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An Evolutionary Algorithm for Multi and Many-Objective Optimization With Adaptive Mating and Environmental Selection

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ABSTRACT Multi-objective evolutionary algorithms (MOEAs) have received immense recognition due to their effectiveness and efficiency in tackling multi-objective optimization problems (MOPs). Recently, numerous studies on MOEAs revealed that when handling many-objective optimization problems (MaOPs) that have more than three objectives, MOEAs encounter challenges and the behavior of MOEAs resembles a random walk in search space as the proportion of nondominated solutions increases subsequently. This phenomenon is commonly observed in most classical Pareto-dominance-based MOEAs (PDMOEAs) such as NSGA-II, SPEAII, as these algorithms face difficulties in guiding the search process towards the optimal Pareto front due to lack of selection pressure. From the literature, it is evident that incorporating sum of normalized objectives into the framework of MOEAs would enhance the converging capabilities. Hence, in this work, we propose a novel multi-objective optimization algorithm with adaptive mating and environmental selection (ad-MOEA) which effectively incorporates the concept of sum of objectives in the mechanisms of mating and environmental selection to control the convergence and diversity adaptively. To demonstrate the effectiveness of the proposed ad-MOEA, we have conducted experiments on 26 test problems that includes DTLZ, WFG and MaOP test suites. Along with the benchmark problem, we have analyzed the performance of the proposed approach on real-world problems. The experimental results demonstrate the effectiveness of the proposed method with respect to the state-of-art methods.

INDEX TERMS Evolutionary computation, multi-objective optimization, many-objective optimization, Pareto-dominance, sum of normalized objectives, crowding distance.

I. INTRODUCTION

Multi-objective optimization problems (MOPs) refer to the optimization problems with more than one objective that are conflicting in nature and are optimized simultaneously [1]. The concept of multi-objective optimization is often investigated in the real-world applications, such as software engineering [2], power distribution networks [3] and industrial scheduling [4]. The conflicting behavior of the objectives in MOPs, leads to obtaining a set of nondominated solutions, termed as Pareto optimal set (PS) that refers to Pareto Front (PF) in objective space [1], [5]. In the literature, it is evident that multi-objective evolutionary algorithms (MOEAs) are effective in solving MOPs due to their ability to obtain the Pareto optimal set in a single individual run. The main aim of MOEAs when solving the MOPs is to obtain balance between the convergence (refers to closeness of the obtained Pareto front to the true Pareto front) and diversity (refers to uniform distribution of the population in the obtained Pareto front) [1], [5]. The ability of providing the trade-off between convergence and diversity in MOEAs mainly depends on the employed selection strategy.

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In literature, a variety of selection strategies were proposed based on which MOEAs are broadly classified as: Pareto-dominance based MOEAs (PDMOEAs) [6]–[12], decomposition-based MOEAs [13]–[17] indicator-based MOEAs [17]–[20], reference-set based MOEAs [8], [21], preference-based MOEAs [22], [23] and Hybrid MOEAs [24], [25]. Among them, the Pareto-dominance MOEAs are popular as the output of the MOEA is a set of nondominated solutions and PDMOEAs have the capability to provide the set of well-distributed nondominated solutions [7].

However, most of the classical Pareto-dominance based MOEAs (PDMOEAs) perform better on the MOPs but fail drastically to handle the problems with more than three objectives termed as many-objective optimization problems (MaOPs) [1], [5]. In other words, the performance of PDMOEAs deteriorate drastically when solving the many-objective optimization problems as they encounter difficulties in achieving the trade-off between convergence and diversity. The main factor responsible for the degradation in the performance of PDMOEAs is the quantity of nondominated solutions increase progressively in accordance to the objectives. This phenomenon decreases the selection pressure and fails to guide the search process towards the convergence [6], [8]. In other words, due to the increasing of dimensionality objective space, the behavior of the MOEAs adopts a random search as most of the individuals become incomparable with respect to the Pareto-dominance [7].

Moreover, as the number of objectives increases, to obtain a trade-off between diversity and convergence becomes a daunting task. In other words, arbitrarily large populations cannot be used in the evolution of MOEAs as the population size used in MOEAs is limited due to the computational complexity issues. Hence, due to the limited population size, the individuals will be eventually far away from each other in the high dimensional space resulting in the inefficiency of offspring generation [8]. Finally, the visualization and representation of the trade-off surface are the challenges faced by MOEAs, which are not directly effecting the evolutionary process, but causes problems in decision-making. To overcome these difficulties, additional diversity metrics are employed along with Pareto-dominance in PDMOEAs that provides importance to the dominance resistant solutions in the mating and environmental selections. However, the prioritization of dominance resistant solutions in the selection process cannot strengthen the selection pressure toward the Pareto Front, and may even impede the evolutionary process to a certain extent [8].

In literature, to improve the performance of PDMOEAs in tackling MaOPs, various modifications have been proposed that can be classified as: a) relaxing dominance relation [26]–[34], b) Pareto-dominance in conjunction with additional metrics [6], [7], [35]–[37] and c) combining dominance and decomposition-based approaches [38], [39]. In the first class of algorithms, the definition of the conventional Pareto-dominance is modified to enhance the comparability of two candidate solutions [26]–[34]. In the second class of approaches, along with the Pareto-dominance relation an additional selection criterion is employed in the PDMOEAs to preserve convergence and diversity [6], [7], [35]–[37]. The last category focus on effectively combining the dominance and decomposition based approaches to benefit from the mutual advantages of two approaches [38], [39].

In this paper, we propose a novel multi-objective evolutionary algorithm with adaptive mating and environmental selections (ad-MOEA) in which the concept of sum of normalized objectives (*SoNB*) is incorporated into the mating and environmental selections. Along with the sum of normalized objectives, the crowding distance metrics (*CWD*) is employed to promote the diversity. The proposed approach falls into the second category mentioned above where the Pareto-dominance in conjunction with additional metrics namely sum of normalized objectives (*SoNB*) and crowding distance (*CWD*) is employed. The main motivation of combining sum of normalized objectives and crowding distance is that if sum of normalized objectives is alone adopted as secondary selection metric, the algorithm will experience loss of diversity and by adapting crowding distance alone as the secondary selection, the algorithm will suffer from the degradation in the performance of the convergence. In other words, employing sum of normalized objectives exclusively would result in a final solution set concentrated on a single region of the PF whereas employing crowding distance metric would result in a set of solutions that are far away from the true PF. Moreover, the solutions that promote convergence are found in the most crowded regions and preserving them in the initial generations would enhance the selection pressure of the algorithm towards the PF. Hence, in the proposed work, to preserve the solutions that accelerate convergence in the initial stages, we assign more preference to the sum of normalized objectives in the early generations and as the evolution progresses the more importance is assigned to the crowding distance.

In addition, the application of *SoNB* and *CWD* is adaptively controlled to achieve a proper balance between convergence and diversity depending on the characteristics of optimization problem. The sum of objective concept in the proposed ad-MOEA approach is entirely different from the weighted sum concept which is included in popular decomposition based approach, MOEA-D [13]. In MOEA-D approach, the MOPs are converted into several single-objective problems (SOPs) with the help of weighted sum. To evaluate the weighted sum, there is necessity for initializing weights and updating them as the evolution progress. The sum of normalized objectives concept adopted in the proposed approach is entirely different from the weighted sum concept, where the objective function are first normalized in the range of [0, 1] and then the summation of normalized objective values is determined. The sum of normalized objectives doesn't require the initialization of weights and updating weights.

The main contributions of this paper are summarized as follows:

a) In the proposed ad-MOEA the concept of sum of normalized objectives is employed in the mating and environmental selections in conjunction with Pareto-dominance and crowding distance to adaptively emphasize the converging capabilities by prioritizing the individuals that improve convergence.
b) In mating selection, the concept of weighted rank is employed so that depending on the stage of the evolution (exploration or exploitation) the appropriate solutions can be assigned probability to participate in the offspring population. In other words, each solution in the population will be ranked with respect to the convergence and diversity criteria and a weighted rank is assigned using a self-adaptive parameter 'w' that emphasis convergence or diversity depending on the state of evolution.
c) In environmental selection, after the employing the Pareto-dominance criteria, the solutions are chosen until the critical front. Then, in the critical front, at first, the solutions are selected based on the sum of normalized objectives with a certain probability and then the remaining solution are selected based on crowding distance. The probability with which the individuals are selected based on sum of normalized objectives is determined by the self-adaptive parameter 'w'. To adapt the parameter 'w', the information related to the number of nondominated solutions in the population is used as the final aim of MOEAs is to obtain a set of nondominated solutions that spread over the entire Pareto-front.

The rest of this paper is organized as follows. In Section II, we have presented the preliminaries, related work and motivation behind the proposed method and a brief description of the ad-MOEA is presented in the Section III. Simulation results with discussion is presented in Section IV and finally, Section V concludes the paper

II. PRELIMINARIES AND RELATED WORK

In this section, basic definitions related to the multi-objective optimization are presented in the preliminaries along with the related work.

A. PRELIMINARIES

Without loss of generality, a multi-objective optimization problem can be formulated as follows:

$$\begin{aligned} &\min f(x) = (f_1(x), f_2(x), \ldots, f_M(x)) \\ &s.t.\ x \epsilon S \subset R^n \end{aligned} \tag{1}$$

where x represents n-dimensional decision vector in space and M is the number of objectives. When $M > 3$ Ratios can be represented as then the problem represents a MaOP.

Definition 1: For any two different solutions, x and $y \epsilon$S, if $\forall m = 1, 2, \ldots, M, f_m(x) \leq f_m(y), \exists i = 1, 2, \ldots, M$, and $f_i(x) \leq f_i(y)$ then the solution x dominates the solution y denoted as $x < y$.

Definition 2: A decision vector, $x \epsilon S \subset R^n$, can be considered as a Pareto-optimal, if and only if there exists no other decision vector, $x^* \epsilon S \subset R^n$ that dominates x i.e., $x \leq x^*$.

Definition 3: The set consists of all the Pareto-optimal solutions is termed as the Pareto-optimal set (PS) and the set of all the Pareto-optimal objective vectors is called as the Pareto-optimal Front.

Definition 4: To evaluate, sum of normalized objectives, at first, the objective values are converted into ratios, using the best and worst objective values of each objective function in the current population [40]. Then, for a solution i, the sum of normalized objectives can be calculated as the summation of ratios.

Mathematically, the Ratios can be represented as the

$$f_i'(x) = \left(\frac{f_i(x) - \min(f(x))}{\max(f(x)) - \min(f(x))} \right) \tag{2}$$

$$f_{sonb}(i) = \sum_{i=1}^{M} f_i'(x) \tag{3}$$

where M denotes the number of objectives. The main advantage of the sum of normalized objectives concepts is that it removes the range-dependence of the solutions.

B. RELATED WORK

In this section, a detailed review on the various modifications proposed for PDMOEAs in the literature to enhance the performance of PDMOEAs in handling MaOPs is presented. The proposed modifications to the PDMOEAs can be divided into three categories, a) relaxation of dominance relation; b) Pareto-dominance in conjunction with additional metrics; c) combining dominance and decomposition-based approaches.

1) RELAXATION OF DOMINANCE RELATION

The first category of ideas focus on relaxing the dominance relation, which refers to modifying the definition of conventional Pareto-dominance by introducing new dominance relations. The main intention of introducing modified dominance relation is to enhance the probability of two individuals being comparable on MaOPs. In [26], a novel method to control the dominance area of individuals (CDAS) in order to enhance the performance of the MOEAs was proposed. By controlling the dominance area, CDAS algorithm induces appropriate ranking for the individuals and enhance the selection process. This approach employs an external user–defined parameter S that effectively controls the degree of contraction and expansion of the dominance area of the individuals [26]. In [27], a modified version to CDAS approach that self-adapts external parameter S and self-controls the dominance area for each individual (S-CDAS) was proposed to tackle MaOPs. The main improvement that was proposed in S-CDAS algorithms is that a fine-grained ranking is employed in S-CDAS that includes the extreme solutions always in the top front [27].

In [28], a generalized Pareto-optimality (GPO) to deal with the scalability issues of PDMOEAs was proposed. The main

idea of the GPO is that the dominance area of the solutions expands progressively as the dimensionality of the problem increases. The generalized Pareto optimality approach is associated with a parameter called as expanding angle that controls the degree of expansion of dominance area. In [29], a fuzzification of the Pareto-dominance relation was proposed and its applicability in design of MOEAs is analyzed. Along with that, a generic ranking scheme is depicted which allocates any set of vectors with dominance degrees in a set-dependent, nonsymmetrical and scale-independent manner [29]. In [30], a new fitness evaluation mechanism was introduced that differentiates solutions continuously into different degrees of optimality. Along with the fitness evaluation mechanism, based on the fuzzy logic, a fuzzy Pareto-dominance (FD) was proposed and integrated into the popular NSGA-II and SPEA2 to analyze the performance on MaOPs [30]. In [31], a new dominance relation based on the reference solution termed as r-dominance that can create strict partial order among Pareto-equivalent solutions was proposed.

