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Active Vibration Suppression of Uncertain Hose and Drogue Systems in the Presence of Actuator Nonlinearities

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ABSTRACT This paper investigates vibration suppression of uncertain hose and drogue systems in the presence of actuator nonlinearities. Firstly, a previously presented model of the hose and drogue systems is extended to describe how the hose and drogue systems restrain the vibration, while the accompanying unknown aerodynamic coefficients are estimated by invoking the parameter projection method. Subsequently, for the actuator nonlinearities of dead-zone and saturation, a smooth dead-zone approximate function is constructed to design the dead-zone compensation method, based upon which the proposed control scheme can handle actuator dead-zone and saturation simultaneously while improving the output efficiency of the actuator. Next, for the actuator nonlinearities of backlash and saturation, a smooth backlash inverse is constructed based upon which the presented control scheme can cope with the both actuator nonlinearities simultaneously. Finally, by utilizing backstepping method and hyperbolic tangent function, the proposed control schemes can also achieve the control objectives of vibration suppression and external disturbance attenuation. Simulation examples are included to demonstrate the validity of the proposed control schemes.

INDEX TERMS Adaptive control, backlash, dead-zone, distributed parameter system, uncertain nonlinear system.

NOMENCLATURE

$A(t)$	Aerodynamic force generated by the elevators
A_0, F_θ	Coefficients of $A(t)$
d_h	Diameter of the hose
d_{drog}	Diameter of the drogue
f_t	Skin friction drag of the hose
C_{f_t}	Coefficient of f_t
f_n	Pressure drag of the hose in the normal direction
C_{f_n}	Coefficient of f_n
f_{drog}	Drag of the drogue
$C_{f_{drog}}$	Coefficient of f_{drog}
g	Acceleration of gravity
L	Length of the hose
m	Mass of the drogue and elevators
$P(z)$	Tension of the HDS
ρ	Linear density of the hose
ρ_{air}	Air density
$\theta(t)$	Angle of the elevators
$\bar{\theta}, \underline{\theta}$	Upper and lower bounds of $\theta(t)$
θ_0	Constant angle of the HDS
V_0	Constant velocity of the air-tanker
$w(z,t)$	Transverse displacement of the HDS
$\mathfrak{N}(x)$	Actuator dead-zone or backlash
$N(x)$	Approximation of actuator dead-zone $\mathfrak{N}(x)$
$\hat{N}(x)$	Estimate of $N(x)$
$\tilde{N}(x)$	Estimate error of $N(x)$
$\dot{(*)}$	Partial derivative of $(*)$ with respect to t
$\gamma_{\mathfrak{N}l}, \gamma_{\mathfrak{N}r}$	Slopes of actuator dead-zone
γ_l, γ_r	Approximations of $\gamma_{\mathfrak{N}l}, \gamma_{\mathfrak{N}r}$
$\hat{\gamma}_l(t), \hat{\gamma}_r(t)$	Estimates of γ_l, γ_r
$\tilde{\gamma}_l(t), \tilde{\gamma}_r(t)$	Estimate errors of γ_l, γ_r
$\bar{\gamma}, \underline{\gamma}$	Upper and lower bounds of $\gamma_{\mathfrak{N}l}$ and $\gamma_{\mathfrak{N}r}$
$\bar{\hat{\gamma}}, \underline{\hat{\gamma}}$	Upper and lower bounds of $\hat{\gamma}_l(t)$ and $\hat{\gamma}_r(t)$
$a_{\mathfrak{N}l}, a_{\mathfrak{N}r}$	Breakpoints of actuator dead-zone

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a_l, a_r	Approximations of $a_{\mathfrak{N}l}, a_{\mathfrak{N}r}$
$\hat{a}_l(t), \hat{a}_r(t)$	Estimates of a_l, a_r
$\tilde{a}_l(t), \tilde{a}_r(t)$	Estimate errors of a_l, a_r
$\underline{a}_l, \bar{a}_r$	Lower bound of $a_{\mathfrak{N}l}$ and upper bound of $a_{\mathfrak{N}r}$
$\underline{\hat{a}}_l, \bar{\hat{a}}_r$	Lower bound of $\hat{a}_l(t)$ and upper bound of $\hat{a}_r(t)$
$\widehat{\gamma_l a_l}(t), \widehat{\gamma_r a_r}(t)$	Estimates of $\gamma_l a_l$ and $\gamma_r a_r$
$\widetilde{\gamma_l a_l}(t), \widetilde{\gamma_r a_r}(t)$	Estimate errors of $\gamma_l a_l$ and $\gamma_r a_r$
ζ	Slope of actuator backlash
$\hat{\zeta}(t)$	Estimate of ζ
$\tilde{\zeta}(t)$	Estimate error of ζ
$\bar{\zeta}, \underline{\zeta}$	Upper and lower bounds of ζ
$\bar{\hat{\zeta}}, \underline{\hat{\zeta}}$	Upper and lower bounds of $\hat{\zeta}(t)$
h_l, h_r	Breakpoints of actuator backlash
$\bar{h}$	Upper bound of $-h_l$ and h_r
$\widehat{\zeta h_l}(t), \widehat{\zeta h_r}(t)$	Estimates of ζh_l and ζh_r
$\widetilde{\zeta h_l}(t), \widetilde{\zeta h_r}(t)$	Estimate errors of ζh_l and ζh_r
$\bar{\widehat{\zeta h_l}}$	Upper bound of $\widehat{\zeta h_l}(t)$
$\underline{\widehat{\zeta h_r}}$	Lower bound of $\widehat{\zeta h_r}(t)$
$(*)'$	Partial derivative of $(*)$ with respect to z

I. INTRODUCTION

The hose and drogue system (HDS) is vital equipment in aerial refueling, which can transfer fuel from the air-tanker to the receiver [1]. As depicted in Fig. 1, the HDS is composed of a hose, a drogue at the end of the hose, and a set of active control surfaces (elevators) mounted on the drogue. The active control surfaces were developed in the last decade [2], to restrain the vibration of the HDS by generating additional aerodynamic force. It is noteworthy that the vibration of the HDS is ineluctable due to the intrinsic flexible nature of the HDS [3], and this phenomenon lengthens the docking process as well as increasing the risk of docking failure [4]–[6]. Therefore, vibration suppression is mandatory for the HDS to work effectively.

In the last few decades, vibration suppression of flexible systems has been vastly investigated, and a plethora of research advances have been documented [7]–[12], [16]. For instance, an overhead crane with flexible cable was studied based upon a backstepping-approach-based controller [7]. Two control schemes respectively based on active disturbance rejection control and sliding mode control were proposed for a one-dimensional Euler-Bernoulli beam equation, to cope with the external disturbance flowing to the control end [8]. In three-dimensional space, an effective control strategy was developed for nonlinear slender beams with large translational and rotational motions [9]. A boundary controller for an axially moving string was proposed to suppress the vibration of the system [10]. And the boundary control of a robotic aircraft with articulated flexible wings was investigated in [11]. With respect to the HDS investigated in this paper, Liu *et al.* established a novel dynamic model by utilizing the partial differential equation (PDE), and developed several control strategies to suppress the vibration of the HDS as well as achieving additional objectives [12]–[15]. However, it is noteworthy that the model developed by Liu *et al.* does not consider how the active control surfaces (elevators) generate the control force. Furthermore, the uncertainties of the HDS are also neglected in the above model which will influence the control performance of the closed-loop system [16]–[18], [34]–[36]. Accordingly, challenges still remain regarding vibration suppression of the HDS.

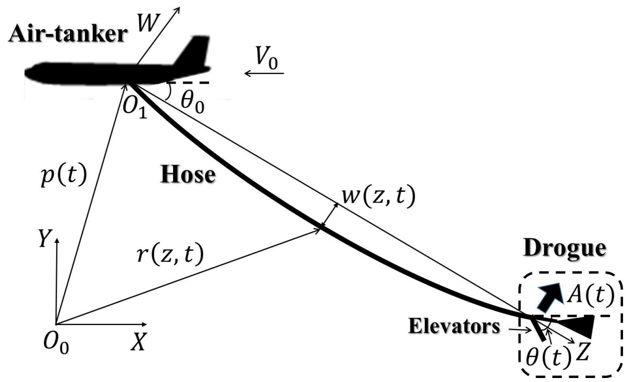

FIGURE 1. The hose and drogue system.

The dead-zone or backlash usually appears in the actuator of mechanical equipment, the HDS is no exception [13], [23]. These nonlinearities degrade the control performance of mechanical equipment, and there have been amounts of control schemes developed to handle them [19]–[25]. For dead-zone nonlinearity, the control problem of uncertain systems with actuator dead-zone was investigated [19], and the effect of dead-zone nonlinearity was eliminated by designing a novel smooth dead-zone inverse. Further, two control schemes were developed by utilizing the fuzzy control method, in which both schemes compensate the dead-zone in the actuator successfully [20], [21]. A neural-network-based control strategy was presented to cope with actuator dead-zone for a vibrating string system [22]. For backlash nonlinearity, an adaptive backlash inverse scheme was developed for a known linear plant with unknown backlash in the actuator [23]. Furthermore, two smooth backlash inverses were employed to cope with unknown backlash for nonlinear systems [24], [25]. However, it is noteworthy that the above works do not involve actuator saturation, which is also a common actuator nonlinearity that degrades the control performance of mechanical equipment [26], [27], [36]–[39]. Accordingly, it is meaningful to study the control scheme which can cope with actuator dead-zone and saturation or actuator backlash and saturation simultaneously.

In this paper, two novel control schemes are presented for the uncertain HDS with actuator nonlinearities. The contributions of this paper are summarized as follows.

1) Compared with the traditional model presented in [12], [13], our extended model considers how the active control surfaces (elevators) generate the aerodynamic force

to suppress the vibration of the HDS, which increases the design difficulty of the controller. The unknown aerodynamic coefficients of the extended model are estimated by the parameter projection method, which will improve the control performance of the closed-loop system.

2) Compared with the traditional control schemes, our first control scheme will handle the actuator nonlinearities of dead-zone and saturation simultaneously. Furthermore, it is noted that the output efficiency of the actuator will decline if we handle the two aforementioned actuator nonlinearities. To address it, a novel dead-zone approximate function is constructed, such that our first control scheme will improve the output efficiency of the actuator while handling the two aforementioned actuator nonlinearities simultaneously.

3) Compared with the traditional control schemes, our second control scheme will handle the actuator nonlinearities of backlash and saturation simultaneously. It is noteworthy that the two aforementioned actuator nonlinearities affect each other, thus the control difficulty here is how to cope with them simultaneously. To overcome it, a novel smooth backlash inverse is constructed, based upon which our second control scheme will resolve this problem properly.

The remainder of this paper is organized as follows: the extended model of the HDS is established in Section II, Section III designs the novel dead-zone approximate function which is the basis of our first control scheme. And then our two control schemes are developed in Section IV, followed by illustrative examples in Section V. Conclusions are drawn in Section VI.

II. PROBLEM FORMULATION

In this paper, we only investigate the vibration of the HDS in the vertical plane, and its axial motion is ignored, as advocated in [2], [12].

The HDS is illustrated in Fig. 1. The Earth-fixed coordinate system is (O_0XY). The air-tanker keeps a level flight with a constant velocity V_0. The HDS is released from the wings of the air-tanker [1], and (O_1ZW) is the body-fixed coordinate system attached to the HDS. θ_0 is the constant angle between X axis and Z axis, $w(z,t)$ is the transverse displacement of the HDS. The elevators mounted on the drogue are the actuator of the HDS, $\theta(t)$ is the elevators' angle, $A(t)$ is the aerodynamic force generated by the elevators. Let $p(t) = [p_X(t), p_Y(t)]^T$ be the position vector of (O_1ZW) relative to (O_0XY), $r(z,t) = [r_X(z,t), r_Y(z,t)]^T$ be the position vector of the HDS relative to (O_0XY), and can be expressed as:

$$r(z,t) = \begin{bmatrix} zcos\theta_0 + w(z,t)sin\theta_0 + p_X(t) \\ -zsin\theta_0 + w(z,t)cos\theta_0 + p_Y(t) \end{bmatrix}. \tag{1}$$

A. TRADITIONAL MODEL

The traditional model of the HDS presented in [12], [13] is expressed as [30]:

$$\rho\ddot{w}(z,t) = \left[P(z)w'(z,t)\right]' + Q, \tag{2}$$
$$Q = f_n - \rho g cos\theta_0, \tag{3}$$

and the boundary conditions of (2) are obtained as:

$$\begin{aligned} m\ddot{w}(L,t) &= -mgcos\theta_0 - P(L)w'(L,t) \\ &\quad + f_{drog}sin\theta_0 + A(t) + d_{L_1}(t), \end{aligned} \tag{4}$$
$$w(0,t) = 0, \tag{5}$$

where ρ is the linear density of the hose, g is the acceleration of gravity, m is the mass of the drogue and elevators, L is the length of the hose, $d_{L_1}(t)$ is the disturbance, $P(z)$ is the tension of the HDS expressed as [12], [28], [29]:

$$P(z) = [m + \rho(L-z)]gsin\theta_0 + f_t + f_{drog}cos\theta_0, \tag{6}$$
$$f_t = C_{f_t}\rho_{air}(V_0cos\theta_0)^2\pi d_h/2, \tag{7}$$
$$f_{drog} = C_{f_{drog}}\rho_{air}V_0^2\pi d_{drog}^2/8, \tag{8}$$
$$f_n = C_{f_n}\rho_{air}(V_0sin\theta_0)^2 d_h/2, \tag{9}$$

f_t is the skin friction drag of the hose, f_{drog} is the drag of the drogue, f_n is the pressure drag of the hose in the normal direction, C_{f_t}, $C_{f_{drog}}$, and C_{f_n} are the corresponding coefficients, ρ_{air} is the air density, d_h and d_{drog} are the diameters of the hose and drogue, respectively. Furthermore, $P(z)$ in (6) satisfies the following property.

Lemma 1: For any $z \in [0, L]$, there exist constants P_{min}, P_{max}, and P'_{min} such that the following inequalities hold:

$$\begin{aligned} 0 &\le P_{min} \le P(z) \le P_{max}, \\ P'_{min} &\le P'(z) \le 0. \end{aligned} \tag{10}$$

Proof: Notice that V_0 is a constant parameter, thus from (6)–(8), we derive that (10) holds. This completes the proof. □

B. EXTENDED MODEL

We found that the traditional model (2)–(5) regards the aerodynamic force $A(t)$ as the input, and neglects how $A(t)$ is generated. From Fig. 1, it is seen that $A(t)$ is generated by the elevators, thus we can utilize the linearization approach [31] to obtain the following equations:

$$\begin{aligned} A(t) &= F_\theta\theta(t) + A_0 + d_{L_2}(t), \\ \theta(t) &= Sat(\mathfrak{N}(u(t))), \end{aligned} \tag{11}$$

where $F_\theta > 0$ and A_0 are the unknown coefficients of the aerodynamic force $A(t)$, $d_{L_2}(t)$ is the disturbance induced by linearization, $\theta(t)$ is the actuator output (i.e., elevators' angle), $u(t)$ is the actuator input (i.e., controller to be designed), $Sat(*)$ is the actuator saturation defined as:

$$Sat(*) = \begin{cases} \bar{\theta}, & * > \bar{\theta} \\ *, & \underline{\theta} \le * \le \bar{\theta} \\ \underline{\theta}, & * < \underline{\theta} \end{cases} \tag{12}$$

$-\underline{\theta}$, $\bar{\theta}$ are positive constants, $\mathfrak{N}$ is the actuator nonlinearity which can be dead-zone or backlash, the expression of $\mathfrak{N}$ can be found in the following subsection.

