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Trialogue on the number of fundamental constants

Michael J. Duff

Michigan Center for Theoretical Physics, Randall Laboratory
Ann Arbor, MI, USA
E-mail: mduff@umich.edu

Lev B. Okun

ITEP
Moscow, 117218, Russia

E-mail: okun@heron.itep.ru

Gabriele Veneziano

Theory Division, CERN
CH-1211 Geneva 23, Switzerland, and
Laboratoire de Physique Theorique, Universite Paris Sud
91405, Orsay, France
E-mail: Gabriele.Veneziano@cern.ch

Abstract: This paper consists of three separate articles on the number of fundamental dimensionful constants in physics. We started our debate in summer 1992 on the terrace of the famous CERN cafeteria. In the summer of 2001 we returned to the subject to find that our views still diverged and decided to explain our current positions. LBO develops the traditional approach with three constants, GV argues in favor of at most two (within superstring theory), while MJD advocates zero.

Keywords: Models of Quantum Gravity, Standard Model.

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Part I
Fundamental constants: parameters and units

Lev B. Okun

Abstract. There are two kinds of fundamental constants of Nature: dimensionless (like $\alpha \simeq 1/137$) and dimensionful (c — velocity of light, $\hbar$ — quantum of action and angular momentum, and G — Newton's gravitational constant). To clarify the discussion I suggest to refer to the former as fundamental parameters and the latter as fundamental (or basic) units. It is necessary and sufficient to have three basic units in order to reproduce in an experimentally meaningful way the dimensions of all physical quantities. Theoretical equations describing the physical world deal with dimensionless quantities and their solutions depend on dimensionless fundamental parameters. But experiments, from which these theories are extracted and by which they could be tested, involve measurements, i.e. comparisons with standard dimensionful scales. Without standard dimensionful units and hence without certain conventions physics is unthinkable.

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1. Introduction: parameters and units

There is no well established terminology for the fundamental constants of Nature. It seems reasonable to consider as fundamental the dimensionless ratios, such as the famous $\alpha = e^2/\hbar c \simeq 1/137$ and similar gauge and Yukawa couplings in the framework of standard model of elementary particles or its extensions.

It is clear that the number of such constants depends on the theoretical model at hand and hence depends on personal preferences and it changes of course with the evolution of physics. At each stage of this evolution it includes those constants which cannot be

expressed in terms of more fundamental ones, because of the absence of the latter [1]. At present this number is a few dozens, if one includes neutrino mixing angles. It blows up with the inclusion of hypothetical new particles.

On the other hand the term "fundamental constant" is often used for such dimensionful constants as the velocity of light c, the quantum of action (and of angular momentum) $\hbar$, and the Newton gravitational coupling constant G. This article is concerned with these dimensionful constants which I propose to call fundamental (or basic) units.

Physics consists of measurements, formulas and "words". This article contains no new formulas, it deals mainly with "words" because, unlike many colleagues of mine, I believe that an adequate language is crucial in physics. The absence of accurately defined terms or the uses (i.e. actually misuses) of ill-defined terms lead to confusion and proliferation of wrong statements.

2. Stoney's and Planck's units of L, T, M

The three basic physical dimensions: length L, time T, and mass M with corresponding metric units: cm, sec, gram, are usually associated with the name of C.F. Gauss. In spite of tremendous changes in physics, three basic dimensions are still necessary and sufficient to express the dimension of any physical quantity. The number three corresponds to the three basic entities (notions): space, time and matter. It does not depend on the dimensionality of space, being the same in spaces of any dimension. It does not depend on the number and nature of fundamental interactions. For instance, in a world without gravity it still would be three.

In the 1870's G.J. Stoney [2], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M: $l_S = e\sqrt{G}/c^2$, $t_S = e\sqrt{G}/c^3$, $m_S = e/\sqrt{G}$. The expression for m_S has been derived by equating the Coulomb and Newton forces. The expressions for l_S and t_S has been derived from m_S, c and e on dimensional grounds: $m_S c^2 = e^2/l_S$, $t_S = l_S/c$.

When M. Planck discovered in 1899 $\hbar$, he introduced [3] as universal units of Nature for L, T, M: $l_P = \hbar/m_P c$, $t_P = \hbar/m_P c^2$, $m_P = \sqrt{\hbar c/G}$.

One can easily see that Stoney's and Planck's units are numerically close to each other, their ratios being $\sqrt{\alpha}$.

3. The physical meaning of units

The Gauss units were "earth-bound" and "hand-crafted". The cm and sec are connected with the size and rotation of the earth.[1] The gram is the mass of one cubic cm of water.

An important step forward was made in the middle of XX century, when the standards of cm and sec were defined in terms of of wave-length and frequency of a certain atomic line.

Enormously more universal and fundamental are c and $\hbar$ given to us by Nature herself as units of velocity $[v] = [L/T]$ and angular momentum $[J] = [MvL] = [ML^2/T]$ or action $[S] = [ET] = [Mv^2T] = [ML^2/T]$. (Here [ ] denotes dimension.)

3.1 The meaning of c

It is important that c is not only the speed of light in vacuum. What is much more significant is the fact that it is the maximal velocity of any object in Nature, the photon being only one of such objects. The fundamental character of c would not be diminished in a world without photons. The fact that c is the maximal v leads to new phenomena, unknown in newtonian physics and described by relativity. Therefore Nature herself suggests c as fundamental unit of velocity.

In the Introduction we defined as fundamental those constants which cannot be calculated at our present level of fundamental knowledge (or rather ignorance). This "negative" definition applies equally to parameters and to units (to α and to c). At first sight α looks superior to c because the value of α does not depend on the choice of units, whereas the numerical value of c depends explicitly on the units of length and time and hence on conventions. However c is more fundamental than α because its fundamental character has not only a "negative" definition, but also a "positive" one: it is the basis of relativity theory which unifies space and time, as well as energy, momentum and mass.

[1] metre was defined in 1791 as a 1/40,000,000 part of Paris meridian.

By expressing v in units of c (usually it is defined as $\beta = v/c$) one simplifies relativistic kinematics. On the other hand the role of c as a conversion factor between time and distance or between mass and rest-energy is often overstated in the literature. Note that in spite of the possibility of measuring, say, distance in light-seconds, the length does not become identical to time, just as momentum is not identical to energy. This comes from the pseudoeuclidian nature of four-dimensional space-time.

3.2 The meaning of $\hbar$

Analogously to c, the quantity $\hbar$ is is also fundamental in the "positive" sense: it is the quantum of the angular momentum J and a natural unit of the action S. When J or S are close to $\hbar$, the whole realm of quantum mechanical phenomena appears.

Particles with integer J (bosons) tend to be in the same state (i.e. photons in a laser, or Rubidium atoms in a drop of Bose-Einstein condensate). Particles with half-integer J (fermions) obey the Pauli exclusion principle which is so basic for the structure of atoms, atomic nuclei and neutron stars.

Symmetry between fermions and bosons, dubbed supersymmetry or SUSY, is badly broken at low energies, but many theorists believe that it is restored near the Planck mass (in particular in superstrings and M-theories).

The role of $\hbar$ as a conversion factor between frequency and energy or between wave-length and momentum is often overstated.

It is natural when dealing with quantum mechanical problems to use $\hbar$ as the unit of J and S.

3.3 The status of G

The status of G and its derivatives, m_P, l_P, t_P, is at present different from that of c and $\hbar$, because the quantum theory of gravity is still under construction. The majority of experts connect their hopes with extra spatial dimensions and superstrings.[2] But the bridge between superstrings and experimental physics exists at present only as wishful thinking. Recent surge of interest to possible modifications of Newton's potential at sub-millimetre distances demonstrates that the position of G is not as firm as that of c and $\hbar$.