In [32], a modification to the conventional Pareto-dominance, called g-dominance was proposed that can be incorporated in the design of any MOEA. The g-dominance utilizes the information that is incorporated in a reference point and without the help of any scalarizing function approximates the efficient set around the most preferred area. In [33], the authors have proposed ε-MOEA that employs a new variant of Pareto-dominance, ε-dominance along with archiving/selection strategies to improve the search towards the Pareto-optimal set. The ε-MOEA algorithms preserve an archive of nondominated solutions of finite-size and updates the archive iteratively based on ε-dominance when a new solution is generated. In [34], the authors have proposed strengthened dominance relation (SDR), a new modification to Pareto-dominance relation that employs a niching method based on the angles between the individuals. In each niche, the SDR maintains the best converged solution and size of each niche is determined adaptively by the distribution of the individuals which is measured by the angles between them [34]. However, the PDMOEAs with modified dominance relationships demonstrated improved performance in handling MaOPs but tends to get trapped in the sub-regions of the Pareto front.

2) PARETO-DOMINANCE IN CONJUNCTION WITH ADDITIONAL METRICS

This category of ideas focus on employing an additional metrics in conjunction with Pareto-dominance to promote convergence in PDMOEAs. Recently, many works have concentrated on developing effective additional metrics to provide proper equilibrium between the convergence and diversity. In [6], a knee-point driven evolutionary algorithm (KnEA) was proposed that employs the concept of knee-points and weighted distance to promote convergence and diversity simultaneously. In order to find the knee-points, an adaptive strategy is employed in KnEA that identifies the knee-points in small neighborhood. In the mating selection and environmental selection of KnEA, more importance is given to the knee-points [6]. The main reason for prioritizing the knee-points in KnEA is that they provide bias towards the larger hypervolume and promotes convergence and achieves diversity simultaneously. However, in identifying the knee-points, to determine the size of neighborhood, a parameter T is required which has to be designed manually [6]. In [8], the authors have proposed a MOEA (NSGA-III) that generates a set of uniformly distributed reference points and the nondominated solutions with least perpendicular distance from the reference points are given priority so as to maintain diversity. In other words, in NSGA-III, preservation of diversity mechanism is aided by a set of reference points that are well distributed. In the environmental selection of NSGA-III, each individual of the combined population is associated with a reference point and the solutions with the least distance to the reference points will be selected [8].

In [35], the authors have introduced a diversity mechanism, shift-based density estimation (SDE) metrics that aims to maintain diversity without the loss of convergence. The basic idea in SDE is to estimate the density of a candidate solution, by shifting the positions of other individuals with respect to the convergence comparison on each objective. As a result, the individuals with poor convergence performance will have high-density values [35]. In [7], Pareto-dominance based MOEA with ranking methods was proposed in which each candidate solution in the population is assigned a rank referred to as priority rank. The priority rank is assigned to each candidate solution in accordance to – a) Pareto rank of solutions that is obtained through nondominated sorting; b) sum of normalized objectives or sum of average ranks; and c) niche radius. The individuals with the least priority rank are preferred during the mating and environmental selection as they commendably accelerate the convergence towards the Pareto front and simultaneously maintains diversity. In [41], an extension to this work was reported, in which both the average rank and sum of normalized objectives are incorporated into a unique framework of PDMOEA to exploit mutual advantages of sum of normalized objectives and sum of average ranks on a common platform.

In [36], the authors have proposed a vector angle-based EA for MaOPs that employs Pareto-dominance in combination with the maximum-vector-angle-first principle. Along with these metrics, a worse-elimination principle was also adopted which replaces the worse solutions in accordance to convergence (determined by the sum of normalized objectives) conditionally with the other candidate solutions. On the other hand, the maximum-vector-angle-first principle concentrates on achieving the distribution of the solution set [36]. In [37], based on favorable convergence (FC) and directional diversity (DD), a many-objective evolutionary algorithm (MOEA-DDFC) was proposed. In MOEA-DDFC, favorable convergence and directional diversity were incorporated to promote both the convergence and diversity simultaneously. In the mating selection, along with Pareto-dominance,

favorable convergence was adopted to generate offspring population from the well-converged individuals [37]. In the environmental selection, after the Pareto-dominance, FC and DD are combined in a tournament like manner in order to provide good trade-off between convergence and diversity [37].

3) COMBINING DOMINANCE AND DECOMPOSITION-BASED APPROACHES

The third category focus on combining the dominance based approaches with the decomposition-based algorithms. In [38], the authors have proposed a unified paradigm in which the dominance-based approach is combined with the decomposition-based approach to tackle the MaOPs (MOEA-DD). In MOEA-DD, a set of weights are used to segregate the objective space into different sub-regions and each weight vector defines a sub-problem for fitness evaluation [38]. However, the parent population is updated through a steady-state evolution where at a time only one offspring individual is considered. In the steady-state evolution, efficient non-domination level update approach [42] was adopted to update the parent population's nondomination level structure after inclusion of offspring solution. In [39], a bi-criterion evolution (BCE) framework was proposed that combines the Pareto-criterion (PC) methods with non Pareto-criterion (NPC) approaches to enhance the convergence and diversity. The bi-criterion approach attempts to combine the advantages of PC and NPC approaches through a collaborative manner with ample exchange of information to facilitate each other's evolution. On the other hand, the bi-criterion evolution tries to compensate for each other's weaknesses [39].

III. PROPOSED ad-MOEA

In this section, a detailed explanation of the general framework of ad-MOEA is presented. From the literature, it is evident that sum of normalized objectives improves the converging capabilities of the algorithm. Hence, in the proposed approach, we aim to utilize the advantage provided by sum of the normalized objectives by incorporating in mating and environmental selection along with Pareto-dominance and crowding distance to balance the convergence and diversity adaptively.

A. GENERAL FRAMEWORK OF PROPOSED ad-MOEA

The general framework of the proposed algorithm is similar to classical NSGA-II [12]. In proposed approach, an initial parent population of size N is randomly generated and evaluated. After Initialization, Pareto rank and sum of the normalized objectives (*SoNB*) as described in section 2 and crowding distance (*CWD*) are obtained. Then, based on these criteria a weighted rank for each candidate solution is calculated. Then mating selection procedure is performed where two parent solutions are randomly selected and compared based on the weighted rank, and one solution with least weighted rank is chosen for the offspring generation. After mating selection, an offspring population of size N is generated with the help of mutation and recombination operators. The offspring population is then combined with the parent population and for each individual in the combined population, Pareto rank and sum of the normalized objectives (*SoNB*) for each candidate solution are obtained. In the environmental selection procedure, after Pareto-dominance, the individuals are chosen until the critical front.

In the critical front, with a certain probability, candidate solutions are selected based on sum of objectives and the remaining candidate solutions will be selected based on crowding distance. The probability for selecting the candidate solutions is determined by a self-adaptive parameter 'w'. Initially, the value of the parameter 'w' is set to one, as convergence is necessary in the initial generations. Then depending on the characteristics of the problem, the value of parameter 'w' is self-adapted. However, the value of 'w' is expected to decrease as the evolution reaches the final stages where diversity needs to be enhanced. Therefore, in initial generations more preference is assigned to the individuals that promote convergence and as the evolution progress, the focus adaptively shift towards the individuals that are diverse. The framework of the proposed approach is depicted in the figure 1.

Algorithm 1 Mating_Selection (P, F, *SoNB*, *CWD*)

Require: P (population)
Calculate Weighted Rank (*WR*) for each individual
$S \leftarrow \emptyset$
while $|S| < N$ **do**
randomly choose x and y from P
if $WR(x) \prec WR(y)$ **then**
    $S \leftarrow S \cup \{x\}$
else if $WR(y) \prec WR(x)$ **then**
    $S \leftarrow S \cup \{y\}$
else
if *rand (1)* < 0.5 **then**
    $S \leftarrow S \cup \{x\}$
else
    $S \leftarrow S \cup \{y\}$
end if
end if
end while
return S

B. MATING SELECTION

The mating selection procedure plays an important role in the evolution of the MOEAs; as producing, the promising offspring solutions would improve the performance of the algorithms over the generations. In the proposed ad-MOEA, the concept of weighted rank is employed in the mating selection to prioritize the better solutions to generate offspring population. The procedure of the mating selection is depicted in the algorithm 1. To obtain the weighted rank, at first, individuals are sorted based on the Pareto rank and sum of normalized objectives in ascending order, and each individual is assigned with a rank in accordance to the sorted order.

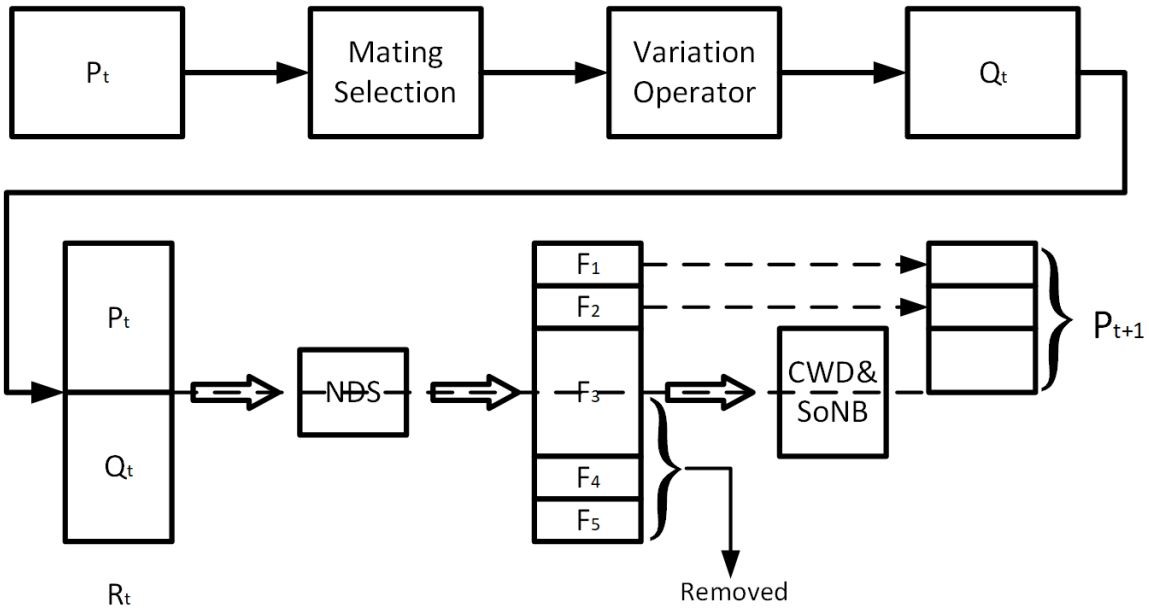

FIGURE 1. Framework of proposed NSGA-II$^+$ (# SoNB – Sum of Normalized Objectives & # CWD – Crowding Distance).

TABLE 1. Example to demonstrate the sorting procedure in mating selection.

#	*PR*	*SoNB*	*CWD*	Instance #1 Sorting based on *PR* and *SoNB*			Instance #2 Sorting based on *PR* and *CWD*		
				PR	*SoNB*	Sorted order	*PR*	*CWD*	Sorted order
S_1	2	0.11	0.48	1	0.09	S_2	1	0.77	S_5
S_2	1	0.09	0.28	1	0.13	S_8	1	0.69	S_{10}
S_3	1	0.74	0.37	1	0.74	S_3	1	0.56	S_8
S_4	3	0.29	0.61	1	0.8	S_{10}	1	0.37	S_3
S_5	1	0.9	0.77	1	0.9	S_5	1	0.28	S_2
S_6	3	0.42	0.95	2	0.11	S_1	2	0.78	S_7
S_7	2	0.56	0.78	2	0.48	S_9	2	0.65	S_9
S_8	1	0.13	0.56	2	0.56	S_7	2	0.48	S_1
S_9	2	0.48	0.65	3	0.29	S_4	3	0.95	S_6
S_{10}	1	0.8	0.69	3	0.42	S_6	3	0.31	S_4

#	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Assigned Rank with respect to Instance #1 (r_1)	6	1	3	9	5	10	8	2	7	4
Assigned Rank with respect to Instance #2 (r_2)	8	5	4	10	1	9	6	3	7	2

 # *PR*—Pareto Rank; # *SoNB*—Sum of Normalized Objectives; # *CWD*—Crowding Distance

The individuals are sorted again according to the Pareto rank and crowding distance in ascending and descending order respectively and another rank is assigned to each individual. The sorting procedure for each individual is demonstrated with the example presented in the Table 1.

