Energy-Efficient Noncooperative Power Control in Small-Cell Networks

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***Abstract*—In this paper, energy-efficient power control for small cells underlaying a macrocellular network is investigated. We formulate the power control problem in self-organizing small-cell networks as a noncooperative game and propose a distributed energy-efficient power control scheme, which allows the small base stations (SBSs) to take individual decisions for attaining the Nash equilibrium (NE) with minimum information exchange. In particular, in the noncooperative power control game, a nonconvex optimization problem is formulated for each SBS to maximize their energy efficiency (EE). By exploiting the properties of parameter-free fractional programming and the concept of perspective function, the nonconvex optimization problem for each SBS is transformed into an equivalent constrained convex optimization problem. Then, the constrained convex optimization problem is converted into an unconstrained convex optimization problem by exploiting the mixed penalty function method. The inequality constraints are eliminated by introducing the logarithmic barrier functions, and the equality constraint is eliminated by introducing the quadratic penalty function. We also theoretically show the existence and the uniqueness of the NE in the noncooperative power control game. Simulation results show remarkable improvements in terms of EE by using the proposed scheme.**

***Index Terms*—Energy efficiency, non-cooperative game theory, mixed penalty function, power control, perspective function.**

I. Introduction

WITH the rapid expansion of wireless communications networks, tremendous spectrum efficiency (SE) and energy efficiency (EE) improvement is required for 5G mobile communication systems. As an expected feature of 5G systems, small cell networks (SCNs) [1], composed of a large number of densely deployed small cells with small coverage area and low transmission power, are expected to offer higher SE and EE. SCNs require self-organization, self-learning and intelligent decision making at small cell base stations (SBSs), which can be randomly deployed by the operators or by the users in the hot-spot areas of city or rural locations.

Due to the large number of SBSs deployed in SCNs, conventional centralized power control schemes which require cooperation among all base stations (BSs) may not be practical. Most of the existing works on distributed power control for SCNs aimed at improving the sum throughput or decreasing the interference [2]–[7]. In [2], the authors proposed a distributed scheme based on pricing mechanism to maximize the sum-rate. A distributed utility was proposed to alleviate the cross-tier interference at the macrocell from cochannel femtocells in [3]. In [4], a payoff function was formulated to improve the fairness. In [5], the authors formulated a net utility function considering both the gain of throughput and the punishment of interference. In [6], the authors proposed a novel power control scheme to guarantee the target signal to interference plus noise ratios (SINRs) of the macrocell users, and make as many femtocell users as possible to achieve their target SINRs. In [7], cooperative game theory was exploited to deal with the co-tier interference of the small cells.

With the explosive growth in data traffic, the energy consumption of wireless infrastructures and devices has increased greatly. The growing energy consumption further brings about large electricity and maintenance bills for network operators, and tremendous carbon emissions into the environment. Consequently, improving the EE has become an important and urgent task. A large amount of works has recently been devoted to investigating the maximization of EE in wireless communications systems [8]–[11]. In general, the EE maximization problems are non-convex optimization problems, which can be transformed into equivalent convex optimization problems by using fractional programming [14]. In [8], the authors unified various approaches for the EE optimization problem for a typical scenario in wireless communications systems, and discussed three types of fractional programming solutions in detail, including parametric convex program, parameter-free convex program and dual program. In [9], the resource allocation for energy

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efficient secure communication in an orthogonal frequency-division multiple-access (OFDMA) downlink network was investigated, and the considered non-convex optimization problem was transformed into a convex optimization problem by exploiting the properties of parametric fractional programming. In [10], the authors proposed an energy efficient power control scheme for a multi-carrier link over a frequency selective fading channel with a delay-outage probability constraint, and derived the global optimum solution using parameter-free fractional programming. In [11], by using fractional programming and exterior penalty function, an efficient iterative joint resource allocation and power control scheme was proposed to maximize the EE of device-to-device (D2D) communications. However, all the above research works on EE maximization have mainly focused on non-SCNs considering only one BS, which may not be applicable in SCNs. Furthermore, the sheer number of densely deployed SBSs makes distributed power control more desirable. For the existing research works on power control for SCNs, there are only a few studies concerning the optimization of EE. In [12], the authors introduced a Stackelberg game-theoretic framework for heterogeneous networks which enables both the small cells and the macro cells to strategically decide on their downlink power control policies. In [13], a bargaining cooperative game framework for interference-aware power coordination was proposed.