4. The cube of theories

The epistemological role of $c, \hbar, G$ units in classifying theories was first demonstrated in a jocular article by G. Gamov, D. Ivanenko and L. Landau [4], then quite seriously by M. Bronshtein [5, 6], A. Zelmanov [8, 7] and others (see e.g. [9, 10]); and it is known now as the cube of theories.

The cube is located along three orthogonal axes marked by c (actually by $1/c$), $\hbar$, G. The vertex (000) corresponds to non-relativistic mechanics, (c00) — to special relativity, (0$\hbar$0) — to non-relativistic quantum mechanics, ($c\hbar$0) — to quantum field theory, (c0G)

[2] The characteristic length of a superstring $\lambda_s = l_P\sqrt{\alpha_{GUT}}$, where $\alpha_{GUT} = \alpha(q^2 = M^2_{GUT})$. (As is well known, the fundamental parameters are "running": their values depend on q^2.)

— to general relativity, $(c\hbar G)$ — to futuristic quantum gravity and the Theory of Everything, TOE. There is a hope that in the framework of TOE the values of dimensionless fundamental parameters will be ultimately calculated.

5. The art of putting $c = 1, \hbar = 1, G = 1$

The universal character of $c, \hbar, G$ and hence of m_P, l_P, t_P makes natural their use in dealing with futuristic TOE. (In the case of strings the role of l_P is played by the string length λ_s.) In such natural units all physical quantities and

variables become dimensionless. In practice the use of these units is realized by putting $c = 1, \hbar = 1, G$ (or λ_s) $= 1$ in all formulas. However one should not take these equalities too literally, because their left-hand sides are dimensionful, while the right-hand sides are dimensionless. It would be more proper to use arrows "$\to$" (which mean "substituted by") instead of equality signs "=".

The absence of $c, \hbar, G$ (or any of them) in the so obtained dimensionless equations does not diminish the fundamental character of these units. Moreover it stresses their universality and importance.

It is necessary to keep in mind that when comparing the theoretical predictions with experimental results one has anyway to restore ("$\leftarrow$") the three basic units $c, \hbar, G$ in equations because all measurements involve standard scales.

The above arguments imply what is often dubbed as a "moderate reductionism", which in this case means that all physical phenomena

can be explained in terms of a few fundamental interactions of fundamental particles and thus expressed in terms of three basic units and a certain number of fundamental dimensionless parameters.

6. International system of units

An approach different from the above underlies the International System of Units (Système Internationale d'Unitées — SI) [11, 12]. This System includes 7 basic units (metre, second, kilogram, ampere, kelvin, mole, candela) and 17 derivative ones. The SI might be useful from the point of view of technology and metrology, but from the point of view of pure physics four out of its seven basic units are evidently derivative ones. Electric current is number of moving electrons per second. Temperature is up to a conversion factor

(Boltzman constant $k = 1.38 \times 10^{-23}$ joules/kelvin) is the average energy of an ensemble of particles. Mole is trivially connected with the number of molecules in one gram-molecule, called Avogadro's number $N_A = 6.02 \times 10^{23}$/mole. As for unit of optical brightness or illumination (candela), it is obviously expressed in terms of the flux of photons.

It is interesting to compare the character of k with that of $c, \hbar, m_P$. The Boltzman constant is an important conversion factor which signals the transition from a few (or one) particle systems to many particle systems. However it radically differs from $c, \hbar, m_P$, as there is no physical quantity with the dimension of k, for which k is a critical value.

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The role of conversion factor is only a secondary one for $c, \hbar, m_P$, whereas for k it is the only one.

In the framework of SI vacuum is endowed with electric permittivity $\varepsilon_0 = 8.85 \times 10^{-12}$ farad/m and magnetic permeability $\mu_0 = 12.57 \times 10^{-7}$ newton/(ampere)^2, whereas $\varepsilon_0\mu_0 = 1/c^2$. This is caused by electrodynamic definition of charge, which in SI is secondary with respect to the current. In electrostatic units $\varepsilon_0 = \mu_0 = 1$. According to the SI standard this definition is allowed to use in scientific literature, but not in text-books (see critical exposition of SI in ref. [13]).

7. Remarks on Gabriele's part II

I note with satisfaction that some of the original arguments and statements do not appear in his part of this Trialogue II. Among them there are the following statements: 1. that in string theory there is room only for two and not three dimensionful constants [14, 15]; 2. that units of action are arbitrary [which means that $\hbar$ is not a fundamental unit (LO)]; 3. that masses unlike length and time intervals are not measured directly [16]. Gabriele admits in section 6 that his two units can be "pedagogically confusing" and the set $c, \hbar, \lambda_s$ is "most practical", but he considers the latter "not economical" and in other parts of the part II he insists on using λ_s^2 instead of $\hbar$.

Of course, if you forget about the pedagogical and practical sides of physics, the most economical way is not to have fundamental units at all, like Mike, but that is a purely theoretical approach ("hep-th"), and not physical one ("physics", "hep-ph").

It seems to me inconsistent to keep two units (c, λ_s) explicitly in the equations, while substituting by unity the third one ($\hbar$), as Gabriele is doing in part II and refs. [14, 15, 16]. According to my section 5 above, this corresponds to using $\hbar$ as a unit of J and S, while not using c and λ_s as units of velocity and length.

I also cannot agree that the electron mass, or G_F are as good for the role of fundamental unit as the Planck mass or G.

8. Remarks on Mike's part III

In section 4 of Mike's part III he introduces a definition of fundamental constants with the help of an alien with whom it is possible to exchange only dimensionless numbers. According to Mike, only those constants are fundamental the values of which can be communicated to the alien. Thus Mike concludes that there exist no fundamental units. According to my section 5 above, this actually corresponds to the use of $c, \hbar, G$ as fundamental units.

In fact, at the end of section 2 Mike writes "that the most economical choice is to use natural units where there are no conversion factors at all." Mike explained to me that his natural units are $c = \hbar = G = 1$. As these equalities cannot be considered literally, I believe that Mike uses the same three units as I do. However he concludes section 2 with a statement: "Consequently, none of these units or conversion factors is fundamental."

(In response to the above paragraph Mike added a new paragraph to his section 2, in which he ascribed to me the view that one cannot put $c = 1$. According to my section 5, one can (and should!) put $c = 1$ in relativistic eqations, but must understand that this means that c is chosen as the unit of velocity.)

The “alien definition” of fundamental constants is misleading. We, theorists, communicate not with aliens, but with our experimental colleagues, students, and non-physicists. Such communication is impossible and physics is unthinkable without standardized dimensionful units, without conventions..

Concerning Mike's criticism of my article [10], I would like to make the following remark. The statement that only dimensionless variables, functions and constants have physical meaning in a theory does not mean that every problem should be explicitly presented in dimensionless form. Sometimes one can use dimensionful units and compare their ratios with ratios of other dimensionful units. This approach was used in ref. [10], where entertaining stories by O. Volberg [17] and G. Gamov [18] were critically analyzed. In these stories, in order to demonstrate the peculiarities of relativistic kinematics, the velocity of light was assumed to be of the order of that of a car, or even bicycle, while the everyday life remained the same as ours. In ref. [10] I have shown that if c is changed, while dimensions of atoms are not changed (mass and charge of electron as well as $\hbar$, are the same as in our world), then electromagnetic and optical properties of atoms (and hence the everyday life) would change drastically because of change of α, which is the ratio of electron velocity in hydrogen atom to that of light. It is not clear to me why in section 5 of his paper Mike disagrees with these considerations.

