A Distributed Admission Control Algorithm for Multicell MISO Downlink Systems

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***Abstract*—The problem of admission control in a multicell downlink multiple-input single-output system is considered. The objective is to maximize the number of admitted users subject to a signal-to-interference-plus-noise ratio constraint at each admitted user and a transmit power constraint at each base station. We cast the admission control problem as an ℓ_0 minimization problem. This problem is known to be combinatorial, NP-hard. Hence, we have to rely on suboptimal algorithms to solve it. We approximate the ℓ_0 minimization problem via a non-combinatorial one. Then, we propose a distributed algorithm to solve the non-combinatorial problem. The proposed algorithm is derived by using alternating direction method of multipliers in conjunction with sequential convex programming. We show numerically that the proposed algorithm achieves a near-to-optimal performance.**

***Index Terms*—Admission control, alternating direction method of multipliers (ADMM), distributed algorithm, ℓ_0 minimization.**

I. Introduction

The problem of user admission control in wireless networks is a difficult combinatorial optimization problem, and it is known to be NP-hard [1], [2]. The exhaustive search method is one approach to find the global optimal solution of the admission control problem. However, the computational complexity of the exhaustive search method increases exponentially with the number of users. Systematic approaches like branch and bound has been proposed to optimally solve this problem [2]. Although the solution in [2] is optimal, it is not suitable for practical scenarios due to the complexity of this method [3]. Hence, fast suboptimal algorithms are desirable in practice [1].

For multi-input single-output (MISO) systems, the centralized implementation of the admission control problem has been studied in [1]. Specifically, the authors in [1] have formulated this problem as an integer nonlinear optimization problem for a single cell. Then, two approximate solutions are proposed via semidefinite-relaxation (SDR) method [4] and second order cone programming [5, Ch. 4.4]. The work in [1] can be directly applied to a multicell scenario, however, its distributed implementation is not straightforward, and there is no reported work on that. In the context of multicell systems, this problem has been cast as an ℓ_0 minimization problem in [6]. Then, the ℓ_1-norm relaxation technique [7] in conjunction with SDR method is used to provide both centralized and distributed algorithms. In particular, the distributed algorithm is based on block coordinate descent method, where the subproblems associated with the base stations (BSs) are solved in a cyclic order. Both algorithms in [1] and [6] are derived by using the deflation based approach, which relies on dropping users at each iteration of the algorithm.

In this paper we propose a distributed algorithm to solve the admission control problem for the downlink of a multicell MISO system. The distributed algorithm is derived by using the consensus-based alternating direction method of multipliers (ADMM) [8] in conjunction with *sequential convex programming* [9], [10]. In contrast to the existing distributed algorithm in [6] (that solves the subproblems in a cyclic order), our proposed algorithm solves the subproblems independently in parallel at each BS. Numerically, we show that the proposed algorithm achieves a near-to-optimal performance.

II. System model and Problem Formulation

We consider the downlink of a multicell MISO system with K BSs. The set of all BSs is denoted by $\mathcal{K}$ and we label them with the integer values $k = 1, \ldots, K$. We assume that each BS is equipped with T transmit antennas. We denote the set of all users associated with kth BS by $\mathcal{U}_k$, and we label them with the integer values $i = 1, \ldots, I_k$.

The received signal-to-interference-plus-noise ratio (SINR) of ith user of BS k is given by

$$\Gamma_{ki}(\mathbf{m}) = \frac{|(\mathbf{h}_{ki}^k)^{\mathrm{H}}\mathbf{m}_{ki}|^2}{\sum\limits_{j\in\mathcal{U}_k, j\neq i}|(\mathbf{h}_{ki}^k)^{\mathrm{H}}\mathbf{m}_{kj}|^2 + \sum\limits_{l\in\mathcal{K}\setminus\{k\}}\sum\limits_{j\in\mathcal{U}_l}|(\mathbf{h}_{ki}^l)^{\mathrm{H}}\mathbf{m}_{lj}|^2 + \sigma^2}, \quad (1)$$

where $\mathbf{m}_{ki} \in \mathbb{C}^T$ is the transmit beamformer associated with ith user of BS k, $\mathbf{h}_{ki}^l \in \mathbb{C}^T$ is the channel vector from BS l to ith user of BS k, and σ^2 is the noise power at the receiver. In expression (1), we use the notation $\mathbf{m}$ to denote a vector obtained by stacking $\mathbf{m}_{ki}$ for all $i \in \mathcal{U}_k$ and $k \in \mathcal{K}$ on top of each other, i.e., $\mathbf{m} = [\mathbf{m}_{11}^{\mathrm{T}}, \mathbf{m}_{12}^{\mathrm{T}}, \ldots, \mathbf{m}_{KI_K}^{\mathrm{T}}]^{\mathrm{T}}$.

