Competitive Online Algorithm for Leasing Wireless Channels in 3-Tier Sharing Framework

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Abstract*— To meet the ever growing need for wireless spectrum, the Federal Communication Commision (FCC) introduced a spectrum sharing model called the *3-Tier Sharing Framework*. In this model, under-utilized federal spectrum will be released for shared use where the highest preference will be given to Tier-1 followed by Tier-2 and then Tier-3. In this paper, we present a model where a wireless operator, who is interested in maximizing its profit, can operate as a Tier-2 and/or a Tier-3 user. Tier-2 is characterized by *paid* but "almost" guaranteed and interference free channel access while Tier-3 access is *free* but has lesser guarantee and also faces channel interference. So the operator has to optimally decide between paid but better channel quality and free but degraded channel quality. Also, the operator has to make these decisions without knowing future market parameters like customer demands or channel availability. We use tools from *ski-rental* literature to design a deterministic online algorithm for leasing channels which does not rely on the knowledge of market statistics. The efficiency of the online algorithm is analyzed by deriving its competitive ratio (CR) and by conducting simulations. The mathematical model for leasing channels is a novel generalization of the classical ski-rental problem. We therefore make fundamental contribution to ski-rental literature which may have diverse applications beyond the problem considered in this paper.

I. Introduction

The demand for wireless Internet access is ever growing and the wireless spectrum is getting scarce. The President's Council of Advisors on Science and Technology (PCAST), in their report [1], calls the notion of spectrum scarcity a *"fundamental misunderstanding"* arising due to under-utilization of spectrums. In support of PCAST report [1], the FCC decided to release 150 MHz of federal spectrum (from 3.55 to 3.7 GHz) . The shared use of these federal spectrums should follow the *3-Tier Sharing Framework* [2] (refer Figure 1): *Tier-1* is called the "Incumbent tier" consisting of federal users who have the highest priority access to any channel and are guaranteed interference protection from lower tiers. *Tier-2* is called the "Priority Access Licenses (PAL) tier". PAL users can lease the channels by participating in auctions. They can use the leased channels whenever Tier-1 users are not using it. Priority Access Users are gauranteed interference protection from Tier-3 users. *Tier-3* is called the "Generalized Authorized Access (GAA) tier". GAA users can opportunistically use a channel for *free* provided that it is not used by Tier-1 or Tier-2 users. A Tier-3 user is not gauranteed interference protection from Tier-1, Tier-2 or even other Tier-3 users. The Spectrum Access System (SAS) is a central database which keep record of channel states. It is also a policy engine which enforces the 3-Tier hierarchy.

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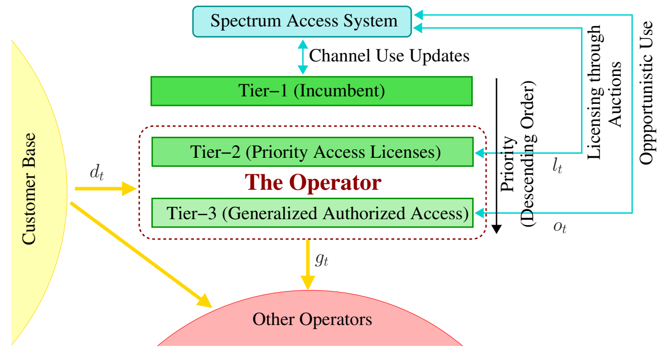

Fig. 1. The 3-Tier Sharing Framework and our System Model.

In this paper, we consider a market consisting of many operators. These operators use wireless spectrum to serve customer demands. We consider one such operator, labelled "The Operator" in Figure 1. The objective is to *maximize the profit* of the operator. In our model, the operator can work as Tier-2 and/or Tier-3 user. We consider a time slotted model where at the t^{th} epoch, the operator must make the following three decisions upon receiving d_t customer demands:

1) Amount of customer demand g_t to reject. As shown in Figure 1, the rejected customer demands gets served by "other operators" in the market.
2) Number of channels l_t to lease (Tier-2) in order to serve the accepted customer demands.
3) Number of channels o_t to use opportunistically (Tier-3) in order to serve the accepted customer demands.2

The decision process of rejecting demands, leasing channels and opportunistic channel use relies on customer demand pattern and channel availability trends. The operator has to make decisions without the future knowledge of these variables. *Online nature* of the problem leads to the following uncertainties when an operator wants to lease a channel:

1) *Uncertainty in Customer Demand:* Leasing a channel is profitable only if the customer demand in future epochs is consistently high.
2) *Uncertainty in channel availability for opportunistic use:* Leasing a channel is not profitable if there are enough channels for opportunistic use in future epochs.
3) *Uncertainty in channel availability for leasing:* All the channel leases may get sold out in future.

Along with the online nature of the problem, there is an additional issue that serving customer demands using opportunistic channels may lead to lower Quality of Service (QoS) to the customers because such channels may suffer from harmful interference. This is a challenging online optimization problem which we address in this paper.

To the best of authors knowledge, the only work in spectrum sharing literature which resembles our problem is addressed in [3]. In [3], the authors modelled the customer demand and channel availability statistics as Discrete Time Markov Chain. It then used tools from Stochastic Dynamic Programming to design an online algorithm for leasing channels. Apart from the problem considered in this paper, there are other bodies of work which is of importance to the 3-TSF. In [4], the authors designed a network protocol and a SAS which implements the rules of the 3-TSF. The work done in [5] considers a market where an operator can operate in either Tier-2 or Tier-3. It investigates the incentive of an operator to enter such a market in presence of competition. Other allied areas of research can be of significant importance to the 3-TSF though they are not directly related. From economic standpoint, research in the field of spectrum contracts ([6], [7]), auctions and pricing ([8]) will help understand if the 3-Tier Sharing Framework is economically attractive for potential investors. From technical standpoint, dynamic channel allocation is of significant importance to 3-TSF. While doing dynamic channel allocation it is crucial to consider blocking probability [9] and co/adjacent channel interference [10].

This paper makes three contributions. *First,* we present a system model in which an operator using 3-TSF can maximize its profit by strategically operating as a Tier-2 and/or a Tier-3 user. Our model is novel as it captures key elements of 3-TSF.

