# Wilson loops in large N field theories

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## Abstract

We propose a method to calculate the expectation values of an operator similar to the Wilson loop in the large N limit of field theories. We consider  $\mathcal{N}=4$  3+1 dimensional super-Yang-Mills. The prescription involves calculating the area of a fundamental string worldsheet in certain supergravity backgrounds. We also consider the case of coincident M-theory fivebranes where one is lead to calculating the area of M-theory two-branes. We briefly discuss the computation for 2+1 dimensional super-Yang-Mills with sixteen supercharges which is non-conformal. In all these cases we calculate the energy of quark-antiquark pair.

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## 1. Introduction

It has been expected for some time that the 't Hooft limit [1] of large N gauge theories is related to a string theory (see [2] and references therein). In [3] a precise string theory was proposed for the 't Hooft limit of  $\mathcal{N}=4$  super-Yang-Mills in 3+1 dimensions, based on earlier studies [4]. The 't Hooft limit is defined as the limit of  $N \to \infty$  keeping  $g_{YM}^2 N$ fixed. In this limit we get weakly coupled string theory on  $AdS_5 \times S^5$  where the radius of the five-sphere and the curvature radius of anti-de Sitter are proportional to  $(g_{YM}^2N)^{1/4}$  in string units. There is also a flux of the Ramond-Ramond self dual five-form field strength on the five-sphere. The string coupling is  $g \sim g_{YM}^2$  and goes to zero in the t' Hooft limit. In general we do not know how to solve free string theory on  $AdS_5 \times S^5$ . However, when the qN is large the radius of curvature is large and we can use the string in background fields approximation. In [5,6,7] it was shown how to calculate conformal dimensions of operators and correlators in conformal field theory in terms of supergravity when qN is large. In this paper we consider the problem of calculating the expectation values of Wilson loop operators. The proposal is that these expectation values correspond to the area of a worldsheet whose boundary is the loop in question. We will further consider similar observables for the M5-brane theory (the conformal (0,2) six dimensional theory). We also discuss Wilson loops in non-conformal theories associated with D-twobranes.

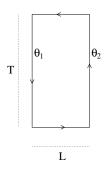
## 2. The Wilson loop

Consider a Yang-Mills theory. The Wilson loop operator is

$$W(\mathcal{C}) = \frac{1}{N} Tr P e^{i \oint_{\mathcal{C}} A}$$
 (2.1)

where  $\mathcal C$  denotes a closed loop in spacetime and the trace is over the fundamental representation. We will be considering mostly the Euclidean field theory. We can view the Wilson loop as the phase factor associated to the propagation of a very massive quark in the fundamental representation of the gauge group. A loop that is often considered is a rectangle as indicated in figure 1. From the expectation value of this rectangular Wilson loop it is possible to read off the energy of a quark-antiquark pair. Namely, in the limit  $T \to \infty$  the expectation value of the Wilson loop is

$$\langle W(\mathcal{C}) \rangle = A(L)e^{-TE(L)}$$
 (2.2)



**Figure 1:** Contour used to extract the quark-antiquark force from the Wilson loop. The vertical direction indicates Euclidean time and the horizontal direction indicates one of the spatial coordinates. This contour lives in four dimensional Euclidean space. The parameters  $\theta_{1,2}$  are for later reference.

where E(L) is the energy of the quark-antiquark pair.

