The rotary allowance
	F_{out}	The traction or braking force
	F_{basic}	Davis resistance formula
	$F_g(x)$	Gradient resistance at location x
	$\theta(x)$	Gradient at location x.
	$F_{\mathrm{c}}(x)$	Curve resistance
	$F_{\mathrm{trac}}(v)$	Maximum available traction force
	$F_{\mathrm{rb}}(v)$	Maximum available regenerative braking force
	$F_{\mathrm{mb}}(v)$	The mechanical braking
	P^j_{aux}	The power of auxiliary on-board equipment
	m	Fleet size
	ω_{maint}	Maintenance cost per car kilometer
Metro timetable	Δt	The length of Minimal Time interval
	T_c	Cycle time for each train travelling a round-trip
	T_p	Planning horizon
	TD	Effective dwell time that considers both spatial and temporal constrains on passenger boarding
	TC^i_j	The time point that train j is full loaded at station i or reaches scheduled departure time
	TD^i_j	The time point that passenger's boarding process can be finished within dwell time
	DM^i_j	The time point that passenger's boarding and alighting process is finished
	DT_{loss}	The door opening, door closing and safety check time
	$DL_{\max}$	The boundary in prolonging dwell time
	DS^i_j	The scheduled departure of train j from station i
	AR^i_j	The scheduled arrival of train j at station i
	DP^i_j	The practical departure of train j from station i
	AP^i_j	The practical arrival time of train j at station i
	ir	The station in which the scheduled departure time is delayed
	idr	The station in which delayed time is fully recovered
	$T^{\max}_i$	Maximum operating time in segment i
	$T^{\min}_i$	Minimum operating time in segment i
	H_j	Headway time between train j and train $j+1$
	H_{ave}	Average headway within the planning horizon
	ρ	Maximum deviation from average headway
Passenger related	C	Train loading capacity
	PA^i_j	The number of alighting passengers per door at station i from train j
	PB^i_j	The number of boarding passengers per door at station i to train j
	SD^i_j	The number of standee per door at station i in train j
	PV^i_j	The number of in-vehicle passengers in train j in the i-th segment
	R^i_j	The number of passengers waiting at station i after the departure of train j due to limited boarding time
	C_{time}	The total travel time of passengers
	$P^{i,v}(t)$	The number of passengers who arrive station i at time t heading to station v

Energy consumption	$E_{trac}^{i,j}$	Traction energy consumption of train j in segment i
	$E_{re\_all}^{t,PSI_n}$	Total amount of the regenerative energy in PSIn at time t
	$E_{aux\_self}^{t,PSI_n}$	Auxiliary energy consumed by braking trains in PSI_n at time t
	$E_{non\_brak}^{t,PSI_n}$	Total energy consumed by non-braking trains in PSI_n at time t
	$E_{aux\_other}^{t,PSI_n}$	Auxiliary energy consumed by non-braking trains in PSI_n at time t
	E_{re}	Utilized regenerative energy
	C_{energy}	Energy consumption cost
	ω_e	The unit price of energy

REFERENCES

[1] L. Zamparini and A. Reggiani, "Meta-analysis and the value of travel time savings: A transatlantic perspective in passenger transport," *Netw. Spatial Econ.*, vol. 7, no. 4, pp. 377–396, Sep. 2007.

[2] H. Niu and X. Zhou, "Optimizing urban rail timetable under time-dependent demand and oversaturated conditions," *Transp. Res. C, Emerg. Technol.*, vol. 36, pp. 212–230, Nov. 2013.

[3] G. F. Newell, "Dispatching policies for a transportation route," *Transp. Sci.*, vol. 5, no. 1, pp. 91–105, Feb. 1971.

[4] A. Ceder, "Bus frequency determination using passenger count data," *Transp. Res. A, Gen.*, vol. 18, nos. 5–6, pp. 439–453, Oct. 1984.

[5] A. Ceder, "Bus timetables with even passenger loads as opposed to even headways," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1760, no. 1, pp. 3–9, Jan. 2001.

[6] C. M. Kwan and C. S. Chang, "Application of evolutionary algorithm on a transportation scheduling problem—The mass rapid transit," in *Proc. IEEE Congr. Evol. Comput.*, vol. 2, Oct. 2005, pp. 987–994.

