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Prefitem

A TACHYONIC EXTENSION OF THE STRINGY NO-GO THEOREM

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ABSTRACT

We investigate the tachyon-dilaton-metric system to study the "graceful exit" problem in string theoretic inflation, where tachyon plays the role of the scalar field. ¿From the phase space analysis, we find that the inflationary phase does not smoothly connect to a Friedmann-Robertson-Walker (FRW) expanding universe, thereby providing a simple tachyonic extension of the recently proved stringy no-go theorem.

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The expansion of the Universe including an inflationary phase is considered to be an interesting and successful model in solving various problems of standard cosmology [1]. The various variants of the inflationary scenarios are based on the existence of a generic scalar field [2]. A standard problem in the inflationary cosmology is the "graceful exit" problem, which relates to the fact that inflation never ends since the percolation of the true vacuum cannot exceed the expansion of the false vacuum [3]. Analysis of standard problems in inflationary cosmology in a (super)string theory has led to interesting results. Superstring theory contains a scalar field known as dilaton in its spectrum, which is supposed to provide the necessary dynamics for inflation. A lot of work has been done in past few years to investigate the role of dilaton in string inflationary cosmology [4] and the conclusion seems to be that the presence of dilaton alone does not provide an adequate inflationary model. In recent times, use of duality symmetry in string cosmology has led to interesting results [5]. Using this symmetry, Brustein and Veneziano have analyzed the "graceful exit" problem in the dilaton-gravity system in the string frame [6]. They have found that the transition from an accelerated inflationary phase to a standard FRW cosmological phase is not possible in the weak curvature regime. In two recent papers [7], the authors have reexamined the graceful exit problem in string cosmology by including an axion field and a general axion- dilaton potential. Their phase space analysis leads to a no-go theorem, which rules out the possibility of a branch change necessary for graceful exit. In this paper, we have considered the effect of tachyon field (which arises in the spectrum of bosonic string theory) on the string theoretic inflation. We have analysed the tachyon-dilaton-metric system to examine the graceful exit problem, that is to understand whether tachyon can play a role in allowing the inflationary phase to end and connect smoothly to a Friedmann-Robertson-Walker (FRW) expanding universe. Our negative conclusion

gives a tachyonic extension of the stringy no-go theorem established by Kaloper etal [7].

The beta function analysis including the tachyon field has been considered before [8]. It is known that tachyon destabilizes the canonical 26 dimensional vacuum. But it is quite possible that the collective effects may stabilize the closed bosonic string in another ground state containing a nonzero tachyon expectation value. Upto cubic order in tachyon field, the string field theory gets a contribution to the tachyon potential of the form,

$$V(\hat{T}) = -\frac{2}{\alpha'}\hat{T}^2 + \frac{\hat{g}}{6}\hat{T}^3$$
(1)

Here, \hat{T} is the tachyon field appearing in string field theory and \hat{g} is the three point tachyon coupling at zero momentum.

We start with the low energy string effective action involving the metric $G_{\mu\nu}$, dilaton Φ and tachyon T. For the time dependent backgrounds, the equations of motion become quadratic so that solutions come in pairs corresponding to the two signs or branches and these two solutions are related through duality transformations. The positive branch solution describes accelerated expansion (contraction) corresponding to the pre big-bang phase and the negative branch solution describes deccelerated expansion (contraction) corresponding to the post big-bang phase of decreasing curvature. So the question is to see if the dynamics allows the two branches to join smoothly, so that at some later time, the universe expands as a regular FRW universe. This picture has been analysed for the dilaton driven inflation. It has been observed in ref.[6] that though branch change is possible in this context, a second branch change also occurs which forbids complete graceful exit from the accelerated inflation. This result was again confirmed by establishing an exact no-go theorem [7]. From the phase space analysis, we find that the exact no-go theorem is also valid in our case and hence tachyon can not induce a smooth branch change necessary for a successful graceful exit.