We assume that the power allocation is subject to a maximum transmit power constraint at each BS, i.e.,

$$\textstyle\sum_{i\in\mathcal{U}_k} \| \mathbf{m}_{ki} \|_2^2 \leq P_k^{\max}, \quad k \in \mathcal{K}, \quad (2)$$

where $P_k^{\max}$ is the maximum transmit power of kth BS. Furthermore, we assume that the QoS of ith user of BS k is assured if its SINR is greater than a threshold γ_{ki}, i.e.,

$$\Gamma_{ki}(\mathbf{m}) \geq \gamma_{ki}. \quad (3)$$

Let $\tilde{\mathcal{U}}_k(\mathbf{m})$ denote a generic set of admissible users at kth BS. Specifically, $\tilde{\mathcal{U}}_k(\mathbf{m})$ denotes a set of users who satisfy their SINR thresholds under the power constraint, i.e.,

$$\tilde{\mathcal{U}}_k(\mathbf{m}) = \{ki \,|\, \Gamma_{ki}(\mathbf{m}) \geq \gamma_{ki}, \sum_{i\in\mathcal{U}_k} \|\mathbf{m}_{ki}\|_2^2 \leq P_k^{\max}, i \in \mathcal{U}_k\}, \quad (4)$$

for all $k \in \mathcal{K}$. Our goal is to maximize the number of admitted users to the system, i.e., to maximize the sum of the cardinalities of $\tilde{\mathcal{U}}_k(\mathbf{m})$ for all $k \in \mathcal{K}$. We now formulate this design problem as a mathematical optimization problem. To do this, let us introduce the nonnegative auxiliary variables s_{ki} for all $k \in \mathcal{K},\ i \in \mathcal{U}_k$, and consider a set of relaxed SINR constraints as follows:

$$\Gamma_{ki}(\mathbf{m}) \geq \gamma_{ki} - s_{ki}, \quad i \in \mathcal{U}_k, k \in \mathcal{K}. \tag{5}$$

In (5), when $s_{ki} = 0$ we recover constraint (3), i.e., the SINR constraint of ith user of kth BS is satisfied. Furthermore, by making s_{ki} large enough the set of relaxed SINR constraints in (5) can be always made feasible.

Note that maximizing the number of admitted users that satisfy the SINR constraints (3) is equivalent to minimizing the number of users that requires a strictly positive value of s_{ki} that satisfy constraint (5). Hence, by using expressions (2) and (5) the problem of admission control can be expressed as

$$\begin{array}{lll} \text{minimize} & \sum_{k\in\mathcal{K}} \| \mathbf{s}_k \|_0 & \\ \text{subject to} & \Gamma_{ki}(\mathbf{m}) \geq \gamma_{ki} - s_{ki}, \quad i \in \mathcal{U}_k, k \in \mathcal{K} & \text{(6a)} \\ & \sum_{i\in\mathcal{U}_k} \| \mathbf{m}_{ki} \|_2^2 \leq P_k^{\max}, \quad k \in \mathcal{K} & \text{(6b)} \\ & s_{ki} \geq 0, \quad i \in \mathcal{U}_k, k \in \mathcal{K}, & \text{(6c)} \end{array}$$

where $\mathbf{s}_k = [s_{k1}, \ldots, s_{kI_k}]^{\mathrm{T}}$, variables are $\{s_{ki}, \mathbf{m}_{ki}\}_{k\in\mathcal{K}, i\in\mathcal{U}_k}$.

III. Algorithm Derivation

Problem (6) is a difficult combinatorial optimization problem [7] due to the ℓ_0 objective function. In fact, this problem is known to be NP-hard [7], [11]. Therefore, we have to rely on suboptimal algorithms to solve it. In this section, we provide a distributed algorithm to solve problem (6).

We start by approximating the objective function of problem (6) with a concave function $\sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{U}_k} \log(s_{ki} + \epsilon)$, where ϵ is a small positive constant and $s_{ki} \geq 0$ for all $i \in \mathcal{U}_k,\ k \in \mathcal{K}$ [12]. Let us now define a new variable β_{ki} to denote the interference-plus-noise experienced by ith user of kth BS, i.e., $\beta_{ki} = \sum_{j\in\mathcal{U}_k, j\neq i} |(\mathbf{h}_{ki}^k)^{\mathrm{H}} \mathbf{m}_{kj}|^2 + \sum_{l\in\mathcal{K}\setminus\{k\}} \sum_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}} \mathbf{m}_{lj}|^2 + \sigma_{ki}^2$ for all $i \in \mathcal{U}_k,\ k \in \mathcal{K}$. Then, a solution of problem (6) can be approximated by solving the following optimization problem:

$$\begin{array}{lll} \text{minimize} & \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{U}_k} \log(s_{ki} + \epsilon) & \\ \text{subject to} & \gamma_{ki} - s_{ki} - \dfrac{|(\mathbf{h}_{ki}^k)^{\mathrm{H}} \mathbf{m}_{ki}|^2}{\beta_{ki}} \leq 0,\ i \in \mathcal{U}_k, k \in \mathcal{K} & \text{(7a)} \\ & \sum\limits_{\substack{j\in\mathcal{U}_k,\\ j\neq i}} |(\mathbf{h}_{ki}^k)^{\mathrm{H}} \mathbf{m}_{kj}|^2 + \sum\limits_{l\in\mathcal{K}\setminus\{k\}} \sum\limits_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}} \mathbf{m}_{lj}|^2 & \\ & + \sigma_{ki}^2 \leq \beta_{ki}, \quad i \in \mathcal{U}_k, k \in \mathcal{K} & \text{(7b)} \\ & \text{constraints } (6b),\ (6c), & \text{(7c)} \end{array}$$

with variables $\{s_{ki}, \mathbf{m}_{ki}, \beta_{ki}\}_{k\in\mathcal{K}, i\in\mathcal{U}_k}$. In constraint (7a), we have replaced the interference-plus-noise term of $\Gamma_{ki}(\mathbf{m})$ with β_{ki}. Note that the objective function of problem (7) is increasing in s_{ki}, hence, it can be shown (e.g., by contradiction) that constraint (7b) holds with equality at the optimal point.

