Is the Low-ℓ Microwave Background Cosmic?

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The large-angle (low-ℓ) correlations of the cosmic microwave background exhibit several statistically significant anomalies compared to the standard inflationary cosmology. We show that the quadrupole plane and the three octopole planes are far more aligned than previously thought (99.9% C.L.). Three of these planes are orthogonal to the ecliptic at 99.1% C.L., and the normals to these planes are aligned at 99.6% C.L. with the direction of the cosmological dipole and with the equinoxes. The remaining octopole plane is orthogonal to the supergalactic plane at 99.6% C.L.

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Much effort is currently being devoted to examining the cosmic microwave background (CMB) temperature anisotropies measured with the Wilkinson Microwave Anisotropy Probe (WMAP) [1–4] and other CMB experiments [5]. While the data are regarded as a dramatic confirmation of standard inflationary cosmology, anomalies exist. In particular the correlations at large angular separations, or low ℓ, exhibit several peculiarities.

Most prominent among the "low-ℓ anomalies" is the near vanishing of the two-point angular correlation function $C(\theta)$ at angular separations greater than about 60 degrees. This was first measured using the Cosmic Background Explorer's differential microwave radiometer [6] and recently confirmed by observations with WMAP [4]. This anomalous lack of large-angle correlation is connected to the low value of the quadrupole contribution C_2 in a spherical harmonic expansion of the CMB sky—

$$\Delta T(\theta, \phi) \equiv \sum_{\ell=1}^{\infty} T_\ell \equiv \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \qquad (1)$$

[with $(2\ell + 1)C_\ell = \sum_m |a_{\ell m}|^2$]—although the smallness of C_2 does not fully account for the shape of $C(\theta)$. The significance of the large-angle behavior of $C(\theta)$, especially in light of the large cosmic variance in C_2, is a matter of some controversy. The comparisons are, moreover, confused by the fact that where one author may calculate only the probability of the low value of C_2, others, such as the WMAP team, calculate the probability of $C(\theta)$ being as low as it is over some range of angles. The issue of what prior probabilities and estimators [7–9] to use further complicates the statistical situation.

While the overall absence of large-scale power has attracted the most attention, several other large-angle anomalies have been pointed out. de Oliveira-Costa *et al.* [10] have shown that the octopole is unusually planar, meaning that the hot and cold spots of the octopolar anisotropies lie nearly in a plane. The same authors found that the axes $\mathbf{n_2}$ and $\mathbf{n_3}$ about which the "angular momentum" dispersion $\sum_m m^2 |a_{\ell m}|^2$ of the quadrupole and octopole are maximized are unusually aligned, $|\mathbf{n_2} \cdot \mathbf{n_3}| = 0.9838$. Eriksen *et al.* [11] found that the deficit in large-scale power is due to a systematic deficit in power between $\ell = 2$ and 40 in the north ecliptic hemisphere compared to the south ecliptic hemisphere. Some of us [12] have shown that the $\ell = 4$ to 8 multipoles exhibit an odd, but very unlikely ($\sim$1% probability), correlation with $\ell = 2$ and $\ell = 3$. These low-ℓ anomalies (and others [13]) have all been pointed out before, but no simple connection has been made between them. Here we remedy that situation.

By far the largest signal in the CMB anisotropy is the dipole, recently measured by WMAP [1] to be (3.346 ± 0.017) mK in the direction $(l = 263.85° \pm 0.1°,\ b = 48.25° \pm 0.04°)$ in galactic coordinates. This is caused by the motion of the Sun with respect to the rest frame defined by the CMB. As shown by Peebles and Wilkinson [14], the dipole induced by a velocity v is $\bar{T}(v/c)\cos\theta$, where θ is measured from the direction of motion. Given $\bar{T} = (2.725 \pm 0.002)$ K [15], one infers that $v \simeq 370$ km/s.