In order to perform this calculation for the cases of interest it will be necessary to introduce massive quarks. To this effect consider breaking  $U(N+1) \to U(N) \times U(1)$  by giving some expectation value  $\vec{\Phi}$  to a Higgs field. Then the massive W-bosons have a mass proportional to  $|\vec{\Phi}|$  and transform in the fundamental representation of U(N). So in the limit  $|\vec{\Phi}| \to \infty$  they provide the very massive quarks necessary to compute Wilson loops in the U(N) theory. Notice that we are interested in physics for energy scales much lower than  $|\vec{\Phi}|$  so that that the U(N) theory is effectively decoupled from the U(1) theory. Consider the equation of motion for the massive W boson. Extracting the leading time dependence as  $W = e^{-i|\Phi|t}\tilde{W}$  we get an equation for  $\tilde{W}$  which to first order in  $1/|\vec{\Phi}|$  reads

$$(\partial_0 - iA_0 - i\theta^I X^I)\tilde{W} = 0 (2.3)$$

where we have defined  $\theta^I \equiv \frac{\Phi^I}{|\vec{\Phi}|}$ . Notice that  $A_0$  and  $X_I$  are matrices in the adjoint of U(N). This implies that if we consider this massive W boson describing a closed loop  $\mathcal{C}$  its interaction with the U(N) gauge field will lead to the insertion of the operator

$$W(\mathcal{C}) = \frac{1}{N} Tr P e^{i \oint ds [A_{\mu}(\sigma)\dot{\sigma}^{\mu} + \theta^{I}(s)X^{I}(\sigma)\sqrt{\dot{\sigma}^{2}}]}$$
(2.4)

The difference with (2.1) is the fact that we have an extra coupling to  $X^I$ . The operator in (2.4) is determined by the contour  $\mathcal{C}$  (or  $\sigma^{\mu}(s)$ ) as well as a function  $\vec{\theta}(s)$  mapping each point on the loop to a point on the five-sphere. We are interested in this operator because it is the one that naturally arises when we consider the propagation of a massive W-boson. The appearance of  $X^I$  might seem surprising at first sight, but it is obvious when we remember that a string ending on a p-brane is not only a source of electric field but it also carries "scalar" charge for the fields  $X^I$  since it is pulling the brane. In fact this coupling is crucial to understand the BPS bound for strings stretching between different branes [8]. In the calculations below  $\theta(s)$  will be basically constant.

# 3. Relation to supergravity

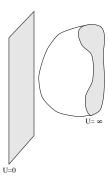
A natural proposal for the expectation value of the Wilson loop is

$$\langle W(\mathcal{C}) \rangle \sim e^{-S}$$
 (3.1)

where, in the large gN approximation, S is the proper area of a fundamental string worldsheet which at the boundary of AdS describes the loop C and lies along  $\theta^I(s)$  on  $S^5$ . See figure 2. In general we should consider the full partition function of string theory on  $AdS_5 \times S^5$  with the condition that a string worldsheet is ending on the loop C and the points  $\theta(s)$  on  $S^5$  at the boundary of AdS. This is a natural proposal in terms of the identification proposed in [7,5] for relating gauge theory observables to calculations on AdS. However the right hand side in (3.1) contains also the contribution from the mass of the W-boson and it is therefore infinity. Subtracting this contribution we find a finite result for the Wilson loop operator

$$\langle W(\mathcal{C}) \rangle \sim \lim_{\Phi \to \infty} e^{-(S_{\Phi} - \ell \Phi)}$$
 (3.2)

Where  $\ell$  is the total length of the Wilson loop, measured with the flat Minkowski metric appropriate to the gauge theory, and  $\Phi$  is the mass of the W-boson. The equation (3.2) is our final recipe for computing the Wilson loop. This result is not "zig-zag" invariant, in the sense of [2], since the operator (2.4) is not invariant, as opposed to (2.1).



**Figure 2:** Proposal to calculate Wilson loop expectation values. We should consider the partition function of string theory on  $AdS_5 \times S^5$  with a string worldsheet ending on the contour C on the boundary of AdS.