In the Table 1, we presented an example that considers the Pareto rank, sum of normalized objectives and crowding distance of each individual. As shown in the Table 1, the solutions S2, S3, S5, S8 and S10 belong to the first S10 belong to the first nondominated front and has Pareto rank '1', the solutions S1, S7 and S9 belong to second front and has Pareto rank '2' and finally, the solutions S4 and S6 belongs to third front and has Pareto rank '3'. From Table 1, we can observe that the sorted order of solutions according to the Pareto rank and sum of normalized objectives is S2, S8, S3, S10, S5, S1, S9, S7, S4 and S6. Then, each solution is assigned with a rank with respect to the sorted order of Pareto rank and sum of normalized objectives. Similarly, the solutions are again sorted according to Pareto rank in ascending order and crowding distance in descending order respectively. From the Table 1, we can observe that the sorted order of solutions according to the Pareto rank and crowding distance is S_5, S_{10}, S_8, S_3, S_2, S_7, S_9, S_1, S_4 and S_6. The main motivation behind sorting the solutions based on Pareto rank and sum of normalized objectives in one instance and Pareto rank and crowding distance is to assign preferences to each solutions according

to convergence and diversity. For example, from Table 1, we can that the solutions S_2, S_3, S_5, S_8 and S_{10} has Pareto rank '1', and hence to establish preferences between them, we sorted the solutions according to both convergence and diversity criteria. Then, the weighted rank for each candidate solution is determined with the help of parameter 'w'. The parameter 'w' is self-adaptive and is communicated through the environmental selection. The process for calculating the weighted rank is depicted in algorithm 2.

Algorithm 2 Weighted Rank

Input: F (Front Number), *SoNB* (Sum of normalized objectives), *CWD* (Crowding Distance)
Output: Weighted Rank *(WR)*
1: Sort the solutions based on F and *SoNB* in ascending order
2: Assign rank to each individual according to the sorted order.
3: Sort the solutions based on F and *CWD* in ascending order and descending order respectively.
4: Assign another rank to each individual according to the sorted order
5: Let a solution i, have ranks r_1 and r_2 corresponding to case 1 and case 2 and w be the weight.
6: weighted rank of the solution i is calculated as

$$WR(i) = r_1 * (1 - w) + r_2 * w$$

7: **return** ***WR***

Weighted Rank: To calculate the weighted rank, let us assume the rank obtained for a solution i with respect to sorted order of Pareto rank and sum of normalized objectives be r_1. The rank obtained in accordance with the sorted order of Pareto rank and crowding distance be r_2. Then the weighted rank for the solution, i can be calculated as

$$WR(i) = r_1 * (1 - w) + r_2 * w \tag{4}$$

The main aim of the weighted rank is to concentrate more on convergence in the initial stages and to promote diversity as the evolution progress. The self-adaptive parameter 'w' assists in promoting the convergence in early stages and diversity in the later stages. The process for obtaining the weighted rank is depicted in the algorithm 2.

After calculating the weighted rank, to generate the offspring population, the solutions with least weighted rank are preferred. A binary tournament selection procedure is adopted in the mating selection where weighted rank is used as a metric to select parent population for generating offspring population. In other words, two individuals A and B are randomly chosen from the parent population. If weighted rank of solution A is less than that of solution B then solution, then solution A is chosen. If both the solutions A and B have same weighted rank, then one solution is chosen is randomly.

C. ENVIRONMENTAL SELECTION

The main aim of the environmental selection is to preserve the elite solutions for the next iterations as parent population. In the environmental selection, at first, the offspring population obtained after mating selection is combined with the parent population. Then Pareto-dominance procedure is adopted on the combined population and the individuals are assigned to different nondominated fronts. Along with the Pareto-dominance, sum of the normalized objectives and crowding distance for each candidate solution is obtained. At first, the individuals from the first nondominated front (F_1) are selected and if ($|F_1| > N$), then the front is considered as critical front and the solutions in F_1 are selected based on the adaptive approach that consider sum of normalized objectives (*SoNB*) and the crowding distance (*CWD*). If the size of solution of individuals in the front (F_1) is less than N, ($|F_1| < N$), then, solutions are selected based from second nondominated front F_2. If ($|F_1 \cup F_2| > N$), then front $(F_2)_{\text{is}}$ considered as critical front and adaptive approach is employed to select the solutions in the second nondominated front (F_2). The same procedure that is followed until a population of size N with elitist solutions is obtained for the following generations.

Adapting the Weight Parameter (w) Based on Sum of Normalized Objectives and Crowding Distance:

In the critical front, to select the candidate solutions, an adaptive approach is adopted in this work. To select the individuals in the critical front, the parameter 'w' is employed which is self-adaptive. To adapt the parameter, the information regarding the number of nondominated solutions in the combined population is utilized. The parameter 'w' is self-adopted as follows

$$w_t = (0.99^* w_{t-1}) + 0.01^* \left(1 - \left(\frac{N_F}{N}\right)^{\frac{1}{M}}\right) \tag{5}$$

where w_t denotes the current generation value of 'w' and w_{t-1} represents the value of 'w' in previous generation and M is number of objectives. N_F denotes number of nondominated solutions in the population and N denotes population size.

At first, all the solutions in the critical front are sorted based on the sum of the normalized objectives and a percentage (w * *100*) of required solutions are chosen based on sum of the normalized objectives. For the remaining solutions, we obtain crowding distance and the remaining solutions are chosen based on crowding distance. In other words, let k be the size of solutions in critical front and l be the size of solutions to be selected from the critical front. At first, the k solutions in the critical front are sorted according to the sum of normalized objectives in ascending order and (w * *100*)% of l solutions are selected according to the sorted order. Let the size of solutions selected based on sum of objectives be t. The selected solutions are ignored and for the remaining solutions of size (k - t), crowding distance is determined. The remaining (k - t) solutions are sorted according to crowding distance in descending order and the required (l - t) solutions be selected based on the sorted order. During the evolution of MOEA, the parameter 'w' plays a key role in environmental selection in adaptively switching

Algorithm 3 Environmental Selection

Input: N (Population size), Q (Combined Population)
Output: P (Population)
1: $(F_1, F_2...F_L) \leftarrow Nondominated\_sort\ (Q)$
2: $P \leftarrow \emptyset, i \leftarrow 1$
3: Calculate sum of normalized objectives (*SoNB*) and crowding distance (*CWD*) for each individual
4: **while** $|P \cup F_i| < N$ **do**
5: $P \leftarrow P \cup F_i, i \leftarrow i + 1$
6: **end while**
7: **if** $|P \cup F_i| = N$ **then**
8: $P \leftarrow P \cup F_i$
9: **else**
10: $P \leftarrow P \cup F_{i-1}$
11: Adaptively determine the parameter 'w'
12: $K \leftarrow (N - |P \cup F_{i-1}|)$ // * Solutions to be selected from last front F_i*//
13: Sort the solutions in front F_i based on *SoNB* in ascending order.
14: Select *((w*100)%)* of K solutions in the sorted order of *SoNB*
16: Determine the crowding distance for the remaining solutions
15: Sort the remaining solutions based on *CWD* in descending order
16: Select *((1-w)*K)* solutions in the sorted order of *CWD*
17: **end if**
18: **return** ***P***

the focus from the convergence to diversity. The procedure for the environmental selection is presented in the algorithm.

IV. EXPERIMENTS SETUP AND DISCUSIONS

In this section, an overview of the experimental design along with the discussion on results is presented which includes the description of algorithms considered for comparison, benchmark problems, general parameters employed and performance evaluation metrics. The simulation results reported in this study are conducted on a PC with 3.40 GHz Intel Core i7-2600QM CPU and Windows 10 SP1 64-bit operating system with MATLAB 2019b version. All the algorithms that are considered for comparison and the proposed approach are simulated with the help of MATLAB software. In this section, to investigate the performance of the proposed ad-MOEA, nine popular MOEAs such as KnEA[1] [6], RVEA [21], S-CDAS[2] [27] NSGA-II [12], r-NSGAII [31], g-NSGAII [32], NSGAII-SDR [34] and NSGA-III [8] and MOEA-D [13] are considered for comparison. The experiments are performed on 26 test problems which includes three benchmark test suites DTLZ [43], WFG [44] and MaOPs [45]. For the DTLZ and WFG benchmark problems 2-, 4-, 6-, 8-, 10- objectives are considered and for MaOP benchmark problems 4-, 6-, 8-, 10- objectives are considered. For a fair comparison, the number of function evaluations allocated for the algorithms under comparison are maintained identical. The population size varies with respect to the objectives. In other words, the population sizes for 2-, 4-, 6-, 8-, 10- objectives are maintained as 100, 120, 132, 156, and 274 respectively. All the algorithms are simulated for 30 times each and the population obtained at the final iteration are saved for comparison. To generate the offspring population, we have employed the simulated binary crossover (SBX) and polynomial mutation as variation operators with distribution indices $n_c = 20$ and $n_m = 20$. The mutation probability and crossover probability used are $p_m = 1/D$ and $p_c = 1.0$ respectively, where D is the number of decision variables. In the MaOP problems, for the KnEA algorithm, the parameter 'T' is set to 0.5 and the rest of the parameters of the algorithms in comparison are maintained according to the original publication.

The DTLZ problem suite consists of seven test problems (DTLZ1-DTLZ7) [43], WFG problem suite consists of nine test problems (WFG1-WFG9) [44], and MaOP problem suite consists of ten test problems (MaOP1- MaOP10) [45] respectively. The DTLZ problem set possess different characteristics and test different capabilities of the MOEAsMainly, the problems, DTLZ1, DTLZ3 and DTLZ6 are multimodal in nature and assess the converging ability of the MOEAs by introducing a large number of local Pareto-optimal fronts. Due to the presence of local Pareto fronts, MOEAs encounter difficulties to converge to the global Pareto-optimal front. The remaining problems such as DTLZ2, DTLZ4-DTLZ7 assess the capability of MOEAs in tackling the problems with different shapes. The WFG benchmark problems are associated with the Pareto-optimal fronts that possess characteristics such as convex, concave, linear, multi-modal, degenerated, biased, and disconnected; and therefore test the different capabilities of MOEAs. The MaOP problem test suite introduces the problems with difficult features such as complicated Pareto-set, objective scalability, disconnected, biased and degeneracy to test the performance of MOEAs. The characteristics and settings for the benchmark problems along with the number of iterations adopted for each problem are presented in the Table 2. In the current work, we have employed the hypervolume indicator (HV) [46] to evaluate the performance of the algorithms. The hypervolume indicator have the ability to evaluate the converging abilities and diversity performances of the MOEAs. To calculate the hypervolume indicator, a reference point is required.

In our experimental setup, we have adopted different reference point techniques for the three-benchmark problem suites, DTLZ, WFG and MaOP. For the DTLZ problems, the final obtained populations of each individual run for all the algorithms that are considered for comparison on a given test instance are combined. Then, from the combined set, we identified the nondominated solutions and the maximum

[1]The code for KnEA and RVEA are downloaded from http://www.soft-computing.de/jin-pub_year.html

[2]The code for S-CDAS, NSGA-II, r-NSGAII, g-NSGAII, NSGAII-SDR, NSGAIII and MOEA-D are downloaded from https://github.com/BIMK/PlatEMO

TABLE 2. Characterisitcs and settings of benchmark problem suites.