Problem (7) is a non-combinatorial optimization problem. However, it is still *nonconvex* because the objective function is concave and constraint function (7a) is not convex. In the sequel, we apply sequential convex programming [9] to solve the problem (7). Here, we approximate the objective function and constraint function (7a) with their best convex approximations. Then, we iteratively solve the approximated convex problem to find a solution for problem (7).

Let the objective function of problem (7) is denoted by $f(\mathbf{s}) = \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{U}_k} \log(s_{ki} + \epsilon)$, where $\mathbf{s} = [s_{11}, \ldots, s_{KI_K}]$. Note that $f(\mathbf{s})$ is a concave function. Hence, the best convex approximation of function $f(\mathbf{s})$ can be obtained by replacing it with its first order approximation [9], near an arbitrary positive point $\hat{\mathbf{s}} = [\hat{s}_{11}, \ldots, \hat{s}_{KI_K}]$ as follows:

$$\hat{f}(\mathbf{s}) = f(\hat{\mathbf{s}}) + \textstyle\sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{U}_k} (s_{ki} - \hat{s}_{ki})/(\hat{s}_{ki} + \epsilon). \tag{8}$$

We now focus on constraint (7a). Let $g_{ki}(\mathbf{m}_{ki}, \beta_{ki})$ be a function defined as $g_{ki}(\mathbf{m}_{ki}, \beta_{ki}) = |(\mathbf{h}_{ki}^k)^{\mathrm{H}} \mathbf{m}_{ki}|^2/\beta_{ki}$. It is easy to see that constraint function (7a) is a difference of the affine function $\gamma_{ki} - s_{ki}$ and the convex function $g_{ki}(\mathbf{m}_{ki}, \beta_{ki})$. The best convex approximation of constraint function (7a) near an arbitrary point $(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki})$ can be obtained by replacing $g_{ki}(\mathbf{m}_{ki}, \beta_{ki})$ with its first order approximation [9] as

$$\begin{aligned} \hat{g}_{ki}(\mathbf{m}_{ki}, \beta_{ki}) &= g_{ki}(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki}) + \\ &\nabla g_{ki}(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki})^{\mathrm{T}}((\mathbf{m}_{ki}, \beta_{ki}) - (\hat{\mathbf{m}}_{ki}, \hat{\beta_{ki}})), \end{aligned} \tag{9}$$

where $\nabla g_{ki}(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki})$ is the gradient of $g_{ki}(\mathbf{m}_{ki}, \beta_{ki})$ evaluated at point $(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki})$, defined as

$$\nabla g_{ki}(\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki}) = \left(2\mathbf{h}_{ki}\mathbf{h}_{ki}{}^{\mathrm{H}}\hat{\mathbf{m}}_{ki}/\hat{\beta}_{ki}, -\hat{\mathbf{m}}_{ki}{}^{\mathrm{H}}\mathbf{h}_{ki}\mathbf{h}_{ki}{}^{\mathrm{H}}\hat{\mathbf{m}}_{ki}/\hat{\beta}_{ki}^2\right). \tag{10}$$

Now by using expressions (8) and (9), we approximate problem (7) near arbitrary positive point $(\hat{s}_{ki}, \hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki})$ for all $i \in \mathcal{U}_k, k \in \mathcal{K}$, as the following convex optimization problem:

$$\begin{array}{ll} \text{minimize} & \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{U}_k} w_{ki} s_{ki} \\ \text{subject to} & \gamma_{ki} - s_{ki} - \hat{g}_{ki}(\mathbf{m}_{ki}, \beta_{ki}) \leq 0, \quad i \in \mathcal{U}_k, k \in \mathcal{K} \\ & \sum\limits_{\substack{j\in\mathcal{U}_k,\\ j\neq i}} |(\mathbf{h}_{ki}^k)^{\mathrm{H}} \mathbf{m}_{kj}|^2 + \sum\limits_{l\in\mathcal{K}\setminus\{k\}} \sum\limits_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}} \mathbf{m}_{lj}|^2 \\ & + \sigma_{ki}^2 \leq \beta_{ki}, \quad i \in \mathcal{U}_k, k \in \mathcal{K} \\ & \text{constraints } (6b),\ (6c), \end{array} \tag{11}$$

where $w_{ki} = 1/(\hat{s}_{ki} + \epsilon)$, variables are $s_{ki}, \mathbf{m}_{ki}, \beta_{ki}$ for all $k \in \mathcal{K}, i \in \mathcal{U}_k$, and w_{ki} acts as a weight associated with user i of kth BS. Note that in the objective function of problem (11) we dropped the constant term $f(\hat{\mathbf{s}}) - \hat{s}_{ki}/(\hat{s}_{ki} + \epsilon)$, since it does not affect the solution of problem (11).