Second, compared to [3], our online algorithm has two novelty. First, unlike [3], our algorithm does not require statistical knowledge of the involved random processes (customer demand and spectrum availability). In the early stages of the deployment of 3-TSF, the operator will have either limited or no knowledge of market statistics. Hence, the algorithm proposed in this paper will be more suitable compared to [3]. Second, the algorithm proposed in [3] has high time complexity (pseudo-polynomial) if the optimization horizon is large. Our algorithm has low time complexity (polynomial) irrespective of the optimization horizon.

Third, this work adds a new application area to the ski-rental problem among the already existing ones [11], [12], [13]. We also make a fundamental contribution to ski-rental literature, i.e. how the classical breakeven algorithm for the ski-rental problem gets modified if the number of skis available for leasing is finite.

The rest of the paper is organized as follows: In section II we mathematically formulate the profit maximization problem of the operator. Section III contains the main result of the paper, a competitive deterministic online algorithm for leasing channels. Simulation results are presented in Section IV. Finally we conclude the paper in Section V with a brief discussion on the immediate extensions to this work.

TABLE I

A TABLE OF FREQUENTLY USED NOTATIONS.

Notation	Description
T	Optimization Horizon.
M	Total number of channels.
d_t	Number of channels needed to serve the customer demand at epoch t. We call this *"demand"* in the rest of the paper.
g_t	Number of demands the operator rejected at epoch t.
p_t	Price of serving a demand at epoch t.
p_M	Upper bound on p_t, $p_t \leq p_M$.
P , τ	Price and duration of a channel lease. In general, $p_t \ll P$.
M_t^l	Number of channels available for leasing at epoch t.
l_t	Number of channels the operator leased at epoch t.
v_t	Number of channels other operators *wants* to lease at epoch t.
w_t	Number of channels leased by all the operators at epoch t.
M_t^o	Number of channels the operator can opportunistically use at epoch t.
A_t	Number of active channel leases the operator has at epoch t.
λ_t	Number of active channel leases which got pre-empted by Tier-1 users at epoch t.
o_t	Number of channels the operator used opportunistically at epoch t.
$f_t\left(o_t\right)$	A function to penalize opportunistic channel use.
r_t	$r_t = g_t + o_t$, the combined action of using channels opportunistically and rejecting demands.
$F_t\left(r_t\right)$	Renting function: A function to penalize the renting action.
φ_t	$\varphi_t = (d_t\,,\, \lambda_t\,,\, p_t\,,\, M_t^o\,,\, v_t\,,\, f_t\left(\cdot\right))$, a tuple which forms the input to $\mathbf{OP1}$.
$(x)^+$	Positivity operator: $(x)^+ = \max\left(0, x\right)$

II. System Model

Notations: Table I lists notations used throughout this paper. Other notations used in this paper are standard.

In this section we formulate the profit maximization problem of the operator. We also discuss the underlying assumptions in our problem formulation. Time is considered to be slotted. In every epoch $t \in \{1, \ldots, T\}$, the decision variables are $g_t\,, o_t,\, l_t \in \{0, \ldots, M\}$. Operator's profit is

$$\mathcal{P} = \sum_{t=1}^{T}\left(p_t\left(d_t - g_t\right) - Pl_t\right) = \underbrace{\sum_{t=1}^{T} p_t d_t}_{1^{st}\text{ term}} - \underbrace{\sum_{t=1}^{T}\left(p_t g_t + Pl_t\right)}_{2^{nd}\text{ term}} \tag{1}$$

where $p_t\left(d_t - g_t\right)$ is the operator's revenue for serving $\left(d_t - g_t\right)$ demands and Pl_t is the operator's expense to buy l_t channels at epoch t. Operator wants to maximize its profit $\mathcal{P}$ which is equivalent to minimizing the 2^{nd} term because in our model, the operator has no control demand d_t and price p_t. To make the model more realistic, we add a function $f_t\left(o_t\right)$ to the 2^{nd} term which penalizes opportunistic use of channels. This leads to the final optimization problem

$$\begin{aligned}
\mathbf{OP1}: \quad & \min_{\{g_t, o_t, l_t\}} \mathcal{C} = \sum_{t=1}^{T}\left(p_t g_t + f_t\left(o_t\right) + Pl_t\right) \\
\text{subject to:} \quad & o_t + g_t + \left(\sum_{i=(t-\tau+1)}^{t} l_i - \lambda_t\right) \geq d_t \\
& 0 \leq o_t \leq M_t^o \;;\; 0 \leq l_t \leq M_t^l \;;\; 0 \leq g_t
\end{aligned}$$

The function $f_t\left(o_t\right)$ is time-varying and is assumed to be *convex* and *increasing* in the range $[0\,,\, M_t^o]$. It has two real world interpretations: *First*, to account for harmful interference in an opportunistic channel, the operator may choose to transmit at a higher power level[1]. In this case $f_t(o_t)$ respresents the cost to transmit at a higher power level. *Second*, the use of opportunistic channels leads to lower QoS to the customers if the channel interference is high. The function $f_t(o_t)$ can be used to capture the monetary loss due to lower QoS. The term $\sum_{i=(t-\tau+1)}^{t} l_i$ in $\mathbf{OP1}$ is the number of active channel leases at epoch t and is denoted as A_t. However λ_t active leases may get interrupted by Tier-1 users[2] leaving effectively $\left(\sum_{i=(t-\tau+1)}^{t} l_i - \lambda_t\right)$ active channel leases. Given that the lease period of a channel is τ, the time evolution of M_t^l is governed by the equation[3]

$$M_t^l = M_{t-1}^l - w_{t-1} + w_{t-\tau} \qquad \text{where} \tag{2}$$

$$w_t = l_t + \min\left(M_t^l - l_t\,,\, v_t\right) \tag{3}$$

where $w_{t-\tau}$ are the number of leases which reappears in the market at t^{th} epoch when its lease period expires. In equation (3), the term $\min\left(M_t^l - l_t\,,\, v_t\right)$ is the number of channels leased by other operators in the market. This term captures an key assumption of our model, i.e. the operator has superior auctioning strategy compared to other operators. Therefore if the operator wants to lease $l_t \leq M_t^l$ channels at epoch t, it will win the bid for l_t channels, leaving $M_t^l - l_t$ channels which can be leased by the other operators.