9. Conclusions

It is obvious that using proper language (terms and semantics) three fundamental units are the only possible basis for a selfconsistent description of fundamental physics. Other conclusions are viable only through the improper usage of terms.

Acknowledgments

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Part II
Fundamental units in physics: how many, if any?

Gabriele Veneziano

Abstract. I summarize my previous work on the question of how many fundamental dimensionful constants (fundamental units) are needed in various theoretical frameworks such as renormalizable QFT + GR, old-fashioned string theory, and modern string/M-theory. I will also try to underline where past and present disagreement on these issues between Lev Okun, Mike Duff, and myself appears to be originating from.

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1. Introductory remarks

Some fifteen years ago I wrote a short letter [14] on the number of (dimensionful) fundamental constants in string theory, where I came to the somewhat surprising conclusion that two constants, with dimensions of space and time, were both necessary and sufficient. Somewhat later, I became aware of S. Weinberg's 1983 paper [1], whose way of looking at the question of defining fundamental constants in physics I tried to incorporate in my subsequent work on the subject [15, 16].

After reading those papers of mine once more, I still subscribe to their content, even if I might have expressed some specific points differently these days. Here, rather than repeating the details of my arguments, I will try to organize and summarize them stressing where, in my opinion, the disagreement between Lev, Mike and myself arises from. I have the impression that, in the end, the disagreement is more in the words than in the physics, but this is what we should try to find out.

The rest of this note is organized as follows: In section 2 I make some trivial introductory statements that are hopefully uncontroversial. In sections 3, 4 and 5 I describe how I see the emergence of fundamental units (the name I will adopt for fundamental dimensionful constants following Lev's suggestion) in QFT+GR, in the old Nambu-Goto formulation of quantum string theory (QST), and in the so-called Polyakov formulation, respectively. In sections 6 I will try to point at the origin of disagreement between myself and Lev while, in section 7, the same will be done w.r.t. Mike. Section 8 briefly discusses the issue of time-varying fundamental units.

2. Three questions and one answer

Let me start with two statements on which we all seem to agree:

- Physics is always dealing, in the end, with dimensionless quantities, typically representing ratios of quantities having the same dimensions, e.g.

$$\alpha = \frac{e^2}{\hbar c}\,; \qquad \frac{m_e}{m_p}\,; \qquad \ldots \tag{2.1}$$

It is customary to introduce "units", i.e. to consider the ratio of any physical quantity q to a fixed quantity u_q of the same kind so that

$$q = (q/u_q)u_q ; \tag{2.2}$$

where u_q is a name (e.g. centimetre or second) and (q/u_q) is a number. Obviously, $q_1/q_2 = (q_1/u_q)/(q_2/u_q)$.

Let us now ask the following three questions

Q_1: are units arbitrary?

Q_2: are there units that are more fundamental than others according to S. Weinberg's definition [1]?

Q_3: How many units (fundamental or not) are necessary?

and try to answer them in the context of different theories of elementary particles and interactions.

I hope we agree that the answer to the first question is yes, since only q_i/q_j matter and these ratios do not depend on the choice of units.

I think that the answer to the other two questions depends on the framework we are considering (Cf. Weinberg, ref. [1]). The next three sections therefore analyze Q_2 and Q_3 within three distinct frameworks, and provide, for each case, answers A_2 and A_3, respectively.

3. Fundamental units in QFT + GR

Quantum Field Theory (QFT) (or more specifically the Standard Model (SM)) plus General Relativity (GR) represent the state of the art in HEP before the string revolution of 1984. Weinberg's 1983 paper [1] reflects therefore the attitude about FC's at the dawn of the string revolution. I would summarize it briefly as follows:

A_2: a qualified yes.

At the QFT level of understanding c and $\hbar$ appear to be more fundamental units of speed and action than any other. In newtonian mechanics only the ratios of various velocities in a given problem matter. By contrast, in (special) relativity the ratio of each velocity appearing in the problem to the (universal) speed of light, c, also matters. Likewise, in classical mechanics only the ratios of various terms in the action matter, the overall normalization being irrelevant while, in QM, the ratio of the total action to the (universal) quantum of action $\hbar$ does matter (large ratios, for instance, correspond to a semiclassical situation). It appears therefore that both c and $\hbar$ have a special status as the most basic units of speed and action.

Indeed, let's apply S. Weinberg's criterion [1] and ask: can we compute c and $\hbar$ in terms of more fundamental units? Within QFT the answer appears to be an obvious no. Had we chosen instead some other arbitrary units of speed and action, then, within a given theory, we would be able to compute them, in principle at least, in terms of c and $\hbar$, i.e. in terms of something more fundamental (and of some specified dimensionless constants such as α).

A_3: most probably three

It is quite clear, I think, that in QFT + GR we cannot compute everything that is observable in terms of c, $\hbar$, and of dimensionless constants, without also introducing some mass or length scale. Hence it looks that the answer to the third question is indeed three. Unlike in the case of c and $\hbar$, it is much less obvious, however, which mass or length scale, if any, is more fundamental in the sense of SW. The Planck mass, M_P, does not look like a particularly good choice since it is very hard, even conceptually, to compute, say, m_e or m_p in terms of M_P in the SM + GR framework. This is a bit strange: we seem to need three units, but we can only identify two fundamental ones. So why three? Why not more? Why not less?

Why not more? This is because it looks unnecessary (and even "silly" according to present understanding of physical phenomena) to introduce a separate unit for temperature, for electric current and resistance, etc., or separate units for distances in the x, y and z directions. I refer to Lev for a discussion about how to go from the seven units of the International System of Units (SI) down to three [13], and for how three fundamental units define the so-called "cube" of physical theories [10].

And why not less, say just two? Well because mass or energy appear as concepts that are qualitatively different from, say, distances or time intervals. Let us recall how mass emerges in classical mechanics (CM). We can base CM on the action principle and get $F = ma$ by varying the action

$$S = \int \left(\frac{1}{2} m \underline{\dot{x}}^2 - V(x)\right) dt \Rightarrow m a = F \equiv -\frac{dV}{dx} ; \tag{3.1}$$

but, as it's well known, classically the action can be rescaled by an arbitrary factor. If we had only one species of particles in Nature we could use, instead of S,

$$\tilde{S} = \int \left(\frac{1}{2} \underline{\dot{x}}^2 - \frac{V(x)}{m}\right) dt \equiv \int \left(\frac{1}{2} \underline{\dot{x}}^2 - \tilde{V}(x)\right) dt \Rightarrow a = \tilde{F} \equiv -\frac{1}{m}\frac{dV}{dx} : \tag{3.2}$$

No physical prediction would change by using units in which masses are pure numbers provided we redefine forces accordingly! In this system of units $\hbar$ would be replaced by $\hbar/m$ and would have dimensions of $v^2 \times t$. If we have already decided for c as unit of velocity, $\hbar$ would define therefore a fundamental unit of time (the Compton wavelength of the chosen particle divided by c). However, in the presence of many particles of different mass, we cannot decide which mass to divide the action by, which choice is most fundamental.

I think there is even a deeper reason why QFT + GR needs a separate unit for mass. QFT is affected by UV divergences that need to be renormalized. This forces us to

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introduce a cut-off which, in principle, has nothing to do with c, $\hbar$ or M_P, and has to be "removed" in the end. However, remnants of the cut-off remain in the renormalized theory. In QCD, for instance, the hadronic mass scale (say m_p) originates from a mechanism known as dimensional transmutation, and is arbitrary. Perhaps one day, through string

theory or some other unified theory of all interactions, we will understand how m_p is related to M_P, but in QFT+GR it is not. We do not know therefore which of the two, M_P or m_p, is more fundamental and the same is true for the electron mass m_e, for G_F etc. etc.