The solar motion also implies [14,16,17] the presence of a kinematically induced Doppler quadrupole (DQ). To second order in $\beta \equiv v/c \simeq 10^{-3}$, an observer moving with respect to the CMB rest frame sees the usual monopole term with a blackbody spectrum, a dipolar term $\propto \beta\cos\theta$ with a dipole spectrum and a quadrupolar term $\propto \beta^2(3\cos^2\theta - 1)$ with a quadrupole spectrum. Higher multipoles are induced only at higher order in β and so can be neglected. To first approximation the quadrupole spectrum differs very little from the dipole spectrum across the frequency range probed by WMAP. The DQ is itself a small contribution to the quadrupole. It has a total band power of only 3.6 μK^2 compared to (154 ± 70) μK^2 from the cut-sky WMAP analysis [3], 195.1 μK^2 extracted [10] from the WMAP internal linear combination (ILC) full-sky map [18], or 201.6 μK^2 from the Tegmark *et al.* full-sky map [19] (henceforward the Tegmark map). Therefore, it is a good approximation to treat the Doppler quadrupole as having a dipole spectrum plus a small spectral distortion which we shall ignore. We can then readily subtract the DQ from the ILC or Tegmark map.

(The ILC and Tegmark maps differ in the amount of spatial filtering used to produce them.)

Meanwhile, some of us showed [12] that the ℓth multipole T_ℓ can instead be written uniquely in terms of a scalar $A^{(\ell)}$ which depends only on the total power in this multipole (i.e. on C_ℓ) and ℓ unit vectors $\{\hat{\mathbf{v}}^{(\ell,i)}|i = 1, \ldots, \ell\}$. These "multipole vectors" are entirely independent of C_ℓ, and instead encode all the information about the phase relationships of the $a_{\ell m}$. Heuristically,

$$T_\ell \approx A^{(\ell)} \prod_{i=1}^{\ell} (\hat{\mathbf{v}}^{(\ell,i)} \cdot \hat{e}), \tag{2}$$

where $\hat{\mathbf{v}}^{(\ell,i)}$ is the ith multipole vector of the ℓth multipole. (In fact the right hand side contains terms with angular momentum $\ell - 2$, $\ell - 4$,... These are subtracted by taking the appropriate traceless symmetric combination, as described in [12].) Note that the signs of all the vectors can be absorbed into the sign of $A^{(\ell)}$. For convenience we take the vectors to point in the north galactic hemisphere. The multipole vectors for $\ell = 2$ and 3 for the DQ-corrected Tegmark map are [in galactic (l, b)]

$$\begin{aligned} &\hat{\mathbf{v}}^{(2,1)} = (11.26°, 16.64°), \qquad \hat{\mathbf{v}}^{(2,2)} = (118.87°, 25.13°),\\ &\hat{\mathbf{v}}^{(3,1)} = (22.63°, 9.18°), \qquad \hat{\mathbf{v}}^{(3,2)} = (86.94°, 39.30°),\\ &\hat{\mathbf{v}}^{(3,3)} = (-44.92°, 8.20°). \end{aligned} \tag{3}$$

(A similar analysis has been done for the ILC map. Results are quoted where instructive.)

As described in [12] there are several ways to compare the multipole vectors, however most striking is to compute for each ℓ the $\ell(\ell-1)/2$ independent cross-products. These are the oriented areas $\mathbf{w}^{(\ell,i,j)} \equiv \pm(\hat{\mathbf{v}}^{(\ell,i)} \times \hat{\mathbf{v}}^{(\ell,j)})$. The overall signs of the area vectors are again unphysical (we take them to point in the north galactic hemisphere), however their magnitudes are not. The area vectors for $\ell = 2, 3$ for the Tegmark map (cf. Figure 1) are

$$\begin{aligned} \mathbf{w}^{(2,1,2)} &= 0.9900(-105.73°, 56.62°),\\ \mathbf{w}^{(3,1,2)} &= 0.9017(-78.38°, 49.76°),\\ \mathbf{w}^{(3,2,3)} &= 0.9072(-141.61°, 38.96°),\\ \mathbf{w}^{(3,3,1)} &= 0.9184(173.77°, 79.54°). \end{aligned} \tag{4}$$

The directions of $\mathbf{w}^{(2,1,2)}$ and of de Oliveira-Costa *et al.*'s $\mathbf{n}_2$ are mathematically equivalent. The small difference is due to the removal of the DQ here in $\mathbf{w}^{(2,1,2)}$.