In our model, the tuple φ_t (defined in Table I) is a time sequence which forms the input to $\mathbf{OP1}$. The cost $\mathcal{C}$ incurred by $\mathbf{OP1}$ is a function of φ_t. All other variables in $\mathbf{OP1}$, except p_t and P, lies in the discrete set $\{0, 1, \ldots, M\}$.

A. Assumptions

The key assumptions in our system model are as follows:

1) The cost of leasing a channel is time varying as it depends on the outcome of the auction. However we assumed it to be a constant P.
2) The *role of competition* among operators to maximize the profit of the operator is not considered., for e.g. we do not design optimal pricing policy for p_t to compete for customer demand nor do we consider optimal bidding strategy to compete in auctions. In our model p_t and d_t are arbitrary time varying sequences which can not be controlled by the operator.

Other than the assumptions on system model, we also need to impose the following assumptions in order to design online algorithms with provable theoretic bounds:

1) Future knowledge of p_t, d_t, λ_t, M_t^o, v_t and $f_t(\cdot)$ is not assumed. However we impose the following constrain

$$0 < \mu_l \leq \frac{\sum_{i=(t-\tau+1)}^{t} M_i^l}{\tau} \;;\; \forall t \tag{4}$$

which says that the moving horizon time average of M_t^l over τ period is at least μ_l.
2) The function $f_t(o_t)$ can be evaluated for any o_t.
3) p_t is upper bounded by p_M, i.e. $p_t \leq p_M\,; \forall t$. Knowledge of p_M is assumed.

[1]The highest power level at which the operator can transmit as Tier-3 user is constrained by FCC rules. This indeed leads to the second interpretation.

[2]The SAS will try to relocate the channel of Tier-2 user if it gets interrupted by Tier-1 user. λ_t models such relocations of channels too.

[3]Equation 2 and 3 is valid even for $t < 1$. However $l_t = 0\,; \forall t < 1$.

III. Deterministic Online Algorithm

In this section we will design a deterministic online algorithm for leasing channels. We approach this in steps. In Section III-A we introduce a theorem which provides better insight into the structure of $\mathbf{OP1}$ and effectively reduces the number of decision variables from three to two. In Section III-B we consider a special case of $\mathbf{OP1}$ called the *Modified Ski-Rental Problem*. It is simpler to analyze but provides useful insights into the online algorithms for $\mathbf{OP1}$. *Finally,* we will design and analyze a deterministic online algorithm for leasing channels in Section III-C.

A. Simplification of $\mathbf{OP1}$

$\mathbf{OP1}$ can be decoupled into two sub-problems, one which decides how many of the available opportunistic channel to use and the other which captures the online nature of leasing channels. The following theorem formalizes this notion.

Theorem 1: Let

$$\mathbf{OP2}: \; \bar{o}_t = \underset{0 \leq o_t \leq M_t^o}{\arg\min} \; -p_t o_t + f_t(o_t)$$

and define the following function

$$F_t(r_t) = p_t\left(r_t - \bar{o}_t\right)^+ + f_t\left(\min\left(r_t\,,\,\bar{o}_t\right)\right) \tag{5}$$

Then the optimal solution g_t^*, o_t^* and l_t^* of $\mathbf{OP1}$ can be obtained by solving the optimization problem

$$\mathbf{OP3}: \quad \min_{\{r_t, l_t\}} \mathcal{C} = \sum_{t=1}^{T}\left[F_t(r_t) + P l_t\right]$$

$$\text{subject to:} \quad r_t + \sum_{i=(t-\tau+1)}^{t} l_i \geq D_t \triangleq (d_t + \lambda_t)$$

$$0 \leq l_t \leq M_t^l \;;\; 0 \leq r_t$$

for the optimal solution $\bar{r}_t$ and $\bar{l}_t$ and then setting

$$g_t^* = \left(\bar{r}_t - \bar{o}_t\right)^+ \;;\quad o_t^* = \min\left(\bar{r}_t\,,\,\bar{o}_t\right) \;;\quad l_t^* = \bar{l}_t \tag{6}$$

Proof: Please refer [14] for the proof.

$\mathbf{OP2}$ decides how many of the available opportunistic channel M_t^o to be used at epoch t. The function $h_t(o_t) = -p_t o_t + f_t(o_t)$ is *unimodal* (refer [14]). We can therefore use tools like *binary search* or *fibonnaci search* [15] to solve $\mathbf{OP2}$ in $\mathcal{O}\left(\log_2\left(M_t^o\right)\right)$ time.

$\mathbf{OP3}$ has two decision variables. The variable l_t as usual implies leasing *(Tier-2)*. The new variable r_t implies *renting*. Renting is the combined action of using channels opportunistically *(Tier-3)* and rejecting demands, i.e. $r_t = o_t + g_t$. So in every epoch the operator has to decide *how much to rent* and *how much to lease* in order to serve the *effective demand* D_t. We will use this terminology quite often in rest of the paper. The function $F_t(r_t)$ is called the *renting function*. It penalizes the renting action and has the following properties:

Property 1: $F_t(r_t)$ is monotonically increasing in r_t. This property suggest that if $\bar{l}_t$ is the optimal solution to $\mathbf{OP3}$, and $\overline{A}_t$ the corresponding sequence of the number of active leases, then the optimal solution $\overline{r}_t$ is given by

$$\overline{r}_t = \left(D_t - \overline{A}_t\right)^+ \tag{7}$$

Property 2: $F_t(r_t)$ is convex in r_t. This implies

$$F_t(r_t) - F_t(r_t - 1) \leq F_t(r_t + 1) - F_t(r_t) \ ; \ \forall r_t \tag{8}$$

Property 3: First derivative of $F_t(r_t)$ is bounded as follows

$$F_t(r_t + 1) - F_t(r_t) \leq p_t \leq p_M \ ; \ \forall r_t \tag{9}$$

The proof of these properties are trivial. It can be found in [14] but has been skipped here for brevity.