# 4. Quark anti-quark potential

In this section we consider the calculation of a rectangular Wilson loop as in figure 1. We take the angle  $\theta^I(s) = \theta^I_0$  to be a constant. We consider the limit  $T \to \infty$ . In this limit the problem becomes translational invariant along the  $\hat{T}$  direction. We put the quark at x = -L/2 and the anti-quark at x = L/2. Here "quark" means an infinitely massive W-boson connecting the N branes with one brane which is far away in the direction  $\vec{\theta}_0$ . The action for the string worldsheet is

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{\det G_{MN} \partial_{\alpha} X^{M} \partial_{\beta} X^{N}}$$
 (4.1)

where  $G_{MN}$  is the Euclidean  $AdS_5 \times S^5$  metric

$$ds^{2} = \alpha' \left[ \frac{U^{2}}{R^{2}} (dt^{2} + dx_{i}dx_{i}) + R^{2} \frac{dU^{2}}{U^{2}} + R^{2} d\Omega_{5}^{2} \right]$$
(4.2)

where  $R = (4\pi g N)^{1/4}$  is the radius in string units and  $U = r/\alpha'$  has dimensions of energy. Notice that the factors of  $\alpha'$  cancel out in (4.1), as they should. Since we are interested in a static configuration we take  $\tau = t$ ,  $\sigma = x$  so that the action becomes

$$S = \frac{T}{2\pi} \int dx \sqrt{(\partial_x U)^2 + U^4/R^4}$$

$$\tag{4.3}$$

We need to solve the Euler-Lagrange equations for this action. Since the action does not depend on x explicitly the solution satisfies

$$\frac{U^4}{\sqrt{(\partial_x U)^2 + U^4/R^4}} = \text{constant} \tag{4.4}$$

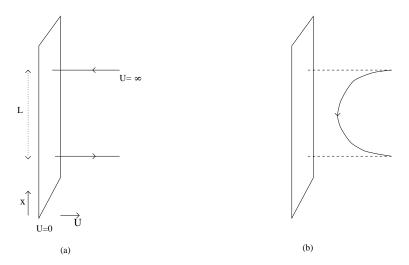
Defining  $U_0$  to be the minimum value of U, which by symmetry occurs at x = 0, we find that the solution is<sup>2</sup>

$$x = \frac{R^2}{U_0} \int_1^{U/U_0} \frac{dy}{y^2 \sqrt{y^4 - 1}}$$
 (4.5)

where  $U_0$  is determined by the condition

$$\frac{L}{2} = \frac{R^2}{U_0} \int_1^\infty \frac{dy}{y^2 \sqrt{y^4 - 1}} = \frac{R^2}{U_0} \frac{\sqrt{2}\pi^{3/2}}{\Gamma(1/4)^2}$$
(4.6)

<sup>&</sup>lt;sup>2</sup> All integrals below can be calculated in terms of Elliptic of Beta functions [9].



**Figure 3:** (a) Initial configuration corresponding to two W-bosons before we turn on their coupling to the U(N) gauge theory. (b) Configuration after we consider the coupling to the U(N) gauge theory. This configuration minimizes the action. The quark-antiquark energy is given by the difference of the total length of the strings in (a) and (b).

The qualitative form of the solution is shown in figure 3. Notice that the string approaches the point x = L/2 quickly for large U

$$\frac{L}{2} - x \sim \frac{1}{U^3} , \qquad U \gg U_0 .$$
 (4.7)

Now we compute the total energy of the configuration. If we just plug in the solution (4.5) in (4.3), we find that the answer is infinity. However as we said above this infinity is simply due to the fact that we are including the mass of the W-boson which corresponds to a string stretching all the way to  $U = \infty$ . We can regularize the expression by integrating the energy only up to  $U_{max}$ . Subtracting the regularized mass of the W-boson which is  $U_{max}/(2\pi)$  we find a finite result<sup>3</sup>

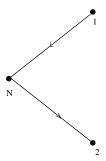
$$E = \frac{2U_0}{2\pi} \left[ \int_1^\infty dy \left( \frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - 1 \right]$$

$$E = -\frac{4\pi^2 (2g_{YM}^2 N)^{1/2}}{\Gamma(\frac{1}{4})^4 L}$$
(4.8)

<sup>&</sup>lt;sup>3</sup> A convenient way to do the integral is to multiply the whole integral by  $y^{\lambda}$ , calculate the two terms independently as a function of  $\lambda$  and then set  $\lambda = 0$ .