Finally, the magnitudes of the dot products between $\mathbf{w}^{(2,1,2)}$ and $\mathbf{w}^{(3,i,j)}$ ordered from largest to smallest are

$$\begin{aligned} A_1 &\equiv |\mathbf{w}^{(2,1,2)} \cdot \mathbf{w}^{(3,1,2)}| = 0.8509,\\ A_2 &\equiv |\mathbf{w}^{(2,1,2)} \cdot \mathbf{w}^{(3,2,3)}| = 0.7829,\\ A_3 &\equiv |\mathbf{w}^{(2,1,2)} \cdot \mathbf{w}^{(3,3,1)}| = 0.7616. \end{aligned} \tag{5}$$

Using instead the normal vectors $\hat{\mathbf{n}}^{(\ell,i,j)} \equiv \mathbf{w}^{(\ell,i,j)}/|\mathbf{w}^{(\ell,i,j)}|$, the dot products are

$$D_1 = 0.9531, \qquad D_2 = 0.8719, \qquad D_3 = 0.8377. \tag{6}$$

One can see from the large values of all three of the D_i that the quadrupole and octopole are aligned, and that the octopole is unusually planar.

The A_i retain information about both the magnitudes and orientations of the $\mathbf{w}^{(\ell,i,j)}$. We have compared their values against 10^5 Monte Carlo (MC) simulations of Gaussian random statistically isotropic skies with pixel noise (as in [12]). The probability that the largest of these dot products is at least A_1, the 2nd largest at least A_2, and the 3rd largest at least A_3 is 0.021%. It is 0.11% without the DQ correction, supporting that this is not just a statistical accident. (The ILC map yields 0.025% and 0.12%.)

Ordering dot products does not induce a well-defined ordering relation for the MC maps [20]. A robust and more conservative statistic is the sum of the dot products, $S = \sum_i A_i$. We find that only 0.128% of the MC maps have a larger S than the DQ-corrected Tegmark map. Thus the quadrupole-octopole correlation is excluded from being a chance occurrence in a gaussian random statistically isotropic sky at >99.87% C.L. This result is statistically independent of (though perhaps not physically unrelated to) the lack of power at large angular scales since all the information about the power is contained in the $A^{(\ell)}$, and not in the multipole vectors.

So far we have looked only at the correlation between the CMB quadrupole and octopole. Motivated by the results of Eriksen *et al.* [11], we next ask whether the quadrupole and octopole correlate with the ecliptic or the galaxy. We notice (see Fig. 1) that three of the four $\hat{\mathbf{n}}^{(\ell,i,j)}$ seem to lie near the ecliptic plane. Their dot products with

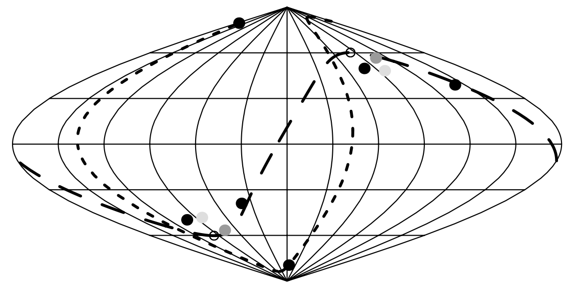

FIG. 1. The normal vectors for $\ell = 2$ (dark gray), and $\ell = 3$ (black), as well as ± the dipole direction (light gray), and the equinoxes (open circles) plotted in sinusoidal projection. The Galactic center is the coordinate origin. Galactic longitude is positive on the left. Grid lines are separated by 30°. The long-dashed line is the ecliptic; the short-dashed line is the supergalactic plane. The clustering of the $\ell = 3$ normal vectors is indicative of the "planarity of the octopole" [10]. (The clustering is clearer to the eye in the south hemisphere because of the projection.) Their closeness to the $\ell = 2$ normal is indicative of the quadrupole-octopole correlation. Note how three normals are close to the dipole, the ecliptic and the equinoxes, while the fourth is on the supergalactic plane.