Then substitute (11) into (4), we can derive the extended model of the HDS as:

$$\rho\ddot{w}(z,t) = \left[P(z)w'(z,t)\right]' + Q,$$

$$
\begin{aligned}
\dot{w}_1 &= w_2, \\
\dot{w}_2 &= \big[-mgcos\theta_0 - P(L)w'(L,t) + A_0 \\
&\quad + f_{drog}sin\theta_0 + F_\theta \theta(t) + d_L(t) \big]/m, \\
\theta(t) &= Sat(\mathfrak{N}(u)), \\
w(0,t) &= 0,
\end{aligned} \tag{13}
$$

where $w_1 = w(L,t)$, $w_2 = \dot{w}(L,t)$, $d_L(t) = d_{L_1}(t) + d_{L_2}(t)$ and satisfies the following assumption.

Assumption 1: The disturbance $d_L(t)$ in (13) satisfies $0 \le |d_L(t)| \le \bar{d}_L$, where $\bar{d}_L$ is a positive constant.

Remark 1: Compared with the traditional model (2)–(5), the extended model (13) considers how the aerodynamic force $A(t)$ is generated, as described by (11). The accompanying unknown parameters F_θ, A_0 and disturbance $d_{L_2}(t)$ increase the controller design difficulty, which will be handled in Section IV.

C. ACTUATOR NONLINEARITIES

In this paper, we consider the following two nonlinearities in the actuator:

1) DEAD-ZONE

The dead-zone nonlinearity $\mathfrak{N}(x)$ is described as [13]:

$$
\mathfrak{N}(x) = \begin{cases} \gamma_{\mathfrak{N}r}(x - a_{\mathfrak{N}r}), & x \ge a_{\mathfrak{N}r} \\ 0, & a_{\mathfrak{N}l} < x < a_{\mathfrak{N}r} \\ \gamma_{\mathfrak{N}l}(x - a_{\mathfrak{N}l}), & x \le a_{\mathfrak{N}l} \end{cases} \tag{14}
$$

where x is the actuator input, $\gamma_{\mathfrak{N}r}$, $\gamma_{\mathfrak{N}l}$, $a_{\mathfrak{N}r}$, and $a_{\mathfrak{N}l}$ are the unknown constant slopes and breakpoints of $\mathfrak{N}(x)$, respectively. Besides, $x(t)$ is a function of t, here we write only x for brevity of notation. In the following equation, $x(t)$ is written as x for the same reason.

2) BACKLASH

The backlash nonlinearity $\mathfrak{N}(x)$ is described as [24]:

$$
\mathfrak{N}(x) = \begin{cases} \zeta(x - h_r), & \text{if } \dot{x} > 0 \text{ and } \mathfrak{N} = \zeta(x - h_r) \\ \zeta(x - h_l), & \text{if } \dot{x} < 0 \text{ and } \mathfrak{N} = \zeta(x - h_l) \\ \mathfrak{N}(t_-), & \text{otherwise} \end{cases} \tag{15}
$$

where x is the actuator input, ζ is the unknown constant slope of $\mathfrak{N}(x)$, h_r, h_l are the unknown constant parameters, $\mathfrak{N}(t_-)$ denotes that there is no change in $\mathfrak{N}$.

The parameters in the above two actuator nonlinearities satisfy the following assumption.

Assumption 2: There exist known positive constants $\underline{\gamma}$, $\bar{\gamma}$, $\bar{a}_r$, $\underline{\zeta}$, $\bar{\zeta}$, and $\bar{h}$ as well as known negative constant $\underline{a}_l$ such that $\gamma_{\mathfrak{N}r}$, $\gamma_{\mathfrak{N}l}$, $a_{\mathfrak{N}r}$, $a_{\mathfrak{N}l}$, ζ, h_r, and h_l satisfy

$$
\gamma_{\mathfrak{N}r}, \gamma_{\mathfrak{N}l} \in [\underline{\gamma}, \bar{\gamma}],\ a_{\mathfrak{N}r} \in [0, \bar{a}_r],\ a_{\mathfrak{N}l} \in [\underline{a}_l, 0], \tag{16}
$$

$$
\zeta \in [\underline{\zeta}, \bar{\zeta}],\ h_r, -h_l \in [0, \bar{h}]. \tag{17}
$$

The control objective of this paper is that design controller $u(t)$ such that the closed-loop system of (13) is stable subject to the actuator dead-zone (14) and saturation (12) or actuator backlash (15) and saturation (12). Furthermore, $w(z,t)$ is uniformly ultimately bounded.

III. DEAD-ZONE APPROXIMATE FUNCTION

To develop the control scheme handling actuator dead-zone and saturation, a novel dead-zone approximate function and its properties are presented in this section.

Our control scheme for actuator dead-zone and saturation requires differentiability of actuator dead-zone (14), which obviously cannot be satisfied. Thus we need to design a differentiable function $N(x)$ to approximate actuator dead-zone (14). The differentiable function $N(x)$ is designed as:

$$
N(x) = \begin{cases} \big[\gamma_r - (\gamma_r - \eta_\gamma)e^{-\frac{x}{\bar{\hat{a}}_r}}\big]\big[x - [a_r - (a_r \\ -\eta_{a1}\eta_{a2}\bar{\hat{a}}_r)e^{-\frac{x}{\bar{\hat{a}}_r}}]tanh(\frac{x}{\eta_{a2}\bar{\hat{a}}_r})\big], & x \ge 0 \\ \big[\gamma_l - (\gamma_l - \eta_\gamma)e^{-\frac{x}{\underline{\hat{a}}_l}}\big]\big[x + [a_l + (-a_l \\ +\eta_{a1}\eta_{a2}\underline{\hat{a}}_l)e^{-\frac{x}{\underline{\hat{a}}_l}}]tanh(\frac{-x}{\eta_{a2}\underline{\hat{a}}_l})\big], & x < 0 \end{cases} \tag{18}
$$

where η_γ, η_{a1}, η_{a2} are positive constants, $\bar{\hat{a}}_r$ and $\underline{\hat{a}}_l$ are constants satisfying $\bar{\hat{a}}_r > \bar{a}_r$, $\underline{\hat{a}}_l < \underline{a}_l$; γ_r, γ_l, a_r, a_l are unknown parameters denoted as:

$$
\begin{aligned}
\gamma_r &= \gamma_{\mathfrak{N}r}, \quad \gamma_l = \gamma_{\mathfrak{N}l}, \\
a_r &= \begin{cases} a_{\mathfrak{N}r}, & a_{\mathfrak{N}r} \ge \underline{a}_r \\ \underline{a}_r, & 0 \le a_{\mathfrak{N}r} < \underline{a}_r \end{cases} \\
a_l &= \begin{cases} \bar{a}_l, & \bar{a}_l < a_{\mathfrak{N}l} \le 0 \\ a_{\mathfrak{N}l}, & a_{\mathfrak{N}l} \le \bar{a}_l \end{cases}
\end{aligned} \tag{19}
$$

$\underline{a}_r$, $\bar{a}_l$ are small known constants.

Remark 2: It is noteworthy that the actuator dead-zone (14) equals a linear function minus a saturation function, and can be approximated by a linear function minus a hyperbolic tangent function. Inspired by this property, the differentiable function (18) is designed to approximate the dead-zone nonlinearity (14).

The following lemma presents the properties of the differentiable function (18).

Lemma 2: Assume that (16)–(17) hold, then $N(x)$ satisfies:
(i) $N(x)$ is differentiable in **R**.
(ii) There exists a constant $\bar{\delta}_{\mathfrak{N}} > 0$ such that

$$
|\delta_{\mathfrak{N}}(x)| \le \bar{\delta}_{\mathfrak{N}}, \quad \forall x \in \mathbf{R}, \tag{20}
$$

where

$$
\delta_{\mathfrak{N}}(x) \triangleq \mathfrak{N}(x) - N(x). \tag{21}
$$

Proof: (i) With the expression of $N(x)$ in (18), it is apparent $N(x)$ is differentiable in **R**.

(ii) From (14) and (18), one can rapidly find that (20) can be ensured if $\lim_{x\to\infty} |\delta_{\mathfrak{N}}(x)|$ is bounded. Then notice that $\lim_{x\to\infty} |\delta_{\mathfrak{N}}(x)| \le \bar{\gamma}max\{\underline{a}_r, -\bar{a}_l\}$, thus (20) holds. This completes the proof. □

It is noteworthy that (18) is an unknown function, we cannot utilize it to design our control scheme directly. To address this problem, let $\hat{\gamma}_r(t)$, $\hat{\gamma}_l(t)$, $\hat{a}_r(t)$, $\hat{a}_l(t)$, $\widehat{\gamma_r a_r}(t)$, and $\widehat{\gamma_l a_l}(t)$ be the estimates of the unknown parameters γ_r, γ_l, a_r, a_l,

$\gamma_r a_r$, and $\gamma_l a_l$, respectively. Then we can present the following piecewise function to estimate $N(x)$:

$$\hat{N}(x) = \begin{cases} b_{1r}x - b_{2r}tanh(\dfrac{x}{\eta_{a2}\bar{\hat{a}}_r}), & x \geq 0 \\ b_{1l}x - b_{2l}tanh(\dfrac{-x}{\eta_{a2}\underline{\hat{a}}_l}), & x < 0 \end{cases} \quad (22)$$

where

$$\begin{aligned} b_{1r} &= \hat{\gamma}_r(1 - e^{-\frac{x}{\bar{a}_r}}) + \eta_\gamma e^{-\frac{x}{\bar{a}_r}}, \\ b_{2r} &= (\eta_\gamma \hat{a}_r + \eta_{a1}\eta_{a2}\bar{\hat{a}}_r\hat{\gamma}_r)e^{-\frac{x}{\bar{a}_r}}(1 - e^{-\frac{x}{\bar{a}_r}}) \\ &\quad + \widehat{\gamma_r a_r}(1 - e^{-\frac{x}{\bar{a}_r}})^2 + \eta_\gamma\eta_{a1}\eta_{a2}\bar{\hat{a}}_r e^{-\frac{2x}{\bar{a}_r}}, \\ b_{1l} &= \hat{\gamma}_l(1 - e^{-\frac{x}{\bar{a}_l}}) + \eta_\gamma e^{-\frac{x}{\bar{a}_l}}, \\ b_{2l} &= (-\eta_\gamma \hat{a}_l - \eta_{a1}\eta_{a2}\underline{\hat{a}}_l\hat{\gamma}_l)e^{-\frac{x}{\bar{a}_l}}(1 - e^{-\frac{x}{\bar{a}_l}}) \\ &\quad - \widehat{\gamma_l a_l}(1 - e^{-\frac{x}{\bar{a}_l}})^2 - \eta_\gamma\eta_{a1}\eta_{a2}\underline{\hat{a}}_l e^{-\frac{2x}{\bar{a}_l}}. \end{aligned} \quad (23)$$

It is seen that $\hat{N}$ is a function of seven arguments: x, $\hat{\gamma}_r$, $\hat{\gamma}_l$, $\hat{a}_r$, $\hat{a}_l$, $\widehat{\gamma_r a_r}$, and $\widehat{\gamma_l a_l}$, but we denote it as $\hat{N}(x)$ for brevity of notation. Similarly, we omit the independent variable t of $\hat{\gamma}_r$, $\hat{\gamma}_l$, $\hat{a}_r$, $\hat{a}_l$, $\widehat{\gamma_r a_r}$, and $\widehat{\gamma_l a_l}$.

To proceed, we define

$$\Omega = [\underline{\hat{\gamma}}, \bar{\hat{\gamma}}]^2 \times [0, \bar{\hat{a}}_r] \times [\underline{\hat{a}}_l, 0] \times [0, \bar{\hat{\gamma}}\bar{\hat{a}}_r] \times [\bar{\hat{\gamma}}\underline{\hat{a}}_l, 0], \quad (24)$$

where $\bar{\hat{\gamma}}$, $\underline{\hat{\gamma}}$ are positive constants satisfying $[\underline{\hat{\gamma}}, \bar{\hat{\gamma}}] \supset [\underline{\gamma}, \bar{\gamma}]$, $\bar{\hat{a}}_r$, $\underline{\hat{a}}_l$ are defined below (18). Then the properties of the novel dead-zone approximate function (22) can be presented in the following lemma.

Lemma 3: (i) $\hat{N}(x)$ is differentiable respect to x in **R**.
(ii) For any $(x, \hat{\gamma}_r, \hat{\gamma}_l, \hat{a}_r, \hat{a}_l, \widehat{\gamma_r a_r}, \widehat{\gamma_l a_l}) \in \mathbf{R} \times \Omega$, if positive constants $\bar{\hat{\gamma}}$, $\underline{\hat{\gamma}}$, η_γ, η_{a1}, η_{a2} satisfy:

$$\eta_{a1} < 1, \quad \eta_\gamma < \underline{\hat{\gamma}}, \quad (25)$$

$$2\bar{\hat{\gamma}}/\underline{\hat{\gamma}} < \eta_{a2}(1 + \eta_{a1} - \eta_\gamma/\underline{\hat{\gamma}}), \quad (26)$$

$$(1 + \eta_{a2} - 2\eta_{a1}\eta_{a2})\eta_\gamma/\underline{\hat{\gamma}} < \eta_{a2} - \eta_{a1}\eta_{a2}, \quad (27)$$

then we can have

$$\frac{\partial \hat{N}(x)}{\partial x} > 0.$$

(iii) Define $\tilde{N}(x, \tilde{\gamma}_r, \tilde{\gamma}_l, \tilde{a}_r, \tilde{a}_l, \widetilde{\gamma_r a_r}, \widetilde{\gamma_l a_l})$ as (for brevity of notation, we write only $\tilde{N}(x)$ in the remainder of this paper):

$$\tilde{N}(x) = N(x) - \hat{N}(x), \quad (28)$$

then the following equation always holds:

$$\tilde{N}(x) = \frac{\partial \tilde{N}}{\partial \tilde{\gamma}_r}\tilde{\gamma}_r + \frac{\partial \tilde{N}}{\partial \tilde{\gamma}_l}\tilde{\gamma}_l + \frac{\partial \tilde{N}}{\partial \tilde{a}_r}\tilde{a}_r + \frac{\partial \tilde{N}}{\partial \tilde{a}_l}\tilde{a}_l + \frac{\partial \tilde{N}}{\partial \widetilde{\gamma_r a_r}}\widetilde{\gamma_r a_r} + \frac{\partial \tilde{N}}{\partial \widetilde{\gamma_l a_l}}\widetilde{\gamma_l a_l}, \quad (29)$$

where $\tilde{\gamma}_r(t) = \gamma_r - \hat{\gamma}_r(t)$, $\tilde{\gamma}_l(t) = \gamma_l - \hat{\gamma}_l(t)$, $\tilde{a}_r(t) = a_r - \hat{a}_r(t)$, $\tilde{a}_l(t) = a_l - \hat{a}_l(t)$, $\widetilde{\gamma_r a_r}(t) = \gamma_r a_r - \widehat{\gamma_r a_r}(t)$, $\widetilde{\gamma_l a_l}(t) = \gamma_l a_l - \widehat{\gamma_l a_l}(t)$.

Proof: (i) Property (i) is apparent.

(ii) Owing to the proofs of $x \geq 0$ and $x < 0$ are similar, we only discuss the case of $x \geq 0$.