A. An equivalent reformulation

In this subsection, we equivalently reformulate problem (11) in a *global consensus* form. We start by introducing an auxiliary variable z_{ki}^l to denote the interference generated by lth BS to ith user of BS k, i.e., $z_{ki}^l = \sum_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}} \mathbf{m}_{lj}|^2$ for all $i \in \mathcal{U}_k$, $k \in \mathcal{K}$, and $l \in \mathcal{K}\setminus\{k\}$. Then problem (11) can be equivalently written as

$$
\begin{array}{ll}
\text{minimize} & \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{U}_k} w_{ki}s_{ki} \\
\text{subject to} & \gamma_{ki} - s_{ki} - \hat{g}_{ki}(\mathbf{m}_{ki},\beta_{ki}) \leq 0,\ i\in\mathcal{U}_k,\ k\in\mathcal{K} \\
& \sum\limits_{j\in\mathcal{U}_k, j\neq i} |(\mathbf{h}_{ki}^k)^{\mathrm{H}}\mathbf{m}_{kj}|^2 + \sum\limits_{l\in\mathcal{K}\setminus\{k\}} z_{ki}^l + \sigma_{ki}^2 \\
& \qquad\qquad\qquad \leq \beta_{ki},\quad i\in\mathcal{U}_k,\ k\in\mathcal{K} \\
& \sum\limits_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}}\mathbf{m}_{lj}|^2 \leq z_{ki}^l,\ i\in\mathcal{U}_k, k\in\mathcal{K}, l\in\mathcal{K}\setminus\{k\} \\
& \text{constraints } (6b),\ (6c),
\end{array}
\tag{12}
$$

with optimization variables $\{s_{ki},\mathbf{m}_{ki},\beta_{ki}\}_{k\in\mathcal{K},i\in\mathcal{U}_k}$ and $\{z_{ki}^l\}_{i\in\mathcal{U}_k,k\in\mathcal{K},l\in\mathcal{K}\setminus\{k\}}$. Note that problem (11) and (12) are equivalent as it can be easily shown (e.g., by contradiction) that third inequality of problem (12) holds with equality at the optimal point.

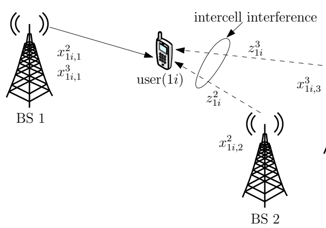

Fig. 1. Illustration of BS coupling, and introducing local copies to decouple a problem. BS2 and BS3 are coupled with BS1 by coupling variables z_{1i}^2 and z_{1i}^3, respectively. To distribute the problem, local copy $x_{1i,1}^2$ of z_{1i}^2 at BS1 and local copy $x_{1i,2}^2$ of z_{1i}^2 at BS 2 are introduced. Similarly, local copy $x_{1i,1}^3$ of z_{1i}^3 at BS1 and local copy $x_{1i,3}^3$ of z_{1i}^3 at BS3 are introduced.

In problem (12), variable z_{ki}^l represents the power of intercell interference caused by lth BS to ith user of BS k; thus variable z_{ki}^l couples BS l and k. We use consensus technique [8, Ch.7] to distribute problem (12) over the BSs. The method consists of introducing local copies of coupling variables z_{ki}^l for all $i\in\mathcal{U}_k, k\in\mathcal{K}$, and $l\in\mathcal{K}\setminus\{k\}$, at each BS (see Fig. 1).

Let us define $x_{ki,k}^l$ as the local copy of z_{ki}^l saved at kth BS, and $x_{ki,l}^l$ as the local copy of z_{ki}^l saved at lth BS. Thus for each z_{ki}^l we make two local copies (see Fig. 1). Then, problem (12) can be written equivalently in *global consensus* form as

$$
\begin{array}{lll}
\text{minimize} & \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{U}_k} w_{ki}s_{ki} & \\
\text{subject to} & \gamma_{ki} - s_{ki} - \hat{g}_{ki}(\mathbf{m}_{ki},\beta_{ki}) \leq 0,\ i\in\mathcal{U}_k, k\in\mathcal{K} & \text{(13a)} \\
& \sum_{j\in\mathcal{U}_k, j\neq i} |(\mathbf{h}_{ki}^k)^{\mathrm{H}}\mathbf{m}_{kj}|^2 + \sum_{l\in\mathcal{K}\setminus\{k\}} x_{ki,k}^l + & \\
& \sigma_{ki}^2 \leq \beta_{ki},\quad i\in\mathcal{U}_k,\ k\in\mathcal{K} & \text{(13b)} \\
& \sum_{j\in\mathcal{U}_l} |(\mathbf{h}_{ki}^l)^{\mathrm{H}}\mathbf{m}_{lj}|^2 \leq x_{ki,l}^l,\ i\in\mathcal{U}_k,\ k\in\mathcal{K}, & \\
& l\in\mathcal{K}\setminus\{k\} & \text{(13c)} \\
& x_{ki,k}^l = z_{ki}^l,\ i\in\mathcal{U}_k,\ k\in\mathcal{K},\ l\in\mathcal{K}\setminus\{k\} & \text{(13d)} \\
& x_{ki,l}^l = z_{ki}^l,\quad i\in\mathcal{U}_k,\ k\in\mathcal{K},\ l\in\mathcal{K}\setminus\{k\} & \text{(13e)} \\
& \text{constraints } (6b),\ (6c), & \text{(13f)}
\end{array}
$$