the north ecliptic pole are (for the Tegmark map) in order of size: 0.0271, 0.0450, 0.1786, and 0.5233. This means that three of the four planes defined by the quadrupole and octopole are nearly orthogonal to the ecliptic. The probability that a MC map has the same four dot products smaller than these is 0.104% (for the ILC map 0.193%). The sum of the four dot products, S, of MC maps exceeds that of the DQ-corrected Tegmark map only in 0.925%, so a chance alignment of the normals with the ecliptic plane is excluded at >99% C.L. A similar comparison to the galactic plane is unremarkable—the normal closest to the galactic pole, $n^{(3,3,1)}$ (the normal not near the ecliptic), could be even closer to the pole at 6.6% probability; however, $n^{(3,3,1)}$ is only 0.07° off the supergalactic plane (see Fig. 1). Consequently, the likelihood of the dot product of one $n^{(\ell,i,j)}$ with the supergalactic pole being as small as observed is just 0.449%.

The three normals that are near the ecliptic also lie very near the axis of the dipole. The likelihood of this alignment with the dipole is 0.041% (0.098%) for the Tegmark (ILC) map and 0.405% with respect to the S statistics. Since the dipole axis lies so close to the equinoxes, this may also be viewed as an alignment with the equinoxes. The angular difference in ecliptic longitude of the normals from the equinox is $<1^{\mathrm{d}}$, 13^{d}, 13^{d}, and 44^{d}. The probability of these four headless vectors having ecliptic longitudes this close to the equinoxes is 0.039% (0.005%) for the Tegmark (ILC) map, 0.368% for the S statistics. This is statistically independent from their near orthogonality to the ecliptic poles.

Two different statistical tests (ordered dot products and sum of dot products) show evidence for low-ℓ anomalies in the Tegmark map (and consistent results for the ILC map). These anomalies could be due to a wrong treatment of foregrounds, although these full-sky maps do not refer to a foreground model. However, they employ a linear combination of frequencies to produce a minimum variance map. They are susceptible to bias because chance alignments between CMB and Galactic signals tend to cancel in order to produce minimum variance. Error estimates for the ILC and Tegmark maps have not been published, nevertheless we try to estimate the errors on the correlations induced by errors in the full-sky maps. As we add (*ad hoc*) Gaussian noise of 10 μK to a_{20} (which has a value of 9.3 μK in the Tegmark map), the evidence (from ordered dot products) for the quadrupole-octopole alignment is slightly reduced, from 99.97% C.L. to 99.57% C.L. All other correlations remain at >99% C.L. Adding the same error to all $a_{\ell m}$ with $\ell = 2$ and 3 preserves all correlations at >95% C.L.

In Fig. 2, we plot a combined Doppler-subtracted quadrupole-plus-octopole map, T^{DS}_{2+3}. The dashed line is the ecliptic. Strikingly, the ecliptic threads its way along the node line separating one of the hot spots from one of the cold spots, tracking the node over a third of the sky (and avoiding the extrema over the rest). The alignment is even better in the ILC map (not shown) than in the Tegmark map, which is not surprising given the spatial filter used by Tegmark *et al.* Replotting in other projections (not shown) does not noticeably alter the apparent correlation. A second look at Fig. 2 reveals a north-south ecliptic asymmetry—the three extrema in the north are visibly weaker than those in the south. One is cautioned that, given the observed planarity of the quadrupole-plus-octopole, one expects some hemispheric asymmetry because of the parity of the octopole. This may explain the asymmetry found in [11].

Interestingly, the correlation with the ecliptic is stronger in the combined quadrupole-octopole map, than in either separately, and the north-south ecliptic asymmetry is visible in neither alone. The correlation (but not the asymmetry) appears to be weakened if higher multipoles are added in, however a more thorough analysis is merited. (For example, in T_4 the extrema seem to define both the ecliptic and the plane of the quadrupole-octopole, and the dipole direction lies near the center of a cold spot.) These trends are exhibited in Fig. 3.