From (22), $\frac{\partial \hat{N}(x)}{\partial x}$ can be denoted as

$$\frac{\partial \hat{N}(x)}{\partial x} = b_{1r} - \frac{b_{2r}}{\eta_{a2}\bar{\hat{a}}_r}\frac{1}{cosh^2(\frac{x}{\eta_{a2}\bar{\hat{a}}_r})} + \eta_{a2}\bar{\hat{a}}_r\frac{\partial b_{1r}}{\partial x}\frac{x}{\eta_{a2}\bar{\hat{a}}_r} - \frac{\partial b_{2r}}{\partial x}tanh(\frac{x}{\eta_{a2}\bar{\hat{a}}_r}). \quad (30)$$

Thus $\frac{\partial \hat{N}(x)}{\partial x} > 0$ can be ensured by the following inequalities:

$$\eta_{a2}\bar{\hat{a}}_r\frac{\partial b_{1r}}{\partial x} > \frac{\partial b_{2r}}{\partial x}, \quad (31)$$

$$\eta_{a2}\bar{\hat{a}}_r b_{1r}\big|_{x=0} > b_{2r}\big|_{x=0}, \quad (32)$$

$$\frac{\partial b_{1r}}{\partial x} > 0, \quad b_{1r}\big|_{x=0} > 0. \quad (33)$$

According to (23), one has

$$\frac{\partial b_{1r}}{\partial x} = \frac{e^{-\frac{x}{\bar{a}_r}}}{\bar{\hat{a}}_r}(\hat{\gamma}_r - \eta_\gamma), \quad (34)$$

$$b_{1r}\big|_{x=0} = \eta_\gamma, \quad b_{2r}\big|_{x=0} = \eta_\gamma\eta_{a1}\eta_{a2}\bar{\hat{a}}_r. \quad (35)$$

Then in view of $(\hat{\gamma}_r, \hat{\gamma}_l, \hat{a}_r, \hat{a}_l, \widehat{\gamma_r a_r}, \widehat{\gamma_l a_l}) \in \Omega$ and (24), we can derive that (32) and (33) are ensured by (25).

Define $\Phi_1 = \frac{\bar{\hat{a}}_r}{e^{-\frac{x}{\bar{a}_r}}}(\eta_{a2}\bar{\hat{a}}_r\frac{\partial b_{1r}}{\partial x} - \frac{\partial b_{2r}}{\partial x})$. Then from (23), (31) can be ensured if the following inequality holds:

$$\begin{aligned} \Phi_1 &= 2\widehat{\gamma_r a_r}(e^{-\frac{x}{\bar{a}_r}} - 1) + \eta_\gamma\hat{a}_r(1 - 2e^{-\frac{x}{\bar{a}_r}}) \\ &\quad + \eta_{a1}\eta_{a2}\bar{\hat{a}}_r\hat{\gamma}_r(1 - 2e^{-\frac{x}{\bar{a}_r}}) + \eta_{a2}\bar{\hat{a}}_r\hat{\gamma}_r \\ &\quad - \eta_\gamma\eta_{a2}\bar{\hat{a}}_r + 2\eta_\gamma\eta_{a1}\eta_{a2}\bar{\hat{a}}_r e^{-\frac{x}{\bar{a}_r}} > 0. \end{aligned} \quad (36)$$

In view of (36), one derives

$$\frac{\partial \Phi_1}{\partial \hat{\gamma}_r} \geq 2\eta_{a1}\eta_{a2}\bar{\hat{a}}_r(1 - e^{-\frac{x}{\bar{a}_r}}), \quad (37)$$

$$\frac{\partial \Phi_1}{\partial \widehat{\gamma_r a_r}} = 2(e^{-\frac{x}{\bar{a}_r}} - 1), \quad (38)$$

$$\frac{\partial \Phi_1}{\partial \hat{a}_r} = \eta_\gamma(1 - 2e^{-\frac{x}{\bar{a}_r}}). \quad (39)$$

Next, consider the following two cases.

1) $0 \leq x < -\bar{\hat{a}}_r ln(0.5)$:

From (37)–(39), it is apparent that Φ_1 is monotonous increase respect to $\hat{\gamma}_r$, and monotonous decrease respect to $\hat{a}_r$ and $\widehat{\gamma_r a_r}$. Then recalling $(\hat{\gamma}_r, \hat{\gamma}_l, \hat{a}_r, \hat{a}_l, \widehat{\gamma_r a_r}, \widehat{\gamma_l a_l}) \in \Omega$ and (24), we can have that (36) holds if the following inequality holds:

$$\begin{aligned} \Phi_2 &= 2\big[\bar{\hat{\gamma}}/\underline{\hat{\gamma}} - (1 - \eta_{a1}\eta_{a2})\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2}\big]e^{-\frac{x}{\bar{a}_r}} \\ &\quad - 2\bar{\hat{\gamma}}/\underline{\hat{\gamma}} + (1 - \eta_{a2})\eta_\gamma/\underline{\hat{\gamma}} + \eta_{a1}\eta_{a2} \\ &\quad + \eta_{a2} > 0. \end{aligned} \quad (40)$$

Next, consider the following two subcases.

1.1) $\bar{\hat{\gamma}}/\underline{\hat{\gamma}} - (1 - \eta_{a1}\eta_{a2})\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2} > 0$:

In this subcase, Φ_2 is monotonous decrease respect to x. Notice that $0 \leq x < -\bar{\hat{a}}_r ln(0.5)$, so we can deduce that (40) holds if the following inequality holds:

$$\bar{\hat{\gamma}}/\underline{\hat{\gamma}} < \eta_{a2}(1 - \eta_\gamma/\underline{\hat{\gamma}} + \eta_{a1}\eta_\gamma/\underline{\hat{\gamma}}), \tag{41}$$

which can be ensured by (26) and the following inequality

$$\eta_{a2}(1 + \eta_{a1} - \eta_\gamma/\underline{\hat{\gamma}}) \leq 2\eta_{a2}(1 - \eta_\gamma/\underline{\hat{\gamma}} + \eta_{a1}\eta_\gamma/\underline{\hat{\gamma}}). \tag{42}$$

If $\eta_\gamma/\underline{\hat{\gamma}} > 0.5$, then (25) implies that

$$\eta_{a2}(1 - \eta_\gamma/\underline{\hat{\gamma}}) + \eta_{a1}\eta_{a2}(2\eta_\gamma/\underline{\hat{\gamma}} - 1) \geq 0. \tag{43}$$

This confirms (42).

If $\eta_\gamma/\underline{\hat{\gamma}} \leq 0.5$, then in view of (25), it is apparent that either $\eta_{a1} \leq 0.5$ or $\eta_{a1} > 0.5$, the following inequality

$$\eta_{a2}[1 - \eta_{a1} + \eta_\gamma/\underline{\hat{\gamma}}(2\eta_{a1} - 1)] \geq 0 \tag{44}$$

always holds, which means that (42) always holds.

1.2) $\bar{\hat{\gamma}}/\underline{\hat{\gamma}} - (1 - \eta_{a1}\eta_{a2})\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2} \leq 0$:

In this subcase, Φ_2 is monotonous increase respect to x. Then notice $0 \leq x < -\bar{\hat{a}}_r ln(0.5)$, we can have that (40) can be ensured by (27).

2) $x \geq -\bar{\hat{a}}_r ln(0.5)$:

From (37)–(39), Φ_1 is monotonous increase respect to $\hat{\gamma}_r$ and $\hat{a}_r$, and monotonous decrease respect to $\widehat{\gamma_r a_r}$. Then recalling $(\hat{\gamma}_r, \hat{\gamma}_l, \hat{a}_r, \hat{a}_l, \widehat{\gamma_r a_r}, \widehat{\gamma_l a_l}) \in \Omega$ and (24), we can derive that (36) holds if the following inequality holds:

$$\begin{aligned}\Phi_2 = 2\big[\bar{\hat{\gamma}}/\underline{\hat{\gamma}} + \eta_{a1}\eta_{a2}\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2}\big]e^{-\frac{x}{\bar{\hat{a}}_r}} \\ - 2\bar{\hat{\gamma}}/\underline{\hat{\gamma}} - \eta_{a2}\eta_\gamma/\underline{\hat{\gamma}} + \eta_{a1}\eta_{a2} + \eta_{a2} > 0.\end{aligned} \tag{45}$$

If $\bar{\hat{\gamma}}/\underline{\hat{\gamma}} + \eta_{a1}\eta_{a2}\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2} > 0$, then Φ_2 is monotonous decrease respect to x. Notice that $x \geq -\bar{\hat{a}}_r ln(0.5)$, thus (45) can be ensured by (26).

If $\bar{\hat{\gamma}}/\underline{\hat{\gamma}} + \eta_{a1}\eta_{a2}\eta_\gamma/\underline{\hat{\gamma}} - \eta_{a1}\eta_{a2} \leq 0$, then Φ_2 is monotonous increase respect to x. Notice that $x \geq -\bar{\hat{a}}_r ln(0.5)$, thus (45) can be ensured by (41). From the above proof in subcase 1.1), it has been proven that (41) can be ensured by (25) and (26).

Thus we conclude that $\frac{\partial \hat{N}(x)}{\partial x} > 0$ if (25)–(27) hold.

(iii) From (18), (22), and (23), we can have (29) readily. This completes the proof. □

Remark 3: It is seen that (25)–(27) are always feasible if $\eta_{a1} > 0.5$ and η_{a2} is sufficiently large. Nevertheless, a excessively large η_{a2} may deteriorate the performance of the closed-loop system. Thus η_{a2} should be chosen properly.

Remark 4: Constructing a dead-zone approximate function which can satisfy the property of $\frac{\partial \hat{N}(x)}{\partial x} > 0$ is the design difficulty in this section. Due to the unknown parameters of the dead-zone nonlinearity, the designed dead-zone approximate function must have time-varying estimate parameters, which will increase the difficulty proving the aforementioned property.

IV. CONTROL SCHEMES DESIGN

In Subsection IV.A, the control scheme coping with actuator dead-zone and saturation is developed based upon the dead-zone approximate function (22). Then the control scheme handling actuator backlash and saturation is designed in Subsection IV.B. Besides, the following two lemmas are useful for our proof.

Lemma 4 [32]: For any constants $\epsilon > 0$ and $\eta \in \mathbb{R}$, the following inequality always holds,

$$0 \leq |\eta| - \eta tanh(\eta/\epsilon) \leq k_p\epsilon = 0.2785\epsilon, \tag{46}$$

where $tanh(*)$ is the hyperbolic tangent function.

Lemma 5 [33]: Given function $\phi(t)$, constants ϕ_0, ϕ_l, ϕ_r satisfying $\phi_l < \phi_r$, nonempty set Π satisfying $\Pi \subset [\phi_l, \phi_r]$. Then the projection operator (47) defined at the bottom of the next page has the following properties:
(i) $\phi(t)$ remains in $[\phi_l, \phi_r]$, if $\dot{\phi}(t) = \mathbf{Proj}(\tau(t), \phi(t), \phi_l, \phi_r)$ and $\phi(0) \in [\phi_l, \phi_r]$,
(ii) $-[\phi_0 - \phi(t)] \cdot \mathbf{Proj}(\tau(t), \phi(t), \phi_l, \phi_r) \leq -[\phi_0 - \phi(t)] \cdot \tau(t)$, $\forall \phi(t) \in [\phi_l, \phi_r]$, $\phi_0 \in \Pi$.

Lemma 6 [41]: Given function $\phi(z, t) \in \mathbb{R}$ with $(z, t) \in [0, L] \times [0, +\infty)$, and it satisfies $\phi(0, t) = 0$. Then the following inequality always holds:

$$\phi^2 \leq L \int_0^L \phi'^2 \, dz.$$

A. CONTROL SCHEME FOR ACTUATOR DEAD-ZONE AND SATURATION

The control scheme is developed by utilizing the backstepping method. Thus we introduce the change of coordinate as:

$$\begin{aligned} w_{e1} &= w_1 = w(L, t), \\ w_{e2} &= w_2 = \dot{w}(L, t), \\ w_{e3} &= \hat{N}(u(v)) - \alpha, \end{aligned} \tag{48}$$

where

$$\alpha = \hat{F}_{\theta_{inv}}(t)\alpha_0, \tag{49}$$

$$\begin{aligned} \alpha_0 = {} & mgcos\theta_0 + P(L)w'(L, t) - f_{drog}sin\theta_0 - \hat{A}_0(t) \\ & - \frac{m\beta_2 L}{\beta_1}\dot{w}'(L, t) - k_1(\dot{w}_1 + \frac{\beta_2 L}{\beta_1}w'(L, t)) \\ & - tanh\Big(\frac{\beta_1\bar{d}_L(\dot{w}_1 + \frac{\beta_2 L}{\beta_1}w'(L, t))}{\delta_{d_L}}\Big)\bar{d}_L, \end{aligned} \tag{50}$$

β_1, β_2, k_1, and δ_{d_L} are positive constants, $\hat{F}_{\theta_{inv}}(t)$ and $\hat{A}_0(t)$ are estimates of $1/F_\theta$ and A_0, respectively.

1) STEP 1

Consider the following Lyapunov function candidate:

$$V_1 = \frac{\beta_1 m}{2}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1}w'(L, t))^2. \tag{51}$$

Then from (13), one has

$$\dot{V}_1 = \beta_1\big[\dot{w}_1 + \frac{\beta_2 L}{\beta_1}w'(L, t)\big]\big[-mgcos\theta_0 + A_0$$

$$-P(L)w'(L,t)+f_{drog}sin\theta_0+d_L(t)$$
$$+\frac{m\beta_2 L}{\beta_1}\dot{w}'(L,t)+F_\theta Sat(\mathfrak{N}(u))\big]. \quad (52)$$

To proceed, we choose controller u (i.e., actuator input) as:

$$u(v)=\begin{cases}\frac{\bar{\theta}}{\bar{\gamma}}tanh(\frac{\bar{\gamma}}{\bar{\theta}}v(t)), & v(t)\geq 0\\ \frac{\underline{\theta}}{\bar{\gamma}}tanh(\frac{\bar{\gamma}}{\underline{\theta}}v(t)), & v(t)<0\end{cases} \quad (53)$$

where $v(t)$ will be designed later. Then, we obtain the following lemma.

Lemma 7: Consider controller (53), then $\mathfrak{N}(u)$ satisfies

$$Sat(\mathfrak{N}(u))=\mathfrak{N}(u). \quad (54)$$

Proof: In light of (53), one deduces

$$\frac{\underline{\theta}}{\bar{\gamma}}<u(t)<\frac{\bar{\theta}}{\bar{\gamma}}, \quad (55)$$

then in view of (14) and (16), one has

$$\underline{\theta}=\bar{\gamma}\frac{\underline{\theta}}{\bar{\gamma}}<\mathfrak{N}(u(t))<\bar{\gamma}\frac{\bar{\theta}}{\bar{\gamma}}=\bar{\theta}, \quad (56)$$

finally, recalling (12), (56) yields (54). This completes the proof. □

Now substitute (53) into (52). Then in light of Lemma 7 and (48), we deduce

$$\dot{V}_1=\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]\big[-mgcos\theta_0+A_0$$
$$-P(L)w'(L,t)+f_{drog}sin\theta_0+d_L(t)$$
$$+\frac{m\beta_2 L}{\beta_1}\dot{w}'(L,t)+F_\theta(\delta_{\mathfrak{N}}(u)+\tilde{N}(u)$$
$$+w_{e3}+\alpha)\big], \quad (57)$$

where we have utilized the following equation based upon (21) and (28):

$$\mathfrak{N}(u)=\hat{N}(u)+\delta_{\mathfrak{N}}(u)+\tilde{N}(u). \quad (58)$$

To proceed, substitute (49)–(50) into (57). Then in view of Assumption 1 and Lemma 4, we derive

$$\dot{V}_1\leq\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]\big[\tilde{A}_0+d_L(t)$$
$$+F_\theta(\delta_{\mathfrak{N}}(u)+\tilde{N}(u)+w_{e3}-\tilde{F}_{\theta_{inv}}\alpha_0)$$
$$-tanh(\frac{\beta_1\bar{d}_L(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))}{\delta_{d_L}})\bar{d}_L\big]$$
$$-k_1\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))^2$$
$$\leq-k_1\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))^2+k_p\delta_{d_L}$$
$$+\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]\big[\tilde{A}_0+F_\theta(\delta_{\mathfrak{N}}(u)$$
$$+\tilde{N}(u)+w_{e3}-\tilde{F}_{\theta_{inv}}\alpha_0)\big], \quad (59)$$

where $\tilde{F}_{\theta_{inv}}(t)=1/F_\theta-\hat{F}_{\theta_{inv}}(t)$, $\tilde{A}_0(t)=A_0-\hat{A}_0(t)$.