with optimization variables $\{s_{ki},\mathbf{m}_{ki},\beta_{ki}\}_{k\in\mathcal{K},i\in\mathcal{U}_k}$, and $\{x_{ki,k}^l, z_{ki}^l\}_{i\in\mathcal{U}_k,k\in\mathcal{K},l\in\mathcal{K}\setminus\{k\}}$. Note that in constraint (13b) we have replaced z_{ki}^l by the local copy $x_{ki,k}^l$. In constraints (13c), we have replaced z_{ki}^l by the local copy $x_{ki,l}^l$. Constraints (13d) and (13e) are called *consistency constraints* and they enforce the local copies $x_{ki,k}^l$ and $x_{ki,l}^l$ to be equal to the corresponding global variable z_{ki}^l.

We now express problem (13) more compactly. To do this, we first express the consistency constraints of problem (13) more compactly by using vector notation. The set of local variables associated with kth BS includes two components: 1) copies of the interference experienced by all the users associated with kth BS from all the other BSs, i.e., $\{x_{ki,k}^l\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_k}$, and 2) copies of the interference generated by kth BS to all the other users, i.e., $\{x_{li,k}^k\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_l}$. Thus, we can compactly write the local copies of intercell interference terms associated with kth BS as

$$
\mathbf{x}_k = \left(\{x_{ki,k}^l\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_k}, \{x_{li,k}^k\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_l}\right). \tag{14}
$$

Similarly, we can compactly write the global variables associated with kth BS as

$$
\mathbf{z}_k = \left(\{z_{ki}^l\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_k}, \{z_{li}^k\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_l}\right). \tag{15}
$$

It is worth noting that some components of $\mathbf{z}_k$ are also in the components of $\mathbf{z}_l$ for all $l\in\mathcal{K}\setminus\{k\}$. For example, components $\{z_{ki}^l\}_{i\in\mathcal{U}_k}$ and $\{z_{lj}^k\}_{j\in\mathcal{U}_l}$ are common to both $\mathbf{z}_k$ and $\mathbf{z}_l$. With the compact notations introduced above we equivalently write the equality constraints (13d) and (13e) as

$$
\mathbf{x}_k = \mathbf{z}_k,\quad k\in\mathcal{K}. \tag{16}
$$

Now, for the sake of brevity, let us define the following set

$$
\mathcal{O}_k = \{\mathbf{s}_k,\mathbf{m}_k,\boldsymbol{\beta}_k,\mathbf{x}_k \mid \text{constraints (13a)–(13c), (13f)}\}, \tag{17}
$$

where $\boldsymbol{\beta}_k = [\beta_{k1},\ldots,\beta_{kI_k}]$, and the following function

$$
f_k(\mathbf{s}_k,\mathbf{m}_k,\boldsymbol{\beta}_k,\mathbf{x}_k) = \left\{\begin{array}{ll} \sum_{i\in\mathcal{U}_k} w_{ki}s_{ki} & (\mathbf{s}_k,\mathbf{m}_k,\boldsymbol{\beta}_k,\mathbf{x}_k)\in\mathcal{O}_k \\ \infty & \text{otherwise.} \end{array}\right. \tag{18}
$$

Then by using notations in (14), (15), (17), and (18), consensus problem (13) can be compactly written as

$$
\begin{array}{ll}
\text{minimize} & \sum_{k\in\mathcal{K}} f_k(\mathbf{s}_k,\mathbf{m}_k,\boldsymbol{\beta}_k,\mathbf{x}_k) \\
\text{subject to} & \mathbf{x}_k = \mathbf{z}_k,\quad k\in\mathcal{K},
\end{array}
\tag{19}
$$

with variables $\mathbf{s}_k,\mathbf{m}_k,\boldsymbol{\beta}_k$, $\mathbf{x}_k$, and $\mathbf{z}_k$ for all $k\in\mathcal{K}$.

B. Distributed algorithm via ADMM

In this section, we derive a distributed algorithm for problem (19). The proposed algorithm is based on ADMM [8]. Let $y_{ki,k}^l$ and $y_{ki,l}^l$ for all $i\in\mathcal{U}_k, k\in\mathcal{K}, l\in\mathcal{K}\setminus\{k\}$, be the dual variables associated with constraints (13d) and (13e) of problem (13). Specifically, the dual variables associated with kth BS in constraints (13d) and (13e) can be compactly written as

$$
\mathbf{y}_k = \left(\{y_{ki,k}^l\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_k}, \{y_{li,k}^k\}_{l\in\mathcal{K}\setminus\{k\},i\in\mathcal{U}_l}\right). \tag{20}
$$

Now we write the *augmented Lagrangian* for problem (19) as

$$
\begin{aligned}
L_\rho(\{\mathbf{s}_k,\boldsymbol{\beta}_k,\mathbf{x}_k,\mathbf{m}_k,\mathbf{z}_k\}_{k\in\mathcal{K}}) = \sum_{k\in\mathcal{K}} \Big(& f_k(\mathbf{s}_k,\boldsymbol{\beta}_k,\mathbf{x}_k,\mathbf{m}_k) \\
& + {\mathbf{y}_k}^{\mathrm{T}}(\mathbf{x}_k - \mathbf{z}_k) + \frac{\rho}{2}\parallel \mathbf{x}_k - \mathbf{z}_k \parallel_2^2 \Big),
\end{aligned}
\tag{21}
$$

where $\{\mathbf{y}_k\}_{k\in\mathcal{K}}$ are the dual variables associated with the equality constraint of (19).