Motivated by the aforementioned discussions, we propose a non-cooperative game theory based power control scheme for SCNs, where each SBS maximizes its own EE by performing power control independently and distributively. By exploiting the properties of parameter-free fractional programming and the concept of perspective function, the non-convex optimization problem for each SBS player is transformed into an equivalent constrained convex optimization problem. Then, this constrained convex optimization problem is further converted into an unconstrained convex optimization problem by exploiting the mixed penalty function method. The inequality constraints are eliminated by introducing the logarithmic barrier functions and the equality constraint is eliminated by introducing a quadratic penalty function. The existence and uniqueness of the Nash equilibrium (NE) in the non-cooperative power control game are further proved theoretically.

II. System Model

We consider a two-tier downlink cellular network consisting of one macrocell and K small cells as illustrated in Fig. 1.

Let $\mathcal{K} = \{1, 2, \ldots, K\}$ denote the set of the SBSs. Let $\mathcal{N}_0 = \{1, 2, \ldots, N_0\}$ and $\mathcal{N}_k = \{1, 2, \ldots, N_k\}$ denote the set of the macrocell user equipments (MUEs) and the set of the small cell user equipments (SUEs) served by the kth SBS, respectively. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of the resource blocks (RBs). Here, one RB refers to one time-frequency resource block which includes one time slot and 12 subcarriers [11]. For efficient spectrum sharing, we assume that both the macrocell and small cells can utilize all the available RBs. We also assume that the

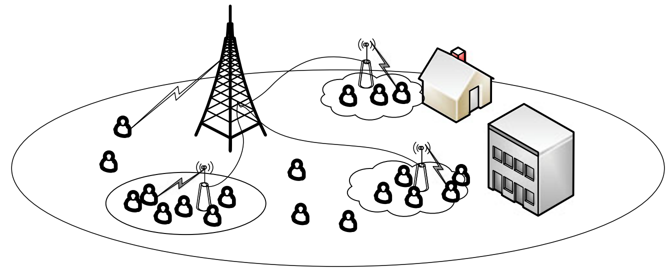

Fig. 1. System model of SCNs.

resource allocation of all the MUEs and SUEs has already been carried out.

Let P_k^i denote the transmit power of the kth SBS to its corresponding SUE over the ith RB, P_0^i the transmit power of the MBS to its corresponding MUE over the ith RB, and P_l^i the transmit power of the lth SBS to its corresponding SUE over the ith RB. Let $\boldsymbol{p}_k = [P_k^1, P_k^2, \ldots, P_k^i, \ldots, P_k^N]^T$. Let $R_k(\boldsymbol{p}_k)$ denote the transmission rate of the kth SBS. Then, it can be expressed as,

$$R_k(\boldsymbol{p}_k) = W \sum_{i=1}^{N} \log_2 \left(1 + \frac{H_{k,k}^i P_k^i}{H_{0,k}^i P_0^i + \sum_{l \neq k, l=1}^{K} H_{l,k}^i P_l^i + N_0} \right), \quad (1)$$

where W denotes the bandwidth of one RB, $H_{k,k}^i$ the channel gain from the kth SBS to its corresponding SUE over the ith RB, $H_{0,k}^i$ the interference channel gain from the MBS to the corresponding SUE over the ith RB associated with the kth SBS, $H_{l,k}^i$ the interference channel gain from the lth SBS to the corresponding SUE over the ith RB associated with the kth SBS, and N_0 the noise power.

Let EE_k denote the EE of the kth SBS. Then, it can be expressed as,

$$\mathrm{EE}_k = \frac{R_k}{P_k^c + \frac{1}{\sigma} \sum\limits_{i=1}^{N} P_k^i}, \quad (2)$$

where P_k^c denotes the circuit power consumption of the kth SBS, and it is a power offset that is independent of the radiated power, but derived from signal processing, and the related circuit power, etc [11]. σ denotes the inefficiency of the power amplifier, and $0 < \sigma \leq 1$. Correspondingly, the EE of the entire system can be expressed as [11]

$$\mathrm{EE}_s = \frac{\sum_{k=1}^{K} R_k}{\sum_{k=1}^{K} \left(P_k^c + \frac{1}{\sigma} \sum_{i=1}^{N} P_k^i \right)}. \quad (3)$$

It should be pointed out here that we use the heterogeneous feature of SCNs and consider multiple small cells and multiple users in the system model. Both the cross-tier interference from the macrocell to the considered small cell and the co-tier interference from the other small cells to the considered small cell are taken into account.

III. Proposed Energy-Efficient Noncooperative Power Control Scheme

In this section, we propose a distributed energy efficient power control scheme for SCNs based on non-cooperative game theory [15]. In the formulated non-cooperative power control game, each SBS is allowed to take individual decisions for attaining the NE with minimum information exchange. The fractional objective function in (3) is a non-concave function which is hard to solve directly. Just as we pointed out in [11], a brute force approach is generally required for obtaining a global optimal solution. However, such a method has an exponential complexity with respect to the number of RBs and the number of cells. Therefore, we propose an efficient method to solve this challenging problem.