2) STEP 2

Consider the following Lyapunov function candidate:

$$V_2=V_1+\frac{1}{2}w_{e3}^2+\frac{1}{2}\tilde{F}_\theta^2+\frac{1}{2}\tilde{A}_0^2+\frac{F_\theta}{2}(\tilde{F}_{\theta_{inv}}^2+\tilde{\gamma}_r^2$$
$$+\tilde{\gamma}_l^2+\tilde{a}_r^2+\tilde{a}_l^2+\widetilde{\gamma_r a_r}^2+\widetilde{\gamma_l a_l}^2), \quad (60)$$

where $\tilde{F}_\theta(t)=F_\theta-\hat{F}_\theta(t)$, $\hat{F}_\theta(t)$ is the estimate of F_θ. Then differentiating V_2, and in view of (48), we deduce

$$\dot{V}_2=\dot{V}_1+w_{e3}(\frac{d\hat{N}(u)}{dt}-\dot{\alpha})-\tilde{F}_\theta\dot{\hat{F}}_\theta-\tilde{A}_0\dot{\hat{A}}_0$$
$$-F_\theta\big(\tilde{F}_{\theta_{inv}}\dot{\hat{F}}_{\theta_{inv}}+\tilde{\gamma}_r\dot{\hat{\gamma}}_r+\tilde{\gamma}_l\dot{\hat{\gamma}}_l+\tilde{a}_r\dot{\hat{a}}_r$$
$$+\tilde{a}_l\dot{\hat{a}}_l+\widetilde{\gamma_r a_r}\dot{\widehat{\gamma_r a_r}}+\widetilde{\gamma_l a_l}\dot{\widehat{\gamma_l a_l}}\big). \quad (61)$$

It is noteworthy that $\hat{N}(u)$ presented in (22) has seven arguments, thus from (53), we have

$$\frac{d\hat{N}(u)}{dt}=\frac{\partial\hat{N}}{\partial u}\frac{du}{dv}\dot{v}+\frac{\partial\hat{N}}{\partial\hat{\gamma}_r}\dot{\hat{\gamma}}_r+\frac{\partial\hat{N}}{\partial\hat{\gamma}_l}\dot{\hat{\gamma}}_l+\frac{\partial\hat{N}}{\partial\hat{a}_r}\dot{\hat{a}}_r$$
$$+\frac{\partial\hat{N}}{\partial\hat{a}_l}\dot{\hat{a}}_l+\frac{\partial\hat{N}}{\partial\widehat{\gamma_r a_r}}\dot{\widehat{\gamma_r a_r}}+\frac{\partial\hat{N}}{\partial\widehat{\gamma_l a_l}}\dot{\widehat{\gamma_l a_l}}. \quad (62)$$

Substituting (62) into (61), we deduce

$$\dot{V}_2=\dot{V}_1+w_{e3}\big[\frac{\partial\hat{N}}{\partial u}\frac{du}{dv}\dot{v}+\frac{\partial\hat{N}}{\partial\hat{\gamma}_r}\dot{\hat{\gamma}}_r+\frac{\partial\hat{N}}{\partial\hat{\gamma}_l}\dot{\hat{\gamma}}_l+\frac{\partial\hat{N}}{\partial\hat{a}_r}\dot{\hat{a}}_r$$
$$+\frac{\partial\hat{N}}{\partial\hat{a}_l}\dot{\hat{a}}_l+\frac{\partial\hat{N}}{\partial\widehat{\gamma_r a_r}}\dot{\widehat{\gamma_r a_r}}+\frac{\partial\hat{N}}{\partial\widehat{\gamma_l a_l}}\dot{\widehat{\gamma_l a_l}}-\dot{\alpha}\big]$$
$$-\tilde{F}_\theta\dot{\hat{F}}_\theta-\tilde{A}_0\dot{\hat{A}}_0-F_\theta\big(\tilde{F}_{\theta_{inv}}\dot{\hat{F}}_{\theta_{inv}}+\tilde{\gamma}_r\dot{\hat{\gamma}}_r+\tilde{\gamma}_l\dot{\hat{\gamma}}_l$$
$$+\tilde{a}_r\dot{\hat{a}}_r+\tilde{a}_l\dot{\hat{a}}_l+\widetilde{\gamma_r a_r}\dot{\widehat{\gamma_r a_r}}+\widetilde{\gamma_l a_l}\dot{\widehat{\gamma_l a_l}}\big). \quad (63)$$

To proceed, we design $v(t)$ and update laws of $\hat{\gamma}_r$, $\hat{\gamma}_l$, $\hat{a}_r$, $\hat{a}_l$, $\widehat{\gamma_r a_r}$, $\widehat{\gamma_l a_l}$, $\hat{F}_\theta$, $\hat{A}_0$, and $\hat{F}_{\theta_{inv}}$ as:

$$\dot{v}(t)=\big(\frac{\partial\hat{N}(u)}{\partial u}\big)^{-1}(\tau_v(t)-k_v v(t)), \quad (64)$$

$$\mathbf{Proj}(\tau(t),\phi(t),\phi_l,\phi_r)=\begin{cases}\big[1-sg(\phi_r)\frac{\phi(t)^2-(\phi_r^2-sg(\phi_r)\epsilon_r)}{\epsilon_r}\big]\tau(t), & \phi(t)>sg(\phi_r)\sqrt{\phi_r^2-sg(\phi_r)\epsilon_r}\ and\ \tau(t)>0\\ \big[1-sg(-\phi_l)\frac{\phi(t)^2-(\phi_l^2-sg(-\phi_l)\epsilon_l)}{\epsilon_l}\big]\tau(t), & \phi(t)<-sg(-\phi_l)\sqrt{\phi_l^2-sg(-\phi_l)\epsilon_l}\ and\ \tau(t)<0\\ \tau(t), & else\end{cases} \quad (47)$$

where ϵ_r, ϵ_l are positive constants satisfying $[\phi_l+\epsilon_l,\phi_r-\epsilon_r]=\Pi$; $sg(*)=1$, if $*>0$, $sg(*)=-1$, if $*\leq 0$.

$$
\begin{cases}
\dot{\hat{\gamma}}_r = \mathbf{Proj}(\tau_{\gamma_r}, \hat{\gamma}_r, \underline{\hat{\gamma}}, \bar{\hat{\gamma}}), \\
\dot{\hat{\gamma}}_l = \mathbf{Proj}(\tau_{\gamma_l}, \hat{\gamma}_l, \underline{\hat{\gamma}}, \bar{\hat{\gamma}}), \\
\dot{\hat{a}}_r = \mathbf{Proj}(\tau_{a_r}, \hat{a}_r, 0, \bar{\hat{a}}_r), \\
\dot{\hat{a}}_l = \mathbf{Proj}(\tau_{a_l}, \hat{a}_l, \underline{\hat{a}}_l, 0), \\
\dot{\widehat{\gamma_r a_r}} = \mathbf{Proj}(\tau_{\gamma_r a_r}, \widehat{\gamma_r a_r}, 0, \bar{\hat{\gamma}}\bar{\hat{a}}_r), \\
\dot{\widehat{\gamma_l a_l}} = \mathbf{Proj}(\tau_{\gamma_l a_l}, \widehat{\gamma_l a_l}, \bar{\hat{\gamma}}\underline{\hat{a}}_l, 0), \\
\dot{\hat{F}}_\theta = \mathbf{Proj}(\tau_{F_\theta}, \hat{F}_\theta, \underline{\hat{F}}_\theta, \bar{\hat{F}}_\theta), \\
\dot{\hat{A}}_0 = \mathbf{Proj}(\tau_{A_0}, \hat{A}_0, \underline{\hat{A}}_0, \bar{\hat{A}}_0), \\
\dot{\hat{F}}_{\theta_{inv}} = \mathbf{Proj}(\tau_{F_{\theta_{inv}}}, \hat{F}_{\theta_{inv}}, \underline{\hat{F}}_{\theta_{inv}}, \bar{\hat{F}}_{\theta_{inv}}),
\end{cases} \tag{65}
$$

where $\tau_v(t)$ will be designed later, $k_v > 0$ is a constant,

$$
\begin{aligned}
\tau_{\gamma_r} &= \beta_1 \frac{\partial \tilde{N}}{\partial \tilde{\gamma}_r}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{\gamma_r}(\hat{\gamma}_r(t) - \gamma_{r0}), \\
\tau_{\gamma_l} &= \beta_1 \frac{\partial \tilde{N}}{\partial \tilde{\gamma}_l}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{\gamma_l}(\hat{\gamma}_l(t) - \gamma_{l0}), \\
\tau_{a_r} &= \beta_1 \frac{\partial \tilde{N}}{\partial \tilde{a}_r}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{a_r}(\hat{a}_r(t) - a_{r0}), \\
\tau_{a_l} &= \beta_1 \frac{\partial \tilde{N}}{\partial \tilde{a}_l}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{a_l}(\hat{a}_l(t) - a_{l0}), \\
\tau_{\gamma_r a_r} &= \beta_1 \frac{\partial \tilde{N}}{\partial \widetilde{\gamma_r a_r}}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{\gamma_r a_r}(\widehat{\gamma_r a_r}(t) \\
&\quad - \gamma_r a_{r0}), \\
\tau_{\gamma_l a_l} &= \beta_1 \frac{\partial \tilde{N}}{\partial \widetilde{\gamma_l a_l}}(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{\gamma_l a_l}(\widehat{\gamma_l a_l}(t) \\
&\quad - \gamma_l a_{l0}), \\
\tau_{F_\theta} &= \beta_1(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) w_{e3} - k_{F_\theta}(\hat{F}_\theta(t) - F_{\theta 0}), \\
\tau_{A_0} &= \beta_1(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) - k_{A_0}(\hat{A}_0(t) - A_1), \\
\tau_{F_{\theta_{inv}}} &= -\beta_1(\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t))\alpha_0 - k_{F_{\theta_{inv}}}(\hat{F}_{\theta_{inv}}(t) \\
&\quad - F_{\theta_{inv} 0}),
\end{aligned}
$$

$\underline{\hat{F}}_\theta$ and $\bar{\hat{F}}_\theta$ are positive constants satisfying $(\underline{\hat{F}}_\theta, \bar{\hat{F}}_\theta) \ni F_\theta$; $\underline{\hat{F}}_{\theta_{inv}}, \bar{\hat{F}}_{\theta_{inv}}, \underline{\hat{A}}_0$, and $\bar{\hat{A}}_0$ are constants satisfying $(\underline{\hat{F}}_{\theta_{inv}}, \bar{\hat{F}}_{\theta_{inv}}) \ni 1/F_\theta$, $(\underline{\hat{A}}_0, \bar{\hat{A}}_0) \ni A_0$; k_{γ_r}, k_{γ_l}, k_{a_r}, k_{a_l}, $k_{\gamma_r a_r}$, $k_{\gamma_l a_l}$, k_{F_θ}, k_{A_0}, $k_{F_{\theta_{inv}}}$, $F_{\theta 0}$, and $F_{\theta_{inv} 0}$ are positive constants, γ_{r0}, γ_{l0}, a_{r0}, a_{l0}, $\gamma_r a_{r0}$, $\gamma_l a_{l0}$, and A_1 are constants. Then we can have the following lemma.

Lemma 8: Consider controller (64) and update laws (65). If (25)–(27) and the following initial conditions

$$
(\hat{\gamma}_r(0), \hat{\gamma}_l(0), \hat{a}_r(0), \hat{a}_l(0), \widehat{\gamma_r a_r}(0), \widehat{\gamma_l a_l}(0)) \in \Omega. \tag{66}
$$

$$
\begin{cases}
\hat{F}_\theta(0) \in [\underline{\hat{F}}_\theta, \bar{\hat{F}}_\theta], \quad \hat{A}_0(0) \in [\underline{\hat{A}}_0, \bar{\hat{A}}_0], \\
\hat{F}_{\theta_{inv}}(0) \in [\underline{\hat{F}}_{\theta_{inv}}, \bar{\hat{F}}_{\theta_{inv}}],
\end{cases} \tag{67}
$$

hold, then we can have that
(i) Controller (64) exists for all $t \geq 0$.
(ii) The estimate parameters satisfy

$$
\begin{aligned}
&-\tilde{F}_\theta \dot{\hat{F}}_\theta - \tilde{A}_0 \dot{\hat{A}}_0 - F_\theta \big(\tilde{F}_{\theta_{inv}} \dot{\hat{F}}_{\theta_{inv}} + \tilde{\gamma}_r \dot{\hat{\gamma}}_r \\
&\qquad + \tilde{\gamma}_l \dot{\hat{\gamma}}_l + \tilde{a}_r \dot{\hat{a}}_r + \tilde{a}_l \dot{\hat{a}}_l + \widetilde{\gamma_r a_r} \dot{\widehat{\gamma_r a_r}} + \widetilde{\gamma_l a_l} \dot{\widehat{\gamma_l a_l}}\big) \\
&\leq -\beta_1 \big[\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)\big]\big[\tilde{A}_0 + F_\theta(\tilde{N}(u) \\
&\qquad - \tilde{F}_{\theta_{inv}} \alpha_0) + w_{e3} \tilde{F}_\theta\big] + c_0,
\end{aligned} \tag{68}
$$

where

$$
\begin{aligned}
c_0 &= k_{F_\theta} \tilde{F}_\theta (\hat{F}_\theta - F_{\theta 0}) + k_{A_0} \tilde{A}_0 (\hat{A}_0 - A_1) \\
&\quad + F_\theta \big[k_{F_{\theta_{inv}}} \tilde{F}_{\theta_{inv}} (\hat{F}_{\theta_{inv}} - F_{\theta_{inv} 0}) \\
&\quad + k_{\gamma_r} \tilde{\gamma}_r (\hat{\gamma}_r - \gamma_{r0}) + k_{\gamma_l} \tilde{\gamma}_l (\hat{\gamma}_l - \gamma_{l0}) \\
&\quad + k_{a_r} \tilde{a}_r (\hat{a}_r - a_{r0}) + k_{a_l} \tilde{a}_l (\hat{a}_l - a_{l0}) \\
&\quad + k_{\gamma_r a_r} \widetilde{\gamma_r a_r} (\widehat{\gamma_r a_r} - \gamma_r a_{r0}) \\
&\quad + k_{\gamma_l a_l} \widetilde{\gamma_l a_l} (\widehat{\gamma_l a_l} - \gamma_l a_{l0})\big].
\end{aligned} \tag{69}
$$

Proof: See APPENDIX A. □

Now substitute (64)–(65) into (63). And in view of Lemma 8 and (59), we can have

$$
\begin{aligned}
\dot{V}_2 &\leq -k_1 \beta_1 (\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t))^2 + k_p \delta_{d_L} + \beta_1 \big[\dot{w}_1 \\
&\quad + \frac{\beta_2 L}{\beta_1} w'(L,t)\big] F_\theta (\delta_{\mathfrak{N}}(u) + w_{e3}) + w_{e3} \big[\frac{du}{dv}(\tau_v(t) \\
&\quad - k_v v(t)) + \frac{\partial \hat{N}}{\partial \hat{\gamma}_r} \dot{\hat{\gamma}}_r + \frac{\partial \hat{N}}{\partial \hat{\gamma}_l} \dot{\hat{\gamma}}_l + \frac{\partial \hat{N}}{\partial \hat{a}_r} \dot{\hat{a}}_r + \frac{\partial \hat{N}}{\partial \hat{a}_l} \dot{\hat{a}}_l \\
&\quad + \frac{\partial \hat{N}}{\partial \widehat{\gamma_r a_r}} \dot{\widehat{\gamma_r a_r}} + \frac{\partial \hat{N}}{\partial \widehat{\gamma_l a_l}} \dot{\widehat{\gamma_l a_l}} - \dot{\alpha}\big] - \beta_1 \big[\dot{w}_1 \\
&\quad + \frac{\beta_2 L}{\beta_1} w'(L,t)\big] w_{e3} \tilde{F}_\theta + c_0.
\end{aligned} \tag{70}
$$

Then we design $\tau_v(t)$ as

$$
\tau_v = U(\chi) \tau_{v0}, \tag{71}
$$

where $U(\chi)$ is a Nussbaum function presented in [34], and can be expressed as

$$
U(\chi) = \chi^2 cos(\chi), \quad \dot{\chi} = k_\chi w_{e3} \tau_{v0}, \tag{72}
$$

$$
\begin{aligned}
\tau_{v0} &= -\beta_1 (\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)) \hat{F}_\theta - k_2 w_{e3} + \dot{\alpha} \\
&\quad + k_v \frac{du}{dv} v - \frac{\partial \hat{N}}{\partial \hat{\gamma}_r} \dot{\hat{\gamma}}_r - \frac{\partial \hat{N}}{\partial \hat{\gamma}_l} \dot{\hat{\gamma}}_l - \frac{\partial \hat{N}}{\partial \hat{a}_r} \dot{\hat{a}}_r \\
&\quad - \frac{\partial \hat{N}}{\partial \hat{a}_l} \dot{\hat{a}}_l - \frac{\partial \hat{N}}{\partial \widehat{\gamma_r a_r}} \dot{\widehat{\gamma_r a_r}} - \frac{\partial \hat{N}}{\partial \widehat{\gamma_l a_l}} \dot{\widehat{\gamma_l a_l}},
\end{aligned} \tag{73}
$$

$k_\chi > 0$ and $k_2 > 0$ are constants.

