Are physical objects necessarily burnt up by the blue sheet inside a black hole?

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Abstract

The electromagnetic radiation that falls into a Reissner-Nordström black hole develops a "blue sheet" of infinite energy density at the Cauchy horizon. We consider classical electromagnetic fields (that were produced during the collapse and then backscattered into the black hole), and investigate the blue-sheet effects of these fields on infalling objects within a simplified model. These effects are found to be finite and even negligible for typical parameters.

The Reissner-Nordström spacetime is the unique electrically charged, spherically symmetric, static vacuum solution of the Einstein-Maxwell equations. Although astrophysical black holes are not likely to be significantly charged, the Reissner-Nordström solution (which is simple to deal with due to its spherical symmetry) can serve as a toy model for more realistic black holes,

such as Kerr. This may be physically justified by the similarity of the inner causal structures of the Reissner-Nordström and Kerr solutions – as expressed, e.g., in similar conformal Penrose diagrams [1].

Even though the causal structure of a Reissner-Nordström black hole admits a hypothetical journey into another asymptotically flat Universe [2], it turned out that undertaking such a journey might actually be dangerous The "tunnel" inside the black hole is crossed by a null hypersurface known as the inner horizon. This null hypersurface is also a Cauchy horizon, i.e., it is the boundary of the domain of dependence for initial data specified on spacelike hypersurfaces in the external Universe. Electromagnetic (or gravitational) radiation falling into the black hole becomes infinitely blueshifted at the Cauchy horizon, typically causing the energy density to blow up [3, 4, 5]. This could cause two kinds of problems: First, the divergent flux of energy carried by the electromagnetic waves might heat any infalling physical object unboundedly, thus burning it up. Second, acting as a source term in the Einstein equations, the divergent energy density leads to a divergent curvature at the Cauchy horizon. The infalling object would then experience an unlimited tidal force, which might lead to its ultimate destruction due to the tidal distortion. (A more direct cause for the diverging curvature at

the Cauchy horizon is the infinite blue-shift of the gravitational waves, which leads to the divergence of the gradient of the metric perturbations.)

Recently, there has been growing evidence [6], that the divergence of the curvature at the Cauchy horizon is rather weak. Namely, the actual tidal deformation suffered by a physical object as it reaches the Cauchy horizon is finite, and (for typical parameters) even negligible. If this is indeed the case, the ability of physical objects to traverse the Cauchy horizon may depend crucially on the other potential problem, i.e., the possible annihilation due to the divergent electromagnetic radiation. The main goal of this letter is to investigate this issue within a simplified model.

In what follows, we consider an isolated charged black hole, surrounded by electromagnetic waves, which we treat as a linear perturbation. (In fact, because of the non-vanishing electric field of the background, this linear perturbation consists of both electromagnetic and gravitational waves [4, 5].) First, we calculate the asymptotic behavior of the electromagnetic perturbation near the Cauchy horizon. Then, we use a simplified model to evaluate the possible effects of this field on (test-) infalling objects. Let us denote by M, Q_* the mass and charge of the black hole, and by r the radial Schwarzschild coordinate. Let $\Delta = r^2 - 2Mr + Q_*^2$. The horizons of the Reissner-Nordström

black hole are the event horizon r_+ and the inner horizon r_- , which are located at the roots of Δ , namely, at $r_{\pm} = M \pm (M^2 - Q_*^2)^{1/2}$. We define the null co-ordinates $u = r_* - t$ and $v = r_* + t$, where r_* is the Regge-Wheeler "tortoise" co-ordinate defined by $d/dr_* = (\Delta/r^2)d/dr$. The co-ordinate t is spacelike between the event and the Cauchy horizons, and we take $t = +\infty$ at the event horizon. In this letter we are interested in the section $u=-\infty$ of the event horizon and the section $v = +\infty$ of the inner horizon. (These are the sections which intersect in the standard Penrose diagram at future timelike infinity of the external Universe.) We assume that the object moves along a typical radial world-line that intersects the event horizon and the Cauchy horizon at some finite values $v = v_0$ and $u = u_0$, respectively. Accordingly, the trajectory of the object can be described by the function $r(\tau)$ and by u_0 , where τ is the proper time of the infalling object. We set $\tau(r=r_{-})=0$. The details of $r(\tau)$ are unimportant to our discussion. (The only piece of information that enters the calculations is the value of \dot{r} near r_{-} , where a dot denotes differentiation with respect to proper time. However, our results are not sensitive to this parameter.)