[7] C. Liebchen, "The first optimized railway timetable in practice," *Transp. Sci.*, vol. 42, no. 4, pp. 420–435, Nov. 2008.

[8] L. Sun, J. G. Jin, D.-H. Lee, K. W. Axhausen, and A. Erath, "Demand-driven timetable design for metro services," *Transp. Res. C, Emerg. Technol.*, vol. 46, pp. 284–299, Sep. 2014.

[9] H. Niu, X. Tian, and X. Zhou, "Demand-driven train schedule synchronization for high-speed rail lines," *IEEE Trans. Intell. Transport. Syst.*, vol. 16, no. 5, pp. 2642–2652, Oct. 2015.

[10] E. Barrena, D. Canca, L. C. Coelho, and G. Laporte, "Single-line rail rapid transit timetabling under dynamic passenger demand," *Transp. Res. B, Methodol.*, vol. 70, pp. 134–150, Dec. 2014.

[11] Y. Wang, B. De Schutter, T. van den Boom, B. Ning, and T. Tang, "Real-time scheduling for single lines in urban rail transit systems," in *Proc. IEEE Int. Conf. Intell. Rail Transp.*, Beijing, Chian, Aug. 2013, pp. 1–6.

[12] P. Shang, R. Li, and L. Yang, "Optimization of urban single-line metro timetable for total passenger travel time under dynamic passenger demand," *Procedia Eng.*, vol. 137, pp. 151–160, Jan. 2016.

[13] T. Zhang, D. Li, and Y. Qiao, "Comprehensive optimization of urban rail transit timetable by minimizing total travel times under time-dependent passenger demand and congested conditions," *Appl. Math. Model.*, vol. 58, pp. 421–446, Jun. 2018.

[14] Y. Shen, G. Ren, and Y. Liu, "Timetable design for minimizing passenger travel time and congestion for a single metro line," *PROME Traffic Transp.*, vol. 30, no. 1, pp. 21–33, Feb. 2018.

[15] M. T. Claessens, N. M. van Dijk, and P. J. Zwaneveld, "Cost optimal allocation of rail passenger lines," *Eur. J. Oper. Res.*, vol. 110, no. 3, pp. 474–489, Nov. 1998.

[16] T. Lindner and U. T. Zimmermann, "Cost optimal periodic train scheduling," *Math. Methods Oper. Res.*, vol. 62, no. 2, pp. 281–295, Nov. 2005.

[17] A. Tirachini, D. A. Hensher, and S. R. Jara-Díaz, "Comparing operator and users costs of light rail, heavy rail and bus rapid transit over a radial public transport network," *Res. Transp. Econ.*, vol. 29, no. 1, pp. 231–242, Jan. 2010.

[18] Y. Wang, Z. Liao, T. Tang, and B. Ning, "Train scheduling and circulation planning in urban rail transit lines," *Control Eng. Pract.*, vol. 61, pp. 112–123, Apr. 2017.

[19] G. Laporte, F. A. Ortega, M. A. Pozo, and J. Puerto, "Multi-objective integration of timetables, vehicle schedules and user routings in a transit network," *Transp. Res. B, Methodol.*, vol. 98, pp. 94–112, Apr. 2017.

[20] K. Ghoseiri, F. Szidarovszky, and M. J. Asgharpour, "A multi-objective train scheduling model and solution," *Transp. Res. B, Methodol.*, vol. 38, no. 10, pp. 927–952, Dec. 2004.

[21] X. Li, D. Wang, K. Li, and Z. Gao, "A green train scheduling model and fuzzy multi-objective optimization algorithm," *Appl. Math. Model.*, vol. 37, no. 4, pp. 2063–2073, Feb. 2013.

[22] X. Xu, K. Li, and X. Li, "A multi-objective subway timetable optimization approach with minimum passenger time and energy consumption," *J. Adv. Transp.*, vol. 50, no. 1, pp. 69–95, Jan. 2016.

[23] X. Yang, B. Ning, X. Li, and T. Tang, "A two-objective timetable optimization model in subway systems," *IEEE Trans. Intell. Transport. Syst.*, vol. 15, no. 5, pp. 1913–1921, Oct. 2014.

[24] S. Yang, J. Wu, H. Sun, X. Yang, Z. Gao, and A. Chen, "Bi-objective nonlinear programming with minimum energy consumption and passenger waiting time for metro systems, based on the real-world smart-card data," *Transportmetrica B, Transp. Dyn.*, vol. 6, no. 4, pp. 302–319, Apr. 2017.