We see that the energy goes as 1/L, a fact which is determined by conformal invariance. Notice that the energy goes as  $(gN)^{1/2}$  as opposed to gN which is the perturbative result. This indicates some screening of the charges. The above calculation makes sense for all distances L when gN is large independently of the value of g, this suggest that one could define a magnetic Wilson loop operator which for large gN would be determined in terms of classical D-string solutions with prescribed boundary conditions at infinity. In the standard 't Hooft limit the interaction between Wilson loops is governed by g which goes as 1/N.

## 4.1. Case of non-constant angle



**Figure 4:** Configuration of a U(N+2) gauge theory Higgsed to  $U(N) \times U(1)_1 \times U(1)_2$ . This is a view in the transverse space. We also show two massive W-bosons they are characterized by the the angle of the Higgs expectation value of the U(1) factor that they are associated to.

Now we consider the case where the "angle" of the two quarks is different. This arises when we break  $U(N+2) \to U(N) \times U(1)_1 \times U(1)_2$  by giving expectation values  $\vec{\Phi}_1, \vec{\Phi}_2$  to the two U(1) factors as indicated in figure 4. Then the angles are  $\vec{\theta}_i = \vec{\Phi}_i/|\vec{\Phi}|$ . So we consider a W-boson described by a string going between the N branes and the brane associated to  $U(1)_1$  and a W-boson going between the brane associated to  $U(1)_2$  and the N branes as indicated in figure 4. Notice that the orientation of the string determines whether we have a quark (transforming in the fundamental of U(N)) or an anti-quark (transforming in the antifundamental of U(N)). The potential for this configuration can be calculated in terms of the large T limit of the expectation value of the Wilson loop shown in figure 1, with different values of  $\vec{\theta}$  on each vertical line. So we should consider a string worldsheet which at x = L/2 goes to  $U = \infty$  and to the point  $\vec{\theta}_1$  of the five-sphere and at x = -L/2 goes to  $u = \infty$  and to the point u = 0 the five-sphere. The action for a time independent configuration is

$$S = \frac{T}{2\pi} \int dx \sqrt{(\partial_x U)^2 + U^2(\partial_x \vec{\theta})^2 + U^4/R^4} \ . \tag{4.9}$$

From the symmetries of the problem we see that the string will lie along a great circle of the sphere. So if we call  $\theta$  the angle along this great circle we can choose  $\theta_{1,2} = \pm \Delta \theta/2$ . The problem then becomes symmetric around x = 0. We can solve the Euler-Lagrange equations as above by using the fact that the Lagrangian (4.9) is independent of x and  $\theta$  so that we have conserved quantities associated to "energy" and "angular momentum" (interpreting x as time). Solving these equations we find

$$x = \frac{R^2}{U_0} \sqrt{1 - l^2} \int_1^{U/U_0} \frac{dy}{y^2 \sqrt{(y^2 - 1)(y^2 + 1 - l^2)}} ,$$

$$\theta = l \int_1^{U/U_0} \frac{dy}{\sqrt{(y^2 - 1)(y^2 + 1 - l^2)}} ,$$
(4.10)

and the parameters  $U_0$ , l are determined by the conditions

$$\frac{L}{2} = x(U = \infty) = \frac{R^2}{U_0} \sqrt{1 - l^2} I_1(l) ,$$

$$\frac{\Delta \theta}{2} = \theta(U = \infty) = l I_2(l) ,$$
(4.11)

where

$$I(l)_{1} = \frac{1}{(1-l^{2})\sqrt{2-l^{2}}} \left[ (2-l^{2})E\left(\frac{\pi}{2}, \sqrt{\frac{1-l^{2}}{2-l^{2}}}\right) - F\left(\frac{\pi}{2}, \sqrt{\frac{1-l^{2}}{2-l^{2}}}\right) \right] ,$$

$$I_{2}(l) = \frac{1}{\sqrt{2-l^{2}}} F\left(\frac{\pi}{2}, \sqrt{\frac{1-l^{2}}{2-l^{2}}}\right) ,$$