The class of perturbations that we consider here is the one which is inherent to any non-spherical gravitational-collapse; these are the electromagnetic perturbations which result from the evolution of non-vanishing electromagnetic multipole moments (in the star) during the collapse. When these perturbations propagate outwards, some fraction of them is backscattered off the spacetime curvature and captured by the black hole. This process leads to a "tail" of infalling radiation at the event horizon which, at late times $(v \gg M)$, decays like $(v/M)^{-(2l+2)}$, where l is the multipole order of the mode [7]. This electromagnetic field can be treated by the formalism developed in [4, 5, 8]. We studied the asymptotic behavior of infalling electromagnetic perturbations at the Cauchy horizon for polar modes [9]. Assuming $v_0 \gg M$ (which also implies $-u_0 \gg M$), the divergent components of the Maxwell field strength tensor, as expressed in the rest frame of an infalling observer, are given by the following approximate expression to the leading order in $\kappa_-\tau$ and $(\kappa_-u_0)^{-1}$:

$$E = -B \approx C'(\kappa_{-}\tau)^{-1} \left(\ln|\kappa_{-}\tau| + \frac{1}{2}\kappa_{-}u_{0} + \ln|\dot{r}| \right)^{-(2l+3)}, \tag{1}$$

where $\kappa_{-} = (r_{+} - r_{-})/r_{-}^{2}$ is the surface gravity of the Cauchy horizon; E(B) is the electric (magnetic) field, which points toward the $\partial / \partial \theta$ ($\partial / \partial \phi$) direction; C' is a slowly varying function of θ, ϕ (through the Legendre polynomials) and is also proportional to the initial value of the perturbing fields on the surface of the collapsing star (or on the event horizon). The choice of the

polar modes is not expected to cause any loss of generality, since similar qualitative behavior is to be anticipated for axial modes too.

The next stage of our analysis is to consider the interaction of the divergent electromagnetic field (1) with the matter comprising the infalling object. We assume that the object is much smaller than the radius of curvature between the event and the inner horizons, and hence the effects of curvature are negligible. Consequently, we can construe the object as being at rest in its locally co-moving Minkowski frame when an electromagnetic impulse of the shape (1) comes from null infinity and interacts with it. Even in flat spacetime, the interaction of matter with such an electromagnetic impulse is enormously complicated. There are many types of radiative processes, which may depend on the details of the specific matter intricately. We therefore ignore all the details of these radiative processes, and use a simplified toymodel to describe the radiation–matter interaction. Imagine that the object is made of classical "atoms". (Later, we also consider a quantum analogue.) Each "atom" consists of two electrically charged structureless particles with charges +e and -e, separated from each other by some internal force (e.g., a "spring"). With the lack of external forces, the system is static. In our case, the Lorentz force induced by the blue-shifted electromagnetic field changes

the separation between the two particles. Having in mind a small deviation from equilibrium (which is justified a posteriori), we assume a linear restoring force $F = -\mu\omega^2 X$, where X is the deviation (of the particles' separation) from equilibrium, ω is the resonance frequency, and μ is the reduced mass. (The phrase "atom" refers here to an elementary unit of matter; we do not consider a solar-type system here.) The dipole is chosen to be aligned in the $\partial/\partial\theta$ direction (to allow for a maximum interaction with the field). We take the initial conditions to be X=0 and $\dot{X}=0$. The system's energy absorption is described by its "excitation", i.e., by the gain in kinetic and potential energy. Although this model is extremely simplified, it may provide some insight into the interaction of classical radiation with matter.