A. Problem Formulation

Using microeconomic concepts, we assume that all the considered SBSs participate in a K-player non-cooperative power control game $G = [\mathcal{K}, \{\boldsymbol{p}_k\}, \{u_k(\cdot)\}]$, where $\{\boldsymbol{p}_k\}$ denotes the set of the transmit power vectors of the kth SBS, and $u_k(\cdot)$ denotes the utility function of the kth SBS to be maximized. Let $\boldsymbol{p}_{-k} = [\boldsymbol{p}_1^T, \boldsymbol{p}_2^T, \ldots, \boldsymbol{p}_{k-1}^T, \boldsymbol{p}_{k+1}^T, \ldots, \boldsymbol{p}_K^T]^T$ denote the transmit power vector of the $K-1$ other SBSs. Then, for all the considered SBSs, the non-cooperative power control game can be formulated as follows,

$$\max_{\boldsymbol{p}_k \in C_k} u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}\right), \text{ for each SBS in } \mathcal{K}, \tag{4}$$

where C_k is the feasible region of $\boldsymbol{p}_k$. In our case, the utility function that we consider is EE, i.e., $u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}\right) = \mathrm{EE}_k$. Let P_t denote the maximum power consumption of each SBS, and R_t the minimum rate requirement of each SBS. By considering the transmit power constraint and the transmission rate constraint, the above optimization problem for the kth SBS can be expressed in the following equivalent form,

$$\begin{aligned} &\max_{\boldsymbol{p}_k} u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}\right) \\ \text{s.t. } &C_{k,1}: P_k^i \geq 0, i = 1, 2, \ldots, N \\ &C_{k,2}: \sum_{i=1}^{N} P_k^i \leq P_t \\ &C_{k,3}: R_k(\boldsymbol{p}_k) \geq R_t. \end{aligned} \tag{5}$$

where $C_{k,1}$ and $C_{k,2}$ are the transmission power constraints for the kth SBS, and $C_{k,3}$ guarantees the target rate requirement of the kth SBS. In the following, where no confusion is caused, we also use $C_{k,1}, C_{k,2}$, and $C_{k,3}$ to denote the sets of transmit powers which meet the corresponding constraints. Therefore, $C_{k,1} \cap C_{k,2} \cap C_{k,3} = C_k$. Note that, as in [11], all the subcarriers within one RB are characterized by the same channel gains in our formulated EE optimization problem, and hence, no single-user diversity is exploited. However, the assumption of constant channel gain in the allocated subcarriers in one RB makes the power control problem more practical, despite it representing a suboptimal solution.

Algorithm 1 The iterative algorithm based on the non-cooperative power control game

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- Step 1: Initialization: $n = 1$, ϵ, $\boldsymbol{p}_k(1)$, and $\boldsymbol{p}_{-k}(1)$.
- Step 2: For each SBS (i.e., $k = 1, 2, \ldots, K$), calculate $\boldsymbol{p}_k(n+1) = \arg\max_{\boldsymbol{p}_k \in C_k} u_k\left[\boldsymbol{p}_k, \boldsymbol{p}_{-k}(n)\right], \ k \in \mathcal{K}$.
- Step 3: Set $\beta = \sum_{k=1}^{K} \left| u_k\left[\boldsymbol{p}_k(n+1), \boldsymbol{p}_{-k}(n+1)\right] - u_k\left[\boldsymbol{p}_k(n), \boldsymbol{p}_{-k}(n)\right] \right|$. If $\beta \geq \epsilon$, set $n = n+1$.
- Step 4: Repeat the steps $2 \sim 3$ until $\beta < \epsilon$.

In the above non-cooperative power control game, each SBS optimizes its individual utility that depends on the transmit powers of the $K-1$ other SBSs. It is necessary to calculate the equilibrium point wherein each SBS achieves its maximum utility conditioned on the transmit powers of the $K-1$ other SBSs. Such an operating point in the optimization problem (4) is called an NE [15]. Let $\boldsymbol{p}_k^*$ denote the kth SBS's best response to the strategy $\boldsymbol{p}_{-k}^*$ specified for the $K-1$ other SBSs at the NE. Then, $\boldsymbol{p}_k^*$ can be expressed as follows,

$$\boldsymbol{p}_k^* = \arg\max_{\boldsymbol{p}_k \in C_k} u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}^*\right). \tag{6}$$

It means that no SBS can obtain a unilateral profit by deviating from the NE, i.e.,

$$u_k\left(\boldsymbol{p}_k^*, \boldsymbol{p}_{-k}^*\right) \geq u_k\left(\boldsymbol{p}_k, \boldsymbol{p}_{-k}^*\right), \quad \text{for every feasible strategy } \boldsymbol{p}_k \text{ in } C_k. \tag{7}$$

Accordingly, the goal of our considered optimization problem is to find the NE in the non-cooperative power control game G.