Now substituting (71) into (70), we deduce

$$
\begin{aligned}
\dot{V}_2 &\leq -k_1 \beta_1 (\dot{w}_1 + \frac{\beta_2 L}{\beta_1} w'(L,t))^2 + k_p \delta_{d_L} + \beta_1 \big[\dot{w}_1 \\
&\quad + \frac{\beta_2 L}{\beta_1} w'(L,t)\big] F_\theta \delta_{\mathfrak{N}}(u) + \frac{\dot{\chi}}{k_\chi}(\frac{du}{dv} U(\chi) - 1) \\
&\quad - k_2 w_{e3}^2 + c_0.
\end{aligned} \tag{74}
$$

Then we obtain the following result.

Theorem 1: Consider the uncertain HDS (13) satisfying Assumptions 1 and 2, with controllers (49)–(50), (53), (64), (71)–(73), and parameter update laws (65). Then if (25)–(27) and (66)–(67) hold, the closed-loop system of (13) is stable subject to the actuator dead-zone (14) and saturation (12). Furthermore, $w(z,t)$ is uniformly ultimately bounded.

Proof: See APPENDIX B. □

Remark 5: The control scheme presented in this subsection can be summarized as follows: 1) For actuator dead-zone: $\mathfrak{N}(u)$ is firstly approximated and estimated by $\hat{N}(u)$, then $\hat{N}(u)$ is transformed to $\frac{d\hat{N}(u)}{dt}$ by the change of variables, in the end, $\frac{d\hat{N}(u)}{dt}$ is compensated by controllers (64) and (73). 2) For actuator saturation: $Sat(\mathfrak{N}(u))$ is firstly transformed to $\mathfrak{N}(u)$ by adopting controller (53), then the hyperbolic tangent functions existed in controller (53) are handled by controllers (71)–(73). 3) For unknown parameters and disturbance $d_L(t)$: They are resolved by parameter update laws (65) and controller (50), respectively.

Remark 6: Compared with the Lyapunov functions in ordinary differential equation (ODE) systems, the ones in PDE systems do not need to include all states. For our system, the Lyapunov function (103) does not comprise $w(z,t)$, but we can still obtain the stability of the closed-loop system by invoking Lemma 6, as discussed in APPENDIX B.

B. CONTROL SCHEME FOR ACTUATOR BACKLASH AND SATURATION

In this subsection, $\mathfrak{N}(u)$ indicates the actuator backlash expressed in (15), and $u(t)$ is the corresponding controller that will be designed later. Besides, we denote $\hat{\zeta}$, $\widehat{\zeta h_r}$, $\widehat{\zeta h_l}$ as the estimates of the unknown backlash parameters ζ, ζh_r, ζh_l, respectively, and define a set Ω_{re} as:

$$\Omega_{re} = [\underline{\hat{\zeta}}, \bar{\hat{\zeta}}] \times [\underline{\widehat{\zeta h_r}}, \bar{\hat{\zeta}}\bar{h}] \times [-\bar{\hat{\zeta}}\bar{h}, \bar{\widehat{\zeta h_l}}], \tag{75}$$

where $\underline{\widehat{\zeta h_r}} < 0$, $\bar{\widehat{\zeta h_l}} > 0$ are constants with sufficiently small magnitudes, $\bar{\hat{\zeta}}$, $\underline{\hat{\zeta}}$ are positive constants which satisfy $[\underline{\hat{\zeta}}, \bar{\hat{\zeta}}] \supset [\underline{\zeta}, \bar{\zeta}]$ and the following assumption.

Assumption 3: The following inequality always holds:

$$\frac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\underline{\hat{\zeta}}} < min\{\bar{\theta}, -\underline{\theta}\}. \tag{76}$$

Now we can develop the control scheme for actuator backlash and saturation. It is also developed by using the backstepping method. Thus we introduce the change of coordinate as:

$$\begin{aligned} w_{e1re} &= w_1 = w(L,t), \\ w_{e2re} &= w_2 = \dot{w}(L,t), \\ w_{e3re} &= u_1 - \alpha, \end{aligned} \tag{77}$$

where u_1 will be designed later, α is defined in (49).

1) STEP 1

Consider the Lyapunov function defined in (51), then from (52), we have

$$\begin{aligned} \dot{V}_1 = \beta_1\Big[\dot{w}_1 + &\frac{\beta_2 L}{\beta_1}w'(L,t)\Big]\Big[-mgcos\theta_0 + A_0 \\ &- P(L)w'(L,t) + f_{drog}sin\theta_0 + d_L(t) \\ &+ \frac{m\beta_2 L}{\beta_1}\dot{w}'(L,t) + F_\theta Sat(\mathfrak{N}(u))\Big]. \end{aligned} \tag{78}$$

To proceed, we design controller u (i.e., actuator input) as:

$$u = \frac{1}{\hat{\zeta}}[u_1 + \widehat{\zeta h_r}u_2 - \widehat{\zeta h_l}u_3], \tag{79}$$

where

$$u_1 = \begin{cases} \underline{\hat{\zeta}}(\bar{\theta} - \dfrac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\underline{\hat{\zeta}}})tanh\left(\dfrac{\bar{\zeta}v_{re}(t)}{\underline{\hat{\zeta}}(\bar{\theta} - \frac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\underline{\hat{\zeta}}})}\right)/\bar{\zeta}, & v_{re}(t) \geq 0 \\ \underline{\hat{\zeta}}(-\underline{\theta} - \dfrac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\underline{\hat{\zeta}}})tanh\left(\dfrac{\bar{\zeta}v_{re}(t)}{\underline{\hat{\zeta}}(-\underline{\theta} - \frac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\underline{\hat{\zeta}}})}\right)/\bar{\zeta}, & v_{re}(t) < 0 \end{cases} \tag{80}$$

$$u_2 = \begin{cases} 1, & \dot{v}_{re}(t) \geq \dfrac{1}{k_u} \\ \dfrac{1}{2}sin(\dfrac{\pi}{2}k_u\dot{v}_{re}(t)) + \dfrac{1}{2}, & -\dfrac{1}{k_u} \leq \dot{v}_{re}(t) < \dfrac{1}{k_u} \\ 0, & \dot{v}_{re}(t) < -\frac{1}{k_u} \end{cases} \tag{81}$$

$$u_3 = u_2 - 1, \tag{82}$$

$v_{re}(t)$ will be designed later, k_u is a positive constant. Then we can have the following lemma.

Lemma 9: Consider controller (79). Then if $\hat{\zeta}$, $\widehat{\zeta h_r}$, and $\widehat{\zeta h_l}$ satisfy the following condition:

$$(\hat{\zeta}, \widehat{\zeta h_r}, \widehat{\zeta h_l}) \in \Omega_{re}, \tag{83}$$

the following properties always hold:
(i) $\mathfrak{N}(u)$ satisfies

$$Sat(\mathfrak{N}(u)) = \mathfrak{N}(u). \tag{84}$$

(ii) Define $\delta_{\mathfrak{N}_{re}}(u)$ as

$$\delta_{\mathfrak{N}re}(u) = \mathfrak{N}(u) - u_1 - \tilde{\zeta}u + \widetilde{\zeta h_r}u_2 - \widetilde{\zeta h_l}u_3, \tag{85}$$

where $\tilde{\zeta}(t) = \zeta - \hat{\zeta}(t)$, $\widetilde{\zeta h_r}(t) = \zeta h_r - \widehat{\zeta h_r}(t)$, $\widetilde{\zeta h_l}(t) = \zeta h_l - \widehat{\zeta h_l}(t)$. Then we have

$$|\delta_{\mathfrak{N}re}(u)| \leq \bar{\delta}_{\mathfrak{N}re}, \tag{86}$$

where $\bar{\delta}_{\mathfrak{N}re}$ is a positive constant.

Proof: See APPENDIX C. □

Now we suppose that (83) holds (this will be proved in Lemma (10)). Then substituting (79) into (78), and in view of Lemma (9) and (77), we have

$$\begin{aligned} \dot{V}_1 = \beta_1\Big[\dot{w}_1 + &\frac{\beta_2 L}{\beta_1}w'(L,t)\Big]\Big[-mgcos\theta_0 + A_0 \\ &- P(L)w'(L,t) + f_{drog}sin\theta_0 + d_L(t) \\ &+ \frac{m\beta_2 L}{\beta_1}\dot{w}'(L,t) + F_\theta(\delta_{\mathfrak{N}re}(u) + \tilde{\zeta}u \\ &- \widetilde{\zeta h_r}u_2 + \widetilde{\zeta h_l}u_3 + w_{e3re} + \alpha)\Big]. \end{aligned} \tag{87}$$

To proceed, using the similar procedures presented in Subsection IV.A.1, we deduce

$$\dot{V}_1 \leq -k_1\beta_1(\dot{w}_1 + \frac{\beta_2 L}{\beta_1}w'(L,t))^2 + k_p\delta_{d_L}$$

$$+\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]\big[\tilde{A}_0+F_\theta(\delta_{\mathfrak{N}_{re}}(u)+\tilde{\zeta}u-\widetilde{\zeta h_r}u_2+\widetilde{\zeta h_l}u_3+w_{e3re}-\tilde{F}_{\theta_{inv}}\alpha_0)\big]. \quad (88)$$

2) STEP 2

Consider the following Lyapunov function candidate:

$$V_{2re}=V_1+\frac{1}{2}w_{e3re}^2+\frac{1}{2}\tilde{F}_\theta^2+\frac{1}{2}\tilde{A}_0^2+\frac{F_\theta}{2}(\tilde{F}_{\theta_{inv}}^2+\tilde{\zeta}^2+\widetilde{\zeta h_r}^2+\widetilde{\zeta h_l}^2), \quad (89)$$

then differentiating V_{2re}, and in view of (77) and (80), we deduce

$$\dot{V}_{2re}=\dot{V}_1-\tilde{F}_\theta\dot{\hat{F}}_\theta-\tilde{A}_0\dot{\hat{A}}_0-F_\theta\big(\tilde{F}_{\theta_{inv}}\dot{\hat{F}}_{\theta_{inv}}+\tilde{\zeta}\dot{\hat{\zeta}}+\widetilde{\zeta h_r}\dot{\widehat{\zeta h_r}}+\widetilde{\zeta h_l}\dot{\widehat{\zeta h_l}}\big)+w_{e3re}(\frac{du_1}{dv_{re}}\dot{v}_{re}-\dot{\alpha}). \quad (90)$$

To proceed, we design v_{re} as:

$$\dot{v}_{re}(t)=\tau_{v_{re}}(t)-k_v v_{re}(t), \quad (91)$$

$$\tau_{v_{re}}=U(\chi)\tau_{v0re}, \quad (92)$$

$$\tau_{v0re}=-\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))\hat{F}_\theta-k_2 w_{e3re}+\dot{\alpha}+k_v\frac{du_1}{dv_{re}}v_{re}, \quad (93)$$

and design update laws of $\hat{\zeta}$, $\widehat{\zeta h_r}$, $\widehat{\zeta h_l}$, $\hat{F}_\theta$, $\hat{A}_0$, and $\hat{F}_{\theta_{inv}}$ as:

$$\begin{cases}\dot{\hat{\zeta}}=\mathbf{Proj}(\tau_\zeta,\hat{\zeta},\underline{\hat{\zeta}},\bar{\hat{\zeta}}),\\ \dot{\widehat{\zeta h_r}}=\mathbf{Proj}(\tau_{\zeta h_r},\widehat{\zeta h_r},\underline{\widehat{\zeta h_r}},\bar{\hat{\zeta}}\bar{h}),\\ \dot{\widehat{\zeta h_l}}=\mathbf{Proj}(\tau_{\zeta h_l},\widehat{\zeta h_l},-\bar{\hat{\zeta}}\bar{h},\bar{\widehat{\zeta h_l}}),\\ \dot{\hat{F}}_\theta=\mathbf{Proj}(\tau_{F_\theta},\hat{F}_\theta,\underline{\hat{F}}_\theta,\bar{\hat{F}}_\theta),\\ \dot{\hat{A}}_0=\mathbf{Proj}(\tau_{A_0},\hat{A}_0,\underline{\hat{A}}_0,\bar{\hat{A}}_0)\\ \dot{\hat{F}}_{\theta_{inv}}=\mathbf{Proj}(\tau_{F_{\theta_{inv}}},\hat{F}_{\theta_{inv}},\underline{\hat{F}}_{\theta_{inv}},\bar{\hat{F}}_{\theta_{inv}}),\end{cases} \quad (94)$$

where

$$U(\chi)=\chi^2 cos(\chi), \quad \dot{\chi}=k_\chi w_{e3re}\tau_{v0re},$$

$$\tau_\zeta=\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))u-k_\zeta(\hat{\zeta}(t)-\zeta_0),$$

$$\tau_{\zeta h_r}=-\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))u_2-k_{\zeta h_r}(\widehat{\zeta h_r}(t)-\zeta h_{r0}),$$

$$\tau_{\zeta h_l}=\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))u_3-k_{\zeta h_l}(\widehat{\zeta h_l}(t)-\zeta h_{l0}),$$

τ_{F_θ}, τ_{A_0}, and $\tau_{F_{\theta_{inv}}}$ can be found below (65), $k_\zeta>0$, $k_{\zeta h_r}>0$, $k_{\zeta h_l}>0$, ζ_0, ζh_{r0}, and ζh_{l0} are constants. Then we can have the following lemma.

Lemma 10: Consider controllers (91)–(93) and update laws (94). If (67) and the following initial condition

$$(\hat{\zeta}(0),\widehat{\zeta h_r}(0),\widehat{\zeta h_l}(0))\in\Omega_{re} \quad (95)$$

hold, then we can have that

(i) Condition (83) always holds.