Characteristics and settings of DTLZ problem suite					
Problem	M	Parameter K	Number of Variables (D)	Generations	Characteristics
DTLZ1	2, 4, 6, 8, 10	5	$M-1+k$	700	Linear, Multimodal
DTLZ2	2, 4, 6, 8, 10	10	$M-1+k$	250	Concave
DTLZ3	2, 4, 6, 8, 10	10	$M-1+k$	1000	Concave, Multimodal
DTLZ4	2, 4, 6, 8, 10	10	$M-1+k$	250	Concave, Biased
DTLZ5	2, 4, 6, 8, 10	10	$M-1+k$	250	Degenerate
DTLZ6	2, 4, 6, 8, 10	10	$M-1+k$	250	Multi-modal, Degenerate, Disconnected
DTLZ7	2, 4, 6, 8, 10	20	$M-1+k$	250	Mixed, Disconnected, Multimodal, Scaled

Characteristics and settings of WFG problem suite						
Problem	M	Parameter		Number of Variables (D)	Generations	Characteristics
		K	L			
WFG1	2, 4, 6, 8, 10	*For $M=2$; $k=4$*	10	$l+k$	1000	Mixed, Biased, Scaled
WFG2	2, 4, 6, 8, 10		10	$l+k$	250	Convex, Disconnected, Multi-modal, Non-separable, Scaled
WFG3	2, 4, 6, 8, 10	*For $M=4$; $k=6$*	10	$l+k$	700	Linear, Degenerate, Non-separable, Scaled
WFG4	2, 4, 6, 8, 10		10	$l+k$	250	Concave, Multi-modal, Scaled
WFG5	2, 4, 6, 8, 10	*For $M=6$; $k=10$*	10	$l+k$	250	Concave, Deceptive, Scaled
WFG6	2, 4, 6, 8, 10		10	$l+k$	250	Concave, Non-separable, Scaled
WFG7	2, 4, 6, 8, 10	*For $M=8$; $k=7$*	10	$l+k$	250	Concave, Biased, Scaled
WFG8	2, 4, 6, 8, 10		10	$l+k$	250	Concave, Biased, Non-separable, Scaled
WFG9	2, 4, 6, 8, 10	*For $M=10$; $k=9$*	10	$l+k$	250	Concave, Biased, Multi-modal, Deceptive, Non-separable, Scaled

Characteristics and settings of MaOP problem suite				
Problem	M	Number of Variables (D)	Generations	Characteristics
MaOP1	4, 6, 8, 10	20	250*M	Inverse of simplex, Objective scales, Multi-modal
MaOP2	4, 6, 8, 10	20	250*M	Complicated PS
MaOP3	4, 6, 8, 10	20	250*M	Complicated PS, Biased
MaOP4	4, 6, 8, 10	20	250*M	Complicated PS, Biased
MaOP5	4, 6, 8, 10	20	250*M	Complicated PS, Degeneracy
MaOP6	4, 6, 8, 10	20	250*M	Complicated PS, Degeneracy
MaOP7	4, 6, 8, 10	20	250*M	Complicated PS, Local degeneracy
MaOP8	4, 6, 8, 10	20	250*M	Complicated PS, Local degeneracy
MaOP9	4, 6, 8, 10	20	250*M	Complicated PS, Local degeneracy
MaOP10	4, 6, 8, 10	20	250*M	Complicated PS, Local degeneracy

values in each objective are considered as reference point. For the WFG and MaOP problem suites, the range of each objective varies and hence each objective is normalized into a uniform range [0,1] and the reference point considered for WFG and MaOP is $(1, 1... 1)^M$. To normalize the objective values of WFG and MaOP problems, approximated ideal point (z^{ideal}) and approximated nadir point (z^{nadir}) are obtained from the nondominated solution set of the populations combined from each algorithm on a given test instance. The ideal point, z^{ideal} and the nadir point, z^{nadir} are the minimum and maximum values in each objective of the obtained nondominated solution set. To approximate the hypervolume indicator, Monte Carlo method is employed where 1,000,000 sampling points are used. The algorithm with larger hypervolume is referred

TABLE 3. Mean and standard deviation values of Hypervolume results of proposed method with state of art algorithms on DTLZ problems.

#	M	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]	ad-MOEA
DTLZ1	2	0.4900(0.0065)+	0.4946(0.0006)-	0.4934(0.0007)=	0.4933(0.0006)=	0.4935(0.0008)=	0.4931(0.0008)=	0.2042(0.0712)+	0.4948(0.0004)-	0.0000(0.0000)+	0.4927(0.0007)
	4	0.6415(0.1292)+	0.9123(0.0004)-	0.8644(0.1020)+	0.7437(0.2704)+	0.8372(0.1798)+	0.0000(0.0000)+	0.8492(0.0237)+	0.9121(0.0005)-	0.0225(0.0686)+	0.8844(0.0047)
	6	0.5193(0.1018)+	0.9806(0.0002)-	0.1097(0.2322)+	0.1671(0.2962)+	0.1751(0.2967)+	0.0000(0.0000)+	0.8749(0.0399)+	0.9803(0.0003)=	0.0000(0.0000)+	0.9647(0.0020)
	8	0.3259(0.1058)+	0.9948(0.0001)-	0.0000(0.0000)+	0.0000(0.0000)+	0.0000(0.0000)+	0.0000(0.0000)+	0.9021(0.0291)+	0.9405(0.1728)+	0.0000(0.0000)+	0.9554(0.0183)
	10	0.5169(0.2190)+	0.9990(0.0000)+	0.0000(0.0000)+	0.0000(0.0000)+	0.0000(0.0000)+	0.0000(0.0000)+	0.9244(0.0243)-	0.9216(0.1697)-	0.0000(0.0000)+	0.9026(0.0438)
DTLZ2	2	0.1723(0.0151)+	0.2091(0.0010)-	0.2094(0.0005)-	0.2093(0.0004)-	0.1055(0.0053)+	0.1702(0.0003)+	0.0102(0.0163)+	0.2094(0.0007)-	0.2055(0.0032)+	0.2075(0.0005)
	4	0.5594(0.0044)-	0.5886(0.0005)-	0.4812(0.0094)+	0.4802(0.0101)+	0.0746(0.0012)+	0.0483(0.0108)+	0.1071(0.1167)+	0.5875(0.0009)-	0.4593(0.0169)+	0.4870(0.0127)
	6	0.9328(0.0024)-	0.9434(0.0002)-	0.3403(0.1082)+	0.3808(0.1494)+	0.0749(0.0045)+	0.0038(0.0030)+	0.8637(0.0744)+	0.9427(0.0003)-	0.9023(0.0152)-	0.8872(0.0083)
	8	0.9980(0.0002)-	0.9984(0.0000)-	0.4076(0.1224)+	0.4124(0.1098)+	0.0710(0.0061)+	0.0084(0.0026)+	0.9974(0.0022)-	0.9980(0.0008)-	0.9969(0.0003)-	0.9674(0.0165)
	10	0.9990(0.0007)-	1.0000(0.0000)-	0.6583(0.0768)+	0.6620(0.0977)+	0.0792(0.0075)+	0.0162(0.0023)+	1.0000(0.0000)-	0.9999(0.0001)-	0.9999(0.0000)-	0.9613(0.0205)
DTLZ3	2	0.0565(0.0097)+	0.2081(0.0014)=	0.2084(0.0011)=	0.2082(0.0014)=	0.2081(0.0014)=	0.1439(0.0588)+	0.0254(0.0238)+	0.2080(0.0014)=	0.0000(0.0000)+	0.2079(0.0008)
	4	0.4427(0.0835)+	0.6243(0.0052)-	0.5226(0.0995)+	0.5417(0.0159)=	0.5464(0.0122)=	0.0000(0.0000)+	0.1509(0.0213)+	0.6124(0.0653)-	0.0000(0.0000)+	0.5469(0.0100)
	6	0.9967(0.0032)+	1.0000(0.0000)-	0.8351(0.1371)+	0.8402(0.1622)+	0.8204(0.2968)+	0.0000(0.0000)+	1.0000(0.0000)-	1.0000(0.0000)-	0.3042(0.2810)+	0.9996(0.0009)
	8	0.8073(0.3770)+	1.0000(0.0000)-	0.5108(0.1472)+	0.5677(0.1486)+	0.8276(0.3309)+	0.0000(0.0000)+	1.0000(0.0000)-	1.0000(0.0000)-	0.9993(0.0019)+	0.9999(0.0003)
	10	0.9998(0.0010)-	1.0000(0.0000)-	0.4789(0.1014)+	0.4777(0.1071)+	0.7752(0.3721)-	0.0000(0.0000)+	1.0000(0.0000)-	1.0000(0.0000)-	1.0000(0.0001)-	0.9993(0.0006)
DTLZ4	2	0.1583(0.0807)=	0.2095(0.0009)-	0.1326(0.1026)+	0.1396(0.1004)+	0.1137(0.0460)+	0.1646(0.0311)-	0.1250(0.0626)+	0.1371(0.0999)+	0.1805(0.0621)-	0.1453(0.0967)
	4	0.5780(0.0053)-	0.6022(0.0005)-	0.5033(0.0067)+	0.4893(0.0927)+	0.1116(0.0062)+	0.1118(0.1346)+	0.0965(0.0721)+	0.4622(0.1201)+	0.4778(0.0701)+	0.5185(0.0068)
	6	0.9934(0.0003)-	0.9930(0.0047)-	0.7556(0.1469)+	0.7126(0.1329)+	0.2210(0.0100)+	0.6082(0.1082)+	0.8613(0.1475)+	0.9824(0.0118)+	0.9874(0.0103)+	0.9885(0.0017)
	8	1.0000(0.0000)-	1.0000(0.0000)-	0.8924(0.0323)+	0.9014(0.0353)+	0.4649(0.0163)+	0.9346(0.0187)+	0.9946(0.0142)+	1.0000(0.0000)-	1.0000(0.0000)-	0.9996(0.0002)
	10	1.0000(0.0000)-	1.0000(0.0000)-	0.9507(0.0171)+	0.9492(0.0105)+	0.4248(0.0157)+	0.9659(0.0193)+	0.9996(0.0013)=	1.0000(0.0000)-	1.0000(0.0000)-	0.9989(0.0009)
DTLZ5	2	0.1717(0.0155)+	0.2090(0.0009)-	0.2094(0.0005)-	0.2094(0.0005)-	0.1039(0.0050)+	0.1703(0.0004)+	0.0110(0.0152)+	0.2093(0.0021)-	0.2057(0.0012)+	0.2076(0.0005)
	4	0.7675(0.0052)+	0.7545(0.0030)+	0.7794(0.0008)+	0.7795(0.0016)+	0.1343(0.0083)+	0.0861(0.0002)+	0.7762(0.0014)+	0.7722(0.0018)+	0.7588(0.0282)+	0.7809(0.0014)
	6	0.8662(0.0040)-	0.8442(0.0090)+	0.8391(0.0083)+	0.8399(0.0083)+	0.3159(0.0187)+	0.0440(0.0126)+	0.8569(0.0048)=	0.8353(0.0074)+	0.8326(0.0133)+	0.8592(0.0042)
	8	0.8760(0.0037)-	0.8396(0.0086)+	0.8224(0.0173)+	0.8274(0.0136)+	0.3567(0.0242)+	0.0189(0.0046)+	0.8749(0.0037)-	0.8523(0.0072)+	0.8452(0.0155)+	0.8565(0.0068)
	10	0.8800(0.0032)-	0.8564(0.0080)=	0.8341(0.0131)+	0.8292(0.0165)+	0.3621(0.0439)+	0.0069(0.0012)+	0.8827(0.0037)+	0.8768(0.0061)-	0.8694(0.0082)-	0.8594(0.0077)
DTLZ6	2	0.1263(0.0135)+	0.2101(0.0004)-	0.2089(0.0004)-	0.2091(0.0005)-	0.1875(0.0077)+	0.1703(0.0004)+	0.1925(0.0224)+	0.1890(0.0123)+	0.1997(0.0377)+	0.2078(0.0004)
	4	0.9284(0.0079)+	0.9282(0.0025)+	0.8973(0.0433)+	0.8925(0.0560)+	0.2310(0.0059)+	0.0359(0.0189)+	0.9287(0.0011)+	0.9355(0.0005)-	0.9116(0.0259)+	0.9323(0.0011)
	6	0.9872(0.0029)-	0.9792(0.0033)+	0.5763(0.0773)+	0.5485(0.0653)+	0.5889(0.0867)+	0.0692(0.0186)+	0.9726(0.0032)+	0.9868(0.0028)-	0.9672(0.0085)+	0.9815(0.0023)
	8	0.9884(0.0024)-	0.9831(0.0027)+	0.5503(0.0469)+	0.5270(0.0460)+	0.6124(0.1784)+	0.0831(0.0148)+	0.9822(0.0018)=	0.9888(0.0027)-	0.9781(0.0064)+	0.9829(0.0027)
	10	0.9897(0.0012)-	0.9827(0.0121)=	0.5789 (0.0417)+	0.5918(0.0511)+	0.6674(0.1649)+	0.0653(0.0112)+	0.9835(0.0012)+	0.9890(0.0015)=	0.9845(0.0028)=	0.9853(0.0024)
DTLZ7	2	0.1401(0.0007)=	0.1335(0.0022)+	0.1414(0.0004)=	0.1398(0.0090)=	0.0343(0.0117)+	0.0829(0.0003)+	0.1404(0.0005)=	0.1410(0.0003)=	0.0968(0.0422)+	0.1393(0.0088)
	4	0.1917(0.0090)-	0.1643(0.0040)-	0.1567(0.0043)+	0.1552(0.0060)+	0.0061(0.0036)+	0.0218(0.0008)+	0.1866(0.0028)-	0.1880(0.0020)-	0.0713(0.0380)+	0.1619(0.0042)
	6	0.1746(0.0108)-	0.1081(0.0202)=	0.0393(0.0107)+	0.0409(0.0130)+	0.0012(0.0002)+	0.0025(0.0005)+	0.1694(0.0037)-	0.1413(0.0105)-	0.0153(0.0177)+	0.1141(0.0079)
	8	0.0696(0.0225)-	0.0762(0.0142)-	0.0005(0.0005)+	0.0008(0.0009)+	0.0001(0.0000)+	0.0000(0.0000)+	0.1191(0.0080)-	0.0995(0.0215)-	0.0023(0.0044)+	0.0574(0.0093)
	10	0.2505(0.1593)+	0.6033(0.0055)-	0.0240(0.0092)+	0.0230(0.0115)+	0.0000(0.0000)+	0.0001(0.0000)+	0.5853(0.1530)+	0.4612(0.1259)+	0.0062 (0.0129)+	0.5968(0.0032)
+/=/-		15/2/18	8/4/23	29/3/3	28/4/3	31/3/1	33/1/1	21/4/10	9/4/22	26/1/8	

as the better performing algorithm when compared with other algorithms. In addition, to assess the statistical significance between two algorithms the Wilcoxon's rank sum test is adopted.