Each iteration of the ADMM algorithm consists of the following three steps [8, Ch. 3]:

$$\mathbf{s}_k^{q+1}, \boldsymbol{\beta}_k^{q+1}, \mathbf{x}_k^{q+1}, \mathbf{m}_k^{q+1} = \underset{\mathbf{s}_k,\boldsymbol{\beta}_k,\mathbf{x}_k,\mathbf{m}_k}{\operatorname{argmin}} \; L_\rho(\mathbf{s}_k, \boldsymbol{\beta}_k, \mathbf{x}_k, \mathbf{m}_k, \mathbf{z}_k^q, \mathbf{y}_k^q), \quad k \in \mathcal{K} \quad (22a)$$

$$\{\mathbf{z}_k^{q+1}\}_{k\in\mathcal{K}} = \underset{\mathbf{z}_k\in\mathcal{K}}{\operatorname{argmin}} \, L_\rho(\{\mathbf{s}_k^{q+1}, \boldsymbol{\beta}_k^{q+1}, \mathbf{x}_k^{q+1}, \mathbf{m}_k^{q+1}, \mathbf{z}_k, \mathbf{y}_k^q\}_{k\in\mathcal{K}}) \quad (22b)$$

$$\mathbf{y}_k^{q+1} = \mathbf{y}_k^q + \rho(\mathbf{x}_k^{q+1} - \mathbf{z}_k^{q+1}), \quad k \in \mathcal{K}, \quad (22c)$$

where the superscript q is the iteration counter. Note that steps (22a) and (22c) can be carried out independently in parallel at each BS. Recall that components of $\mathbf{z}_k$ couple with two local variables that are associated with the interfering BSs (see constraints (13d) and (13e)). Thus, step (22b) requires gathering the updated local and dual variables from all BSs. In the sequel, we first explain the way to solve steps (22a) and (22b). Then, we present the proposed ADMM based distributed algorithm.

We start by providing a method to compute step (22a). The local variables update ($\mathbf{s}_k^{q+1}$, $\boldsymbol{\beta}_k^{q+1}$, $\mathbf{x}_k^{q+1}$, $\mathbf{m}_k^{q+1}$) in (22a) is a solution of the following optimization problem:

$$\begin{array}{ll} \text{minimize} & f_k(\mathbf{s}_k, \boldsymbol{\beta}_k, \mathbf{x}_k, \mathbf{m}_k) + \mathbf{y}_k{}^{q\mathrm{T}}(\mathbf{x}_k - \mathbf{z}_k^q) \\ & \quad + \frac{\rho}{2} \parallel \mathbf{x}_k - \mathbf{z}_k^q \parallel_2^2, \end{array} \quad (23)$$

with variables $\mathbf{s}_k, \boldsymbol{\beta}_k, \mathbf{x}_k, \mathbf{m}_k$. Here we use $\mathbf{y}_k^{q\mathrm{T}}$ instead of $(\mathbf{y}_k^q)^{\mathrm{T}}$ to lighten the notation. Let $\mathbf{v}_n$ be a scaled dual variable, i.e., $\mathbf{v}_n = (1/\rho)\mathbf{y}_n$. By using the notations (17) and (18), problem (23) can be equivalently written as

$$\begin{array}{ll} \text{minimize} & \sum_{i\in\mathcal{U}_k} w_{ki}s_{ki} + \frac{\rho}{2} \parallel \mathbf{x}_k - \mathbf{z}_k^q + \mathbf{v}_k^q \parallel_2^2 \\ \text{subject to} & \gamma_{ki} - s_{ki} - \hat{g}_{ki}(\mathbf{m}_{ki}, \beta_{ki}) \leq 0, \quad i \in \mathcal{U}_k \\ & \sum_{j\in\mathcal{U}_k, j\neq i} |(\mathbf{h}_{ki}^k)^{\mathrm{H}}\mathbf{m}_{kj}|^2 + \sum_{l\in\mathcal{K}\setminus\{k\}} x_{ki,k}^l \\ & +\sigma_{ki}^2 \leq \beta_{ki}, \quad i \in \mathcal{U}_k \\ & \sum_{i\in\mathcal{U}_k} |(\mathbf{h}_{lj}^k)^{\mathrm{H}}\mathbf{m}_{ki}|^2 \leq x_{lj,k}^k, \;\; j \in \mathcal{U}_l, l \in \mathcal{K}\setminus\{k\} \\ & \sum_{i\in\mathcal{U}_k} \parallel \mathbf{m}_{ki} \parallel_2^2 \leq P_k^{\max} \\ & s_{ki} \geq 0, \quad i \in \mathcal{U}_k, \end{array} \quad (24)$$