B. Iterative Algorithm for EE Maximization

In the following, we propose an iterative power tuning strategy to reach the NE based on (6) in the considered non-cooperative power control game. Let n denote the number of iterations and ϵ denote the convergence threshold. Let $\boldsymbol{p}_k(n)$ and $\boldsymbol{p}_{-k}(n)$ denote the transmission vectors of the kth SBS and the $K-1$ other SBSs for the nth iteration, respectively. Then, the iterative algorithm based on the non-cooperative power control game can be summarized in Algorithm 1. In each iteration, the transmission power strategy $\boldsymbol{p}_k(n+1)$ of the kth SBS is a best response to the transmission power strategy $\boldsymbol{p}_{-k}(n)$ of the $K-1$ other SBSs. The iterative algorithm converges to the only solution if and only if there exists one unique NE in the non-cooperative power control game, and the NE can be reached when each SBS's transmission power strategy is sufficiently close to that in the previous iteration.

C. Problem Equivalence

The optimization problem in (5) involves a fractional objective function which is non-convex, and it requires great computational burden even for a small sized system. Therefore, we resort to a more computationally efficient scheme below to solve this challenging problem.

1) Convex Transformation: Exploiting the properties of parameter-free fractional programming [16] and the concept of perspective function [17], we propose to transform the original non-convex optimization problem in (5) into an equivalent convex optimization problem. Apply the variable transformation

$$y_k^0 = \frac{1}{P_k^c + \sum_{i=1}^N P_k^i},$$

$$y_k^i = \frac{P_k^i}{P_k^c + \sum_{i=1}^N P_k^i}.$$

Define

$$\boldsymbol{y}_k^{'} = \left[\frac{y_k^1}{y_k^0}, \frac{y_k^2}{y_k^0}, \cdots, \frac{y_k^i}{y_k^0}, \cdots, \frac{y_k^N}{y_k^0}\right]^T,$$

$$\boldsymbol{y}_k = \left[y_k^0, y_k^1, \cdots, y_k^i, \cdots, y_k^N\right]^T.$$

Let $R_k^{'}(\boldsymbol{y}_k) = R_k(\boldsymbol{y}_k{}^{'})$, and define $\zeta(\boldsymbol{y}_k) = y_k^0 R_k^{'}(\boldsymbol{y}_k)$. Then, the following optimization problem can be formulated,

$$\begin{aligned}
&\max_{\boldsymbol{y}_k} \; \zeta(\boldsymbol{y}_k) \\
&\text{s.t. } S_{k,1}^i : y_k^i \geq 0, \; i = 0, 1, 2, \ldots, N \\
&\quad S_{k,2} : y_k^0 P_t - \sum_{i=1}^N y_k^i \geq 0 \\
&\quad S_{k,3} : R_k^{'} - R_t \geq 0 \\
&\quad S_{k,4} : P_k^c y_k^0 + \sum_{i=1}^N y_k^i - 1 = 0.
\end{aligned} \tag{8}$$

We are now ready to present the following Theorem, which verifies the equivalence of the optimization problems theoretically.

Theorem 1: The optimization problem in (8) is a convex optimization problem and equivalent to the optimization problem in (5).

Proof: From (1), (2), and (5), let

$$\begin{aligned}
f_k(\boldsymbol{p}_k) &= \frac{1}{W} R_k(\boldsymbol{p}_k) \\
&= \sum_{i=1}^N \log_2\left(1 + \frac{H_{k,k}^i P_k^i}{N_0 + H_{0,k}^i P_0^i + \sum\limits_{l \neq k, l=1}^K H_{l,k}^i P_l^i}\right), \\
h_k(\boldsymbol{p}_k) &= P_k^c + \frac{1}{\sigma}\sum_{i=1}^N P_k^i.
\end{aligned}$$

Taking the second order derivative of $f_k(\boldsymbol{p}_k)$ and $h_k(\boldsymbol{p}_k)$ with respect to P_k^i, we have

$$\begin{aligned}
\frac{\partial^2 f_k(\boldsymbol{p}_k)}{\partial (P_k^i)^2} &= -\frac{(H_{kk}^i)^2}{\left(N_0 + H_{kk}^i P_k^i + H_{0k}^i P_0^i + \sum_{l \neq k, l=1}^K H_{l,k}^i P_l^i\right)^2} \\
&< 0, \\
\frac{\partial^2 h_k(\boldsymbol{p}_k)}{\partial (P_k^i)^2} &= 0.
\end{aligned}$$

We can also readily establish that $\frac{\partial^2 f_k(\boldsymbol{p}_k)}{\partial P_k^i \partial P_k^j} = \frac{\partial^2 h_k(\boldsymbol{p}_k)}{\partial P_k^i \partial P_k^j} = 0$, $\forall i \neq j$. Therefore, the Hessian matrix of $f_k(\boldsymbol{p}_k)$ is negative definite and $f_k(\boldsymbol{p}_k)$ is concave, and the Hessian matrix of $h_k(\boldsymbol{p}_k)$ is positive definite and $h_k(\boldsymbol{p}_k)$ is convex.