The four dimensional low energy string effective action is given by,

$$S_{eff} = -\frac{1}{16\pi\alpha'} \int d^4x \sqrt{G} e^{\Phi} \left[-R - (\nabla\Phi)^2 + (\nabla T)^2 + 2V(T) \right]$$
(2)

The one equations of motion for the metric, tachyon and dilaton are respectively given by,

$$R_{\mu\nu} = \nabla_{\mu}\nabla_{\nu}\Phi + \nabla_{\mu}T\nabla_{\nu}T \tag{3}$$

$$\nabla^2 T + \nabla_\mu \Phi \nabla^\mu T = V'(T) \tag{4}$$

$$R - (\nabla \Phi)^2 - 2\nabla^2 \Phi - (\nabla T)^2 - 2V(T) = 0$$
(5)

where, $V'(T) = \frac{\partial V(T)}{\partial T}$ and V(T) is the tachyon potential. The cosmological evolution is described through solutions of the dilaton-tachyon-gravity equations of motion [9]. We look for solutions of the equations of motion by starting with an ansatz for a flat, isotropic, FRW type of metric given by,

$$ds^2 = -dt^2 + a^2(t)dx_i dx^i \tag{6}$$

where, a(t) is the scale factor and i = 1, 2, 3. With this ansatz, the tt and ii

components of the metric equation are respectively given by,

$$3\frac{\ddot{a}}{a} + \ddot{\Phi} + (\dot{T})^2 = 0 \tag{7}$$

$$a\ddot{a} + 2\dot{a}^2 + a\dot{a}\dot{\Phi} = 0 \tag{8}$$

Tachyon and dilaton equations are respectively given by,

$$\ddot{T} + 3\frac{\dot{a}}{a}\dot{T} + \dot{\Phi}\dot{T} + V'(T) = 0$$
(9)

$$(\dot{\Phi})^2 + \ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} - 2V(T) = 0$$
(10)

The extremum of the tachyon potential (V'(T) = 0) admits $T = T_0$ (constant) as a consistent solution apart from the case of T = 0 [9]. We note that, for $T = T_0$, $V(T_0) \neq 0$ and $V''(T_0)$ is positive which ensures a stable solution. The Hubble parameter is defined as $H = \frac{\dot{a}}{a}$.

Now we rewrite the above equations in terms of Hubble parameter H and H. After some algebraic manipulation, equations(7), (8) and (9) give the following two equations:

$$\dot{H} = \pm H\sqrt{3H^2 + 2V(T) + \dot{T}^2} \tag{11}$$

$$\dot{\Phi} = -3H \mp \sqrt{3H^2 + 2V(T) + \dot{T}^2} \tag{12}$$

Note that these two equations are analogous to the previous axion-dilaton case (except for the derivative of the potential with respect to dilaton term and a term linear in ρ in \dot{H} equation as in our case, the derivative term in tachyon in the action does not have a dilaton factor in front and tachyon potential is independent of Φ) by redefining our $\Phi = -2\Phi$. Finally, T equation is given by,

$$\ddot{T} \pm \left[\sqrt{3H^2 + 2V(T) + \dot{T}^2}\right]\dot{T} + V'(T) = 0$$
(13)

Now the solutions exhibit two different branches, called the positive (+) branch and the negative (-) branch, according to the sign chosen (simultaneously for all the equations). We also get two more solutions due to the sign ambiguity of the initial value of H. The solutions obtained in this way, are related to each other by time reversal and scale factor duality (SFD) [10]. The equations of motion remain invariant under these symmetries. SFD transformations on various fields are given by,

$$\Phi \to \Phi + 6 \ln a; \qquad a \to \frac{1}{a}; \qquad H \to -H; \qquad T \to T.$$
 (14)

The solutions in the negative branch describes deccelerated expansion $(H > 0, \dot{H} < 0)$ or deccelerated contraction $(H < 0, \dot{H} > 0)$, depending on the initial sign of H. This branch can be joined smoothly to a standard radiation dominated FRW expanding universe and the solution is a stable one. On the other hand, the solution in the positive branch describes either accelerated expansion $(H > 0, \dot{H} > 0)$ or accelerated contraction $(H < 0, \dot{H} < 0)$, again depending on the initial sign of H. We see that since this branch accomodates only accelerated solutions, it does not join with the FRW expanding universe. A smooth joining in this case can happen only if the dynamics allows the positive branch to change over to the negative branch.