with variables $\mathbf{s}_k$, $\boldsymbol{\beta}_k$, $\mathbf{x}_k$, and $\mathbf{m}_k$. Note that the second term of the objective function of problem (24) is obtained by: 1) combining the linear and quadratic terms of the objective function of problem (23) as $\mathbf{y}_k{}^{q\mathrm{T}}(\mathbf{x}_k - \mathbf{z}_k^q) + \frac{\rho}{2} \parallel \mathbf{x}_k - \mathbf{z}_k^q \parallel_2^2 = \frac{\rho}{2} \parallel \mathbf{x}_k - \mathbf{z}_k^q + \mathbf{v}_k^q \parallel_2^2 - \frac{\rho}{2} \parallel \mathbf{v}_k^q \parallel_2^2$, and 2) dropping the constant term $\frac{\rho}{2} \parallel \mathbf{v}_k^q \parallel_2^2$. This constant term is dropped since it does not affect the solution of the problem. Let us denote by $\mathbf{s}_k^\star$, $\boldsymbol{\beta}_k^\star$, $\mathbf{x}_k^\star$, and $\mathbf{m}_k^\star$ a solution of problem (24); and we set $\mathbf{s}_k^{q+1} = \mathbf{s}_k^\star$, $\boldsymbol{\beta}_k^{q+1} = \boldsymbol{\beta}_k^\star$, $\mathbf{x}_k^{q+1} = \mathbf{x}_k^\star$, and $\mathbf{m}_k^{q+1} = \mathbf{m}_k^\star$.

We now consider the second step of ADMM algorithm, i.e., (22b), and provide a solution for global variable update. The update $\{\mathbf{z}_k^{q+1}\}_{k\in\mathcal{K}}$ is a solution of the following problem:

$$\text{minimize} \; \sum_{k\in\mathcal{K}} \mathbf{y}_k^{q\mathrm{T}}(\mathbf{x}_k^{q+1} - \mathbf{z}_k) + \frac{\rho}{2} \parallel \mathbf{x}_k^{q+1} - \mathbf{z}_k \parallel_2^2, \quad (25)$$

with variables $\{\mathbf{z}_k\}_{k\in\mathcal{K}}$. The solution of problem (25) is

$$z_{ki}^{l[q+1]} = (x_{ki,l}^{l[q+1]} + x_{ki,k}^{l[q+1]})/2, \;\; i\in\mathcal{U}_k, k\in\mathcal{K}, l\in\mathcal{K}\setminus\{k\}, \quad (26)$$

Due to space limitation we have omitted detailed derivation in obtaining expression (26); we refer the interested reader to [13] for more details.

Finally, we summarize the proposed distributed algorithm for problem (7) in *Algorithm 1*.

Algorithm 1 Distributed algorithm for solving problem (7)

1: **initialization**: $\{s_{ki}^0, \mathbf{m}_{ki}^0, \beta_{ki}^0\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$, $\{w_{ki} = 1\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$, iteration index $q = 0$.

repeat

2: Set $\{\hat{\mathbf{m}}_{ki} = \mathbf{m}_{ki}^q, \hat{\beta}_{ki} = \beta_{ki}^q\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$. Form $\{\hat{g}_{ki}(\mathbf{m}_{ki}, \beta_{ki})\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$ using expression (9).

3: ADMM Algorithm:

repeat

a. Each BS $k \in \mathcal{K}$ updates the local variables $(\mathbf{s}_k^{q+1}, \boldsymbol{\beta}_k^{q+1}, \mathbf{x}_k^{q+1}, \mathbf{m}_k^{q+1})$ by solving (24).

b. BS k and l exchange local copies $x_{ki,k}^l$ and $x_{ki,l}^l$ for all $i \in \mathcal{U}_k$, $k \in \mathcal{K}$, $l \in \mathcal{K}\setminus\{k\}$.

c. Each BS k updates global variable $\mathbf{z}_k^{q+1}$ using (26).

d. Each BS k updates dual variable $\mathbf{y}_k^{q+1}$ by solving (22c). Set $q = q + 1$.

until stopping criterion is satisfied

4: Update $w_{ki} = 1/\,(s_{ki}^q + \epsilon)\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$.

until stopping criterion is satisfied

We refer to each execution of the steps 2-4 as an outer iteration and to each execution of the ADMM algorithm (i.e., steps 3a-3d) as an inner iteration. In each outer iteration, the algorithm approximates problem (7) as a convex function at step 2, and this can be done individually in parallel at each BS. In the inner iteration, steps 3a, 3c, and 3d are decentralized over the BSs. Step 3b requires a coordination between the BSs to exchange the updated values of the local variables. In ADMM algorithm, the standard stopping criteria is to check the primal and dual residuals [8, Ch. 3.3]. As ADMM can produce acceptable results of practical use within a few iterations, a finite number of iterations is used to stop the ADMM algorithm in step 3 [8].

IV. Numerical Results

In our simulations, we consider a setup with $K = 3$ BSs, each one consists of $T = 4$ transmit antennas. The BSs are placed in such a way that the distance between any two adjacent BSs is equal, and we denote this distance by D_{BS}. We assume circular cells, where the radius of each one is denoted by R_{BS}. Furthermore, we consider each BS serves four users.