The equation of motion is $\mu \ddot{X} + \mu \omega^2 X = eE(\tau)$, where $E(\tau)$ is the divergent component of the electric field (1). (The contribution of the magnetic field is neglected, as the ratio of the electric and the magnetic terms in the expression for the Lorentz force is proportional to the system's internal velocity \dot{X} , which is taken to be small – a presumption which is justified a posteriori.) The solution of this equation is

$$X(\tau) = -\frac{1}{2i\omega}e^{-i\omega\tau} \int_{-T}^{\tau} \frac{e}{\mu} E(\tau')e^{i\omega\tau'} d\tau' + \text{c.c.}$$
 (2)

where T is the time of infall from the event horizon to the Cauchy horizon. It can be shown that the total absorbed mechanical energy of the system \mathcal{E}_{c} up to proper time τ is

$$\mathcal{E}_{c}(\tau) = \frac{1}{2}\mu \left| \int_{-T}^{\tau} \frac{e}{\mu} E(\tau') e^{i\omega\tau'} d\tau' \right|^{2}. \tag{3}$$

We assume that ω is of the order of magnitude of typical molecular or atomic frequencies (or higher). For typical astrophysical black holes we have $\omega M \gg 1$. This implies $T \gg \omega^{-1}$. Hence, to evaluate the integral in (2) and (3) we divide the infall period into three qualitatively different regions, denoted as regions a, b, and c, respectively. In region a, defined by $-T < \tau \ll -\omega^{-1}$, the slowly varying electric field can be taken outside the integration, and therefore the absolute value of the integral follows the electric field adiabatically. This property means that the system has no records of its past, so the results are insensitive to the behavior of $E(\tau)$ at $\tau \ll -\omega^{-1}$.] In region c, defined by $-\omega^{-1} \ll \tau < 0$, the exponent in the integrand can be taken outside the integral, and the remaining integration is easily solvable. In between (region b), our assumption $\omega^{-1} \ll M \ll -u_0$ implies that the variation in the logarithmic term in (1) is negligible throughout the region. Taking this logarithmic term to be constant, the integral is easily solvable for region b too. Matching the solutions for the different regions, it can be shown [10] that the total contribution of all three regions to \mathcal{E}_{c} , X and \dot{X} on the Cauchy horizon is, to the leading order in $(\kappa_{-}u_{0})^{-1}$,

$$\mathcal{E}_{\rm c}(\tau=0) \approx \frac{1}{2(2l+2)^2} \frac{C'^2}{\kappa_-^2} \mu \left(\frac{e}{\mu}\right)^2 \left(\frac{1}{2} \kappa_- u_0\right)^{-2(2l+2)},$$

$$\dot{X}(\tau=0) \approx \frac{C'}{(2l+2)\kappa_{-}} \left(\frac{e}{\mu}\right) \left(\frac{1}{2}\kappa_{-}u_{0}\right)^{-(2l+2)},$$

$$X(\tau=0) \approx -\frac{1}{2} \frac{\pi}{\omega} \frac{C'}{\kappa_{-}} \left(\frac{e}{\mu}\right) \left(\frac{1}{2} \kappa_{-} u_{0}\right)^{-(2l+3)}.$$

Now, it is easy to show that for fixed $r(\tau)$ there is a one-to-one correspondence between v_0 and u_0 . In fact, $du_0/dv_0 = -1$. Therefore, the infalling observer can increase $|u_0|$ by simply waiting outside the black hole before jumping in and thus increasing the value of v_0 . For a sufficiently large $|u_0|$ we find that \mathcal{E}_c , X and \dot{X} are finite and small. Therefore, the behavior of the charged classical system obtained by the above analysis is regular, and although the external force acting on the system diverges on the Cauchy horizon, the energy absorbed by it is finite and negligible for a sufficiently large v_0 .

A quantum analogue can be conceived as a non-degenerate two-level system (the ground state $|\psi_i\rangle$ and the excited state $|\psi_f\rangle$) obeying Schrödinger's equation. We take the system to be initially in its ground state $|\psi_i\rangle$. The excitation of the system can be described by the amplitude a_f of the excited

state $|\psi_f\rangle$ in the wave-function. This amplitude can be given [10] in the Coulomb gauge by first order time-dependent perturbation theory to be

$$a_f(\tau) = -\frac{1}{\hbar\Omega_{fi}} \frac{e}{m} \langle \psi_f | \left(\int_{-T}^{\tau} e^{i\Omega_{fi}\tau'} E \, d\tau' \right) p | \psi_i \rangle,$$

where p is the component of the momentum 3-vector in the direction of $\partial/\partial\theta$, $\hbar\Omega_{fi}$ is the energy gap between the states, and m is the electron mass. The integral here is the same as in eq. (3). It can then be shown that on the Cauchy horizon the absorbed energy $\mathcal{E}_{q}(\tau)$ is, to the leading order in $(\kappa_{-}u_{0})^{-1}$,