[25] H. Sun, J. Wu, H. Ma, X. Yang, and Z. Gao, "A bi-objective timetable optimization model for urban rail transit based on the time-dependent passenger volume," *IEEE Trans. Intell. Transport. Syst.*, vol. 20, no. 2, pp. 604–615, Feb. 2019.

[26] J. Yin, L. Yang, T. Tang, Z. Gao, and B. Ran, "Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: Mixed-integer linear programming approaches," *Transp. Res. B, Methodol.*, vol. 97, pp. 182–213, Mar. 2017.

[27] Y. Huang, L. Yang, T. Tang, F. Cao, and Z. Gao, "Saving energy and improving service quality: Bicriteria train scheduling in urban rail transit systems," *IEEE Trans. Intell. Transport. Syst.*, vol. 17, no. 12, pp. 3364–3379, Dec. 2016.

[28] P. Ngatchou, A. Zarei, and A. El-Sharkawi, "Pareto multi objective optimization," in *Proc. 13th Intern. Conf. Intell. Syst. Appli. Power Syst.*, Arlington, VA, USA, 2005, pp. 84–91.

[29] Z. Jiang, C. Xie, T. Ji, and X. Zou, "Dwell time modelling and optimized simulations for crowded rail transit lines based on train capacity," *PROMET Traffic Transp.*, vol. 27, no. 2, pp. 125–135, Apr. 2015.

[30] P. Kecman and R. M. P. Goverde, "Predictive modelling of running and dwell times in railway traffic," *Public Transp.*, vol. 7, no. 3, pp. 295–319, Dec. 2015.

[31] Y. Wang, B. Ning, T. Tang, T. J. J. van den Boom, and B. De Schutter, "Efficient real-time train scheduling for urban rail transit systems using iterative convex programming," *IEEE Trans. Intell. Transport. Syst.*, vol. 16, no. 6, pp. 3337–3352, Dec. 2015.

[32] S. Li, M. M. Dessouky, L. Yang, and Z. Gao, "Joint optimal train regulation and passenger flow control strategy for high-frequency metro lines," *Transp. Res. B, Methodol.*, vol. 99, pp. 113–137, May 2017.

[33] K. Li, H. Huang, and P. Schonfeld, "Metro timetabling for time-varying passenger demand and congestion at stations," *J. Adv. Transp.*, vol. 2018, pp. 1–26, Jul. 2018. 3690603

[34] P. G. Howlett, P. J. Pudney, and X. Vu, "Local energy minimization in optimal train control," *Automatica*, vol. 45, no. 11, pp. 2692–2698, Nov. 2009.

[35] M. Miyatake and H. Ko, "Optimization of train speed profile for minimum energy consumption," *IEEJ Trans. Electr. Electron. Eng.*, vol. 5, no. 3, pp. 263–269, May 2010.

[36] Y. Zhou, Y. Bai, J. Li, B. Mao, and T. Li, "Integrated optimization on train control and timetable to minimize net energy consumption of metro lines," *J. Adv. Transp.*, vol. 2018, Apr. 2018. Art. no. 7905820.

[37] W. H. K. Lam, C. Y. Cheung, and Y. F. Poon, "A study of train dwelling time at the Hong Kong mass transit railway system," *J. Adv. Transp.*, vol. 32, no. 3, pp. 285–295, Jun. 1998.

[38] A. Puong. *Dwell Time Model and Analysis for the MBTA Red Line*. [Online]. Available: http://mit.nelc.edu.eg/NR/rdonlyres/Civil-and-Environmental-Engineering/1-258JPublic-Transportation-Service-and-Operations-PlanningFall2003/D9613FBC-9279-4F31-A46D-8DB2E037E9E4/0/a_dwell-tim.pdf

[39] J. Lee, S. Yoo, H. Kim, and Y. Chung, "The spatial and temporal variation in passenger service rate and its impact on train dwell time: A time-series clustering approach using dynamic time warping," *Int. J. Sustain. Transp.*, vol. 12, no. 10, pp. 725–736, Mar. 2018.

[40] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

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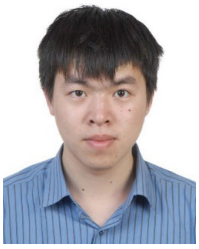

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