We use an exponential path loss model, where the channel gains between BS l and ith user of BS k is modeled as $\mathbf{h}_{ki}^l = \left(d_{ki}^l/d_0\right)^{-\eta/2} \mathbf{c}_{ki}^l$, where d_{ki}^l is the distance from lth BS to ith user of BS k, d_0 is the *far field reference distance*, η is the path loss exponent, and $\mathbf{c}_{ki}^l \in \mathbb{C}^T$ is arbitrarily chosen from the distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. We assume that $\{P_k^{\max} = P_0^{\max}\}_{k\in\mathcal{K}}$, $\{\gamma_{ki} = \gamma\}_{i\in\mathcal{U}_k, k\in\mathcal{K}}$. We define the SNR operating point at a distance d as $\mathrm{SNR}(d) = \left(P_0^{\max}/\sigma^2\right)(d/d_0)^{-\eta}$. In our simulations, we set $\mathrm{SNR}(R_{\mathrm{BS}}) = 5$dB, $d_0 = 1$, $\eta = 4$, $P_0^{\max}/\sigma^2 = 45$dB, and $D_{\mathrm{BS}} = 1.6R_{\mathrm{BS}}$.

Fig. 2 shows the convergence behavior of the distributed *Algorithm 1*. As a benchmark we obtain a centralized solution for problem (6), by iteratively solving problem (11) (we refer the reader to [13] for more detail). Fig. 2 also shows the number of admitted users in each iteration of the distributed *Algorithm 1* compared to that of a centralized algorithm. Here we set the maximum number of ADMM iterations to 10. We denote by $F_{\mathrm{D}}^{q} = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{U}_k} \log(s_{ki}^{q} + \epsilon)$ the objective value of problem (7) calculated at iteration q of *Algorithm 1*, where s_{ki}^{q} is the auxiliary variable of ith user of base station k in qth iteration. The markers "square" and "circle" in Fig. 2 are to indicate F_{D} and the number of admitted users, respectively. Furthermore, these markers represent the start of ADMM algorithm for a new point $\{\hat{\mathbf{m}}_{ki}, \hat{\beta}_{ki}\}_{i \in \mathcal{U}_k, k \in \mathcal{K}}$ that is set at step 2 of the algorithm. Results show that the proposed distributed algorithm converges to the centralized algorithm, and it also admits the same number of users as the centralized algorithm.

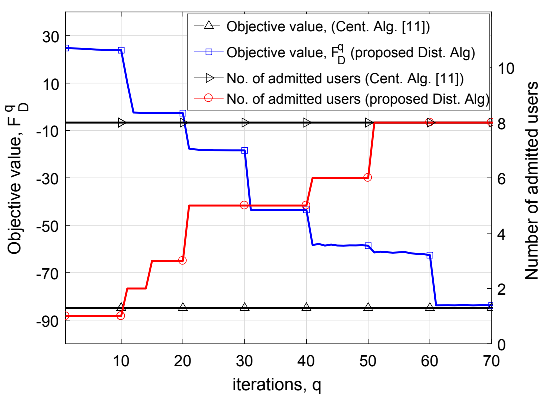

Fig. 2. Objective value F_{D}^{q} versus iterations, and the number of admitted users versus iterations for with $\gamma = 9$dB

Next, we evaluate the average number of admitted users versus SINR target γ of the proposed *Algorithm 1* in Fig. 3. As benchmarks, we consider an exhaustive search algorithm, a centralized algorithm, and the distributed algorithm in [6] (we name as Alg. [6]-distributed). The average number of admitted users obtained by distributed *Algorithm 1* is plotted by running ADMM algorithm for $Q = 1, 5, 10$, and 50 iterations. Results show that for $Q = 5, 10$, and 50 proposed *Algorithm 1* outperforms Alg. [6]-distributed for all the considered SINR targets. When $Q = 1$, our *Algorithm 1* slightly performs better than Alg. [6]-distributed at low SINR targets. However, for large SINR targets, proposed *Algorithm 1* outperforms Alg. [6]-distributed when $Q = 1$. Furthermore, results show that when γ is low, a centralized algorithm admits more users on average than that of the distributed *Algorithm 1*. But for all the simulated cases, when γ is high the average number of admitted users obtained by using *Algorithm 1* is getting closer to that of the centralized algorithm, for considered values of Q.

V. Conclusions

We have considered the admission control problem for the downlink of a multicell multiple-input single-output system. We have cast this problem as an ℓ_0 minimization problem,

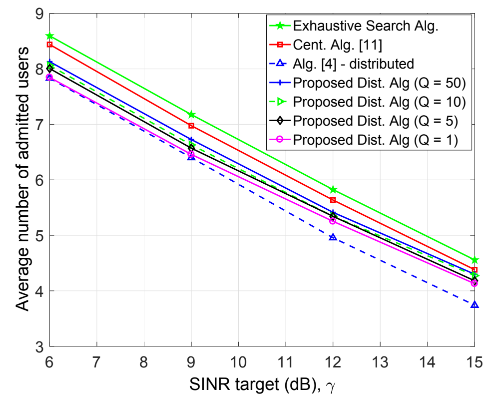

Fig. 3. Distributed algorithm comparison for average number of admitted users versus the SINR target

which is combinatorial, NP-hard. This ℓ_0 minimization problem has then been approximated as a non-combinatorial one. Then, we have proposed a suboptimal distributed algorithms based on sequential convex programming and alternating direction method of multipliers to solve it. Numerically, we show that the proposed algorithm achieves near-to-optimal performance.

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