A. COMPARISON OF ALGORITHMIC PERFORMANCE ON DTLZ PROBLEMS

In this section, we have presented the experimental results of the ad-MOEA with state-of-art algorithms on the DTLZ problems. We have reported the mean and standard deviation results of hypervolume indicator obtained for the algorithms in comparison in Table 3. We have also conducted the significance tests for each algorithm and presented the comparison of proposed approach with the algorithms under consideration with the help of signs ('+', '=', '−'). The sign '+' indicates that the proposed approach is better than the corresponding algorithm, the sign '=' indicates that the proposed ad-MOEA performs significantly equivalent to the corresponding algorithm and the sign '-' indicates that the proposed approach is performing worse than the corresponding algorithm respectively on a specific test instance. From the results presented in Table 3, we can witness that the ad-MOEA when compared with KnEA, out of 35 test instances of DTLZ problems, performs better than KnEA in 15 instances and equal performance in two instances and worse performance in 18 instances. The proposed approach outperforms KnEA in the problems DTLZ1 and DTLZ3, performs competitive in DTLZ5 and DTLZ6 problems, and performs worse in DTLZ2, DTLZ4 and DTLZ7 problems. However, in KnEA, the parameter that controls the number of knee points needs to be tuned for each problems depending on the characteristics of the problem and number of objectives.

The ad-MOEA algorithm is outperformed by the RVEA algorithm in terms of DTLZ problems as the proposed approach performs better, equal and worse than RVEA in eight, four and 23 instances respectively. RVEA algorithm performing consistently better in the DTLZ1, DTLZ2, DTLZ3, and DTLZ4 problems when compared to the ad-MOEA, worse on DTLZ5 and DTLZ6 and competitively on DTLZ7 problems. The performance of the ad-MOEA is consistently better when compared with the algorithms S-CDAS, NSGA-II, r-NSGAII and g-NSGAII. The proposed approach performs better, identical and worse in 29, three and three instances respectively when compared with the S-CDAS. Except for the 2-objective case, the better performance of ad-MOEA can be observed in all the DTLZ problems when compared with the S-CDAS. When compared with the NSGA-II, ad-MOEA performs better in 28 instances and equal performance in four instances and worse performance in three instances. Similar to the algorithm S-CDAS, the NSGA-II performs better in the 2-objective case and performs worse in the remaining for the entire DTLZ problem suite.

The performance of the r-NSGAII and g-NSGAII are even worse when compared to the proposed approach with worse performance in 31 and 33 instances and equal performance in three and one case and better performance in one case each respectively. From the results presented in the Table 3, we can notice a dominating performance of the Ad-MOEA in all DTLZ problems when compared with the r-NSGAII and g-NSGAII. When compared with the algorithm, NSGAII-SDR, the proposed Ad-MOEA, performs

TABLE 4. Mean and standard deviation values of Hypervolume results of proposed method with state of art algorithms on WFG problems.

#	M	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]	ad-MOEA
WFG1	2	0.5746(0.0083)+	0.6170(0.0006)-	0.6162(0.0005)=	0.6161(0.0006)=	0.4050(0.0052)+	0.6161(0.0008)=	0.6141(0.0007)+	0.6168(0.0014)=	0.6057(0.0056)+	0.6163(0.0006)
	4	0.9695(0.0043)=	0.9829(0.0003)-	0.9724(0.0024)=	0.9732(0.0019)=	0.7888(0.0768)+	0.9727(0.0025)=	0.9576(0.0040)+	0.9646(0.0491)+	0.9755(0.0028)-	0.9703(0.0058)
	6	0.9864(0.0025)+	0.9932(0.0016)+	0.9953(0.0007)-	0.9953(0.0005)-	0.9691(0.0064)+	0.9953(0.0006)-	0.9734(0.0054)+	0.9242(0.0833)+	0.9959(0.0006)-	0.9949(0.0006)
	8	0.9879(0.0027)+	0.9916(0.0027)+	0.9987(0.0001)-	0.9986(0.0002)-	0.9888(0.0039)-	0.9986(0.0002)-	0.9542(0.0279)+	0.8615(0.0984)+	0.9976(0.0004)+	0.9982(0.0005)
	10	0.9928(0.0026)+	0.9912 (0.0021)+	0.9991(0.0001)-	0.9991(0.0001)-	0.9975(0.0009)+	0.9992(0.0001)-	0.9654(0.0201)+	0.8714(0.1068)+	0.9985(0.0002)+	0.9988(0.0002)
WFG2	2	0.4505(0.0006)-	0.4426(0.0025)+	0.4503(0.0006)-	0.4502(0.0004)-	0.0449(0.0140)+	0.4502(0.0005)-	0.4475(0.0010)+	0.4350(0.0149)+	0.1310(0.0651)+	0.4497(0.0005)
	4	0.5456(0.0231)+	0.3951(0.1256)+	0.5538(0.0163)+	0.5536(0.0191)+	0.5352(0.0370)+	0.5567(0.0164)+	0.3692(0.0487)+	0.5638(0.0217)=	0.0000(0.0000)+	0.5631(0.0194)
	6	0.4756(0.0305)+	0.3592 (0.1808)+	0.5076(0.0324)+	0.5116(0.0203)+	0.5760(0.0286)-	0.5095(0.0306)+	0.3533(0.0344)+	0.5331(0.0193)+	0.0000(0.0000)+	0.5449(0.0119)
	8	0.5035(0.0317)+	0.3978(0.1693)+	0.5398(0.0232)+	0.5390 (0.0181)+	0.6511(0.0030)-	0.5272 (0.0390)+	0.4773(0.0245)+	0.6014(0.0368)=	0.0000(0.0000)+	0.5812(0.0097)
	10	0.5444(0.0465)+	0.5068(0.0335)+	0.5193(0.0202)+	0.5192(0.0248)+	0.6395(0.0029)-	0.5171(0.0180)+	0.4397(0.0176)+	0.5747(0.0143)=	0.0035(0.0172)+	0.5634(0.0123)
WFG3	2	0.4909(0.0036)=	0.4792(0.0033)+	0.4912(0.0007)+	0.4909(0.0009)+	0.0041(0.0015)+	0.4913(0.0007)+	0.4933(0.0006)-	0.4639(0.0114)+	0.3552(0.0432)+	0.4920(0.0006)
	4	0.2594(0.0034)-	0.2244(0.0069)+	0.2570(0.0030)=	0.2570(0.0024)=	0.1125(0.0069)+	0.2569(0.0024)=	0.2516(0.0036)+	0.2053(0.0527)+	0.1487(0.0303)+	0.2574(0.0021)
	6	0.1405(0.0121)+	0.1072(0.0192)+	0.1624(0.0068)+	0.1639(0.0071)+	0.0409(0.0159)+	0.1630(0.0067)+	0.1231(0.0198)+	0.0738(0.0238)+	0.0544(0.0240)+	0.1751(0.0041)
	8	0.0897(0.0112)+	0.0716(0.0212)+	0.1329(0.0088)+	0.1324(0.0084)+	0.0391(0.0095)+	0.1335(0.0077)+	0.0898(0.0246)+	0.0403(0.0197)+	0.0494(0.0239)+	0.1543(0.0051)
	10	0.0751(0.0121)+	0.0483(0.0238)+	0.1218(0.0059)+	0.1230(0.0056)+	0.0403(0.0058)+	0.1239(0.0053)+	0.1032(0.0177)+	0.0023(0.0036)+	0.0609(0.0275)+	0.1439(0.0038)
WFG4	2	0.2534(0.0016)-	0.2373(0.0031)+	0.2530(0.0008)-	0.2524(0.0019)-	0.1393(0.0096)+	0.2527(0.0013)-	0.2539(0.0008)-	0.2505(0.0038)=	0.2226(0.0129)+	0.2519(0.0009)
	4	0.3745(0.0047)-	0.3521(0.0108)-	0.3087(0.0120)+	0.3010(0.0109)+	0.1941(0.0192)+	0.3082(0.0106)+	0.3631(0.0047)-	0.3270(0.0638)=	0.2808(0.0169)+	0.3243(0.0092)
	6	0.3466(0.0144)-	0.2548(0.0293)+	0.2255(0.0120)+	0.2300(0.0106)+	0.1654(0.0601)+	0.2290(0.0156)+	0.3625(0.0075)-	0.2327(0.0875)+	0.2515(0.0248)+	0.2921(0.0147)
	8	0.4186(0.0193)=	0.4176(0.0363)=	0.3459(0.0188)+	0.3387(0.0208)+	0.2104(0.0437)+	0.3351(0.0167)+	0.5410(0.0093)-	0.4474(0.0431)-	0.3605(0.0242)+	0.4176(0.0125)
	10	0.4468(0.0240)-	0.4183(0.0315)=	0.3257(0.0160)+	0.3213(0.0150)+	0.2156(0.0205)+	0.3195(0.0148)+	0.5739(0.0086)-	0.3819(0.1342)+	0.4077(0.0184)+	0.4106(0.0158)
WFG5	2	0.2740(0.0081)+	0.3011(0.0011)+	0.3046(0.0005)-	0.3049(0.0006)-	0.1203(0.0091)+	0.3015(0.0005)+	0.3054(0.0004)-	0.3060(0.0004)-	0.2863(0.0038)+	0.3040(0.0007)
	4	0.2608(0.0042)-	0.2467(0.0022)-	0.2329(0.0098)=	0.2322(0.0065)+	0.0320(0.0054)+	0.0280(0.0006)+	0.2669(0.0025)-	0.2641(0.0036)-	0.1715(0.0171)+	0.2377(0.0111)
	6	0.1549(0.0186)+	0.1931(0.0049)=	0.1776(0.0116)+	0.1824(0.0137)+	0.0000(0.0000)+	0.1797(0.0158)+	0.2715(0.0018)-	0.2423(0.0050)-	0.1454(0.0139)+	0.1935(0.0131)
	8	0.1927(0.0179)=	0.2183(0.0033)-	0.1875(0.0154)+	0.1881(0.0159)+	0.0000(0.0000)+	0.1900(0.0157)=	0.3262(0.0029)-	0.2718(0.0191)-	0.1750(0.0161)+	0.1980(0.0162)
	10	0.1751(0.0221)+	0.2096(0.0047)-	0.1839(0.0172)=	0.1799(0.0137)+	0.0000(0.0000)+	0.1843(0.0148)=	0.3374(0.0025)-	0.2669(0.0276)-	0.1665(0.0143)+	0.1899(0.0148)
WFG6	2	0.3056 (0.0041)-	0.2891(0.0043)+	0.3032(0.0032)=	0.3040(0.0042)=	0.0000(0.0000)+	0.2985(0.0042)+	0.3053(0.0043)=	0.3019(0.0041)=	0.2558(0.0285)+	0.3029(0.0039)
	4	0.2178(0.0213)+	0.2530(0.0132)-	0.2167(0.0218)+	0.2134(0.0218)+	0.0000(0.0000)+	0.2142(0.0165)+	0.2779(0.0122)-	0.2630(0.0291)-	0.1094(0.0165)+	0.2357(0.0141)
	6	0.0924(0.0284)+	0.1772(0.0335)=	0.1226(0.0291)+	0.1270(0.0273)+	0.0000(0.0000)+	0.1226 (0.0280)+	0.2330(0.0251)-	0.2153(0.0386)-	0.0816(0.0145)+	0.1698(0.0214)
	8	0.1049(0.0212)+	0.1602(0.0287)=	0.1183(0.0294)+	0.1190(0.0270)+	0.0000(0.0000)+	0.1250 (0.0324)+	0.2506(0.0241)-	0.2126(0.0283)-	0.0955(0.0243)+	0.1508(0.0347)
	10	0.0823(0.0197)+	0.1520(0.0369)=	0.1300(0.0235)=	0.1139(0.0271)+	0.0000(0.0000)+	0.1142(0.0255)+	0.2348(0.0247)-	0.2049(0.0263)-	0.0944(0.0226)+	0.1346(0.0287)
WFG7	2	0.2804(0.0044)=	0.2766(0.0027)+	0.2856(0.0005)-	0.2858(0.0007)-	0.1653(0.0245)+	0.2859(0.0006)-	0.2862(0.0005)-	0.2151(0.0455)+	0.2763(0.0042)+	0.2814(0.0022)
	4	0.4895(0.0037)-	0.4812(0.0067)-	0.4390(0.0059)=	0.4412(0.0083)=	0.2213(0.0267)+	0.4387(0.0103)=	0.4871(0.0042)-	0.4205(0.0790)+	0.4183(0.0214)+	0.4402(0.0100)
	6	0.5415(0.0058)-	0.5161(0.0212)-	0.4655(0.0093)-	0.4689(0.0103)-	0.3920(0.0242)+	0.4643(0.0077)-	0.5325(0.0068)-	0.4139(0.0846)+	0.4203(0.0274)+	0.4488(0.0210)
	8	0.5789(0.0108)-	0.5631(0.0147)-	0.5070(0.0102)=	0.5094(0.0087)=	0.4664(0.0177)+	0.5068(0.0107)=	0.5576(0.0098)-	0.2823(0.1595)+	0.3752(0.0355)+	0.5046(0.0135)
	10	0.5460(0.0374)=	0.5805(0.0173)-	0.5455(0.0084)=	0.5457(0.0078)=	0.5326(0.0110)+	0.5427(0.0098)=	0.5753(0.0112)-	0.3158(0.1179)+	0.4009(0.0284)+	0.5475(0.0114)
WFG8	2	0.5667(0.0179)=	0.5041(0.0133)+	0.5913(0.0173)-	0.5872(0.0113)-	0.2157(0.0600)+	0.5278(0.0112)+	0.5519(0.0167)+	0.4336(0.0193)+	0.2236(0.0870)+	0.5739(0.0138)
	4	0.0258(0.0186)+	0.0118 (0.0127)+	0.0811(0.0312)=	0.0870(0.0290)=	0.0056(0.0017)+	0.0000(0.0000)+	0.0951(0.0236)=	0.1704(0.0209)-	0.0075(0.0074)+	0.0928(0.0137)
	6	0.0299(0.0113)+	0.0064(0.0191)+	0.0910(0.0260)=	0.0780(0.0175)=	0.0020(0.0020)+	0.0000(0.0000)+	0.1143(0.0131)-	0.1494(0.0140)-	0.0014(0.0019)+	0.0829(0.0120)
	8	0.0353(0.0081)+	0.0713(0.0363)=	0.0383(0.0166)+	0.0425(0.0143)+	0.0000(0.0000)+	0.0000(0.0000)+	0.1367(0.0150)-	0.1511(0.0147)-	0.0057(0.0043)+	0.0780(0.0087)
	10	0.0226(0.0056)+	0.0956(0.0327)-	0.0283(0.0116)+	0.0297(0.0149)+	0.0000(0.0000)+	0.0000(0.0000)+	0.1128(0.0106)-	0.1200(0.0168)-	0.0036(0.0029)+	0.0762(0.0091)
WFG9	2	0.2138(0.0030)=	0.2075(0.0041)+	0.2146(0.0021)=	0.2144 (0.0021)=	0.1243(0.0084)+	0.2146(0.0025)=	0.2161(0.0028)=	0.2161(0.0023)=	0.2015(0.0055)+	0.2144(0.0025)
	4	0.5091(0.0243)-	0.4667(0.0279)-	0.4038(0.0153)+	0.4039(0.0141)+	0.2063(0.0389)+	0.3154(0.0129)+	0.5114(0.0163)-	0.4648(0.0353)-	0.3544(0.0207)+	0.4490(0.0178)
	6	0.5845(0.0331)-	0.4521(0.0416)+	0.3816(0.0282)+	0.3787(0.0211)+	0.2200(0.0383)+	0.2752 (0.0147)+	0.5846(0.0264)-	0.5468(0.0432)-	0.3628(0.0209)+	0.4682(0.0219)
	8	0.6234(0.0585)-	0.5667(0.0694)-	0.4481(0.0186)+	0.4513(0.0249)+	0.3003(0.0811)+	0.3175 (0.0136)+	0.7040(0.0710)-	0.6648(0.0689)-	0.4522(0.0343)+	0.5359(0.0406)
	10	0.6324(0.0916)-	0.5571(0.0912)+	0.5168(0.0195)+	0.5244(0.0189)+	0.4689(0.0462)+	0.3599(0.0172)+	0.6979(0.0256)-	0.7107(0.0546)-	0.5055(0.0334)+	0.5824(0.0320)
+/=/-		22/8/15	24/7/14	23/13/9	26/10/9	41/0/4	29/9/7	15/3/27	19/8/18	43/0/2	