$$\mathcal{E}_{\mathbf{q}}(0) = \frac{1}{(2l+2)^2} \frac{1}{\hbar \Omega_{fi}} \left(\frac{e}{m}\right)^2 \frac{C'^2}{\kappa_-^2} \left| \langle \psi_f | e^{-i\Omega_{fi}\bar{z}} p | \psi_i \rangle \right|^2 \left(\frac{1}{2} \kappa_- u_0\right)^{-2(2l+2)},$$

where \bar{z} is a spatial Cartesian co-ordinate in the system's rest-frame, pointed in the radial direction. The treatment here can be generalized to a many level system. There is a remarkable correspondence between our results for the classical system and for the quantum system as can be clearly seen from the similarity of the expressions for the absorbed energies in the two cases.

We have shown (in both the classical and the quantum models) that in spite of the divergence of the radiation's energy density (or even its integral over proper time) as a consequence of the infinite blue shift, an infalling observer may experience just a finite effect upon crossing the Cauchy horizon. Moreover, it is possible to reduce the extent of that effect to a negligible impact, depending only on how long after the collapse generating the black hole the observer jumps into it. (Note, however, that physical effects beyond the Cauchy horizon are as yet unknown.)

Obviously, the toy-model used here for the radiation–matter interaction is a very simplified one. We believe, however, that this model captures the main essence of the problem. We also note that our treatment here is based on a first order perturbation analysis; yet we do not expect higher-order contributions to change the qualitative picture significantly.

We have not considered here possible QED effects. In addition, a more realistic model should consider other sources of electromagnetic fields (e.g., the cosmic background radiation). After the completion of this research we became aware of the the possibility that these effects may have a much stronger impact than the classical electrodynamic effects studied here. We are especially worried about pair-production due to the interaction of the infalling matter with the highly blue-shifted cosmic-radiation photons. Preliminary estimates suggest to us that this process could be fatal for a human-being observer (due to his high vulnerability to γ -rays), but typical physical objects of similar or smaller size might survive it. These effects are currently under

investigation.

Further details of this work will be published elsewhere [10].

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References

- [1] S.W. Hawking and G.F.R. Ellis, *The large scale structure of space-time* (Cambridge University Press, Cambridge, 1973).
- [2] I.D. Novikov, Pis'ma Zh. Eksp. Teor. Fiz. 3, 223-227 (1966) [JETP Lett.
 3, 142-144 (1966)]; Astron. Zh. 43, 911-918 (1966) [Sov. Astron. 10, 731-737 (1967)]; V. De La Cruz and W. Israel, Nuovo Cimento 51, 744-759 (1967).
- [3] R. Penrose, in *Battelle Rencontres*, 1967 Lectures in Mathematics and Physics, edited by C.A. DeWitt, and J.A. Wheeler, (W.A. Benjamin, New York, 1968), pp. 121-235; M. Simpson and R. Penrose, Int. J. Theor. Phys. 7, 183-197 (1973).

- [4] Y. Gürsel, V.D. Sandberg, I.D. Novikov and A.A. Starobinsky, Phys. Rev. D 19, 413-420 (1979); Y. Gürsel, I.D. Novikov, V.D. Sandberg and A.A. Starobinsky, *ibid.* 20, 1260-1270 (1979).
- [5] S. Chandrasekhar and J.B. Hartle, Proc. R. Soc. Lond. Ser. A. 384, 301-315 (1982).
- [6] A. Ori, Phys. Rev. Lett. **67**, 789-792 (1991); **68**, 2117-2120 (1992).
- [7] R.H. Price, Phys. Rev. D 5, 2419-2438 (1972); 5, 2439-2454 (1972);
 C. Gundlach, R.H. Price and J. Pullin, *ibid.* 49, 883-889 (1994); 49, 890-899 (1994).
- [8] S. Chandrasekhar, The Mathematical Theory of Black Holes (Oxford University Press, Oxford, 1983).
- [9] The most dominant effects are produced by the l=1 modes. It turns out that this case is not treated appropriately in Ref. [8]. Our treatment of the case l=1 here is based on an extension of the analysis in [8] to the case l=1, made by L.M. Burko (in preparation).
- [10] L.M. Burko and A. Ori, (to be published).