The best we can do, in QFT+GR, is to take any one of these mass scales (be it a particle mass or a mass extracted from the strength of a force) as unit of mass and consider the ratio of any other physical mass to the chosen unit as a pure number that, in general, we have no way to compute, even in principle.

4. Fundamental units in old-fashioned quantum string theory (QST)

A_2: yes, c and λ_s!

With string theory the situation changes because it is as if there were a single particle, hence a single mass. Indeed, a single

classical parameter, the string tension T, appears in the Nambu-Goto (NG) action:

$$S = T \int d(\mathrm{Area}) \,; \qquad \frac{S}{\hbar} = \lambda_s^{2} \int d(\mathrm{Area}) \,; \tag{4.1}$$

where the speed of light c has already been implicitly used in order to talk about the area of a surface embedded in space-time. This fact allows us to replace $\hbar$ by a well defined length, λ_s, which turns out to be fundamental both in an intuitive sense and in the sense of S. Weinberg. Indeed, we should be able, in principle, to compute any observable in terms of c and λ_s (see below for an example). Of course, I could instead compute c and λ_s in terms of two other physical quantities defining more down-to-earth units of space and time, but this would not satisfy SW's criterion of having computed c and λ_s in terms of something more fundamental!

A_3: the above two constants are also sufficient!

This was the conclusion of my 1986 paper: string theory only needs two fundamental dimensionful constants c and λ_s, i.e. one fundamental unit of speed and one of length.

The apparent puzzle is clear: where has our loved $\hbar$ disappeared? My answer was then (and still is): it changed its dress! Having adopted new units of energy (energy being replaced by energy divided by tension, i.e. by length), the units of action (hence of $\hbar$) have also changed. And what about my reasoning in QFT+GR? Obviously it does not hold water any more: For one, QFT and GR get unified in string theory. Furthermore, the absence of UV divergences makes it unnecessary to introduce by hand a cutoff.

And indeed the most amazing outcome of this reasoning is that the new Planck constant, λ_s^2, is the UV cutoff. We can express this by saying that, in string theory, first quantization provides the UV cutoff needed in order to make second quantization well defined. Furthermore, in quantum string theory (QST), there are definite hopes to be able to compute both M_P and m_p (in the above string units, i.e. as lengths) in terms of λ_s, c and of a dimensionless parameter, the string coupling (see below).

The situation here reminds me of that of pure quantum gravity. As noticed by Novikov and Zeldovich [19, part V, ch. 23, par. 19], such a theory would only contain two fundamental units, c, and the Planck length $l_P = \sqrt{G_N \hbar c^{-3}}$, but not $\hbar$ and G_N separately. We may view string theory as an extension of GR that allows the introduction of all elementary particles and all fundamental forces in a geometrical way. No wonder then to find that only geometrical units are necessary.

Let us consider for instance, within the string theory framework, the gravitational acceleration a_2 induced by a string of length L_1 on a string of length L_2 sitting at a distance r from it. A simple calculation gives (for $r \gg L_1, L_2$):

$$a_2 = g_s^2 c^2 \frac{L_1}{r^2} \;, \tag{4.2}$$

where g_s is the (dimensionless!) string coupling discussed in the next section. Clearly, the answer does not contain anything else but geometrical quantities and a pure number.

Another more familiar example is the computation of the energy levels of atoms in terms of the electron mass, its charge, and $\hbar$. These are given, to lowest order in α, by

$$E_n = -\frac{1}{2n^2} m_e \left(\frac{e^2}{\hbar}\right)^2 = -\frac{1}{2n^2}(m_e c^2)\,\alpha^2 \tag{4.3}$$

Weinberg argues, convincingly I think, that the quantities E_n are

less fundamental than the electron charge, mass and $\hbar$. However, if we argue that what we are really measuring are not energies by themselves, but the transition frequencies

$$\omega_{mn} = \frac{1}{\hbar}(E_m - E_n) = \frac{1}{2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\frac{\alpha^2 c}{\lambda_s}\,\mu_e \;, \tag{4.4}$$

we see that, once more, only c and λ_s, and some in principle calculable dimensionless ratios (such as the electron mass in string units, $\mu_e = m_e/M_s$), appear in the answer [14]. Obviously, if we follow Weinberg's definition, λ_s and λ_s/c, and not for instance c/ω_{12} and $1/\omega_{12}$ (which are like the "modern" units of length, and time), play the role of fundamental units of length and time.

5. Fundamental units in modern QST/M-theory

We now turn to the same set of questions within the context of first-quantized string theory in the presence of background fields. Here I will attempt to give A_2 and A_3 together.

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The beautiful feature of this formulation is that all possible parameters of string theory, dimensionful and dimensionless alike, are replaced by background fields whose vacuum expectation values (VEV) we hope to be able to determine dynamically. As a prototype, consider the bosonic string in a gravi-dilaton background. The dimensionless action (i.e. the action divided by $\hbar$ in more conventional notation) reads:

$$S = \frac{1}{2} \int \sqrt{\quad} \quad \partial X \quad \partial X \quad G \quad (X) + R(\ ) \ (X) \quad d^2z \tag{5.1}$$

where $X = X(\ ;\ )$, $= 0;1;\ldots;D-1$, are the string coordinates as functions of the world-sheet coordinates $z = (\ ;\ )$, with respect to which the the partial derivatives are defined. Furthermore, G is the so-called string metric and is the so-called dilaton. Finally, and $R(\ )$ are, respectively, the metric and scalar curvature of the two-dimensional Riemann surface having coordinates and . is clearly dimensionless, while the dimensions of the metric components G are such that $G \ X \ X$ is also dimensionless.

The exponential of the expectation value of gives the dimensionless parameter — known as the string coupling g_s — that controls the strength of all interactions (e.g.) and thus also the string-loop expansion. Instead, the expectation value of G converts lengths and time intervals into pure

numbers. Thus, through its non trivial dimension, the metric G actually provides the metre/clock, i.e. the

fundamental units of space and time that we are after.

If the VEV of G is proportional to , the flat minkowskian metric, then it will automatically introduce the constants c and $_s$ of the previous section via:

$$\langle G \quad (X) \rangle = \mathrm{diag} \quad c^2 \ _s{}^2;\ _s{}^2;\ldots \tag{5.2}$$

The mere finiteness of c and $_s$ is clearly of fundamental importance. However, in our context, the real question is: do the actual values of c and $_s$ mean something (in the same way in which the actual value of $\langle\ \rangle$ does)? What is, in other words, the difference between dimensionful and dimensionless constants? The answer is a bit subtle. String theory should allow to compute in terms of the VEV of . Similarly, it should allow to compute $(\ X)^2 \quad G \quad X \quad X$ for some physical length X (say for the Hydrogen atom). Calling that pure number so many centimetres would fix the string length parameter in cm but, of course, this would be just a convention: the truly convention-independent (physical) quantity is just $(\ X)^2$. Both $\langle\ \rangle$ and $(\ X)^2$ are pure numbers whose possible values distinguish one theory (or one vacuum) from another.

The difference between the two kinds of constants, if any, simply stems from the fact that, while different values of $\langle\ \rangle$ (or) define genuinely different theories, values of $\langle G \quad \rangle$ that are related by a General Coordinate Transformation (GCT) can be compensated by a GCT on X and thus define the same theory as long as $(\ X)^2$ remains the same. In particular, if $\langle G \quad \rangle$ as in the example discussed above, the actual proportionality constants c and $_s$ appearing in (5.2) can be reabsorbed by a GCT. This is why it does not make sense to talk about the absolute values of c and $_s$ or to compare them to those

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of an alien: only the dimensionless numbers $(\Delta X/\lambda_s)^2$, i.e. the values of some physical length or speed in those units are physically relevant and can be compared (see section 7).