(ii) The estimate parameters satisfy

$$-\tilde{F}_\theta\dot{\hat{F}}_\theta-\tilde{A}_0\dot{\hat{A}}_0-F_\theta\big(\tilde{F}_{\theta_{inv}}\dot{\hat{F}}_{\theta_{inv}}+\tilde{\zeta}\dot{\hat{\zeta}}+\widetilde{\zeta h_r}\dot{\widehat{\zeta h_r}}+\widetilde{\zeta h_l}\dot{\widehat{\zeta h_l}}\big)\le-\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]\big[\tilde{A}_0+F_\theta(\tilde{\zeta}u-\widetilde{\zeta h_r}u_2+\widetilde{\zeta h_l}u_3-\tilde{F}_{\theta_{inv}}\alpha_0)+w_{e3re}\tilde{F}_\theta\big]+c_{0re}, \quad (96)$$

where

$$c_{0re}=k_{F_\theta}\tilde{F}_\theta(\hat{F}_\theta-F_{\theta0})+k_{A_0}\tilde{A}_0(\hat{A}_0-A_1)+F_\theta\big[k_{F_{\theta_{inv}}}\tilde{F}_{\theta_{inv}}(\hat{F}_{\theta_{inv}}-F_{\theta_{inv}0})+k_\zeta\tilde{\zeta}(\hat{\zeta}-\zeta_0)+k_{\zeta h_r}\widetilde{\zeta h_r}(\widehat{\zeta h_r}-\zeta h_{r0})+k_{\zeta h_l}\widetilde{\zeta h_l}(\widehat{\zeta h_l}-\zeta h_{l0})\big]. \quad (97)$$

Proof: (i) Lemma 5(i) and (94)–(95) imply that (83) holds.

(ii) The proof is omitted because it is similar to one of Lemma 8(ii). This completes the proof. □

Now substitute (91)–(94) into (90). And in view of Lemma 10 and (88), we can utilize the similar procedures presented in Subsection IV.A.2 to obtain

$$\dot{V}_{2re}\le-k_1\beta_1(\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t))^2+k_p\delta_{d_L}+\beta_1\big[\dot{w}_1+\frac{\beta_2 L}{\beta_1}w'(L,t)\big]F_\theta\delta_{\mathfrak{N}_{re}}(u)+\frac{\dot{\chi}}{k_\chi}(\frac{du_1}{dv_{re}}U(\chi)-1)-k_2 w_{e3re}^2+c_{0re}. \quad (98)$$

Then we can obtain the following result.

Theorem 2: Under Assumptions 1–3, consider the uncertain HDS (13) subject to the actuator backlash (15) and saturation (12), with controllers (49)–(50), (79)–(82), (91)–(93), and parameter update laws (94). Then if (67) and (95) hold, the closed-loop system of (13) is stable. Furthermore, w(z, t) is uniformly ultimately bounded.

Proof: The proof is omitted because it is similar to one of Theorem 1. This completes the proof. □

Remark 7: It is noteworthy that actuator backlash and saturation affect each other, for instance, the anti-windup control cannot be adopted here because actuator backlash is unknown. Thus the control difficulty in this subsection is how to handle actuator backlash and saturation simultaneously. In our control scheme, actuator backlash $\mathfrak{N}(u)$ is firstly compensated by constructing the smooth backlash inverse (79), and then actuator saturation $Sat(\mathfrak{N}(u))$ is also handled by adopting controllers (80) and (91)–(93), the unknown parameters and disturbance $d_L(t)$ are finally resolved by parameter update laws (94) and controller (50), respectively.

Remark 8: In the end of Introduction, we claim that the control scheme presented in Subsection IV.A can handle the actuator nonlinearities of dead-zone and saturation simultaneously while improving the output efficiency of the actuator, here is the explanation. Without the dead-zone approximate function (22), we can utilize the similar idea in Subsection IV.B to develop the proposed control scheme (for narrative convenience, here we assume $\gamma_{\mathfrak{N}r}=\gamma_{\mathfrak{N}l}=\zeta$, $a_{\mathfrak{N}r}=h_r$,

$a_{\mathfrak{N}l} = h_l$). In this way, we can use controller (79) to handle the actuator nonlinearities (here (79) needs to be revised slightly, the independent variable of u_2 and u_3 needs to be turned into u_1). Then, when $|\widehat{\zeta h_r}|$ and $|\widehat{\zeta h_l}|$ are small, the maximum magnitude of controller (79) is close to $(-\underline{\theta} - \bar{\zeta}\bar{\hat{\zeta}}\bar{h}/\underline{\hat{\zeta}})/\bar{\zeta}$ or $(\bar{\theta} - \bar{\zeta}\bar{\hat{\zeta}}\bar{h}/\underline{\hat{\zeta}})/\bar{\zeta}$. As a contrast, the maximum magnitude of controller (53) is close to $-\underline{\theta}/\bar{\zeta}$ or $\bar{\theta}/\bar{\zeta}$, which is larger than the one of (79). Therefore, it is seen that the output efficiency of the actuator is improved by the control scheme presented in Subsection IV.A.

Remark 9: Based upon the computing approach presented in [40], all the variables utilized in our control schemes can be obtained by measuring or computing. It is noteworthy that the measuring or computing errors of these variables are ineluctable, which influences the performance of the closed-loop system.

V. SIMULATION

In this section, two illustrative examples are presented to demonstrate the effectiveness of our control schemes developed in Section IV. The two examples are defined as:

Case I : uncertain HDS (13) subject to actuator dead-zone (14) and saturation (12),

Case II : uncertain HDS (13) subject to actuator backlash (15) and saturation (12).

And they are simulated by utilizing the finite difference method [42].

The system parameters are given in Table 1 [12], [29]. The parameters of the unknown aerodynamic coefficients are $F_\theta = 41.6$, $A_0 = 0.1$, $\underline{\hat{F}}_\theta = 38.6$, $\bar{\hat{F}}_\theta = 44.6$, $\underline{\hat{F}}_{\theta_{inv}} = -1$, $\bar{\hat{F}}_{\theta_{inv}} = 1$, $\underline{\hat{A}}_0 = -1$, $\bar{\hat{A}}_0 = 1$. The parameters of the actuator dead-zone are $\gamma_{\mathfrak{N}r} = 0.85$, $\gamma_{\mathfrak{N}l} = 0.9$, $a_{\mathfrak{N}r} = 0.2$, $a_{\mathfrak{N}l} = -0.15$, $\bar{\gamma} = 1$, $\underline{\gamma} = 0.7$, $\bar{a}_r = -\underline{a}_l = 0.2$, $\underline{a}_r = -\bar{a}_l = 0.01$, $\bar{\hat{\gamma}} = 1.1$, $\underline{\hat{\gamma}} = 0.6$, $\bar{\hat{a}}_r = -\underline{\hat{a}}_l = 0.25$, $\eta_\gamma = 0.4$, $\eta_{a1} = 0.8$, $\eta_{a2} = 60/17$. It is apparent that the above parameters satisfy (16) and (25)–(27). The parameters of the actuator backlash are $\zeta = 0.85$, $h_r = 0.1$, $h_l = -0.09$, $\bar{\zeta} = 1$, $\underline{\zeta} = 0.8$, $\bar{h} = 0.1$, $\bar{\hat{\zeta}} = 1.1$, $\underline{\hat{\zeta}} = 0.7$, $\underline{\widehat{\zeta h_r}} = -0.01$, $\bar{\widehat{\zeta h_l}} = 0.01$. The parameters of actuator saturation are: $\underline{\theta} = -0.43$, $\bar{\theta} = 0.45$. It is apparent that the above parameters satisfy (17) and (76). The disturbance is given as $d_L(t) = -0.1 sin(t)$.

Case I is discussed firstly. In this case, we choose the control gains as: $\beta_1 = 0.995$, $\beta_2 = 0.071$, $k_1 = 289.350$, $k_2 = 0.1$, $k_v = 0.001$, $k_\chi = 10^{-5}$, $k_{F_\theta} = k_{A_0} = k_{F_{\theta_{inv}}} = k_{\gamma_r} = k_{\gamma_l} = k_{a_r} = k_{a_l} = k_{\gamma_r a_r} = k_{\gamma_l a_l} = 1$, $\bar{d}_L = \delta_{d_L} = 0.1$. And we choose the initial conditions as: $w(z, 0) = -z/L$, $\dot{w}(z, 0) = 0$, $\hat{F}_\theta(0) = 41.1$, $\hat{F}_{\theta_{inv}}(0) = 0.1$, $\hat{A}_0(0) = 0$, $\hat{\gamma}_r(0) = \hat{\gamma}_l(0) = 1$, $\hat{a}_r(0) = \hat{a}_l(0) = \widehat{\gamma_r a_r}(0) = \widehat{\gamma_l a_l}(0) = 0$.

The simulation results are shown in Figs. 2–7. Fig. 2 displays the transverse displacement of the HDS without controller. It is seen that owing to the disturbance, the vibration of the HDS is large, which may lead to docking failure in the aerial refueling process.

The control performance of the HDS with our proposed control scheme is shown in Figs. 3 and 6–7. We can see that the vibration of the HDS is suppressed to a small neighborhood of the desired position in the presence of unknown aerodynamic coefficients as well as non-symmetrical actuator dead-zone and saturation.

Compared with the proposed control scheme, most previous works only consider one of the two actuator nonlinearities: dead-zone or saturation (for instance, [12], [13], [19], [20]), and that may degrade the control performance of the closed-loop system, as shown in Figs. 4–6.

TABLE 1. System parameters.

Component	Configuration
ρ_{air}, kg/m^3	0.909
V_0, m/s	100
Hose	
ρ, kg/m	4.09
L, m	14
d_h, m	0.067
C_{f_t}	0.005
C_{f_n}	0.45
θ_0, rad	0.527
Drogue	
m, kg	29.48
d_{drog}, m	0.5688
$C_{f_{drog}}$	0.43

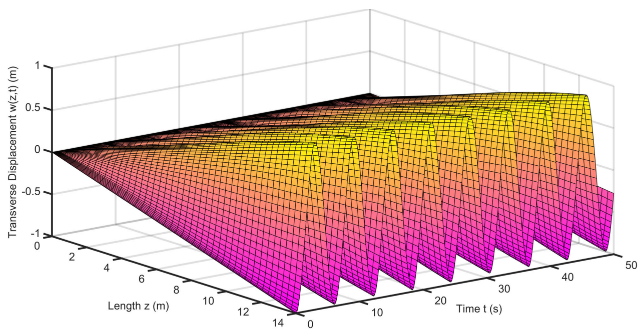

FIGURE 2. Transverse displacement of the HDS without controller.

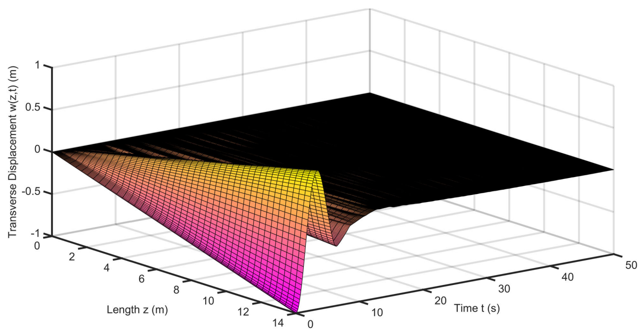

FIGURE 3. Case I: transverse displacement of the HDS with the proposed control scheme.

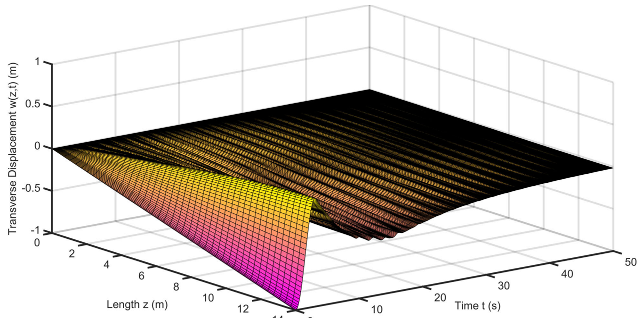

FIGURE 4. Case I: transverse displacement of the HDS with the control scheme neglecting saturation [13].

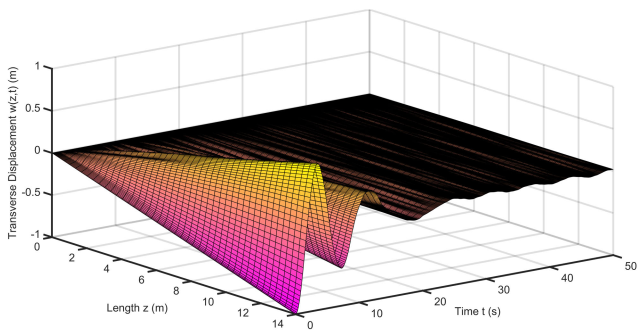

FIGURE 5. Case I: transverse displacement of the HDS with the control scheme neglecting dead-zone [12].

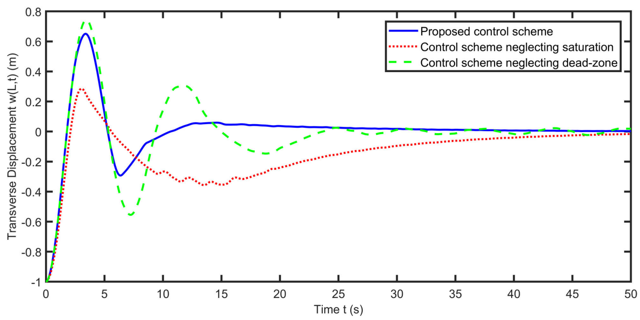

FIGURE 6. Case I: end point deflection of the HDS.

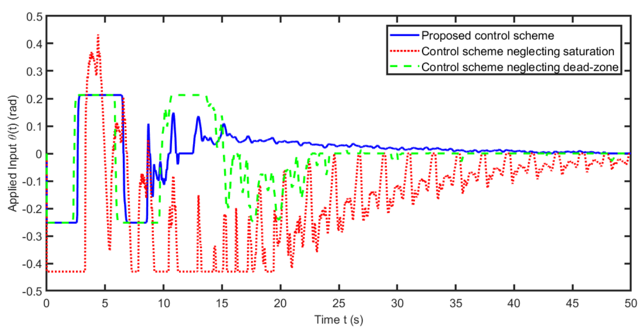

FIGURE 7. Case I: applied input of the HDS.

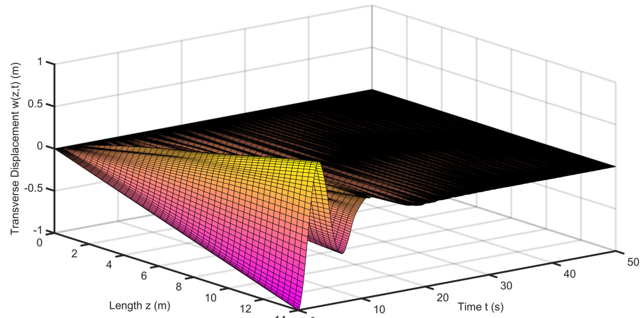

FIGURE 8. Case II: transverse displacement of the HDS with the proposed control scheme.

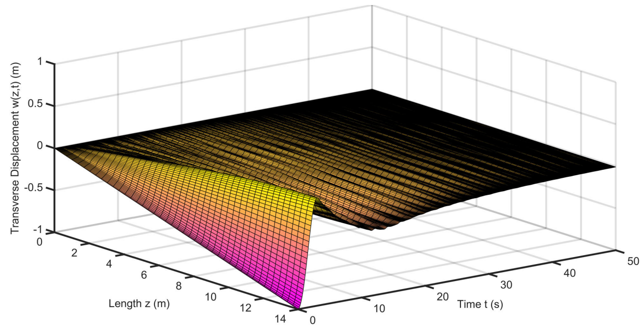

FIGURE 9. Case II: transverse displacement of the HDS with the control scheme neglecting saturation [24].

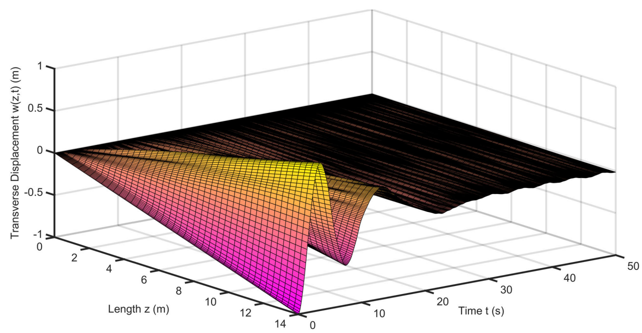

FIGURE 10. Case II: transverse displacement of the HDS with the control scheme neglecting backlash [12].