From (5), let

$$\begin{aligned}
l_{k,1}^i(\boldsymbol{p}_k) &= -P_k^i, \; i = 1, 2, \ldots, N, \\
l_{k,2}(\boldsymbol{p}_k) &= \sum_{i=1}^N P_k^i, \\
l_{k,3}(\boldsymbol{p}_k) &= -\frac{1}{W} R_k(\boldsymbol{p}_k) = -f_k(\boldsymbol{p}_k).
\end{aligned}$$

Taking the second order derivative of $l_{k,1}^i(\boldsymbol{p}_k)$, $l_{k,2}(\boldsymbol{p}_k)$ and $l_{k,3}(\boldsymbol{p}_k)$ with respect to P_k^i, we have

$$\begin{aligned}
\frac{\partial^2 l_{k,1}^i(\boldsymbol{p}_k)}{\partial (P_k^i)^2} &= \frac{\partial^2 l_{k,2}(\boldsymbol{p}_k)}{\partial (P_k^i)^2} = 0, \\
\frac{\partial^2 l_{k,3}(\boldsymbol{p}_k)}{\partial (P_k^i)^2} &= \frac{(H_{kk}^i)^2}{\left(N_0 + H_{kk}^i P_k^i + H_{0k}^i P_0^i + \sum_{l \neq k, l=1}^K H_{l,k}^i P_l^i\right)^2} \\
&> 0.
\end{aligned}$$

We can also readily establish that $\frac{\partial^2 l_{k,1}^i(\boldsymbol{p}_k)}{\partial P_k^i \partial P_k^j} = \frac{\partial^2 l_{k,2}(\boldsymbol{p}_k)}{\partial P_k^i \partial P_k^j} = \frac{\partial^2 l_{k,3}(\boldsymbol{p}_k)}{\partial P_k^i \partial P_k^j} = 0$, $\forall i \neq j$. Therefore, the Hessian matrices of $l_{k,1}^i(\boldsymbol{p}_k)$, $l_{k,2}(\boldsymbol{p}_k)$ and $l_{k,3}(\boldsymbol{p}_k)$ are all positive definite, and $l_{k,1}^i(\boldsymbol{p}_k)$, $l_{k,2}(\boldsymbol{p}_k)$ and $l_{k,3}(\boldsymbol{p}_k)$ are all convex. Correspondingly, the sublevel sets $C_{k,1}$,$C_{k,2}$, $C_{k,3}$ of $l_{k,1}^i(\boldsymbol{p}_k)$, $l_{k,2}(\boldsymbol{p}_k)$ and $l_{k,3}(\boldsymbol{p}_k)$ are convex [17], and the set $C_k = C_{k,1} \cap C_{k,2} \cap C_{k,3}$ is convex.

Consequently, according to [16], the optimization problem in (8) is a convex optimization problem and equivalent to the optimization problem in (5). ■

2) Elimination of Constraints: In general, the exterior penalty function method [11], [18] can be applied to solve convex optimization problems with equality constraints and inequality constraints. However, the solution obtained may not satisfy the constraints. On the other hand, the interior penalty function method [19] can only solve convex optimization problems with inequality constraints, but the solution obtained can always satisfy the constraints. We can see from (8) that the equivalent optimization problem includes both the inequality constraints $S_{k,1}^i$ $(i = 0, 1, 2, \ldots, N)$, $S_{k,2}$, $S_{k,3}$ and the equality constraint $S_{k,4}$. Therefore, we propose to use the mixed penalty function method to transform the constrained convex optimization problem in (8) into an unconstrained one. The inequality constraints $S_{k,1}^i (i = 1, 2, \ldots, N)$, $S_{k,2}$ and $S_{k,3}$ are eliminated by introducing a logarithmic barrier function based on the interior penalty function method, and the equality constraint $S_{k,4}$ is eliminated by introducing a quadratic penalty function based on the exterior penalty function method. Correspondingly, both the disadvantage of the exterior penalty function method and that of the interior penalty function method can be avoided.