The situation would be very different if $\langle G_{\mu\nu}\rangle$ would not be reducible to $\eta_{\mu\nu}$ via a GCT. That would mean a really different world, like one with a different value of α. In ref. [20] I gave the example of $\langle G_{\mu\nu}\rangle$ proportional to the de-Sitter metric, stressing the fact that, in such a vacuum, even λ_s disappears in favour of a dimensionless parameter similar to $\langle\phi\rangle$. Thus, as stressed in [15, 16], my early statement in [14] about having just two constants should be considered valid if the vacuum of QST is minkowskian, in particular in the NG formulation of QST.

To summarize, QM provides, through the string metric $G_{\mu\nu}$, a truly fundamental metre/clock allowing us to reduce space-time distances to pure numbers whose absolute value is physically meaningful. Note, incidentally, that in Classical GR only $g_{\mu\nu}\Delta X^\mu \Delta X^\nu$ is an invariant. However, in the classical case (and even for classical strings), only ratios of quantities of this type matter while in QST, $(\Delta X/\lambda_s)^2$ is, for each single ΔX, a meaningful pure number.

In conclusion, I still stand by my remark in [15] that the fundamental constants of Nature are, in QST, the constants of the vacuum. How many (physically distinct) choices of its VEV's does QST allow? We now believe that all known consistent string theories correspond to perturbations around different vacua of a single, yet unknown, "M-theory". We still do not know, however, how many physically inequivalent non-perturbative vacua M-theory has. Until then, I do not think we can really answer the question of fundamental units in QST, but I would be very surprised if, in any consistent string vacuum, we would find that we need more than one unit of length and one of time.

6. The disagreement with Lev

Lev cannot accept (part I) that $\hbar$ has disappeared from the list. He claims that, without $\hbar$, there is no unit of momentum, of energy, and, especially, of angular momentum. But, as I said in the previous two sections, $\hbar$ has not really disappeared: it has actually been promoted, in string theory, to a grander role, that of providing also, through QM, an UV cutoff that hopefully removes both the infinities of QFT and ordinary Quantum Gravity and the ubiquitous singularities of Classical GR.

I would concede, however, that, given the fact that momentum and energy are logically distinct from lengths and times for ordinary objects, insisting on the use of the same (or of reciprocal) units for both sets can be pedagogically confusing. Therefore I do agree that the set c, $\hbar$, and λ_s define at present, within QST, the most practical (though not the most economical) set of fundamental units.

To rephrase myself: within the NG action there seems to be no reason to introduce a tension T or $\hbar$. The action is naturally the area and the Planck constant is the unit of area needed to convert the action into a number. However, by the standard definition of canonically conjugate variables, this would lead to identical dimensions for momenta and lengths (or for times and energies). For strings that's fine, since we can identify the energy of a string with its length, but when it comes to ordinary objects, i.e. to complicated bound states of fundamental strings or branes, it looks less confusing to give momentum a unit other than length. In order to do that we introduce, somewhat artificially, a conversion factor, the string tension T, so that energies are now measured in ergs, in GeV, or whatever we wish, different choices being related by irrelevant redefinitions of T.

7. The disagreement with Mike

Two issues appear to separate Mike's position from my own:

- The alien story

  Mike quotes an example, due to Feynman, on how we could possibly tell an alien to distinguish left from right. Then he asks: can we similarly communicate to an alien our values for c and α_s and check whether they agree with ours? I claim the answer to be: yes, we can, and, to the same extent that the alien will be able to tell us whether her[3] $\hbar$ agrees with ours, she will also be able to tell us whether her c and α_s agree with ours.

  In order to do that, we "simply" have to give the alien our definitions of cm. and s. in terms of a physical system she can possibly identify (say the H atom) and ask: which are your values of c and α_s in these units? If the alien cannot even identify the system then she lives in a different world/string-vacuum; if she does, then she should come up with the same numbers (e.g. $c = 3 \times 10^{10}$ cm/s) or else, again, her world is not like ours. It thus looks to me that the alien story supports the idea that we do have, in our own world, some fundamental units of length and time. Mike seems to agree with me on the alien's reply, but then concludes that c is not a fundamental unit because a completely rescaled world, in which both c and the velocity of the electron in the H atom are twice as large, is indistinguishable from ours. I conclude, instead, that c is a fundamental unit because the velocity of our electron in units of c is a relevant number to be compared with the alien's.

  Incidentally, the same argument can be applied either to some ancestors (or descendants) of ours, or to inequivalent string vacua. A value of c in cm/s for any of those which differs from ours would really mean different worlds, e.g. worlds with different ratios of the velocity of the electron in the Hydrogen atom and the speed of light. We may either express this by saying that, in the two different worlds, c is different in atomic units, or by saying that c is the same but atomic properties differ. No experimental result will be able to distinguish about these two physically equivalent statements since a rescaling of all velocities is inessential.

- Reducing fundamental units to conversion factors

  Mike's second point is that these units can be used as conversion factors, like k_B, in order to convert any quantity into any other and, eventually, everything into pure

[3] To stress that my alien's reaction is different from that of Mike's alien I have also changed the alien's gender.

numbers. However, I do insist that the point is not to convert degrees Kelvin into MeV, centimetres into seconds, or everything into numbers. The important point is that there are units that are arbitrary and units that are fundamental in the sense that, when a quantity becomes O(1) in the

latter units, dramatic new phenomena occur. It makes a huge difference, for instance, having or not having a fundamental length. Without a fundamental length, properties of physical systems would be invariant under an overall rescaling of their size, atoms would not have a characteristic size, and we would be unable to tell the alien which atom to use as a metre. By contrast, with a fundamental quantum unit of length, we can meaningfully talk about short or large distances (as compared to the fundamental length, of course).

Going back to the discussion at the end of section 5, the pure number $(X)^2$ has a meaning in itself. In the absence of any fundamental units of length and time I would be able to rescale this number arbitrarily (e.g. by rescaling G) without changing physics. Only ratios of two lengths in the problem, like $(X_1)^2=(X_2)^2$ would matter. Because of QM, however, there is a fundamental rod (and clock) that gives, out of any single physical length or time interval, a relevant pure number.

On this particular point, therefore, I tend to agree with Lev. There is, in relativity, a fundamental unit of speed (its maximal value); there is, in QM, a fundamental unit of action (a minimal uncertainty); there is, in string theory, a fundamental unit of length (the characteristic size of strings). QST appears to provide the missing third fundamental unit of the three-constants system. These three units form a very convenient system except that, classically, the units of action are completely arbitrary (and the same is true therefore of mass, energy etc.), while, quantum mechanically, only S=~ matters. In string theory this allows us to identify the Planck constant with the string length eliminating the necessity, but perhaps not the convenience, of a third unit besides those needed to measure lengths and time intervals.

I also agree with Mike that all that matters are pure numbers. As I stressed in section 2, it is easy to convert any quantity into a pure number by choosing arbitrarily some unit. I only add to this the observation that relativity and quantum mechanics provide, in string theory, units of length and time which look, at present, more fundamental than any other. The number of distinct physical quantities (and of corresponding units) is a matter of choice and convenience, and also depends on our understanding of the underlying physical laws. Within QFT + GR it looks most useful to reduce this number to three, but there is no obvious candidate for the third unit after c and ~. With QST, the third unit naturally emerges as being the string length $_s$. However there appears an interesting option to do away with ~. Going further down, say from two to one or to zero, means considering space as being equivalent to time or as both being equivalent to pure numbers, while, keeping the two units c and $_s$, allows to express space and time intervals in terms of pure numbers.