$\mathbf{OP3}$ captures the online nature of leasing channels. Say that the operator has to decide the optimal number of lease $\bar{l}_t$ at epoch t. A *necessary* condition for optimality of $\bar{l}_t$ is

$$\sum_{i=t}^{t+\tau-1} \left[F_i\left((D_i - a_i)^+\right) - F_i\left((D_i - a_i - \bar{l}_t)^+\right)\right] \geq P\bar{l}_t \tag{10}$$

where $a_i = \sum_{j=i-\tau+1}^{t-1} \bar{l}_j \ ; \ \forall i \in \{t, \ldots, t+\tau-1\}$ is the number of active leases in the i^{th} epoch if $\bar{l}_t = 0$. Inequality (10) implies that the net rental cost saved by leasing $\bar{l}_t$ channels should be greater than the cost of leasing $\bar{l}_t$ channels. The operator must know $D_i, F_i(\cdot) \ ; \ \forall i \in \{t, \ldots, t+\tau-1\}$ in order to choose a $\bar{l}_t$ which satisfies inequality (10). To calculate $D_i, F_i(\cdot)$ for $i > t$, the operator needs future knowledge of φ_i (refer Table I). Hence, online information is not enough to find an optimal sequence $\bar{l}_t$.

The operator has to decide $(r_t \, , l_t)$ just based on the knowledge of $\varphi_i \, ; \, \forall i \leq t$ in a certain optimal sense called the competitive ratio. Competitive ratio (CR) is a relative measure of an online algorithm with respect to an optimal algorithm[4]. Define the sequence $\varphi = \{\varphi_1, \varphi_2 \ldots, \varphi_T\}$. Let $\mathcal{C}_{\mathcal{A}}(\varphi)$ and $\mathcal{C}_{OPT}(\varphi)$ be the cost incurred by a deterministic online algorithm $\mathcal{A}$ and the optimal algorithm OPT respectively. $\mathcal{A}$ is called $c-$competitive iff

$$\mathcal{C}_{\mathcal{A}}(\varphi) \leq c \cdot \mathcal{C}_{OPT}(\varphi) \ ; \ \forall \varphi \in \mathcal{S}$$

A smaller c implies a better online algorithm. The set $\mathcal{S}$ contains all possible values of φ. Competitive analysis is often thought of as a two player game between an *adversary* which generates φ to maximize the ratio $\frac{\mathcal{C}_{\mathcal{A}}(\varphi)}{\mathcal{C}_{OPT}(\varphi)}$ and the online algorithm $\mathcal{A}$ which tries to minimize the ratio.

The rest of this section deals with designing an online algorithm for $\mathbf{OP3}$. $\mathbf{OP3}$ has important resemblance with works in ski-rental literature like [12] but with one key difference: the operator cannot lease more than M_t^l channels at epoch t. This additional constrain makes the online algorithm for $\mathbf{OP3}$ non-trivial both in terms of design and analysis.

[4]The optimal algorithm for $\mathbf{OP3}$ is an offline algorithm based on dynamic programming and has pseudo-polynomial time complexity [12].

B. Modified Ski-Rental Problem

In this section we consider a modification of the classical Ski-Rental Problem (SRP) and show that it is a special case of $\mathbf{OP3}$. Then we will design an optimal deterministic online algorithm to solve the Modified Ski-Rental Problem (MSRP) which will give us insights into solving $\mathbf{OP3}$ online.

MSRP can be stated as follows:

1. A skier plans a skiing vacation with a tourism agency which *rents* a ski[5] for p_M dollars per day and *leases* a ski for P dollars (where $p_M \ll P$) with the lease period being $\tau > 1$ days[6]. In context of $\mathbf{OP3}$ it means that $F_t(r_t) = p_M r_t$.

2. The skier needs one ski a day. Skiing vacation is at most τ days (equal to the lease period) but can end on the y^{th} day (where $0 \leq y \leq \tau$) if the skier gets injured while skiing. In context of $\mathbf{OP3}$ it implies the following demand structure : $D_t = 1 \, ; \, 1 \leq t \leq y$ and $D_t = 0 \, ; \, t > y$.

3. The tourism agency has a maximum of M skis to lease. The number of skis available for leasing on the t^{th} day is M_t^l where M_t^l is governed by equation 2 and 3. In context of MSRP, l_t and v_t are the number of skis "the skier" and the other skiers *wanted* to lease in day t respectively.

4. The skier can lease a ski on the first day, i.e. $M_1^l > 0$.

The above four points shows that MSRP is a special case of $\mathbf{OP3}$. If $M = \infty$ then $M_t^l > 0 \, ; \, \forall t$. In this case MSRP reduces to SRP. For SRP, the optimal online deterministic algorithm is the *breakeven algorithm* which states: Say the skier is still skiing on the k^{th} day. If the net renting cost $p_M k \geq P$, the skier should lease a ski on the k^{th} day. Else, the skier should rent. CR of this algorithm is 2.

If M is finite then it is possible that $M_t^l = 0$ for some t. The key difference between SRP and MSRP is the *availability of ski leases*. The skier may decide to lease on the k^{th} day only to find that $M_k^l = 0$. Without any constrain on M_t^l, there may not be any skis available for leasing till the end of skier's vacation. In the worst case scenario, the skier has to keep renting till her vacation ends incurring a cost of τp_M while the offline algorithm which can foresee the future will lease a ski on the 1^{st} day. Hence the CR is $\frac{\tau p_M}{P}$. This suggests that CR for MSRP cannot be better than $\frac{\tau p_M}{P}$ without any constrain on M_t^l. We therefore constrain M_t^l using inequality (4). This leads to the following proposition.

Proposition 1: Say that the skier/operator decides to buy $l_t > 0$ leases at epoch t. The skier/operator buys l_t leases as and how it reappears in the market. Define

$$\eta = \inf\left\{\delta \geq 0 \, | \, \sum_{k=0}^{\delta} M_{t+k}^l \geq l_t\right\}$$

as the *wait time* because the skier/operator has to wait at least till epoch $t + \eta$ to buy all the l_t leases. If M_t^l is constrained by inequality (4), then η can be upper bounded by η_M where

$$\eta_M = \tau\left(1 - \frac{\mu_l}{M}\right) \leq \tau - 1 \tag{11}$$

Proof: Please refer [14] for the proof.

[5]"a ski" implicitly means a pair of skis.

[6]"Renting" and "leasing" are indeed synonyms but in this paper they are differentiated based on price and contract duration.

Proposition 2: Consider the following online algorithm:

1) Keep renting if the net renting cost is less than z_{op}, where z_{op} is the solution to the quadratic equation

$$z_{op}^2 + \tau\left(1 - \frac{\mu_l}{M}\right) p_M z_{op} - P^2 = 0$$

If the net renting cost exceeds z_{op}, the skier must decide to lease a ski.