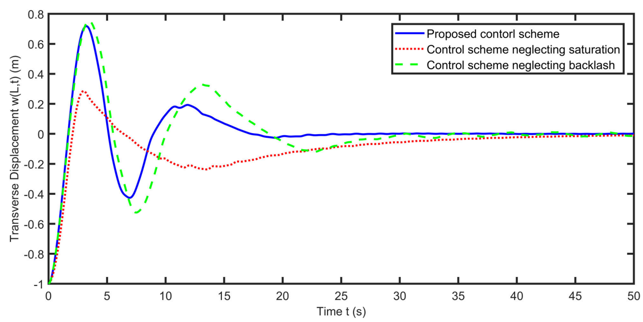

FIGURE 11. Case II: end point deflection of the HDS.

Now we discuss Case II. In this case, we choose the control gains as: $\beta_1 = 0.995$, $\beta_2 = 0.071$, $k_1 = 289.350$, $k_2 = 0.1$, $k_v = 1$, $k_\chi = 10^{-5}$, $k_u = 50$, $k_{F_\theta} = k_{A_0} = k_{F_{\theta_{inv}}} = k_\zeta = k_{\zeta h_r} = k_{\zeta h_l} = 1$, $\bar{d}_L = \delta_{d_L} = 0.1$. And we choose the initial conditions as: $w(z, 0) = -z/L$, $\dot{w}(z, 0) = 0$, $\hat{F}_\theta(0) = 41.1$, $\hat{F}_{\theta_{inv}}(0) = 0.1$, $\hat{A}_0(0) = 0$, $\hat{\zeta}(0) = 1$, $\widehat{\zeta h_r}(0) = \widehat{\zeta h_l}(0) = 0$.

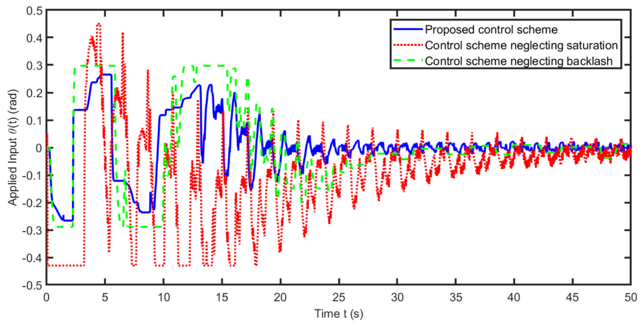

FIGURE 12. Case II: applied input of the HDS.

The simulation results of Case II are shown in Figs. 8–12. Figs. 8 and 11–12 display the transverse displacement and applied input of the HDS with our proposed control scheme, respectively. It is seen that the vibration of the HDS is suppressed to a small neighborhood of the desired position in the presence of unknown aerodynamic coefficients as well as actuator backlash and saturation.

Compared to the proposed control scheme, most traditional control schemes usually neglect one of the two actuator nonlinearities: backlash or saturation (for instance, [12], [23]–[25]). The corresponding results can be found in Figs. 9–11. Evidently, when we overlook backlash or saturation, the vibration of the HDS becomes larger, which means that the control performance of the HDS is degraded.

Therefore, the above simulation results demonstrate that our proposed control schemes are valid for our control problem.

VI. CONCLUSION

This paper investigated vibration control of the uncertain HDS in the presence of actuator nonlinearities. Based upon the linearization approach, a traditional model of the HDS has been extended, to depict how the HDS generate the control force to restrain the vibration of the HDS, while the unknown aerodynamic coefficients of the model have been estimated by invoking the parameter projection method. Subsequently, for actuator dead-zone and saturation, a smooth dead-zone approximate function has been constructed to design the dead-zone compensation method, based upon which the proposed control scheme can handle actuator dead-zone and saturation simultaneously while improving the output efficiency of the actuator. Next, for actuator backlash and saturation, a smooth backlash inverse has been constructed, based upon which the presented control scheme can cope with the both actuator nonlinearities at the same time. Finally, the proposed control schemes have also achieved the control objectives of vibration suppression and external disturbance attenuation. Additionally, since the excessive slack of the HDS may cause the damage of the equipment, our future work will focus on the tension control of the HDS.

APPENDIX A

Proof: (i) Lemma 5(i) and (65)–(66) imply

$$(\hat\gamma_r, \hat\gamma_l, \hat a_r, \hat a_l, \widehat{\gamma_r a_r}, \widehat{\gamma_l a_l}) \in \Omega. \tag{99}$$

Then in light of Lemma 3(ii), and noticing that (25)–(27) hold, we deduce

$$\frac{\partial \hat N(u)}{\partial u} > 0, \tag{100}$$

which guarantees the existence of controller (64).

(ii) Lemma 5(i), (65), and (67) imply

$$\begin{cases} \hat F_\theta \in [\underline{F}_\theta, \bar{\bar{F}}_\theta], \quad \hat A_0 \in [\underline{A}_0, \bar{\bar{A}}_0], \\ \hat F_{\theta_{inv}} \in [\underline{F}_{\theta_{inv}}, \bar{\bar{F}}_{\theta_{inv}}]. \end{cases} \tag{101}$$

Then in light of Lemma 5(ii), (65), (99), (101), and τ_{γ_r}, τ_{γ_l}, τ_{a_r}, τ_{a_l}, $\tau_{\gamma_r a_r}$, $\tau_{\gamma_l a_l}$, τ_{F_θ}, τ_{A_0}, $\tau_{F_{\theta_{inv}}}$ which are defined below (65), we deduce

$$\begin{aligned} &-\tilde F_\theta \dot{\hat F}_\theta - \tilde A_0 \dot{\hat A}_0 - F_\theta \big(\tilde F_{\theta_{inv}} \dot{\hat F}_{\theta_{inv}} + \tilde\gamma_r \dot{\hat\gamma}_r + \tilde\gamma_l \dot{\hat\gamma}_l \\ &\quad + \tilde a_r \dot{\hat a}_r + \tilde a_l \dot{\hat a}_l + \widetilde{\gamma_r a_r}\dot{\widehat{\gamma_r a_r}} + \widetilde{\gamma_l a_l}\dot{\widehat{\gamma_l a_l}}\big) \\ &\le -\beta_1\big[\dot w_1 + \frac{\beta_2 L}{\beta_1} w'(L,t)\big]\big[\tilde A_0 + F_\theta(\frac{\partial \tilde N}{\partial \tilde\gamma_r}\tilde\gamma_r + \frac{\partial \tilde N}{\partial \tilde\gamma_l}\tilde\gamma_l \\ &\quad + \frac{\partial \tilde N}{\partial \tilde a_r}\tilde a_r + \frac{\partial \tilde N}{\partial \tilde a_l}\tilde a_l + \frac{\partial \tilde N}{\partial \widetilde{\gamma_r a_r}}\widetilde{\gamma_r a_r} + \frac{\partial \tilde N}{\partial \widetilde{\gamma_l a_l}}\widetilde{\gamma_l a_l} \\ &\quad - \tilde F_{\theta_{inv}}\alpha_0) + w_{e3}\tilde F_\theta\big] + c_0. \end{aligned} \tag{102}$$

Finally, recalling (29), we can have (68). This completes the proof. □

APPENDIX B

Proof: Consider the following Lyapunov function candidate:

$$\begin{aligned} V_3 = V_2 + \frac{\beta_1}{2}\int_0^L \rho \dot w^2(z,t) + P(z) w'^2(z,t)\, dz \\ + \beta_2 \int_0^L \rho z \dot w(z,t) w'(z,t)\, dz, \end{aligned} \tag{103}$$

where β_1 and β_2 satisfy the following inequality:

$$c_1 \triangleq max\{\frac{\beta_2 L}{\beta_1}, \frac{\beta_2 L \rho}{\beta_1 P_{min}}\} < 1. \tag{104}$$

It is proven in [12] that $V_3 - V_2$ is positive definite if (104) holds, thus V_3 is a proper Lyapunov function candidate.

Utilizing (13) and integration by parts, we can derive the derivative of V_3 as:

$$\begin{aligned} \dot V_3 &\le \dot V_2 + \int_0^L \beta_1 \rho \dot w(z,t)\ddot w(z,t) + \beta_1 P(z) w'(z,t)\dot w'(z,t)\, dz \\ &\quad + \beta_2 \rho \int_0^L z\ddot w(z,t) w'(z,t) + z\dot w(z,t)\dot w'(z,t)\, dz \\ &\le \dot V_2 + \int_0^L \beta_1 \dot w(z,t)[(P(z)w'(z,t))' + Q]\, dz \\ &\quad + \beta_1 P(L)\dot w_1 w'(L,t) \\ &\quad - \int_0^L \beta_1 [P(z)w'(z,t)]'\dot w(z,t)\, dz \\ &\quad + \int_0^L \beta_2 z w'(z,t)[(P(z)w'(z,t))' + Q]\, dz \end{aligned}$$

$$
\begin{aligned}
&+\frac{\beta_2}{2}\rho L\dot{w}_1^2-\frac{1}{2}\int_0^L\beta_2\rho\dot{w}^2(z,t)\,dz\\
\le\ &\dot{V}_2+\int_0^L\beta_1\dot{w}(z,t)Q\,dz\\
&+\int_0^L\frac{\beta_2}{2}zP'(z)w'^2(z,t)+\beta_2zw'(z,t)Q\\
&-\frac{\beta_2\rho}{2}\dot{w}^2(z,t)-\frac{\beta_2}{2}P(z)w'^2(z,t)\,dz\\
&+\frac{\beta_2L}{2}P(L)w'^2(L,t)+\frac{\beta_2\rho L}{2}\dot{w}_1^2\\
&+\beta_1P(L)\dot{w}_1w'(L,t)\\
\le\ &\dot{V}_2-\frac{1}{2}\int_0^L\beta_2\rho\dot{w}^2(z,t)\,dz-\frac{1}{2}\int_0^L\big[\beta_2P(z)\\
&-\beta_2zP'(z)\big]w'^2(z,t)\,dz\\
&+\int_0^L\big[\beta_1\dot{w}(z,t)+\beta_2zw'(z,t)\big]Q\,dz\\
&+\frac{\beta_1^2P(L)}{2\beta_2L}(\dot{w}_1+\frac{\beta_2L}{\beta_1}w'(L,t))^2\\
&-\frac{1}{2}(\frac{\beta_1^2P(L)}{\beta_2L}-\beta_2\rho L)\dot{w}_1^2. \qquad (105)
\end{aligned}
$$

To proceed, we employ Young's inequality to obtain the following inequalities:

$$
\begin{aligned}
&\int_0^L\big[\beta_1\dot{w}(z,t)+\beta_2zw'(z,t)\big]Q\,dz\le\int_0^L(\frac{\beta_1}{2\epsilon_1}\\
&\quad+\frac{\beta_2L}{2\epsilon_2})Q^2+\frac{\beta_1\epsilon_1}{2}\dot{w}^2(z,t)+\frac{\beta_2L\epsilon_2}{2}w'^2(z,t)\,dz, \qquad (106)\\
&\beta_1(\dot{w}_1+\frac{\beta_2L}{\beta_1}w'(L,t))F_\theta\delta_{\mathfrak{N}}(u)\\
&\le\frac{\epsilon_3}{2}\beta_1^2(\dot{w}_1+\frac{\beta_2L}{\beta_1}w'(L,t))^2+\frac{1}{2\epsilon_3}F_\theta^2\bar{\delta}_{\mathfrak{N}}^2, \qquad (107)\\
&\tilde{F}_\theta(\hat{F}_\theta-F_{\theta0})\le-\frac{1}{2}\tilde{F}_\theta^2+\frac{1}{2}(F_\theta-F_{\theta0})^2, \qquad (108)\\
&\left|\int_0^L\beta_2\rho z\dot{w}(z,t)w'(z,t)\,dz\right|\\
&\le\frac{\beta_2L\rho}{2}\int_0^L\dot{w}^2(z,t)\,dz+\frac{\beta_2L\rho}{2}\int_0^Lw'^2(z,t)\,dz, \qquad (109)
\end{aligned}
$$

where $\epsilon_1,\epsilon_2,\epsilon_3$ are positive constants, and note that (108) still holds if we replace F_θ with $F_{\theta_{inv}}, A_0, \gamma_r, \gamma_l, a_r, a_l, \gamma_r a_r$, or $\gamma_l a_l$.

Then in view of (10), (74), and (105)–(108), we derive

$$
\begin{aligned}
\dot{V}_3\le\ &-\frac{c_2}{2}\int_0^L\dot{w}^2(z,t)\,dz-\frac{c_3}{2}\int_0^Lw'^2(z,t)\,dz\\
&-\beta_1c_4(\dot{w}_1+\frac{\beta_2L}{\beta_1}w'(L,t))^2-c_5\dot{w}_1^2-k_2w_{e3}^2\\
&-\frac{k_{F_\theta}}{2}\tilde{F}_\theta^2-\frac{k_{A_0}}{2}\tilde{A}_0^2-F_\theta(\frac{k_{\gamma_r}}{2}\tilde{\gamma}_r^2+\frac{k_{\gamma_l}}{2}\tilde{\gamma}_l^2\\
&+\frac{k_{a_r}}{2}\tilde{a}_r^2+\frac{k_{a_l}}{2}\tilde{a}_l^2+\frac{k_{\gamma_ra_r}}{2}\widetilde{\gamma_ra_r}^2+\frac{k_{\gamma_la_l}}{2}\widetilde{\gamma_la_l}^2\\
&+\frac{k_{F_{\theta_{inv}}}}{2}\tilde{F}_{\theta_{inv}}^2)+c+\frac{1}{k_\chi}(\frac{du}{dv}U(\chi)-1)\dot{\chi}, \qquad (110)
\end{aligned}
$$

where

$$
\begin{aligned}
c_2&=\beta_2\rho-\beta_1\epsilon_1,\\
c_3&=\beta_2P_{min}-\beta_2L\epsilon_2,\\
c_4&=k_1-\frac{\beta_1P(L)}{2\beta_2L}-\frac{\epsilon_3}{2}\beta_1,\\
c_5&=\frac{\beta_1^2P(L)}{2\beta_2L}-\frac{\beta_2\rho L}{2},\\
c&=(\frac{\beta_1}{2\epsilon_1}+\frac{\beta_2L}{2\epsilon_2})LQ^2+\frac{F_\theta^2}{2\epsilon_3}\bar{\delta}_{\mathfrak{N}}^2+\frac{k_{F_\theta}}{2}(F_\theta-F_{\theta0})^2\\
&\quad+\frac{k_{A_0}}{2}(A_0-A_1)^2+\frac{F_\theta}{2}[k_{F_{\theta_{inv}}}(1/F_\theta-F_{\theta_{inv}0})^2\\
&\quad+k_{\gamma_r}(\gamma_r-\gamma_{r0})^2+k_{\gamma_l}(\gamma_l-\gamma_{l0})^2+k_{a_r}(a_r-a_{r0})^2\\
&\quad+k_{a_l}(a_l-a_{l0})^2+k_{\gamma_ra_r}(\gamma_ra_r-\gamma_ra_{r0})^2\\
&\quad+k_{\gamma_la_l}(\gamma_la_l-\gamma_la_{l0})^2]+k_p\delta_{d_L}>0.
\end{aligned}
$$

Now choose appropriate constants β_1, β_2, k_1, ϵ_1, ϵ_2, ϵ_3 to ensure that (104) is feasible and c_2, c_3, c_4, c_5 are positive. Evidently, the appropriate constants always exist. Then from (110) and the following inequality based upon (109):

$$
\begin{aligned}
&|\beta_2\int_0^L\rho z\dot{w}(z,t)w'(z,t)\,dz|\\
&\qquad\le c_1\frac{\beta_1}{2}\int_0^L\rho\dot{w}^2(z,t)+P(z)w'^2(z,t)\,dz,
\end{aligned}
$$

we can derive that

$$\dot{V}_3\le-c_{min}V_3+c+\frac{1}{k_\chi}(\frac{du}{dv}U(\chi)-1)\dot{\chi}, \qquad (111)$$

where $c_{min}=min\{\frac{c_6}{c_1+1},\frac{2c_4}{m},2k_2,k_{F_\theta},k_{A_0},k_{F_{\theta_{inv}}},k_{\gamma_r},k_{\gamma_l},k_{a_r},k_{a_l},k_{\gamma_ra_r},k_{\gamma_la_l}\}>0$, c_1 is defined in (104), $c_6=min\{\frac{c_2}{\beta_1\rho},\frac{c_3}{\beta_1P_{max}}\}$.