This is what distinguishes, in my opinion, fundamental units from conversion factors. While I see no reason to distinguish the units of temperature from those of energy, and thus to introduce Boltzmann's constant, I see every reason to distinguish space from time and to introduce c as a fundamental unit of speed and not as a trivial conversion factor. Another clear difference is that, while the ratio $E(T)/T$ is always the same, we do observe, in Nature, a variety of speeds (all less than c, so far), of lengths, and of frequencies.

8. Time variation of fundamental units?

I think that the above discussion clearly indicates that the "time variation of a fundamental unit", like c, has no meaning, unless we specify what else, having the same units, is kept fixed. Only the time variation of dimensionless constants, such as α or $(\lambda_s/\lambda_X)^2$ for an atom have an intrinsic physical meaning.

We do believe, for instance, that in a cosmological background the variation in time of G_N is accompanied by a corresponding variation of the λ_X of an atom so that $(\lambda_s/\lambda_X)^2$ remains constant. The same is usually assumed to be true for α. However, this is not at all an absolute theoretical necessity (e.g. α can depend on time, in QST, if ϕ does), and should be (and indeed is being) tested. For instance, the same $(\lambda_s/\lambda_X)^2$ is believed to grow with the expansion of the Universe if λ_X represents the wavelength of light coming to us from a distant galaxy. The observed red shift only checks the relative time-dependence of $(\lambda_s/\lambda_X)^2$ for an atom and for the light coming from the galaxy.

However, I claim that, in principle, the time variation of $(\lambda_s/\lambda_X)^2$ has a physical meaning for each one of the two systems separately because it represents the time variation of some physical length w.r.t. the fundamental unit provided by string theory. For instance, in the early Universe, this quantity for the CMBR photons was much smaller than it is today ($O(10^{30})$). If it ever approached values $O(1)$, this may have left an imprint of short-distance physics on the CMBR spectrum.

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Part III

A party political broadcast on behalf of the Zero Constants Party

Michael J. Duff

Abstract. According to the manifesto of Okun's Three Constants Party, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$, the velocity of light, c, and Newton's constant, G. According to Veneziano's Two Constants Party, there are only two: the string length λ_2 and c. Here we present the platform of the Zero Constants Party.

1. The false propaganda of the Three Constants Party

As a young student of physics in high school, I was taught that there were three basic quantities in Nature: Length, Mass and Time [21]. All other quantities, such as electric charge or temperature, occupied a lesser status since they could all be re-expressed in terms of these basic three. As a result, there were three basic units: centimetres, grams and seconds, reflected in the three-letter name "CGS" system (or perhaps metres, kilograms and seconds in the alternative, but still three-letter, "MKS" system).

Later, as an undergraduate student, I learned quantum mechanics, special relativity and newtonian gravity. In quantum mechanics, there was a minimum quantum of action given by Planck's constant $\hbar$; in special relativity there was a maximum velocity given by the velocity of light c; in classical gravity the strength of the force between two objects was determined by Newton's constant of gravitation G. In terms of length, mass, and time their dimensions are

$$\begin{aligned}
[c] &= LT^{-1} \\
[\hbar] &= L^2MT^{-1} \\
[G] &= L^3M^{-1}T^{-2}\,.
\end{aligned} \tag{1.1}$$

Once again, the number three seemed important and other dimensionful constants, such as the charge of the electron e or Boltzmann's constant k, were somehow accorded a less fundamental role. This fitted in perfectly with my high school prejudices and it seemed entirely natural, therefore, to be told that these three dimensionful constants determined three basic units, first identified a century ago by Max Planck, namely the Planck length L_P, the Planck mass M_P and the Planck time T_P:

$$\begin{aligned}
L_P &= \sqrt{G\hbar/c^3} = 1.616 \times 10^{-35}\,\mathrm{m} \\
M_P &= \sqrt{\hbar c/G} = 2.177 \times 10^{-8}\,\mathrm{kg} \\
T_P &= \sqrt{G\hbar/c^5} = 5.390 \times 10^{-44}\,\mathrm{s}
\end{aligned} \tag{1.2}$$

Yet later, researching into quantum gravity which attempts to combine quantum mechanics, relativity and gravitation into a coherent unified framework, I learned about the Bronshtein-Zelmanov-Okun (BZO) cube [10], with axes $\hbar$, c^{-1} and G, which neatly summarizes how classical mechanics, non-relativistic quantum mechanics, newtonian gravity and relativistic quantum field theory can be regarded respectively as the $(\hbar; c^{-1}; G) \to 0$, $(c^{-1}; G) \to 0$, $(\hbar; c^{-1}) \to 0$, and $(G) \to 0$ limits of the full quantum gravity. Note, once again that we are dealing with a three-dimensional cube rather than a square or some figure of a different dimension.

What about Kaluza-Klein theories which allow for $D > 4$ spacetime dimensions? Unlike $\hbar$ and c, the dimensions of G depend on D:

$$[G_D] = M^{-1} L^{D-1} T^{-2} \tag{1.3}$$

and hence (dropping the P subscript), the D-dimensional Planck length L_D, mass M_D and time T_D are given by

$$\begin{aligned} L_D{}^{D-2} &= G_D \hbar c^{-3} \\ M_D{}^{D-2} &= G_D{}^{-1} \hbar^{D-3} c^{5-D} \\ T_D{}^{D-2} &= G_D \hbar c^{-1-D} . \end{aligned} \tag{1.4}$$

After compactification to four dimensions, $G \equiv G_4$ then appears as

$$\frac{1}{G_4} = \frac{1}{G_D} V ; \tag{1.5}$$

where V is the volume of the compactifying manifold. Since V has the four-dimensional interpretation as the vacuum expectation value of scalar modulus fields coming from the internal components of the metric tensor, it depends on the choice of vacuum but does not introduce any more fundamental constants into the lagrangian.

Adherents of this conventional view of the fundamental constants of Nature have been dubbed the "Three Constants Party" by Gabriele Veneziano [16]. Lev Okun is their leader. Until recently I was myself, I must confess, a card-carrying member.[4]

2. The false propaganda of the Two Constants Party

My faith in the dogma was shaken, however, by papers by Gabriele [14, 15, 16], self-styled leader of the rebel Two Constants Party. As a string theorist, Gabriele begins with the two-dimensional Nambu-Goto action of a string. He notes that, apart from the velocity of light still needed to convert the time coordinate t to a length coordinate $x^0 = ct$, the action divided by $\hbar$ requires only one dimensionful parameter, the string length λ_2 (denoted λ_s by Gabriele).

$$\lambda_2{}^2 = \frac{\hbar}{c T_2} ; \tag{2.1}$$

[4] It seems that the choice of length, mass and time as the three basic units is due to Gauss [27], so we could declare him to be the founder of the Three Constants Party, although this was long before the significance of c and $\hbar$ was appreciated.

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where $T_2 = 1/2\pi c\alpha'$ is the tension of the string and α' is the Regge slope. This is because the Nambu-Goto action takes the form

$$\frac{S_2}{\hbar} = \frac{1}{\lambda_2{}^2}\mathrm{Area} \tag{2.2}$$

So if this were to describe the theory of everything (TOE), then the TOE would require only two fundamental dimensionful constants c and λ_2. In superstring theory, the ten-dimensional Planck length is given in terms of the string length λ_2 and the vacuum expectation value of the dilaton field ϕ

$$L_{10}{}^2 = \lambda_2{}^2\langle e^{\phi}\rangle \tag{2.3}$$

Once again, the vev of ϕ will be different in different vacua but does not introduce any new constants into the lagrangian.