better in 21 instances and equal in four instances and worst in 10 instances. The performance of proposed approach is better in the problems DTLZ1 and DTLZ4 when compared with the NSGAII-SDR and a competitive performance is observed in the problems DTLZ2, DTLZ3, DTLZ5 and DTLZ6. The NSGAII-SDR approach is performs better than the proposed ad-MOEA approach on the DTLZ7 problem. The performance of NSGA-III algorithm is better than the proposed algorithm, ad-MOEA in 22 instances and with equal and worse performance in four and nine instances respectively. NSGA-III algorithm performs consistently better in DTLZ1, DTLZ2, DTLZ3, DTLZ6 and DTLZ7 when compared to the proposed approach. The proposed approach performs better than NSGA-III in DTLZ5 problem and competitive in DTLZ4 problem. The proposed approach ad-MOEA performs better than MOEA-D approach on 26 instances, equal performance in one instance and worse in eight instances. The proposed approach performs better on DTLZ1, DTLZ3, and DTLZ5-DTLZ7 problems and competitive on DTLZ2 and DTLZ4 problems when compared with MOEA-D approach.

B. COMPARISON OF ALGORITHMIC PERFORMANCE ON WFG PROBLEMS

In this section, we have presented the experimental results of the ad-MOEA with state-of-art algorithms on the WFG problems. The mean and standard deviation results of hypervolume indicator for the WFG problems are reported in Table 4. Similar to the DTLZ problems, significance tests are conducted and the performance comparison of proposed approach with the state-of-art algorithm are presented in Table 4. From the results presented in the Table 4, we can witness that the performance of ad-MOEA is better when compared with KnEA, RVEA, S-CDAS and NSGA-II and outstanding when compared with the r-NSGAII and g-NSGAII and competitive when compared with NSGA-III and slightly underperforming when compared with NSGAII-SDR. The performance of proposed approach when compared with KnEA, out of 45 test instances of WFG problems, performs better on 22 instances, competitive, and worse on eight and 15 instances respectively. The performance of ad-MOEA when compared with KnEA is better in WFG1, WFG2, WFG3, WFG6 and WFG8 problems and competitive on WFG5 problem. The KnEA algorithm performs better in WFG4, WFG7 and WFG9 problems when compared with proposed approach.

The performance of proposed approach is better in 24 instances, identical in seven instances, and worse in 14.

The algorithm ad-MOEA exhibits a dominating performance when compared with RVEA algorithm for the problems. WFG2 and WFG3 and slightly better performance on

TABLE 5. Mean and standard deviation values of Hypervolume results of proposed method with State of art algorithms on MaOP problems.