2) If a ski is availabe for lease that day, then lease it. Else wait till it is available again.
3) The skier should buy a lease when it is available again only if the wait time $\eta \leq \tau - \frac{(z_{op}+P)}{p_M}$. Else keep renting till the end of skiing vacation.

Among all online algorithms for MSRP which only assumes the knowledge of μ_l and hence the upper bound on wait time, $\eta_M = \tau\left(1 - \frac{\mu_l}{M}\right)$, the above algorithm has the best CR of

$$c_{opt}(\mu_l) = \begin{cases} \left(1 + \frac{z_{op}}{P}\right) + \frac{\tau p_M}{P}\left(1 - \frac{\mu_l}{M}\right) & ; \mu_l \geq \frac{M}{\left(\frac{\tau p_M}{P} - 1\right)} \\ \frac{\tau p_M}{P} & ; \mu_l < \frac{M}{\left(\frac{\tau p_M}{P} - 1\right)} \end{cases} \tag{12}$$

Proof: Please refer [14] for the proof.

Theorem 2: An online algorithm for $\mathbf{OP3}$ which only assumes the knowledge of μ_l and hence the upper bound on wait time, $\eta_M = \tau\left(1 - \frac{\mu_l}{M}\right)$, cannot achieve CR better than $c_{opt}(\mu_l)$.

Proof: This follows from the fact that MSRP is a special case of $\mathbf{OP3}$ and $c_{opt}(\mu_l)$ is the best possible CR we can achieve for MSRP (by *Proposition* 2).

C. Online Algorithm for Leasing Channels

Motivated by the optimal online algorithm to solve MSRP, we suggest a *threshold based* algorithm for leasing channels. There are two threshold criterias:

1) The algorithm decides to lease a channel when the net incremental renting cost exceeds threshold z_{th}.
2) The algorithm rejects the decision to lease a channel if the wait time exceeds the threshold $\tau - \frac{(z_{th}+P)}{p_M}$.

A generic algorithm for any threshold z_{th} is presented in the listing Algorithm 1. However in this paper, we only concentrate on the case when $z_{th} = P$. The working of the algorithm can be divided into five steps.

Step 1 (Learn φ_t)

Recall that the tuple $\varphi_t = (d_t\,,\, \lambda_t\,,\, p_t\,,\, M_t^o\,,\, v_t\,,\, f_t(\cdot))$ is the input to $\mathbf{OP1}$. At epoch t, the operator knows d_t and p_t while λ_t, M_t^o and v_{t-1} can be learned by querying the SAS[7]. The penalty function $f_t(\cdot)$ for opportunistic channel use is estimated, possibly using QoS reviews from the customers.

Step 2 (Calculate $\overline{o}_t$)

The operator computes $\overline{o}_t$ by solving $\mathbf{OP2}$ using binary/ fibonacci search *(line 6)*. The renting function $F_t(r_t)$ is implicitly dependent on $\overline{o}_t$ (refer equation (5)). Hence we need to compute $\overline{o}_t$ in order to evaluate $F_t(r_t)$ in Step 3.

Step 3 (Deciding to lease or not)

The operator maintains two time sequences x_t, the number of virtual active lease the operator has at epoch t and y_t, the virtual number of channels available for leasing at epoch t.

[7]Note that at time t, it is impractical to assume knowledge of v_t however knowledge of v_{t-1} is feasible. In many auction the demand of other bidders may be kept confidential because any asymmetry in information can lead to unfair advantage. However releasing such information after the auction may be feasible. We therefore assume that during time t, the operator can query SAS for the value of $l_{t-1} + v_{t-1}$ (the number of channels the entire market wanted to lease at time t) and hence infer v_{t-1}.

Algorithm 1 $\mathcal{A}_{z_{th}}$: a deterministic online algorithm for leasing channels in 3-Tier Sharing Framework.

1. Let x_t be the number of *virtual* active leases at the t^{th} epoch. Set $x_t = 0\,; t = 1, 2, \ldots, T$
2. Let y_t be the *virtual* number of channels available for leasing at the t^{th} epoch. Set $y_t = 0\,; t = 1, 2, \ldots, T$
3. Initialize an empty FIFO queue which will store timestamps of leasing decisions.
4. Repeat steps 5-13 in all epochs. Let current epoch be t.
5. Learn d_t, p_t, λ_t, M_t^o, v_{t-1} and $f_t(\cdot)$. Set $D_t = d_t + \lambda_t$.
6. Compute $\overline{o}_t$ by solving optimization problem $\mathbf{OP2}$.
7. Set $w_{t-1} = \min(v_{t-1}\,,\, y_{t-1})$, $y_t = y_{t-1} - w_{t-1} + w_{t-\tau}$.
8. Set $\mathcal{R} = z_{th}$.
9. **while** $(\mathcal{R} \geq z_{th})$
  9.a. Check the first epoch after $(t - \tau + 1)$ when the operator could have bought another lease. Mathematically, $t_M = \inf\{k \,|\, y_k > 0\,,\, k \geq t - \tau + 1\}$.
  9.b. Compute the net incremental rental cost $\mathcal{R}$ from t_M to the current epoch t using equation (13).
  9.c. **if** $(\mathcal{R} \geq z_{th})$
   9.c.1. The operator decides to lease a channel. Hence, current epoch t is *pushed* into the FIFO queue.
   9.c.2. Set $x_i = x_i + 1\,; i = t_M, \ldots, t-1$ to update the history of x_i's. This shows that previous mistakes have been accounted.
   9.c.3. Set $w_{t_M} = \min(w_{t_M} + 1\,,\, y_{t_M})$ and $y_i = (y_i - 1)^+\,; i = t_M, \ldots, t$. This shows that a virtual lease was bought at epoch t_M.
   9.c.4. Set $x_i = x_i + 1\,; i = t, \ldots, t + \tau - 1$. This updates future x_i's to show that an additional virtual lease is available in future epochs.
  9.c. **end if**
9. **end while**
10. Set the number of channels to lease to zero : $l_t = 0$.
11. **while** ($M_t^l > 0$ AND "FIFO Queue is Not Empty")
  11.a. *Read* timestamp from the FIFO queue. Let this epoch be t_l. Set wait time $\eta = t - t_l$.
  11.b. **if** $\left(\eta \leq \left(\tau - \frac{(z_{th}+P)}{p_M}\right)\right)$
   11.b.1. Lease a channel: $l_t = l_t + 1$.
  11.b. **end if**
  11.c. *Pop* timestamp from the FIFO queue.
11. **end while**
12. Number of active lease is $A_t = \sum\limits_{i=t-\tau+1}^{t} l_i$. Remaining $r_t = (D_t - A_t)^+$ demands are served by renting.
13. Number of channels to use opportunistically, $o_t = \min(r_t, \overline{o}_t)$. Reject $g_t = (r_t - \overline{o}_t)^+$ demands.