Let

$$\varphi_1(\boldsymbol{y}_k) = \sum_{i=0}^{N} \ln y_k^i,$$

$$\varphi_2(\boldsymbol{y}_k) = \ln\left(y_k^0 P_t - \sum_{\mathrm{i}=1}^{N} y_{\mathrm{k}}^i\right),$$

$$\varphi_3(\boldsymbol{y}_k) = \ln\left(R_k^{'} - R_t\right).$$

Define the logarithmic barrier function with respect to the inequality constraints as follows,

$$\phi_{\mathrm{ie}}(\boldsymbol{y}_k) = -\varphi_1(\boldsymbol{y}_k) - \varphi_2(\boldsymbol{y}_k) - \varphi_3(\boldsymbol{y}_k). \tag{9}$$

According to [17] and [19], it can be readily established that $\varphi_1(\boldsymbol{y}_k)$, $\varphi_2(\boldsymbol{y}_k)$ and $\varphi_3(\boldsymbol{y}_k)$ are concave. Correspondingly, $\phi_{\mathrm{ie}}(\boldsymbol{y}_k)$ is convex. Define the quadratic penalty function with respect to the quality constraint as follows,

$$\phi_{\mathrm{e}}(\boldsymbol{y}_k) = \left(P_k^c y_k^0 + \sum_{i=1}^{N} y_k^i - 1\right)^2. \tag{10}$$

Obviously, $\phi_{\mathrm{e}}(\boldsymbol{y}_k)$ is convex.

Let μ_{ie} and μ_{e} denote the positive penalty factors. μ_{ie} should be set as small as possible and μ_{e} should be set as large as possible. Define

$$\psi(\boldsymbol{y}_k) = -\zeta(\boldsymbol{y}_k) + \mu_{\mathrm{ie}}\phi_{\mathrm{ie}}(\boldsymbol{y}_k) + \mu_{\mathrm{e}}\phi_{\mathrm{e}}(\boldsymbol{y}_k). \tag{11}$$

Then, the constrained convex optimization problem in (8) can be transformed into an equivalent nonconstrained convex optimization problem by using the mixed penalty method as follows,

$$\min_{\boldsymbol{y}_k} \psi(\boldsymbol{y}_k). \tag{12}$$

Correspondingly, we have the following Theorem.

Theorem 2: The non-constrained convex optimization problem in (12) is equivalent to the constrained convex optimization problem in (8).

Proof: From Theorem 1, we know that the optimization problem in (8) is a convex optimization. According to the barrier method in [17] and [19, Th. 4], the optimization problem in (8) can be transformed into an equivalent convex optimization problem as follows,

$$\max_{\boldsymbol{y}_k} \ \psi'(\boldsymbol{y}_k) = -\zeta(\boldsymbol{y}_k) + \mu_{\mathrm{ie}}\phi_{\mathrm{ie}}(\boldsymbol{y}_k)$$
$$\text{s.t. } P_k^c y_k^0 + \sum_{i=1}^{N} y_k^i - 1 = 0. \tag{13}$$

Then, by introducing the augmented Lagrangian terms in [20], the convex optimization problem in (13) can be transformed into an equivalent unconstrained convex optimization problem in (12). Therefore, the non-constrained convex optimization problem in (12) is equivalent to the constrained convex optimization problem in (8). ■

Define

$$\boldsymbol{y}_k^*(n) = \left[y_k^{0,*}(n), y_k^{1,*}(n), \ldots, y_k^{i,*}(n), \ldots, y_k^{N,*}(n)\right]^T,$$

and let $\boldsymbol{y}_k^*(n)$ denote the solution of the above unconstrained optimization problem for the nth iteration in the non-cooperative power control game, which can readily be obtained by using the gradient method. Then, the optimal transmit power can be calculated as follows,

$$P_k^i(n) = \frac{y_k^{i,*}(n)}{y_k^{0,*}(n)}, \ i = 1, 2, \ldots, N. \tag{14}$$

It should be pointed out here that the constraints have been removed by introducing only two parameters in our proposed approach. Correspondingly, the computational complexity of the problem solving can be reduced greatly.

D. The Existence and Uniqueness of the NE

NE offers a predictable and stable outcome about the transmit power strategy that each SBS will choose. For our considered non-cooperative power control game, we have the following theorem.

Theorem 3: There exists one and only one NE in the non-cooperative power control game $G = [\mathcal{K}, \{\boldsymbol{p}_k\}, \{u_k(\cdot)\}]$.