#	M	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]	ad-MOEA
MaOP1	4	0.0255(0.0011)-	0.0024(0.0011)+	0.0152(0.0017)+	0.0148(0.0021)+	0.0000(0.0000)+	0.0146(0.0019)=	0.0039(0.0006)+	0.0090(0.0018)+	0.0004(0.0013)+	0.0186(0.0011)
	6	9.6e-4(0.9e-4)-	0.8e-4(0.3e-4)+	4.6e-4(0.6e-4)+	4.5e-4(0.7e-04)+	0.0000(0.0000)+	4.5e-4(0.6e-4)+	6.7e-4(0.4e-4)=	3.8e-4(0.9e-4)+	1.7e-5(0.3e-5)+	6.8e-4(0.8e-4)
	8	3.2e-5(0.7e-5)-	0.3e-5(0.2e-5)+	1.7e-5(0.4e-5)+	1.9e-5(0.4e-5)+	0.0000(0.0000)+	1.8e-5(0.4e-5)+	3.5e-5(0.6e-5)-	0.1e-5(0.1e-5)+	0.1e-5(0.1e-5)+	2.6e-4(0.5e-5)
	10	2.0e-6(1.7e-6)=	0.1e-6(0.4e-6)+	1.0e-6(1.1e-6)-	0.9e-6(0.9e-6)=	0.0000(0.0000)+	0.7e-6(1.0e-6)=	2.2e-6(1.6e-6)=	0.3e-7(0.2e-6)+	0.3e-7(0.2e-6)+	1.7e-6(1.2e-6)
MaOP2	4	0.9672(0.0117)-	0.9767(0.0041)-	0.8949(0.0483)=	0.9042(0.0431)=	0.9542(0.0188)-	0.0000(0.0000)+	0.9165(0.0214)=	0.9792(0.0066)-	0.8488(0.0667)+	0.9183(0.0331)
	6	1.0000(0.0000)-	0.9990(0.0005)+	0.9424(0.0720)+	0.9407(0.0735)+	0.7557(0.0442)+	0.0000(0.0000)+	0.9755(0.0112)+	0.9947(0.0064)+	0.9974(0.0034)+	0.9997(0.0004)
	8	1.0000(0.0000)=	0.9999(0.0001)+	0.9975(0.0008)+	0.9974(0.0008)+	0.9535(0.0126)+	0.8282(0.0040)+	0.9991(0.0011)+	0.9994(0.0009)+	1.0000(0.0000)=	1.0000(0.0000)
	10	1.0000(0.0000)=	1.0000(0.0000)=	0.9999(0.0000)+	0.9999(0.0000)+	0.9989(0.0003)+	0.9943(0.0001)+	1.0000(0.0000)=	1.0000(0.0000)=	1.0000(0.0000)=	1.0000(0.0000)
MaOP3	4	0.6258(0.0046)-	0.6602(0.0034)-	0.5128(0.0126)+	0.5167(0.0115)+	0.0850(0.0010)+	0.0708(0.0034)+	0.2380(0.0127)+	0.6544(0.0011)-	0.5574(0.0160)-	0.5485(0.0110)
	6	0.8560(0.0080)-	0.8811(0.0003)-	0.4297(0.0989)+	0.4067(0.0816)+	0.0496(0.0026)+	0.0147(0.0014)+	0.5405(0.0067)+	0.8762(0.0224)-	0.7985(0.0113)-	0.7372(0.0168)
	8	0.9801(0.0014)-	0.9875(0.0001)-	0.5480(0.0376)+	0.5361(0.0412)+	0.0309(0.0016)+	0.4563(0.4109)+	0.9418(0.0282)-	0.9827(0.0091)-	0.9740(0.0028)-	0.7735(0.0707)
	10	0.9986(0.0001)-	0.9988(0.0000)-	0.7349(0.0148)+	0.7410(0.0181)+	0.0232(0.0010)+	0.7754(0.1345)+	0.9986(0.0001)-	0.9971(0.0035)-	0.9974(0.0002)-	0.8642(0.0169)
MaOP4	4	0.4083(0.0045)-	0.3418(0.0326)+	0.3917(0.0068)-	0.3936(0.0060)-	0.3553(0.0299)+	0.0000(0.0000)+	0.4134(0.0027)-	0.4022(0.0026)-	0.3341(0.0640)+	0.3870(0.0088)
	6	0.7230(0.0393)+	0.8697(0.0041)-	0.8172(0.0112)+	0.8143(0.0132)+	0.3735(0.0696)+	0.0083(0.0454)+	0.8735(0.0022)-	0.8631(0.0464)-	0.7817(0.1103)+	0.8223(0.0115)
	8	0.9999(0.0003)+	1.0000(0.0000)=	0.9990(0.0011)+	0.9988(0.0014)+	0.7465(0.1325)+	0.0743(0.2362)+	1.0000(0.0000)=	1.0000(0.0000)=	0.9997(0.0017)+	1.0000(0.0000)
	10	0.5957(0.2129)+	1.0000(0.0000)-	0.9850(0.0513)+	0.9971(0.0106)+	0.7857(0.1386)+	0.5611(0.4992)+	1.0000(0.0000)-	0.9997(0.0011)+	1.0000(0.0000)-	0.9999(0.0002)
MaOP5	4	0.2124(0.0343)+	0.1565(0.0574)+	0.3022(0.0037)=	0.3029(0.0024)=	0.2148(0.1055)+	0.0022(0.0063)+	0.2492(0.0269)+	0.1749(0.0288)+	0.1906(0.0140)+	0.3027(0.0015)
	6	0.2296(0.0108)+	0.2901(0.0214)+	0.2882(0.0136)+	0.2921(0.0122)+	0.3664(0.0017)-	0.0320(0.0261)+	0.2542(0.0224)+	0.3068(0.0404)=	0.2129(0.0087)+	0.3268(0.0213)
	8	0.5095(0.0070)+	0.5551(0.0136)=	0.5418(0.0118)+	0.5399(0.0128)+	0.5261(0.1723)+	0.0087(0.0220)+	0.5365(0.0123)+	0.5466(0.0210)+	0.4905(0.0044)+	0.5589(0.0076)
	10	0.8775(0.0640)+	0.9939(0.0001)+	0.9016(0.0787)+	0.9162(0.0712)+	0.9858(0.0265)+	0.5406(0.0706)+	0.9940(0.0001)+	0.9317(0.0788)+	0.7681(0.0005)+	0.9941(0.0001)
MaOP6	4	0.4874(0.0078)+	0.3181(0.0548)+	0.5097(0.0029)+	0.5097(0.0034)+	0.5111(0.0025)+	0.0624(0.0131)+	0.4120(0.0292)+	0.3421(0.1543)+	0.3915(0.0706)+	0.5177(0.0055)
	6	0.8772(0.0096)+	0.9139(0.0083)-	0.8864(0.0185)+	0.8886(0.0171)+	0.9280(0.0014)-	0.4815(0.0569)+	0.9088(0.0077)+	0.8811(0.1209)+	0.7149(0.0628)+	0.9086(0.0097)
	8	0.7999(0.0227)+	0.8951(0.0084)-	0.7743(0.0402)+	0.7840(0.0373)+	0.9014(0.0042)-	0.4101(0.0667)+	0.8849(0.0314)-	0.7334(0.2298)+	0.6893(0.0690)+	0.8679(0.0099)
	10	0.8849(0.0417)+	0.9769(0.0437)+	0.9822(0.0076)+	0.9830(0.0087)+	0.9311(0.0372)+	0.8475(0.0207)+	0.8950(0.0205)+	0.9598(0.0359)+	0.9417(0.0077)+	0.9876(0.0057)
MaOP7	4	0.3571(0.0061)=	0.2895(0.0226)+	0.3583(0.0063)=	0.3570(0.0051)=	0.0096(0.0085)+	0.0007(0.0000)+	0.2856(0.0162)+	0.2717(0.0918)+	0.2416(0.0270)+	0.3573(0.0052)
	6	0.2503(0.0121)+	0.0636(0.0210)+	0.2713(0.0030)+	0.2723(0.0034)+	0.0050(0.0036)+	0.0009(0.0000)+	0.2247(0.0210)+	0.1325(0.1009)+	0.2029(0.0202)+	0.2791(0.0056)
	8	0.1777(0.0119)+	0.0346(0.0138)+	0.2180(0.0027)+	0.2186(0.0030)+	0.0052(0.0034)+	0.0010(0.0000)+	0.1978(0.0247)+	0.1550(0.0524)+	0.1886(0.0172)+	0.2223(0.0061)
	10	0.1287(0.0131)+	0.0324(0.0122)+	0.1898(0.0048)=	0.1887(0.0051)=	0.0053(0.0020)+	0.0011(0.0000)+	0.1960(0.0056)-	0.0820(0.0522)+	0.1783(0.0139)+	0.1908(0.0073)
MaOP8	4	0.3670(0.0047)-	0.3046(0.0241)+	0.3595(0.0045)=	0.3581(0.0050)+	0.0093(0.0088)+	0.0007(0.0000)+	0.2835(0.0235)+	0.2730(0.0902)+	0.2389(0.0310)+	0.3609(0.0047)
	6	0.2673(0.0095)+	0.0750(0.0245)+	0.2792(0.0045)+	0.2795(0.0053)+	0.0050(0.0036)+	0.0008(0.0000)+	0.2355(0.0246)+	0.0982(0.0999)+	0.2063(0.0209)+	0.2889(0.0048)
	8	0.1898(0.0137)+	0.0302(0.0126)+	0.2259(0.0046)+	0.2245(0.0053)+	0.0043(0.0029)+	0.0009(0.0000)+	0.2166(0.0168)+	0.1679(0.0645)+	0.1925(0.0172)+	0.2359(0.0049)
	10	0.1309(0.0151)+	0.0393(0.0123)+	0.1967(0.0050)+	0.1973(0.0059)+	0.0048(0.0019)+	0.0010(0.0000)+	0.2061(0.0043)=	0.0972(0.0471)+	0.1833(0.0131)+	0.2063(0.0034)
MaOP9	4	0.3796(0.0063)-	0.3128(0.0448)+	0.3728(0.0066)-	0.3751(0.0078)-	0.0107(0.0108)+	0.0007(0.0000)+	0.2992(0.0169)+	0.2968(0.0959)+	0.2673(0.0334)+	0.3694(0.0072)
	6	0.2776(0.0110)+	0.0645(0.0235)+	0.3028(0.0079)=	0.3023(0.0051)=	0.0049(0.0044)+	0.0009(0.0000)+	0.2386(0.0362)+	0.2574(0.1148)+	0.2261(0.0311)+	0.3002(0.0059)
	8	0.1855(0.0193)+	0.0319(0.0157)+	0.2493(0.0084)-	0.2498(0.0062)-	0.0043(0.0023)+	0.0011(0.0000)+	0.2265(0.0178)+	0.1585(0.0405)+	0.2075(0.0239)+	0.2421(0.0095)
	10	0.0616(0.0198)+	0.0275(0.0109)+	0.2161(0.0074)=	0.2157(0.0122)=	0.0052(0.0018)+	0.0015(0.0000)+	0.2362(0.0080)-	0.1298(0.0259)+	0.2108(0.0161)=	0.2159(0.0120)
MaOP10	4	0.3855(0.0056)-	0.3237(0.0331)+	0.3794(0.0094)=	0.3821(0.0084)-	0.0112(0.0081)+	0.0007(0.0000)+	0.2979(0.0232)+	0.3408(0.0620)+	0.2618(0.0273)+	0.3743(0.0089)
	6	0.2920(0.0107)+	0.0577(0.0206)+	0.3127(0.0073)-	0.3099(0.0070)=	0.0044(0.0035)+	0.0009(0.0000)+	0.2536(0.0287)+	0.2444(0.1154)+	0.2335(0.0238)+	0.3074(0.0077)
	8	0.1962(0.0202)+	0.0301(0.0138)+	0.2590(0.0074)-	0.2582(0.0077)-	0.0039(0.0025)+	0.0012(0.0000)+	0.2359(0.0247)+	0.1700(0.0272)+	0.2212(0.0278)+	0.2522(0.0107)
	10	0.0935(0.0215)+	0.0273(0.0093)+	0.2297(0.0083)=	0.2271(0.0091)=	0.0060(0.0022)+	0.0016(0.0000)+	0.2402(0.0097)-	0.1282(0.0383)+	0.2128(0.0130)+	0.2263(0.0100)
+/=/-		23/4/13	28/3/9	25/9/6	26/9/5	36/0/4	38/2/0	24/6/10	30/3/7	32/3/5	

WFG1 and WFG8 problems. The proposed approach performs competitively on WFG4, WFG6 and WFG9 problems and worse on WFG5 and WFG7 problems when compared with RVEA algorithm. The performance of the ad-MOEA is better in 23 and equal in 13 and worse in nine instances when compared to S-CDAS and better in 26 instances and equivalent in 10 instances and worse in nine instances respectively when compared with compared with the NSGA-II. The performance of ad-MOEA when compared with S-CDAS and NSGA-II, demonstrates better performance on WFG2, WFG3, WFG4, WFG6 and WFG9, competitive performance on WFG1, WFG7 and WFG8 problems. The proposed approach performs better on the problem WFG5 when compared to NSGA-II and performs competitive when compared with S-CDAS algorithm. The ad-MOEA approach exhibits dominating performance when compared r-NSGAII with better performance in 41 instances, identical performance in zero instance and worse in four instances.

From the comparisons presented in the Table 4, we can notice that for the entire WFG problem suite, the ad-MOEA completely dominates the performance of r-NSGAII algorithm. The algorithm ad-MOEA when compared g-NSGAII illustrates better performance on 29 instances, identical performance in nine instances and worse performance in seven instances. The proposed approach performs consistently better on WFG2, WFG4, WFG5, WFG6, WFG8 and WFG9 problems and competitive on WFG1, WFG3 and WFG9 problems. The performance of ad-MOEA is slightly outperformed by NSGAII-SDR algorithm. The proposed approach depicts better performance in 15 instances and equal performance in three instances and worse performance in 27 instances. The ad-MOEA algorithm, when compared with NSGAII-SDR algorithm performs better on WFG1-WFG3 problems and exhibits worse performance on WFG4-WFG9 problems respectively. When compared with the NSGA-III, ad-MOEA is performing competitively with better performance in 19 instances and equal performance in eight instances and worse performance in 18 instances. The proposed approach performs better on WFG1, WFG3 and WFG7 problems when compared with NSGA-III algorithm and competitively on WFG2 and WFG4 problems. NSGA-III algorithm performs better on the WFG5, WFG6, WFG8 and WFG9 problems when compared with the proposed approach. The proposed ad-MOEA performs better than MOEA-D algorithm on 43 instances, competitive on zero instances and worse on two instances. From the results presented in the Table 4, we can demonstrate that the proposed approach exhibits a dominating performance on entire WFG test suite when compared with the MOEA-D algorithm.

C. COMPARISON OF ALGORITHMIC PERFORMANCE ON MaOP PROBLEMS

In this section, experimental results of the ad-MOEA with state-of-art algorithms on the MaOP problem suite are presented. The mean and standard deviation values of the hypervolume indicator for the MaOP problems are reported in Table 5. From the results presented in the Table 5, we can witness that the proposed approach outperforms all the

TABLE 6. Overall performance comparison of proposed method with state of art algorithms on benchmark problems.

Compared with	Problem Suite	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]
ad-MOEA (+/=/-)	DTLZ	15/2/18	8/4/23	29/3/3	28/4/3	31/3/1	33/1/1	21/4/10	9/4/22	26/1/8
	WFG	22/8/15	24/7/14	23/13/9	26/10/9	41/0/4	29/9/7	15/3/27	19/8/18	43/0/2
	MaOP	23/4/13	28/3/9	25/9/6	26/9/5	36/0/4	38/2/0	24/6/10	30/3/7	32/3/5
	Overall Comparison	**60/14/46**	**60/14/46**	**77/25/18**	**80/23/17**	**108/3/9**	**100/12/8**	**60/13/47**	**58/15/47**	**101/4/15**

algorithms considered for comparison on MaOP problems. The proposed approach when compared with the KnEA algorithm on MaOP benchmark problems, out of 40 test instances performs better in 23 instances and equal performance in four instances and worse performance in 13 instances. ad-MOEA performs better than the KnEA algorithm in MaOP4-MaOP10 problems and competitive performance is witnessed for the MaOP2 problem. KnEA algorithm performs better than proposed approach in the MaOP1 and MaOP3 problems. ad-MOEA in comparison with RVEA performs better in 28 instances, identical in three instances and worse in nine instances. The proposed algorithm performs better in the MaOP1, MaOP5 and MaOP7-MaOP10 problems when compared with the RVEA algorithm. The RVEA algorithm when compared with the ad-MOEA performs better in MaOP3 problem and competitive in the MaOP2, MaOP4 and MaOP6 problems respectively.

When compared with the S-CDAS and NSGA-II approaches, the proposed algorithm performs better in 25 and 26 instances, identical in nine and nine instances, and worse in six and five instances respectively. The ad-MOEA in comparison with S-CDAS and NSGA-II performs better in MaOP1-MaOP6, MaOP8 problems and competitive in the MaOP7, MaOP9-MaOP10 problems. The proposed approach exhibits a dominating performance when compared with the r-NSGAII and gNSGA-II with better performance in 36 and 38 instances and identical performance in zero and two instances and worse performance in four and zero instances respectively. The proposed approach when compared with r-NSGAII performs better on MaOP1-MaOP5 and MaOP7-MaOP10 problems and competitive performance on MaOP6 problem. The algorithm ad-MOEA in comparison with g-NSGAII performs better on the entire MaOP problem suite.

The proposed approach performs better than NSGAII-SDR in 24 instances, identical in six instances, and worse in 10 instances. The proposed approach depicts better performance on MaOP5, MaOP7-MaOP10 problems when compared with NSGAII-SDR. The algorithm ad-MOEA performs competitive on MaOP1, MaOP2 and MaOP6 problems and worse performance on MaOP3 problems. When compared with NSGA-III, ad-MOEA, performs better on 30 instances and identical in three instances and worse on seven instances. The performance of ad-MOEA when compared with NSGA-III is competitive on MaOP2 and MaOP4 problems and worse on MaOP3 problem. ad-MOEA performs better than NSGA-III on MaOP1, MaOP5-MaOP10 problems. The proposed approach perform better than MOEA-D algorithm on 32 instances, identical on three instances and worse on five instances. The proposed approach outperforms MOEA-D on MaOP1 and MaOP4-MaOP10 problems and exhibits competitive performance on MaOP2 problem and worse performance on MaOP3 problem.