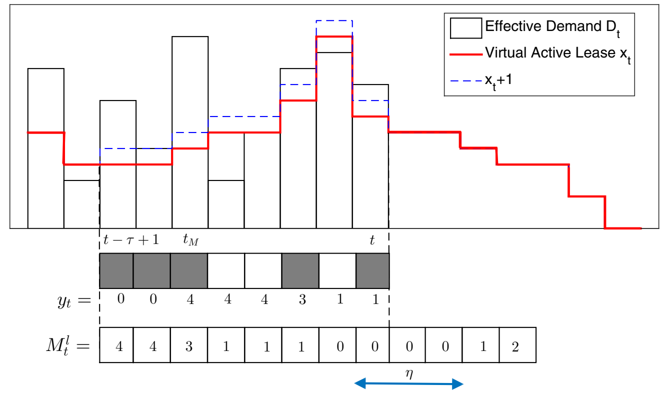

Fig. 2. An illustration of net incremental renting cost and wait time. When $x_t < D_t$ ($x_t \geq D_t$), i.e. the red graph is below (above) the black graph, a non-zero (zero) incremental renting cost is incurred. This is depicted using grey (white) epochs in the upper strip. In this example, $\tau = 8$ and $\eta = 3$.

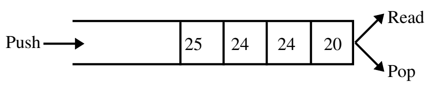

Fig. 3. A FIFO Queue containing time stamps. Timestamps are *pushed* behind the queue while they are *read* and *popped* from front of the queue.

The sequence x_t is virtual in the sense that the operator may not have x_t active leases at epoch t. Rather the operator decides x_t, for a past epoch t, by looking back in time and analyzing the number of additional channels it should have leased at epoch t to minimize the loss. Similarly y_t is the number of channels available for leasing at epoch t if the operator had leased additional channels at past epochs. These two sequences helps the operator decide the number of channel it wants to lease in the current epoch. This can be explained as follows.

At current epoch t, the operator looks back τ epochs and find the first epoch after $t-\tau+1$ when an additional lease could have been bought *(line 9.a.)*. Let this epoch be t_M. The net incremental renting cost $\mathcal{R}$ is the net renting cost which could have been saved in the time period $[t_M, t]$ if one more lease was bought in epoch t_M *(line 9.b.)*. Mathematically,

$$\mathcal{R} = \sum_{i=t_M}^{t} \left[F_i \left((D_i - x_i)^+ \right) - F_i \left((D_i - x_i - 1)^+ \right) \right] \tag{13}$$

where D_i and x_i are the effective demand and the number of virtual active leases respectively in the i^{th} epoch. $F_i\left((D_i - x_i)^+\right)$ and $F_i\left((D_i - x_i - 1)^+\right)$ are the renting cost in the i^{th} epoch to serve the demands above the red and the blue graph respectively in Figure 2.

If $\mathcal{R} \geq P$ then the operator could have minimized the loss by leasing a channel in epoch t_M. To compensate for this mistake the operator decides to lease a channel. The current timestamp t is pushed in the end of the FIFO queue as shown in Figure 3 indicating the decision to buy an additional lease *(line 9.c.1.)*. A virtual lease is bought at epoch t_M to indicate that a corrective measure has been taken for the past mistake. This updates the history of x_i *(line 9.c.2.)* and y_i *(line 9.c.3.)*. Without such updation the operator will take corrective measure for the same mistake multiple times. The future x_i's are also updated assuming that the operator can buy the additional lease in the current epoch *(line 9.c.4.)*.

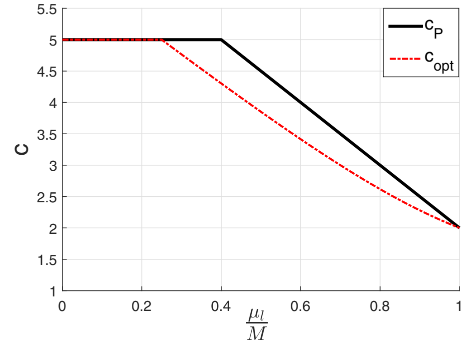

Fig. 4. A typical plot comparing the CR of $\mathcal{A}_P$ with the most optimal CR, $c_{opt}(\mu_l)$, as given by equation (12). In this plot $\frac{\tau p_M}{P} = 5$.

Step 3 is repeated till $\mathcal{R} < P$. When $\mathcal{R} < P$, it indicates that purchasing any more additional lease is costlier than renting. This is a direct consequence of *Property 2*. Hence the operator decides not to buy any additional leases.

Step 4 (To lease or not)

The timestamps of leasing decisions are read from the front of the FIFO queue *(line 11.a.)* as shown in Figure 3. If the wait time η corresponding to the timestamp is lesser than $\tau - \frac{2P}{p_M}$ *(line 11.b)*, an additional channel is leased *(line 11.b.1.)*. Finally the processed timestamp is popped out of the queue *(line 11.c.)*. This step is continued either till the FIFO queue is empty or there are no more channels available for leasing in the current epoch.

A FIFO queue is used so that the timestamps are processed in the order in which they were generated. Otherwise it may happen that the wait time of a timestamp, which could have been below the threshold $\tau - \frac{2P}{p_M}$ gets rejected because it was processed later.

Step 5 (Calculate o_t and g_t)

If there are A_t active leases, then by *Property 1*, $(D_t - A_t)^+$ demands are served by renting *(line 12)*. The number of channels to use opportunistically and the amount of demands to reject is given by equation (6) *(line 13)*.