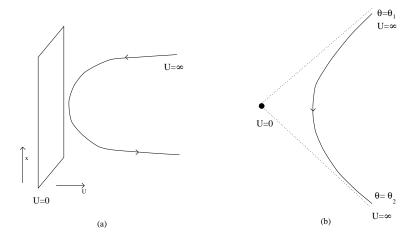
$$(4.12)$$

with F, E are elliptic integrals of the first and second kind. We can also calculate the energy of the system, substracting the mass of the W-bosons and we find

$$E = \frac{2U_0}{2\pi} \left[ \int_1^\infty dy \left( \frac{y^2}{\sqrt{(y^2 - 1)(y^2 + 1 - l^2)}} - 1 \right) - 1 \right]$$

$$= -\frac{2}{\pi} \frac{(2g_{YM}^2 N)^{1/2}}{L} (1 - l^2)^{3/2} I_1^2(l)$$
(4.13)

Where l is a function of the angle (4.11). It is interesting to notice that when  $\Delta\theta \to \pi$  then  $l \to 1$ . Then the solution looks like two straight strings going down to U = 0 and the energy (4.13) goes to zero, as expected since this is a BPS configuration.



**Figure 5:** Solution when the angles associated to the two W-bosons is different. (a) shows the projection on the x, U plane and (b) shows the projection on the  $U, \theta$  plane, where U now is the radial distance.

# 5. M-theory membranes

If we study the theory of coincident M-theory fivebranes, the (0,2) conformal field theory in six dimensions [10], we are led to consider M-theory on  $AdS_7 \times S^4$ . In this case one could define Wilson "surface" observables [11]. Since we do not have an explicit formulation of the theory, we do not have a formula analogous to (2.4). However we could define the Wilson "surfaces" as the phase factor associated to the propagation of a very heavy string on branes (sustracting the part proportional to the free propagation of the heavy string). In order to be more precise, let us suppose that we start with N+1 branes and they we Higgs by separating one of the branes. A membrane stretched between the N five-branes at the origin and the Higgsed five-brane behaves as a string with tension proportional to the separation of the branes. We could consider this heavy string as probe for the unbroken conformal field theory associated with the N branes that are still together. The procedure is analogous to what we saw above. The Wilson "surface" operator is defined to be the extra phase factor associated with the interaction of the heavy string with the N fivebranes. This Wilson "area" operator in the supergravity picture is defined by requiring that a membrane ends at the boundary of  $AdS_7 \times S^4$  on the surface that defines the operator. Notice that we also have to specify a map from the surface to  $S^4$  for the same reasons described above for  $\mathcal{N}=4$  super-Yang-Mills. Again we substract the term corresponding to the free propagation of the heavy string to obtain a finite result. For large N we can trust the supergravity result.

As an example, consider a pair of parallel, infinite strings corresponding to membranes ending on the fivebrane. Let us choose them with opposite orientation but in the same direction on  $S^4$ . This problem is translational invariant along time and the direction of the strings. So the problem of determining the minimal 3-surface reduces, as above, to finding the minimum of the action

$$S = \frac{TL'}{(2\pi)^2} \int dx \sqrt{(\partial V)^2 + V^3/R^3}$$
 (5.1)

where now  $R^3 = \pi N$  and  $V = r/l_p^3$  has dimensions of (energy)<sup>2</sup>. The strings have length L' and are separated by a distance L in the direction  $\hat{x}$ . We obtain the solution

$$x = \frac{R^{3/2}}{V_0^{1/2}} \int_1^{V/V_0} \frac{dy}{y^{3/2} \sqrt{y^3 - 1}}$$
 (5.2)

where

$$\frac{L}{2} = \frac{R^{3/2}}{V_0^{1/2}} \frac{2\sqrt{\pi}\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})}$$
 (5.3)

If we calculate the energy we find

$$\frac{E}{L'} = -\frac{N}{L^2} \frac{8\sqrt{\pi}\Gamma(\frac{2}{3})^3}{\Gamma(\frac{1}{6})^3}$$
 (5.4)

The dependence on L is the one expected from conformal invariance.