Proof: It has been established that the optimization problems in (5), (8) and (12) are equivalent. Accordingly, we can treat $\psi(\boldsymbol{y}_k)$ as the payoff function of the kth SBS in the non-cooperative power control game $G = [\mathcal{K}, \{\boldsymbol{p}_k\}, \{u_k(\cdot)\}]$. From the above description, the payoff function $\psi(\boldsymbol{y}_k)$ is continuous with respect to y_k^i, and $\psi(\boldsymbol{y}_k)$ is convex. Therefore, NE exists in the considered non-cooperative power control game.

According to the definition of $\varphi_1(\boldsymbol{y}_k)$, we can readily establish that

$$\frac{\partial^2 \varphi_1(\boldsymbol{y}_k)}{\left(\partial y_k^i\right)^2} = -\frac{1}{\left(y_k^i\right)^2} < 0,$$

$$\frac{\partial^2 \varphi_1(\boldsymbol{y}_k)}{\partial y_k^i \partial y_k^j} = 0, \ \forall i \neq j.$$

Therefore, the Hessian matrix of $\varphi_1(\boldsymbol{y}_k)$ is negative definite and $\varphi_1(\boldsymbol{y}_k)$ is strictly concave. It is already known that $\varphi_2(\boldsymbol{y}_k)$ and $\varphi_3(\boldsymbol{y}_k)$ are concave, and $\phi_{\mathrm{e}}(\boldsymbol{y}_k)$ are convex. Correspondingly, $\phi_{\mathrm{ie}}(\boldsymbol{y}_k)$ is strictly convex, $\psi(\boldsymbol{y}_k)$ is strictly convex, and the optimization problem in (12) has a unique optimal solution. Therefore, a unique NE exists in the proposed non-cooperative power control game. ■

It should be pointed out here that NE is a combination of strategies such that each participant's strategy at the same time is the optimal response to the other participants' strategies, and that NE, even when it is unique, does not mean that all the participants of the game have achieved their global optimum. One of the metrics used to measure how the efficiency of a system degrades due to the selfish behavior of its participants in economics and game theory is the so-called price of anarchy [21], [22], which is defined as the ratio between the worst NE point and the social optima. According to [23], research over the past seventeen years has provided an encouraging counterpoint to this widespread equilibrium inefficiency: in a number of interesting application domains, game-theoretic equilibria provably

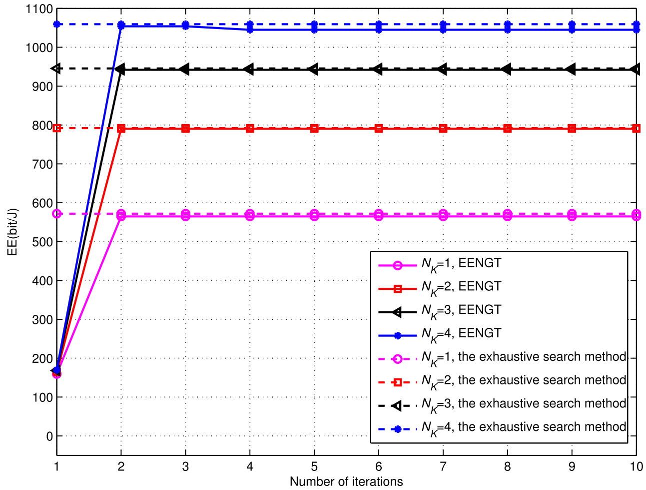

Fig. 2. EE versus the number of iterations with different N_K.

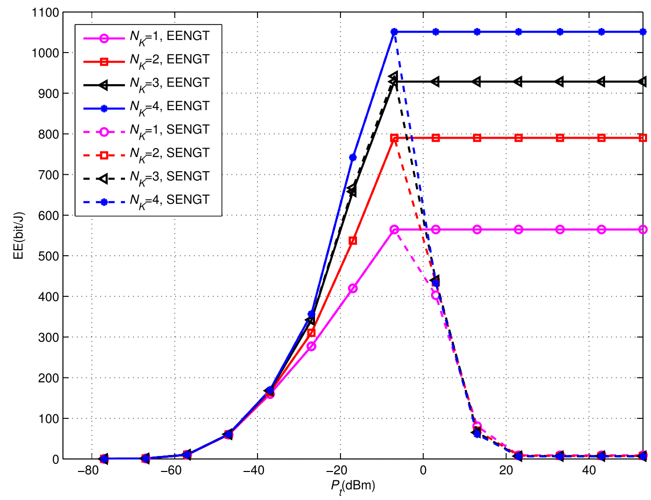

Fig. 3. EE versus the maximum transmit power P_t with different N_K.

approximate the optimal outcome. That is, the price of anarchy is close to 1 in many interesting games.