D. OVERALL PERFORMANCE COMPARISON OF ad-MOEA WITH RESPECT TO STATE-OF-ART ALGORITHMS

In this section, we present the overall comparison of ad-MOEA algorithm with respect to the state-of-art algorithms for DTLZ, WFG and MaOP test problems combined. From the comparisons presented in the Table 6, we can witness that ad-MOEA algorithm when compared with the KnEA algorithm, out of 120-test instance performs better in 60 instances, equal in 14 instances and worse in 46 instances. The KnEA algorithm performs better than ad-MOEA algorithm for DTLZ problems whereas ad-MOEA approach outperforms KnEA algorithm in WFG and MaOP problems. When compared to with RVEA algorithm, the proposed approach performs better in 60 instances, equal in 14 instances and worse in 46 instances. RVEA algorithm outperforms the ad-MOEA in DTLZ problems whereas the ad-MOEA algorithm outperforms RVEA algorithm for WFG and MaOP problems.

The proposed approach performs better than S-CDAS and NSGA-II algorithms in 77 and 80 instances, equal in 25 and 23 instances and worse in 18 and 17 instances respectively. When compared with the r-NSGAII and g-NSGAII algorithm, the proposed approach performs better in 108 and 100 instances and equal in three and 12 instances and worse in nine and eight instances respectively.. When compared to NSGAII-SDR algorithm, the ad-MOEA algorithm performs better in 60 instances and equal in 13 cases and worse in 49 cases. The proposed approach performs better than NSGAII-SDR in DTLZ and MaOP problems and worse in the WFG problems. When compared with NSGA-III algorithm, ad-MOEA approach performs better in 58 instances, equal in 15 instances and worse in 47 cases. The ad-MOEA algorithm outperforms NSGA-III algorithm in MaOP problems and competitive performance of proposed approach is observed in WFG problems. In comparison with MOEA-D algorithm, the proposed approach perform better in 101 instances, competitive on 4 instance and worst on 15 instances. The proposed approach exhibits a dominating performance on the DTLZ. WFG and MaOP problem suites when compared with S-CDAS, NSGA-II, r-NSGAII, g-NSGAII and MOEA-D algorithms.

TABLE 7. Mean and standard deviation values of Hypervolume results of proposed method with state of art algorithms on real-world problems.

Problem	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]	ad-MOEA
Car side impact design	0.4062(0.0308)=	0.3588(0.0310)+	0.4518(0.0079)-	0.4523(0.0076)-	0.1135(0.0316)+	0.2363(0.0178)+	0.3250(0.0177)+	0.4274(0.0143)=	0.2383(0.0593)+	0.4134(0.0140)
Conceptual marine design	0.5183(0.0203)+	0.5350(0.0557)=	0.5415(0.0086)=	0.5444(0.0088)=	0.4700(0.0017)+	0.5188(0.0163)+	0.5928(0.0165)-	0.5206(0.0164)+	0.4702(0.0034)+	0.5393(0.0182)
Water resource planning	0.7916(0.0043)-	0.7573(0.0085)+	0.7666(0.0141)=	0.7681(0.0148)=	0.5750(0.0052)+	0.7131(0.0100)+	0.7540(0.0197)+	0.7988(0.0052)-	0.7431(0.0621)+	0.7650(0.0119)
Car cab design	0.0487(0.0051)=	0.0035(0.0025)+	0.0494(0.0031)=	0.0476(0.0031)=	0.0485(0.0058)=	0.0189(0.0027)+	0.0028(0.0008)+	0.0517(0.0031)=	0.0064(0.0037)+	0.0483(0.0046)
Five degree of freedom vehicle vibration model	0.2878(0.0227)=	0.2543(0.0387)+	0.2959(0.0225)=	0.2917(0.0207)=	0.2818(0.0267)=	0.2886(0.0253)=	0.2944(0.0172)=	0.2941(0.0201)=	0.3742(0.0058)-	0.2920(0.0178)
+/=/-	1/1/3	4/1/0	0/4/1	0/4/1	3/2/0	4/1/0	3/1/1	1/1/3	4/0/1	

TABLE 8. Runtime comparison of proposed method with state of art algorithms on DTLZ2 benchmark problem.

Problem	M	KnEA [6]	RVEA [21]	S-CDAS [27]	NSGA-II [12]	r-NSGAII [31]	g-NSGAII [32]	NSGAII-SDR [34]	NSGA-III [8]	MOEA-D [13]	ad-MOEA
DTLZ2	2	1.1674(0.1460)	0.4509(0.0669)	2.8463(0.6987)	0.2702(0.0329)	0.5492(0.0952)	0.3202(0.0406)	1.2903(0.1292)	1.0322(0.1328)	3.5953(1.4138)	0.3309(0.0469)
	4	2.1025(0.1868)	0.7541(0.2103)	2.9269(0.1621)	0.4645(0.0843)	0.7722(0.1134)	0.5174(0.1201)	1.2709(0.1510)	1.2590(0.1630)	5.9113(1.1005)	0.6541(0.1043)
	6	2.8384(0.2350)	0.8874(0.0567)	3.1139(0.2355)	0.5939(0.0807)	1.0986(0.0885)	0.5894(0.0623)	1.1529(0.0728)	1.3955(0.1024)	8.2410(2.3732)	0.6574(0.0339)
	8	3.7296(0.2924)	1.0202(0.2842)	3.4891(0.3717)	0.9053(0.1393)	1.4259(0.1325)	0.6801(0.0310)	1.4152(0.1485)	1.8369(0.1761)	10.5396(3.0257)	0.9202(0.1312)
	10	12.2751(0.8798)	2.1555(0.3601)	6.1790(0.3935)	1.8282(0.1361)	3.6231(0.2728)	1.2739(0.0887)	3.7384(0.2968)	3.1105(0.2849)	15.0900(3.8757)	2.1555(0.3601)
Car side impact design		2.9300(0.3029)	1.0186(0.5261)	1.8650(0.3855)	0.6219(0.2153)	0.7490(0.1664)	1.1490(0.2285)	1.0240(0.2938)	1.5190(0.4855)	3.4745(0.6127)	0.7767(0.2496)
Conceptual marine design		2.4391(0.3496)	0.8367(0.2446)	2.1511(0.6533)	0.6838(0.1848)	0.7511(0.2158)	2.1390(0.4109)	1.2276(0.4571)	1.5055(0.2804)	3.4898(0.8526)	0.8420(0.2815)
Water resource planning		5.4789(0.9391)	0.9544(0.1529)	2.0601(0.3068)	0.8197(0.2193)	1.1196(0.4190)	1.6543(0.2905)	1.4856(0.5947)	1.8818(0.2957)	4.6160(1.7507)	0.8902(0.2091)
Car cab design		7.4168(1.2505)	0.7736(0.1259)	2.9521(0.6000)	0.6559(0.1437)	1.3966(0.4281)	1.3405(0.4174)	1.7197(0.2220)	1.8531(0.2628)	4.5577(0.4523)	0.8622(0.2058)

E. COMPARISON OF ALGORITHMIC PERFORMANCE ON REAL-WORLD PROBLEMS

In this section, we present performance comparison of proposed ad-MOEA algorithm with state-of-art algorithms on real-world problems. The real world problems considered in our study are Car side impact design [47], Conceptual marine design [47], Water resource planning [47], Car cab design [47] and Five degree of freedom vehicle vibration model [48] problems. The Car side impact design problem formulated in [47] consists of 4-objective functions with 7 decision variables that are continuous in nature. Conceptual marine design problem [47] is a 4-objective problem with 6 design variables that are continuous in nature. The Water resource planning [47] problem consists of 6-objective functions with 3 decision variables with continuous in nature. Car cab design [47] problem is a 9-objective problem with 7 continuous design variables. Five degree of freedom vehicle vibration model [48] consists of 5-objective functions and 7 decision variables. The shape of the true PF is unknown for all the problems considered in this study. The population size and maintained for the Car side impact design [47], Conceptual marine design [47], Water resource planning [47], Car cab design [47], Five degree of freedom vehicle vibration model [48] are 120, 120, 182, 210 and 100 respectively. The number of iterations for the problems Car side impact design [47], Conceptual marine design [47], Water resource planning [47], Car cab design [47] are maintained as 100 and for Five degree of freedom vehicle vibration model [48] problem, the number of iterations considered are 250. To analyze the performance comparison of the proposed approach with the state-of-art algorithms, we have presented the hypervolume results for the real-world problems. The mean and standard deviation results for hypervolume indicator values are presented in Table 7. To evaluate the hypervolume indicator value, we followed the same procedure that is applied to calculate the hypervolume value for WFG and MaOP problems.

From the results presented in Table 7, we can observe that the proposed approach outperforms RVEA, r-NSGAII, g-NSGAII, NSGAII-SDR and MOEA-D algorithms and performs competitive with KnEA and NSGAIII algorithm and worse when compared with S-CDAS and NSGA-II algorithms on Car side impact problem. While handling the Conceptual marine design problem, the proposed ad-MOEA performs better than KnEA, r-NSGAII, g-NSGAII, NSGAIII and MOEA-D algorithm and competitive with RVEA, S-CDAS and NSGAII algorithms and worse when compared with the NSGAII-SDR algorithm. The proposed approach outperforms RVEA, r-NSGAII, g-NSGAII, NSGAII-SDR and MOEA-D algorithms while handling Water resource planning problem and competitive with S-CDAS and NSGA-II algorithms and performs worse when compared with KnEA and NSGAIII. While tackling Car cab design problem, the proposed approach outperforms RVEA, r-NSGAII, g-NSGAII, NSGAII-SDR and MOEA-D algorithms and competitive on KnEA, S-CDAS, NSGAII and NSGAIII algorithms. The proposed approach outperforms RVEA algorithm on Five degree of freedom vehicle vibration model problem and worse when compared with MOEA-D algorithm and performs competitive when compared with KnEA, S-CDAS, NSGA-II, r-NSGAII, g-NSGAII, NSGAII-SDR and NSGAIII.

F. RUNTIME PERFORMANCE COMPARISON OF ad-MOEA WITH RESPECT TO STATE-OF-ART ALGORITHMS

In this section, we have presented the runtime performance comparison of the ad-MOEA algorithm with respect to state-of-art algorithms for the DTLZ2 Problems for 2-, 4-, 6-, 8-, & 10- objectives. Along with that, we have also considered the simulation timing analysis for real-world problems, Car side impact design [47], Conceptual marine design [47], Water resource planning [47], Car cab design [47]. For the fair comparisons of the time, simulations for all the algorithms PC with a 3.40 GHz Intel Core i7-2600QM CPU and Windows 10 SP1 64-bit operating system with MATLAB 2019b version. Based on the results presented in the Table 8, we can witness that the proposed ad-MOEA approach requires less computational time when compared with the rest of the algorithms except NSGA-II and g-NSGAII on the DTLZ2

problems for all the objectives. However, the proposed ad-MOEA requires more computational time when compared with the NSGA-II and g-NSGAII but in terms of the hypervolume results, the proposed approach outperforms the algorithms NSGA-II and g-NSGAII. On the real world problems, the proposed ad-MOEA requires less computational time when compared with KnEA, S-CDAS, g-NSGAII, NSGAII-SDR, NSGAIII and MOEA-D algorithms. The execution time of the proposed ad-MOEA is competitive when compared with r-NSGAII and RVEA algorithm. In other words, the proposed approach ad-MOEA require less time when compared with RVEA on Car side impact and Water resource planning problem and requires more time on Conceptual marine design and car cab design problem. Similarly, when compared with r-NSGAII algorithm, the proposed approach requires less time on Water resource planning and Car cab design problems and more execution time on Car side impact and Conceptual marine design problems. When compared with NSGAII algorithm, the proposed approach execution time is more on the real-world problems.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we have proposed a novel multi-objective evolutionary algorithm with adaptive mating and environmental selections (ad-MOEA) to handle multi and many-objective optimization problems. In the proposed approach, we have incorporated the concept of the sum of the normalized objectives into the mating selection and environmental selection to achieve convergence and diversity simultaneously. In the environmental selection, in the critical front, the solutions are selected based on the sum of the objectives at first and the remaining solutions are selected based on crowding distance. To select the solutions based on sum of normalized objectives, we adopted a certain probability that is determined by a self-adapted parameter. The self-adaptive parameter employed in the current work, assists the evolution process in promoting the convergence in initial stages and diversity in the final stages. Initially, the self-adaptive parameter is set to one in the initial generations, as the evolution progress, the value of self-adaptive parameter is expected to decrease. Hence, more importance is assigned to convergence promoting individuals in initial stages and the focus shifts towards diverse solutions as the evolution progress. Therefore, in initial generations more preference is assigned to the individuals that promote convergence and as the evolution progress, the focus adaptively shift towards the individuals that are diverse. The experimental results demonstrate that the proposed approach have improved the performance when compared with the existing NSGA-II and competitive performance when compared to the other state-of-art algorithms.

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