## 5.1. Wilson loops in non-conformal theories

Consider 2+1 dimensional super-Yang-Mills with sixteen supercharges which is the theory describing coincident D2 branes. We can define the Wilson loop operator as in (2.4). Then we are lead to consider strings in the background of D2 branes. The large N limit of this theory was considered in [12], where it was observed that the supergravity description is valid only in some region of the solution. Therefore the analysis of the Wilson loops will also be a bit more involved. We will find that we can calculate the Wilson loops from supergravity only when the size of the loop is not too small. This is just related to the fact that for small distances we can trust the perturbative super-Yang-Mills theory. The physical result is quite different when the Wilson loop is large. If we consider a string worldsheet, embeded in the p-brane solutions studied in [12] in a configuration appropriate for studing a the quark-antiquark forces we find that we have to minimize the action

$$S = \frac{1}{2\pi} \int dx \sqrt{(\partial_x U)^2 + U^5/R^5}$$
 (5.5)

where  $R^5 = 6\pi^2 g_{YM}^2 N$ . We obtain solutions very similar to (4.5), which lead to the potential

$$E = -\frac{2^{5/3}\sqrt{\pi}\Gamma(\frac{4}{5})^{5/3}}{3^{1/3}\Gamma(\frac{3}{10})^{5/3}} \frac{(g_{YM}^2N)^{1/3}}{L^{2/3}} = -\frac{\Gamma(\frac{4}{5})U_0}{\sqrt{\pi}\Gamma(\frac{3}{10})}$$
(5.6)

between quarks and antiquarks.  $U_0$  is the minimum value of U. Now we perform the analysis of when we can trust (5.6). Let us first consider the large U region. According to [12] we can trust supergravity for  $U \ll g_{YM}^2 N$ . The solutions to (5.5) consist of string worldsheets going all the way to  $U = \infty$ . However the large U behaviour of the solution matches that of the infinitely massive W-boson. So we will require the solution at  $U \sim g_{YM}^2 N$  to be very similar to that of the W-boson, i.e. we require  $x - L/2 \ll L$ . This implies that  $L \gg 1/(g_{YM}^2 N)$ . If the distance between the quarks was much smaller than the above bound then we can apply perturbative Yang-Mills and we would obtain a potential proportional to  $V \sim g_{YM}^2 N \log(Lg_{YM}^2 N)$ . We see that these answers match up to a numerical coefficient with (5.6) when both calculations break down at  $L \sim 1/(g_{YM}^2 N)$ .

Now we need to see if we can trust the behaviour of the solution at small U, which corresponds to large distances. At small U we expect that the worldsheet of the string turns into an M-twobrane wrapped along the eleventh direction. If  $U_0 \gg g_{YM}^2$  then we can trust the above results (5.6). If  $U_0$  is smaller then we have to consider a more complicated situation where we have to solve the equation of the M-twobrane in the background corresponding to a periodic array of M-twobranes as described in [12], this presumably could be done but we will not attempt to do it here.

In summary, for the non-conformal theories one can also use the description of classical string worldsheets embeded in some background supergravity solution to calculate Wilson loops when the the size of the loop is large enough (otherwise we could use the perturbative description). One could consider other p-brane field theories as in [12] and one would find various constraints on the size of the Wilson loop for when one can trust the string worldsheet description. Of course the total size is not the only issue, we also need that the contour does not wiggle too much.

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