A similar argument for reducing the three constants h, c, G to just two was made previously by Zeldovich and Novikov [19] with regard to quantum gravity. The Einstein-Hilbert action divided by $\hbar$ involves G and $\hbar$ only in the combination $G\hbar$ appearing in the square of the Planck length, and so we need only L_P and c. Of course quantum gravity does not pretend to be the TOE and so this argument still leaves open the number of dimensionful constants required for a TOE.

In the light of the 1995 M-theory [22] revolution, we might wish to update Gabriele's argument by starting with the corresponding three-dimensional action for the M2-brane,

$$\frac{S_3}{\hbar} = \frac{1}{\lambda_3{}^3}(\text{3d-volume}); \tag{2.4}$$

where the corresponding parameter is the membrane length λ_3.

$$\lambda_3{}^3 = \hbar/cT_3 \tag{2.5}$$

and where T_3 is the membrane tension. Alternatively, we could start with the six-dimensional action of the dual M5-brane,

$$\frac{S_6}{\hbar} = \frac{1}{\lambda_6{}^6}(\text{6d-volume}) \tag{2.6}$$

where the corresponding parameter is the fivebrane length λ_6

$$\lambda_6{}^6 = \frac{\hbar}{cT_6} \tag{2.7}$$

and where T_6 is the fivebrane tension. Eleven-dimensional M-theory is, in fact, simpler than ten-dimensional superstring theory in this respect, since there is no dilaton. Consequently, the three lengths: membrane length, fivebrane length and eleven-dimensional Planck length are all equal [23] up to calculable numerical factors $\lambda_3 \sim \lambda_6 \sim L_{11}$. So the fundamental length in M-theory is λ_3 rather than λ_2 and will be shorter for string coupling less than unity [26].

However, even if we substitute λ_3 for λ_2, Gabriele would say that we are still left with the number two. This also reduces the number of basic units to just two: length and time.

Gabriele's claim led to many heated discussions in the CERN cafeteria between Lev, Gabriele and myself. We went round and round in circles. Back at Texas A&M, I continued these arguments at lunchtime conversations with Chris Pope and others. There at the College Station Hilton, we eventually reached a consensus and joined what Gabriele would call the Zero Constants Party [16].

Our attitude was basically that $\hbar$, c and G are nothing but conversion factors e.g. mass to length, in the formula for the Schwarzschild radius R_S

$$R_S = \frac{2Gm}{c^2};$$

or energy to frequency

$$E = \hbar\omega$$

energy to mass

$$E = mc^2$$

no different from Boltzmann's constant, say, which relates energy to temperature

$$E = kT.$$

As such, you may have as many so-called "fundamental" constants as you like; the more different units you employ, the more different constants you need.[5] Indeed, no less an authority than the Conference Generale des Poids et Mesures, the international body that administers the SI system of units, adheres to what might be called the Seven Constants Party, decreeing that seven units are "basic": metre (length), kilogram (mass), second (time), ampere (electric current), kelvin (thermodynamic temperature), mole (amount of substance), candela (luminous intensity), while the rest are "derived" [27, 28]. The attitude of the Zero Constants Party is that the most economical choice is to use natural units where there are no conversion factors at all. Consequently, none of these units or conversion factors is fundamental.

Incidentally, Lev (part I) objects in his section 5 that equations such as $c = 1$ cannot be taken literally because c has dimensions. In my view, this apparent

contradiction arises from trying to use two different sets of units at the same time, and really goes to the heart of my disagreement with Lev about what is real physics and what is mere convention. In the units favored by members of the Three Constants Party, length and time have different dimensions and you cannot, therefore, put $c = 1$ (just as you cannot put $k = 1$, if you want to follow the conventions of the Seven Constants Party). If you want to put $c = 1$, you must trade in your membership card for that of (or at least adopt the habits of) the Two Constants Party, whose favorite units do not distinguish length from time.[6] In these units, c is dimensionless and you may quite literally set it equal to one. In the natural units favored by the Zero Constants Party, there are no dimensions at all

[5] In this respect, I take the the number of dimensionful fundamental constants to be synonymous with the number of fundamental (or basic) units.

[6] This $(\hbar, G)$ wing of the Two Constants Party is different from Gabriele's (c, λ_2) wing, which prefers not to introduce a separate unit for mass.

and $\hbar = c = G =$ [illegible] $= 1$ may be imposed literally and without contradiction. With this understanding, I will still refer to constants which have dimensions in some units, such as $\hbar, c, G, k \ldots$, as "dimensionful constants" so as to distinguish them from constants such as α, which are dimensionless in any units.

3. Three fundamental theories?

Lev and Gabriele remain unshaken in their beliefs, however. Lev (part I) makes the, at first sight reasonable, point (echoed by Gabriele in part II) that $\hbar$ is more than just a conversion factor. It embodies a fundamental physical principle of quantum mechanics that there is a minimum non-zero angular momentum. Similarly, c embodies a fundamental physical principle of special relativity that there is a maximum velocity c. If I could paraphrase Lev's point of view it might be to say that there are three "fundamental" units because there are three fundamental physical theories: quantum mechanics, special relativity and gravity. According to this point of view, temperature, for example, should not be included as a basic unit (or, equivalently, Boltzmann's constant should not be included as a fundamental constant.)

However, I think this elevation of $\hbar$, c and G to a special status is misleading. For example, the appearance of c in $x^0 = ct$ is for the benefit of people for whom treating time as a fourth dimension is unfamiliar. But once you have accepted $O(3,1)$ as a symmetry the conversion factor becomes irrelevant. We have become so used to accepting $O(3)$ as a symmetry that we would not dream of using different units for the three space coordinates,[7] but to be perverse we could do so.

To drive this point home, and inspired by the Conference Generale des Poids et Mesures, let us introduce three new superfluous units: xylophones, yachts and zebras to measure intervals along the x, y and z axes. This requires the introduction of three superfluous "fundamental" constants, c_x, c_y and c_z with dimensions length/xylophone, length/yacht and length/zebra, respectively, so that the line element becomes:

$$ds^2 = -c^2 dt^2 + {c_x}^2 dx^2 + {c_y}^2 dy^2 + {c_z}^2 dz^2 . \qquad (3.1)$$

Lev's point is that the finiteness of c ensures that we have $O(3,1)$ symmetry rather than merely $O(3)$. This is certainly true. But it is equally true that the finiteness of c_x, say, ensures that we have $O(3,1)$ rather than merely $O(2,1)$. In this respect, the conversion factors c and c_x are on an equal footing.[8] Both are, in Gabriele's terminology (part II), equally "silly". Both can be set equal to unity and forgotten about.

Similarly, the "fundamental" lengths ℓ_d appearing in brane actions (2.2), (2.4) and (2.6) can be removed from the equations by defining new dimensionless worldvolume coordinates, σ', related to the old ones, σ, by $\sigma = \ell_d \sigma'$.

[7] I am grateful to Chris Pope for this example.

[8] To put this more rigorously, the Poincare group admits a Wigner–Inonu contraction to the Galileo group, obtained by taking the $c \to \infty$ limit. However, this is by no means unique. There are other contractions to other subgroups. For example, one is obtained by taking the $c_x \to \infty$ limit. Although of less historical importance, these other subgroups are mathematically on the same footing as the Galileo group. So, in my opinion, the singling out of c for special treatment has more to do with psychology than physics.