We have shown that the planes defined by the octopole are nearly aligned with the plane of the Doppler-subtracted quadrupole, that three of these planes are orthogonal to the ecliptic plane, with normals aligned with the dipole (or the equinoxes), while the fourth plane is perpendicular to the supergalactic plane. Each of these correlations is unlikely at ≥ 99% C.L., and at least two of them are statistically independent. We have also seen that the ecliptic threads between a hot and a cold spot of the combined Doppler-subtracted-quadrupole and octopole maps—following a node line across about 1/3 of the sky, and separating the three strong extrema from the three weak extrema of the map.

We find it hard to believe that these correlations are just statistical fluctuations around standard inflationary cosmology's prediction of statistically isotropic Gaussian random $a_{\ell m}$'s. That the quadrupole-octopole correlation

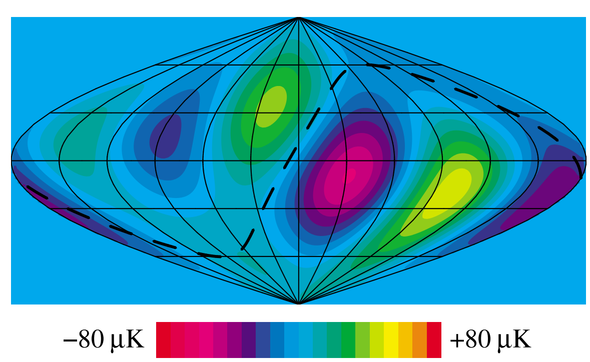

FIG. 2 (color online). Cosmic quadrupole-plus-octopole, T^{DS}_{2+3} (from Tegmark map). Coordinates are as in Fig. 1, background is 0 μK. The ecliptic (dashed line) avoids all extrema.

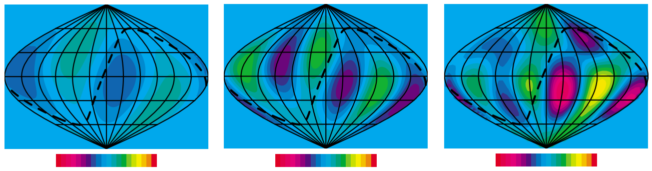

FIG. 3 (color online). Left to right: Cosmic (Doppler-subtracted) quadrupole, octopole, and $T^{\rm DS}_{2+3} + T_4$ from the Tegmark map. The coordinate system and shading scale are as in Fig. 2. The ecliptic (dashed line) is noticeably less correlated with these maps than with the quadrupole-octopole map of Fig. 2.

just happened to increase by $\sim$5 when the quadrupole was Doppler-corrected seems particularly unlikely. The correlation of the normals with the ecliptic poles suggest an unknown source or sink of CMB radiation or an unrecognized systematic. If it is a physical source or sink in the inner solar system it would cause an annual modulation in the time-ordered data. An outer solar-system origin (beyond 100 A.U.) might avoid such a signal. It seems likely that a local source (sink) would also show up in polarization maps, especially since there would be no reason for B modes to be significantly suppressed with respect to E modes. Physical correlation of the CMB with the equinoxes is difficult to imagine, since the WMAP satellite has no knowledge of the inclination of the Earth's spin axis. Alternatively, this normals-ecliptic correlation could be a consequence of the closeness of the dipole to the ecliptic plane and the correlation of the normals to the dipole. But since the dipole is itself believed to be due to local motion, this would suggest noncosmological contamination of $\ell = 2$ and 3, as does the observed correlation with the supergalactic plane.

Although full-sky maps such as the ones we have used are not expected to be reliable at high ℓ and do not agree with each other with respect to the galaxy, for the existing maps (ILC map, Tegmark map and Eriksen *et al.* map [8]) the ecliptic plane does not show up in the difference maps and their power spectra are consistent with each other at low ℓ [8].

If indeed the $\ell = 2$ and 3 CMB fluctuations are inconsistent with the predictions of standard cosmology, then one must reconsider all CMB results within the standard paradigm which rely on low ℓ's, including: the high temperature-polarization correlation C_ℓ^{TE} measured by WMAP [1] at very low ℓ (and hence the inferred redshift of reionization); the normalization of the primordial fluctuations (which relies on the extraction of the optical depth τ from low ℓ); and the running $dn_s/d\log k$ of the spectral index of scalar perturbations (which, as noted in [21], depends on the absence of low-ℓ power in the temperature maps).

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