¿From the above analysis, we see that the equation governing a smooth transition is given by,

$$3H^2 + 2V(T) + \dot{T}^2 = 0 \tag{15}$$

Since H^2 and \dot{T}^2 are positive, eqn.(15)demands that V(T) has to be negative for some value of T. Equation (15)ensures continuity in $\dot{\Phi}$. As we know that \dot{H} remains invariant under the product of scale factor duality and time reversal transformation, it is expected that solutions with same sign of H can change to one another for maintaining continuity of H. Equation (15)describes a curve in phase space and for negative potential, this curve resembles an "Egg", which is assumed to be in the weak coupling region. The egg function is defined as,

$$e = \sqrt{3H^2 + 2V(T) + \dot{T}^2} \tag{16}$$

The egg is the region where $e \leq 0$. The condition for continuity in Φ (eqn. (12)) means that a branch change can occur only when e = 0. In order to investigate the dynamics of the trajectories, we now consider the four dimensional phase space of the problem which consists of H, Φ , T and \dot{T} . Taking a time derivative of the egg function expression and substituting the equations of motion, we obtain,

$$\pm \dot{e} = 3H^2 + \dot{T}^2 = 2H\dot{\Phi} - \dot{H} + \dot{T}^2 \tag{17}$$

for the positive and negative branches respectively (we have already redefined our Φ as -2Φ in order to compare with the axion case). From the above equations, we can immediately see that for the positive branch, the derivative of the egg function is always non-negative (the expression after first equality) and hence there can be no branch change from the positive to the negative one. This gives a simple

proof of the no-go theorem obtained by Kaloper etal. This can also be seen in the following way. As in the axion case, integrating equation (17)w.r.t. time, the trajectory equation can be written as,

$$\pm (e(t_2) - e(t_1)) + H(t_2) - H(t_1) = 2 \int_{t_1}^{t_2} H d\Phi + \int_{t_1}^{t_2} \dot{T}^2 dt$$
(18)

This is the same expression as the axion case (where \dot{T}^2 is replaced by the corresponding axion term ρ without the multiplying dilaton factor), from which a no-go theorem for branch change has been proved. Solving the equation for $\dot{\Phi} = 0$ (eqn.(12)) in association with the fact that H is positive or negative for a point in the $H - \Phi$ plane to be above or below the egg respectively, the first term in eqn.(18) is positive definite as it represents the area of the projection of the Φ trajectories in the $H - \Phi$ plane. The second term is positive by definition. Thus a branch change, which requires l. h. s. to be negative, is ruled out much like the axion case [7]. This implies that a positive trajectory entering the egg from above leaves as (+) since conversion to negative trajectory is ruled out and a (+) trajectory entering the egg from below leaves it as positive since it can not hit the egg at all.

Next, we examine the case when the positive trajectory hits the egg not above or below but at the boundary where H = 0. We note that in this case, $\pm \dot{e} = \rho$ (where $\rho = \dot{T}^2$). As the right hand side is positive by definition, the positive trajectory is repelled away from the egg, thus precluding any chance of a branch change. For $\rho = 0$, however this argument breaks down. The lowest nonvanishing time derivative of the egg function e in this case is the triple derivative given by,

$$\pm e^{(3)} = 2\left(\frac{\partial V}{\partial T}\right)^2,\tag{19}$$

hence, $\pm e^{(3)} \ge 0$. Once again the positive trajectory is repelled from the egg forbiding a branch change. Towards an analysis of the case when the tachyon potential develops inflection points at the boundary of the egg, it is observed that,

$$H^{(n)} = 0, \qquad f^{(6)} = 80 \left(\frac{\partial V}{\partial T}\right)^4 \tag{20}$$

where, (n) stands for the *n*th derivative with respect to time and $f = 2\Lambda + \rho$. So for $\frac{\partial V}{\partial T} = 0$, the trajectory passing through e = H = f = 0 is a constant and does not lead to a branch change. From the above analysis, we note that the egg is incapable of effecting a branch change. Hence tachyon can not induce a graceful exit from the inflationary phase, thereby providing a tachyonic extension of the previously extablished no-go theorem.

To summarize, we have analysed the tachyon-dilaton-gravity system to understand the graceful exit problem in string cosmology. It has been shown in ref.[6] that a satisfactory answer to graceful exit problem can not be obtained in the dilaton-gravity system. This negative result has been confirmed in an axiondilaton-metric system by proving the no-go theorem for the branch change necessary for a successful graceful exit [7]. We have closely followed the approach of Kaloper etal for investigating the dynamics of the trajectory and have extended the the proof of the no-go theorem to the tachyonic case.

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