Theorem 3: The competitive ratio of online algorithm $\mathcal{A}_P$ ($\mathcal{A}_{z_{th}}$ with $z_{th} = P$) is

$$c(\mu_l) = \begin{cases} 2 + \frac{\tau p_M}{P}\left(1 - \frac{\mu_l}{M}\right) & ;\ \mu_l \geq \frac{M}{\left(\frac{\tau p_M}{2P}\right)} \\ \frac{\tau p_M}{P} & ;\ \mu_l < \frac{M}{\left(\frac{\tau p_M}{2P}\right)} \end{cases} \tag{14}$$

Proof: Please refer [14] for the proof.

Figure 4 compares the CR of $\mathcal{A}_P$ with $c_{opt}(\mu_l)$, the best possible CR that any online algorithm for **OP3** (and hence **OP1**) can achieve. Figure 4 shows that the performance of $\mathcal{A}_P$ is satisfactorily close to the optimal CR.

IV. Simulation Results

In this section we carry out simulations using artifically generated traces with two-fold objective. First, is to study the effect of few trace parameters on the online algorithm $\mathcal{A}_P$. Second, we compare $\mathcal{A}_P$ with some benchmark algorithms.

TABLE II

COMMON TRACE PROPERTIES.

1 EPOCH $= 1$ HOUR, $\tau = 1$ YEAR, $T = 10\tau$, $p_M = 1$, $\frac{\tau p_M}{P} = 5$, $M = 50$, $d_M = 15$. '$*$' IMPLIES THAT THE FIELD CAN BE SET TO ANY ACCEPTABLE VALUE.

Trace	# of States	State Space	Mean (μ)	CV ($\frac{\sigma}{\mu}$)
d_t	$d_M + 1$	$\{0, \ldots, d_M\}$	4	0.9
M^o_t	$d_M + 1$	$\{0, \ldots, d_M\}$	2	0.5
p_t	50	$\{0.8p_M, \ldots, p_M\}$	$0.95p_M$	0.05
β_t	50	$\{0.05, \ldots, 1\}$	0.66	0.35
λ_t	$M + 1$	$\{0, \ldots, M\}$	$\frac{3M}{\tau}$	$*$
v_t	3	$\{0, \ldots, 2\}$	$\frac{M}{1.5\tau}$	$*$

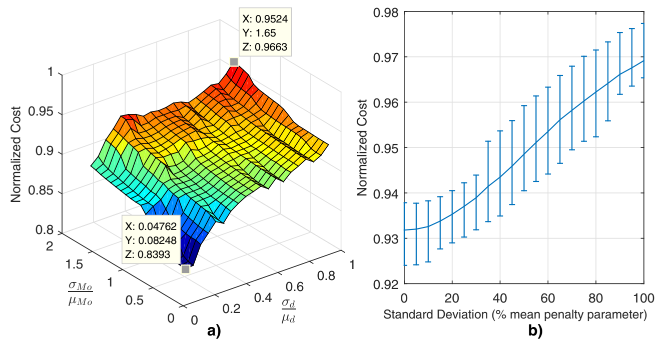

Fig. 5. **a)** The effect of the erratic nature of d_t and M^o_t on the normalized cost. The normalized cost for each pair of $\left(\frac{\sigma_d}{\mu_d}, \frac{\sigma_{Mo}}{\mu_{Mo}}\right)$ is averaged over 100 traces. μ_d and μ_{Mo} was held constant throughout the simulation. **b)** Effect of erroneous β_t on the normalized cost. The normalized cost for each value of standard deviation is averaged over 100 traces.

Setup and trace generation: Recall that d_t, λ_t, p_t, M^o_t, v_t and $f_t(\cdot)$ are the inputs to **OP1**. In all our simulations, $f_t(o) = \frac{p_t}{2M^o_t\beta_t}o^2$ if $M^o_t > 0$ and 0 otherwise. $\beta_t \in (0, 1]$ is the penalty parameter for opportunistic channel use. A lower β_t implies higher penalty. Due to the lack of real-world traces, we had to generate artificial traces for d_t, λ_t, p_t, M^o_t, v_t and β_t. We consider all the six traces to be discrete time markov chain (DTMC). The mean and coefficient of variation (CV[8]) of the stationary distribution of all the six DTMC's (and hence the traces) can be controlled[9]. Table II tabulates common trace properties. These properties will be used in the following simulations unless stated otherwise.

Effect of erratic nature of d_t and M^o_t: The erratic nature of customer demand d_t and opportunistic channel availability M^o_t decides the value of available opportunistic channels. In this regard we study the following normalized cost: Cost incurred by $\mathcal{A}_P$ when it uses the opportunistic channels to the cost incurred by $\mathcal{A}_P$ when it does not use the opportunistic channels. An available opportunistic channel is of value only if there is a demand in that epoch, the probability of which decreases as d_t and M^o_t becomes erratic. Mathematically, the normalized cost should be monotonic increasing in $\frac{\sigma_d}{\mu_d}$ and $\frac{\sigma_{Mo}}{\mu_{Mo}}$ where μ_d and σ_d (μ_{Mo} and σ_{Mo}) represents the mean and standard deviation of d_t (M^o_t). As shown in Figure 5.a., this intuition is verified by simulations too, subjected to simulation errors. There is a saving of 13% between the lowest and the highest $\left(\frac{\sigma_d}{\mu_d}, \frac{\sigma_{Mo}}{\mu_{Mo}}\right)$ pairs.

[8]CV is the ratio of standard deviation to mean. It can be used as a measure of erratic nature of a trace. Higher the CV, more erratic is the trace.

[9]The problem of designing a Markov matrix whose stationary distribution has a given mean and CV can be formulated as a linear program.