In view of (111), we derive

$$V_3(t)\le\big(V_3(0)-\frac{c}{c_{min}}\big)e^{-c_{min}t}+c_U, \qquad (112)$$

where $c_U=\frac{c}{c_{min}}+\frac{1}{k_\chi}\int_0^t(\frac{du}{dv}U(\chi)-1)\dot{\chi}e^{-c_{min}(t-s)}\,ds$. Then from (112) and the Theorem 1 presented in [34], we deduce that $V_3(t)$ is bounded.

Next, in light of Lemma 6, (10), (103), and the inequality above (111), one can derive:

$$\frac{\beta_1P_{min}}{2L}w^2(z,t)\le\frac{\beta_1}{2}\int_0^LP(z)w'^2(z,t)\,dz\le\frac{V_3}{1-c_1},$$

and from (112), one has:

$$|w(z,t)|\le\sqrt{\frac{2L}{\beta_1P_{min}(1-c_1)}\Big[\big(V_3(0)-\frac{c}{c_{min}}\big)e^{-c_{min}t}+c_U\Big]}.$$

With the above inequality and the boundedness of $V_3(t)$, we can rapidly deduce that the closed-loop system of (13) is stable subject to the actuator dead-zone (14) and saturation (12).

Besides, the above inequality further indicates:

$$\lim_{t\to\infty}|w(z,t)|\le\sqrt{\frac{2Lc_U}{\beta_1P_{min}(1-c_1)}}.$$

Therefore, we can obtain that $w(z,t)$ is uniformly ultimately bounded. This completes the proof. $\square$

APPENDIX C

Proof: (i) In view of (75), (81)–(83), and noticing that the magnitudes of $\underline{\widehat{\zeta h_r}}$ and $\widehat{\zeta h_l}$ are sufficiently small, we can have

$$|\widehat{\zeta h_r}u_2 - \widehat{\zeta h_l}u_3| \leq \bar{\hat{\zeta}}\bar{h}. \tag{113}$$

Then from (76) and (80), we deduce

$$-\hat{\underline{\zeta}}(-\underline{\theta} - \frac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\hat{\underline{\zeta}}})/\bar{\zeta} < u_1 < \hat{\underline{\zeta}}(\bar{\theta} - \frac{\bar{\zeta}\bar{\hat{\zeta}}\bar{h}}{\hat{\underline{\zeta}}})/\bar{\zeta}. \tag{114}$$

Next, in light of (75), (79), (83), and (113)–(114), we obtain

$$\frac{\underline{\theta}}{\underline{\zeta}} < u < \frac{\bar{\theta}}{\underline{\zeta}}. \tag{115}$$

Finally, from (15), we derive

$$\underline{\theta} < \mathfrak{N}(u) < \bar{\theta}, \tag{116}$$

which guarantees (84).

(ii) It is presented in [24] that $\mathfrak{N}(u)$ satisfies

$$\mathfrak{N}(u) = \sigma_1(t)\zeta(u(t) - h_r) + \sigma_2(t)\zeta(u(t) - h_l) + \sigma_3(t)\mathfrak{N}_0, \tag{117}$$

where

$$\sigma_1 = \begin{cases} 1, & \dot{\mathfrak{N}} > 0 \\ 0, & \dot{\mathfrak{N}} \leq 0 \end{cases} \tag{118}$$

$$\sigma_2 = \begin{cases} 1, & \dot{\mathfrak{N}} < 0 \\ 0, & \dot{\mathfrak{N}} \geq 0 \end{cases} \tag{119}$$

$$\sigma_3 = 1 - \sigma_1 - \sigma_2, \tag{120}$$

$\mathfrak{N}_0$ is a variable which is invariant and satisfies

$$\mathfrak{N}_0/\zeta + h_l \leq u(t) \leq \mathfrak{N}_0/\zeta + h_r \tag{121}$$

when $\dot{\mathfrak{N}} = 0$.

To proceed, from (79), we have

$$u_1 = \hat{\zeta}u(t) - \widehat{\zeta h_r}u_2(t) + \widehat{\zeta h_l}u_3(t). \tag{122}$$

Then in view of (85), (117)–(120), and (122), we deduce

$$\delta_{\mathfrak{N}re}(u) = \sigma_3(t)(\mathfrak{N}_0 - \zeta u(t)) - \zeta h_r(\sigma_1(t) - u_2(t)) - \zeta h_l(\sigma_2(t) + u_3(t)). \tag{123}$$

Next, consider the following three cases.

1) $\dot{\mathfrak{N}} > 0$

In light of (81)–(82) and (118)–(120), we obtain

$$\begin{aligned} |\delta_{\mathfrak{N}re}(u)| &= |-\zeta h_r(1 - u_2(t)) - \zeta h_l u_3(t)| \\ &= |\zeta(h_r - h_l)u_3(t)| \leq \zeta(h_r - h_l). \end{aligned} \tag{124}$$

2) $\dot{\mathfrak{N}} < 0$

In view of (81)–(82) and (118)–(120), we obtain

$$\begin{aligned} |\delta_{\mathfrak{N}re}(u)| &= |-\zeta h_l(1 + u_3(t)) + \zeta h_r u_2(t)| \\ &= |\zeta(h_r - h_l)u_2(t)| \leq \zeta(h_r - h_l). \end{aligned} \tag{125}$$

3) $\dot{\mathfrak{N}} = 0$

From (121), we have

$$-\zeta h_r \leq \mathfrak{N}_0 - \zeta u(t) \leq -\zeta h_l, \tag{126}$$

then in light of (81)–(82), (118)–(120), and (126), we deduce

$$\begin{aligned} |\delta_{\mathfrak{N}re}(u)| &= |\zeta h_r u_2(t) - \zeta h_l u_3(t) + \mathfrak{N}_0 - \zeta u(t)| \\ &= |\zeta(h_r - h_l)u_2(t) + \zeta h_l + \mathfrak{N}_0 - \zeta u(t)| \\ &\leq \zeta(h_r - h_l). \end{aligned} \tag{127}$$

Therefore, we conclude that (86) is feasible. This completes the proof. $\square$

REFERENCES

[1] J. P. Nalepka and J. L. Hinchman, "Automated aerial refueling: Extending the effectiveness of UAVs," in *Proc. AIAA Modeling Simulation Technol. Conf. Exhibit,* San Francisco, CA, USA, Aug. 2005, p. 6005.

[2] T. Kuk and K. Ro, "Design, test and evaluation of an actively stabilised drogue refuelling system," *Aeronaut. J.*, vol. 117, no. 1197, pp. 1103–1118, Nov. 2013.

[3] M. L. Fravolini, A. Ficola, G. Campa, M. R. Napolitano, and B. Seanor, "Modeling and control issues for autonomous aerial refueling for UAVs using a probe-drogue refueling system," *Aerosp. Sci. Technol.*, vol. 8, no. 7, pp. 611–618, Oct. 2004.

[4] J. Valasek, D. Famularo, and M. Marwaha, "Fault-tolerant adaptive model inversion control for vision-based autonomous air refueling," *J. Guid., Control, Dyn.*, vol. 40, no. 6, pp. 1336–1347, Jun. 2017.

[5] C. Martínez, T. Richardson, P. Thomas, J. L. du Bois, and P. Campoy, "A vision-based strategy for autonomous aerial refueling tasks," *Robot. Auto. Syst.*, vol. 61, no. 8, pp. 876–895, Aug. 2013.

[6] Z. Su, H. Wang, P. Yao, Y. Huang, and Y. Qin, "Back-stepping based anti-disturbance flight controller with preview methodology for autonomous aerial refueling," *Aerosp. Sci. Technol.*, vol. 61, pp. 95–108, Feb. 2017.

[7] B. d'Andréa-Novel and J. M. Coron, "Exponential stabilization of an overhead crane with flexible cable via a back-stepping approach," *Automatica*, vol. 36, no. 4, pp. 587–593, Apr. 2000.

[8] B.-Z. Guo and F.-F. Jin, "The active disturbance rejection and sliding mode control approach to the stabilization of the Euler–Bernoulli beam equation with boundary input disturbance," *Automatica*, vol. 49, no. 9, pp. 2911–2918, Sep. 2013.

[9] K. D. Do, "Modeling and boundary control of translational and rotational motions of nonlinear slender beams in three-dimensional space," *J. Sound Vibrat.*, vol. 389, pp. 1–23, Feb. 2017.

[10] K.-J. Yang, K.-S. Hong, and F. Matsuno, "Robust boundary control of an axially moving string by using a PR transfer function," *IEEE Trans. Autom. Control*, vol. 50, no. 12, pp. 2053–2058, Dec. 2005.

[11] A. A. Paranjape, J. Guan, S.-J. Chung, and M. Krstic, "PDE boundary control for flexible articulated wings on a robotic aircraft," *IEEE Trans. Robot.*, vol. 29, no. 3, pp. 625–640, Jun. 2013.

[12] Z. Liu, J. Liu, and W. He, "Vibration control of a flexible aerial refuelling hose with input saturation," *Int. J. Syst. Sci.*, vol. 48, no. 5, pp. 971–983, Apr. 2017.

[13] Z. Liu, J. Liu, and W. He, "Deadzone compensation based boundary control of a flexible aerial refueling hose with output constraint," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 645–650, Jul. 2017.

[14] Z. Liu, J. Liu, and W. He, "Modeling and vibration control of a flexible aerial refueling hose with variable lengths and input constraint," *Automatica*, vol. 77, pp. 302–310, Mar. 2017.

[15] Z. Liu, X. He, Z. Zhao, C. K. Ahn, and H.-X. Li, "Vibration control for spatial aerial refueling hoses with bounded actuators," *IEEE Trans. Ind. Electron.*, early access, Apr. 13, 2020, doi: 10.1109/TIE.2020.2984442.

[16] M. Ramirez-Neria, G. Ochoa-Ortega, N. Lozada-Castillo, M. A. Trujano-Cabrera, J. P. Campos-Lopez, and A. Luviano-Juarez, "On the robust trajectory tracking task for flexible-joint robotic arm with unmodeled dynamics," *IEEE Access*, vol. 4, pp. 7816–7827, 2016.

[17] Y. Jia, "Robust control with decoupling performance for steering and traction of 4WS vehicles under velocity-varying motion," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 3, pp. 554–569, May 2000.

[18] Y. Jia, "Alternative proofs for improved LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty: A predictive approach," *IEEE Trans. Autom. Control*, vol. 48, no. 8, pp. 1413–1416, Aug. 2003.

[19] J. Zhou, C. Wen, and Y. Zhang, "Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 504–511, Mar. 2006.

[20] J. Yu, P. Shi, W. Dong, and C. Lin, "Adaptive fuzzy control of nonlinear systems with unknown dead zones based on command filtering," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 1, pp. 46–55, Feb. 2018.

[21] T. Senjyu, T. Kashiwagi, and K. Uezato, "Position control of ultrasonic motors using MRAC and dead-zone compensation with fuzzy inference," *IEEE Trans. Power Electron.*, vol. 17, no. 2, pp. 265–272, Mar. 2002.

[22] Z. Zhao, X. Wang, C. Zhang, Z. Liu, and J. Yang, "Neural network based boundary control of a vibrating string system with input deadzone," *Neurocomputing*, vol. 275, pp. 1021–1027, Jan. 2018.

[23] G. Tao and P. V. Kokotović, "Continuous-time adaptive control of systems with unknown backlash," *IEEE Trans. Autom. Control*, vol. 40, no. 6, pp. 1083–1087, Jun. 1995.

[24] J. Zhou, C. Zhang, and C. Wen, "Robust adaptive output control of uncertain nonlinear plants with unknown backlash nonlinearity," *IEEE Trans. Autom. Control*, vol. 52, no. 3, pp. 503–509, Mar. 2007.

[25] V. Agrawal, W. J. Peine, B. Yao, and S. Choi, "Control of cable actuated devices using smooth backlash inverse," in *Proc. IEEE Int. Conf. Robot. Autom.*, Anchorage, AK, USA, May 2010, pp. 1074–1079.

[26] N. H. El-Farra, A. Armaou, and P. D. Christofides, "Analysis and control of parabolic PDE systems with input constraints," *Automatica*, vol. 39, no. 4, pp. 715–725, Apr. 2003.

[27] Y. Su, C. Zheng, and P. Mercorelli, "Nonlinear PD fault-tolerant control for dynamic positioning of ships with actuator constraints," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 3, pp. 1132–1142, Jun. 2017.

[28] W. D. Zhu, J. Ni, and J. Huang, "Active control of translating media with arbitrarily varying length," *J. Vibrat. Acoust.*, vol. 123, no. 3, pp. 347–358, Feb. 2001.

[29] K. Ro and J. W. Kamman, "Modeling and simulation of hose-paradrogue aerial refueling systems," *J. Guid., Control, Dyn.*, vol. 33, no. 1, pp. 53–63, Jan./Feb. 2010.

[30] H. Goldstein, *Classical Mechanics*. Reading, MA, USA: Addison-Wesley, 1951.

[31] R. Brockhaus, W. Alles, and R. Luckner, *Flugregelung*. Berlin, Germany: Springer-Verlag, 2011.

[32] P. Li and G.-H. Yang, "Fault-tolerant control of uncertain nonlinear systems with nonlinearly parameterized fuzzy systems," in *Proc. IEEE Int. Conf. Control Appl.*, St. Petersburg, Russia, Jul. 2009, pp. 382–387.

[33] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.

[34] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1672–1678, Jul. 2011.

[35] Y. Jia, "General solution to diagonal model matching control of multiple-output-delay systems and its applications in adaptive scheme," *Prog. Natural Sci.*, vol. 19, no. 1, pp. 79–90, Jan. 2009.

[36] B. Rui, Y. Yang, and W. Wei, "Nonlinear backstepping tracking control for a vehicular electronic throttle with input saturation and external disturbance," *IEEE Access*, vol. 6, pp. 10878–10885, 2018.

[37] W. He, T. Meng, D. Huang, and X. Li, "Adaptive boundary iterative learning control for an Euler–Bernoulli beam system with input constraint," *IEEE Trans. Neural. Netw.*, vol. 29, no. 5, pp. 1539–1549, Mar. 2018.

[38] B. Homayoun, M. M. Arefi, and N. Vafamand, "Robust adaptive backstepping tracking control of stochastic nonlinear systems with unknown input saturation: A command filter approach," *Int. J. Robust Nonlinear Control*, vol. 30, no. 8, pp. 3296–3313, May 2020.

[39] N. Vafamand, "Adaptive robust neural network-based backstepping control of tethered satellites with additive stochastic noise," *IEEE Trans. Aerosp. Electron. Syst.*, early access, Apr. 20, 2020, doi: 10.1109/TAES.2020.2985276.

[40] K. D. Do and J. Pan, "Boundary control of transverse motion of marine risers with actuator dynamics," *J. Sound Vibrat.*, vol. 318, nos. 4–5, pp. 768–791, Dec. 2008.

[41] M. S. De Queiroz, D. M. Dawson, S. P. Nagarkatti, and F. Zhang, *Lyapunov-Based Control of Mechanical Systems*. Boston, MA, USA: Springer, 2012.

[42] R. D. Richtmyer and K. W. Morton, *Difference Methods for Initial-Value Problems*. New York, NY, USA: Wiley, 1967.

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