IV. Simulation Results

In this section, the performance of the proposed non-cooperative power control scheme is evaluated via simulations. In the simulation, we consider the same number of SUEs in each small cell with a path loss function $H = \kappa d^{-\chi}$ [11], where d denotes the distance between the BS and the UE, $\kappa = 10^{-1}$ and $\chi = 4$ the path loss constant and path loss exponent, respectively. In the simulations, the radiuses of the macrocell and small cells are set to be 1000 meters and 100 meters, respectively. The noise spectral density is set to be -174 dBm/Hz.

In Fig. 2, we illustrate the EE of the proposed scheme optimizing the EE based on non-cooperative game theory (hereafter referred to as EENGT) versus the number of iterations with different N_K for $K = 2$, $R_t = 3$ bit/s and $P_t = 20$ dBm. Also illustrated in the figure as a performance benchmark is the EE of the exhaustive search method, using which the EE of the system in (3) is maximized. We can observe that the larger the number of SUEs, the larger the EE of the proposed scheme. The reason is that different SUEs in the considered same cell do not crosstalk each other, and the SUEs can be allocated the suitable power levels. Then, multiuser diversity can be exploited. We can also observe that only several iterations are required for the proposed scheme to converge to the NE. It can be observed that the EE of the proposed scheme approaches the EE of the exhaustive search method for the considered simulation scenario. However, the complexity can be decreased dramatically by using the proposed scheme. Note here that the complexity of the proposed scheme is $\mathcal{O}(KN^2)$, whereas the exhaustive search method has an exponential complexity with respect to the number of RBs and the number of cells.

In Fig. 3, we show the EE versus the maximum transmit power P_t with different N_K for $K = 2$ and $R_t = 3$ bit/s. Also included in the figure is the EE of the power control scheme optimizing SE based on non-cooperative game theory (hereafter referred to as SENGT) in [5]. For the SENGT, SE instead of EE is maximized.

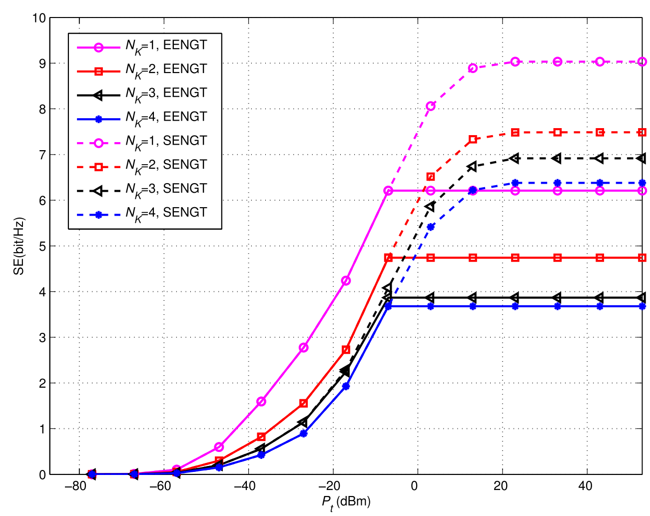

Fig. 4. SE versus the maximum transmit power P_t with different N_K.

It can be observed that the EE of the EENGT increases with P_t when P_t is smaller than a certain threshold value, and that the EE almost remains constant when P_t is large enough. We can readily observe that the EENGT, which optimizes EE, provides an obvious performance improvement in terms of EE over the SENGT in [5], which optimizes SE. The reason is that the latter scheme uses excess power to increase the SE by sacrificing the EE, especially in the high transmit power region.

In Fig. 4, we show the SE versus the maximum transmit power P_t with different N_K for $K = 2$ and $R_t = 3$ bit/s. We compare the system performance of the EENGT again with the SENGT in [5]. It can be obviously observed that the SE of the EENGT increases with the maximum transmit power in the low transmit power region, and that the SE remains constant in the high transmit power region. The reason is that the EENGT clips the transmit power at the SBSs to maximize the system EE. It can also be observed that the SENGT achieves a higher SE than the EENGT. The reason is that the former scheme consumes all the available transmit power in all the considered transmit power region. However, the SE of the baseline scheme comes at the expense of lower EE.

We can observe from Figs. 3 and 4 that both the EE and SE of our proposed scheme increase with the maximum transmit power in the low transmit power region, and they almost remain constant in the high transmit power region. This gives us the insight that only appropriate, rather than exorbitant, transmit power needs to be allocated for each SBS to achieve its maximum utility and reach the NE point.

V. Conclusion

In this paper, we have formulated a non-cooperative power control game for small cells underlaying a macro cellular network. By exploiting the properties of parameter-free fractional programming, the concept of perspective function, and the mixed penalty function, an energy efficient power control scheme has been proposed to maximize the EE. Simulation results have shown that significant improvements in terms of EE are achieved by using the proposed scheme.

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