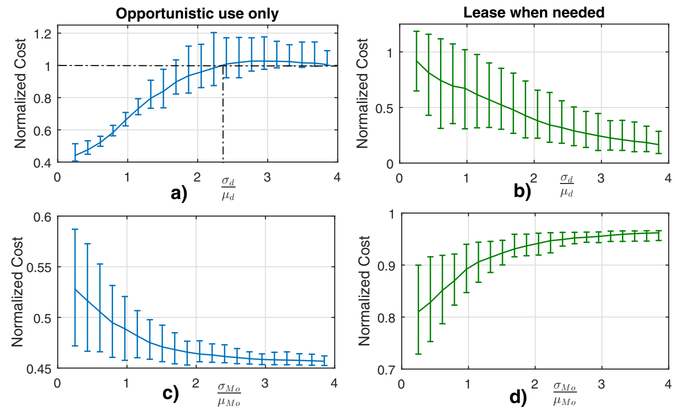

Fig. 6. The performance of our online algorithm with respect to two trivial algorithms. For each value of $\frac{\sigma}{\mu}$, we considered 4 values of μ and for each (σ, μ) pair, we averaged the normalized cost over 100 traces.

Effect of erroneous β_t: Implementation of $\mathcal{A}_P$ relies on computing $f_t(o)$ which in turn relies on the knowledge of β_t. Penalty parameter β_t depends on channel states like number of users in a given channel, the transmission power of individual users etc. The operator does not have direct access to these information, it can only infer it (possibly through customer feedback). Hence β_t is prone to error. Understanding the effect of erroneous β_t on the normalized cost (same as defined before) is important. To do this we add zero mean white gaussian noise to β_t and compute the normalized cost incurred by $\mathcal{A}_P$ as we increase the standard deviation of the gaussian noise. This is shown in Figure 5.b. As expected, the normalized cost increases with increase in standard deviation. More importantly, with standard deviation as high as 100%, the incurred cost can be reduced by 3% if we use the available opportunistic channels.

In the remaining part of this section, we will compare $\mathcal{A}_P$ with some benchmark algorithms. To do this we will use the following definition of normalized cost: Cost incurred by $\mathcal{A}_P$ to the cost incurred by the benchmark algorithm.

Comparison with trivial online algorithms: We compare $\mathcal{A}_P$ with two trivial online algorithms: *i) Opportunistic use only:* This algorithm never leases any channel. It uses available opportunistic channels and reject the remaining demand. *ii) Lease when needed:* This algorithm leases channels whenever the number of active channel leases is less then the demand, provided there are channels available for leasing. Leasing is not advisable if the demand is erratic because there is a high probability that the demand may decrease after we lease a channel. Therefore "opportunistic use only" works better when the demand is erratic (Figure 6.a.) and "lease when needed" works better when demand is smooth (Figure 6.b.). If the number of available opportunistic channel is erratic, it is better to lease a channel because there may not be opportunistic channels available in future. This intuition is validated by Figure 6.c. and 6.d. Figure 6 shows that $\mathcal{A}_P$ outperforms these trivial algorithms except when $\frac{\sigma_d}{\mu_d} \geq 2.3$.

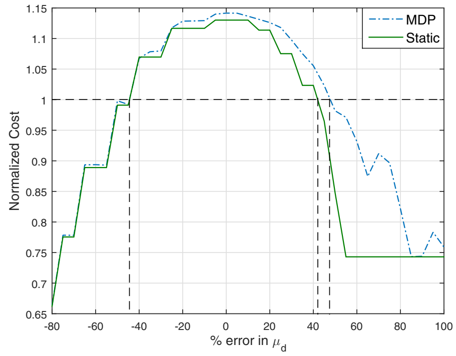

Fig. 7. The performance of our online algorithm with respect to two statistics based algorithms. For each % error in μ_d, the normalized cost has been averaged over 100 traces.

Comparison with statistics based online algorithms: To implement $\mathcal{A}_P$ we do not require any knowledge of the statistics of the six traces. Therefore $\mathcal{A}_P$ will be desirable in the early stages of the deployment of 3-TSF because knowledge of market statistics will be limited to none[10]. We illustrate the advantage of $\mathcal{A}_P$ by comparing it with two statistics based algorithms: *i) Markov Decision Process (MDP):* This algorithm was proposed in [3] and is the *state-of-the-art* work with any resemblance to our problem. It needs complete knowledge of the Markov matrices of all the traces. It can be implemented online only if $T \leq \tau$[11]. In our case $T > \tau$ and hence we use the following heuristic: we divide the optimization horizon T into $\frac{T}{\tau}$ frames and apply the algorithm to each frame seperately. *ii) Static Leasing Strategy:* This algorithm uses the stationary distribution of the traces to compute the number of active leases required to minimize the expected cost. It then tries to maintain the optimal number of active leases subjected to lease availability. Performance of such algorithms are prone to error in the statistical model. Figure 7 shows the normalized cost when μ_d is erroneous. As shown in Figure 7, $\mathcal{A}_P$ performes better than both the algorithms if μ_d is off by around $\pm 50\%$. It is to be noted that in this simulation, all statistical parameter but μ_d was known accurately. Also due to the high time complexity of MDP, we could only simulate for $\tau = 1$ week.

V. Concluding Remarks and Extensions

For a wireless operator who works in Tier-2 and Tier-3 of the 3-TSF, it is important to strategically decide the number of channels to lease (Tier-2), the number of channels to use opportunistically (Tier-3) and the number of customer demands to reject. Such decisions rely on customer demand and channel availability pattern which can be considered as random processes. In this paper, we used tools from ski-rental literature to design an algorithm which makes online decisions without any knowledge of the statistics of the involved random processes. We claim that our algorithm will be of importance in the early stages of the deployment of 3-TSF because the operator will have either limited or no knowledge of market statistics. Our algorithm has bounded competitive ratio which is nearly optimal when compared with the least possible competitive ratio. In the process of designing an online algorithm for leasing channels, we formulated and studied the modified ski-rental problem which is *state-of-the-art* in ski-rental literature.

We are interested in addressing the following three issues in later works. *First*, the online algorithm for leasing channels which we designed has sub-optimal competitive ratio. We are interested in desiging an online algorithm which is optimal in sense of competitive ratio. *Second*, we are interested in designing randomized online algorithms for leasing channels. *Third*, we would like to explore other assumptions, like the lower bound on the time average of the number of channels available for leasing, through which we can derive a better bound on the competitive ratio.

[10]Statistics based algorithms like [3] will outperform $\mathcal{A}_P$ if market statistics is sufficiently accurate. Accurate market statistics will be available after the 3-TSF is in operation for a sufficiently long time.

[11]The MDP based algorithm has a linear time complexity if $T \leq \tau$ and psuedo-polynomial for $T > \tau$. Psuedo-polynomial time complexity is too high to be implementable online.

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