So I would agree with Lev that the finiteness of the conversion factors is important (minimum angular momentum, maximum velocity) but, in my view, no significance should be attached to their value and you can have as many or as few of them as you like.

The reason why we have so many different units, and hence conversion factors, in the first place is that, historically, physicists used different kinds of measuring apparatus: rods, scales, clocks, thermometres, electroscopes etc. Another way to ask what is the mimimum number of basic units, therefore, is to ask what is, in principle, the minimum number of basic pieces of apparatus.[9] Probably Lev, Gabriele and I would agree that $E = kT$ means that we can dispense with thermometers, that temperature is not a basic unit and that Boltzmann's constant is not fundamental. Let us agree with Lev that we can whittle things down to length, mass and time or rods, scales and clocks. Can we go further? Another way to argue that the conversion factor c should not be treated as fundamental, for example, is to point out that once the finiteness of c has been accepted, we do not need both clocks and rulers. Clocks alone are sufficient since distances can be measured by the time it takes light to travel that distance, $x = ct$. We are, in effect, doing just that when we measure interstellar distances in light-years. Conversely, we may do away with clocks in favor of rulers. It is thus superfluous to have both length and time as basic units. Similarly, we can do away with rulers as basic apparatus and length as a basic unit by trading distances with masses using the formula for the Compton wavelength $R_C = h/mc$. Indeed, particle theorists typically express length, mass and time units as inverse mass, mass and inverse mass, respectively. Finally, we can do away with scales by expressing particle masses as dimensionless numbers, namely the ratio of a particle mass to that of a black hole whose Compton wavelength equals its Schwarzschild radius. So in this sense, the black hole acts as our rod, scale, clock, thermometer etc. all at the same time. In practice, the net result is as though we set $\hbar = c = G = \qquad = 1$ but we need not use that language.

J-M. Levy-LeBlond [29] puts it like this: "This, then, is the ordinary fate of universal constants: to see their nature as concept synthesizers be progressively incorporated into the implicit common background of physical ideas, then to play a role of mere unit conversion factors and often to be finally forgotten altogether by a suitable redefinition of physical units."

4. An operational definition

"If, however, we imagine other worlds, with the same physical laws as those of our own world, but with different numerical values for the physical constants determining the limits of applicability of the old concepts, the new and correct concepts of space, time and motion, at which modern science arrives only after very long and elaborate investigations, would become a matter of common knowledge."

George Gamow, Mr. Tompkins in paperback [18]

It seems to me that this issue of what is fundamental will continue to go round and around until we can all agree on an operational definition of "fundamental constants".

[9] I am grateful to Chris Isham for this suggestion.

Weinberg [1] defines constants to be fundamental if we cannot calculate their values in terms of more fundamental constants, not just because the calculation is too hard, but because we do not know of anything more fundamental. This definition is fine, but does not resolve the dispute between Gabriele, Lev and me. It is the purpose of this section to propose one that does. I will conclude that, according to this definition, the dimensionless parameters, such as the fine structure constant, are fundamental, whereas all dimensionful constants, including $\hbar$, c and G, are not.[10]

In physics, we frequently encounter ambiguities such as "left or right" and "matter or antimatter". Let us begin by recalling Feynman's way of discriminating between what are genuine differences and what are mere conventions. Feynman imagines that we can communicate with some alien being [30]. If it were not for the violation of parity in the weak interactions we would have no way of deciding whether what he[11] calls right and left are the same as what we call right and left. However, we can ask him to perform a cobalt 60 experiment and tell him that the spinning electrons determine a left handed thread. In this way we can agree on what is left and right. When we eventually meet the alien, of course, we should beware shaking hands with him if he holds out his left hand (or tentacle). He would be made of antimatter and annihilate with us! Fortunately, after the discovery of CP violation we could also eliminate this ambiguity.

In a similar vein, let us ask whether there are any experiments that can be performed which would tell us whether the alien's universe has the same or different constants of nature as ours. If the answer is yes, we shall define these constants to be fundamental, otherwise not. In particular, and inspired by Gamow's Mr. Tompkins [18], we will ask whether there is in principle any experimental difference that would allow us to conclude unambiguously that his velocity of light, his Planck's constant or his Newton's constant are different from ours. By "unambiguously" I mean that no perceived difference could be explained away by a difference in conventions. (Of course, even Feynman's criterion is not devoid of theoretical assumptions. We have to assume that the cobalt behaves the same way for the alien as for us etc. To be concrete, we might imagine that we are both described by a TOE (perhaps M-theory) in which the fundamental constants are given by vacuum expectation values of scalar fields. The alien and we thus share the same lagrangian but live in possibly different vacua. Let us further assume that both vacua respect $O(3,1)$ symmetry.)

5. The operationally indistinguishable world of Mr. Tompkins

The idea of imagining a universe with different constants is not new, but, in my opinion, the early literature is very confusing. For example, Vol'berg [17] and Gamow [18] imagine a universe in which the velocity of light is different from ours, say by ten orders of magnitude, and describe all sorts of weird effects that would result:

[10] My apologies to those readers to whom this was already blindingly obvious. A similar point of view may be found in [32]. On the other hand, I once read a letter in Physics World from a respectable physicist who believed that a legitimate ambition of a TOE would be to calculate the numerical value of $\hbar$.

[11] I will follow Feynman and assume that the alien is a "he", without resolving the "he or she" ambiguity.

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"The initials of Mr. Tompkins originated from three fundamental physical constants: the velocity of light c; the gravitational constant G; and the quantum constant h, which have to be changed by immensely large factors in order to make their effect easily noticeable by the man on the street."

George Gamow, Mr. Tompkins in paperback [18]

In this one sentence, Gamow manages to encapsulate everything I am objecting to! First, he takes it as axiomatic that there are three

fundamental constants. Second, he assumes a change in these constants can be operationally defined. I for one am mystified by such

comparisons. After all, an inhabitant of such a universe (let us identify him with Feynman's alien) is perfectly free to choose units in which $c = 1$, just as we are. To use the equation

$$k = \frac{E}{c}$$

to argue that in his universe, for the same energy E, the photon emitted by an atom would have a momentum k that is ten orders of magnitude smaller than ours is, to my mind, meaningless. There is no experimental information that we and the alien could exchange that would allow us to draw any conclusion.

By contrast, in his critique of Vol'berg and Gamow, Lev [10] imagines a universe in which the binding energy of an electron in a hydrogen atom $E = me^4/\hbar^2$ exceeds twice the electron rest energy $2mc^2$, where m and e are the electron mass and charge respectively. In such a universe it would be energetically favorable for the decay of the proton to a hydrogen atom and a positron $p \to H + e^+$. This universe is demonstrably different from ours. But, in my opinion, the correct conclusion has nothing to do with the speed of light, but simply that in this universe the dimensionless fine structure constant $\alpha = e^2/\hbar c$ exceeds $\sqrt{2}$.

I believe that these two examples illustrate a general truth: no experimental information that we and the alien could exchange can unambiguously determine a difference in dimensionful quantities. No matter whether they are the $\hbar$, c and G sacred to the Three Constants Party, the λ_2 and c of the Two Constants Party or the seven constants of the Conference Generale des Poids et Mesures. Any perceived difference are all merely differences in convention rather than substance. By contrast, differences in dimensionless parameters like the fine structure constants are physically significant and meaningful.[12] Of course, our current knowledge of the TOE is insufficient to tell us how many such dimensionless constants Nature requires. There are 19 in the Standard model, but the aim of M-theory is to reduce this number. Whether they are all calculable or whether some are the result of cosmological accidents (like the ratios of distances of planets to the sun) remains one of the top unanswered questions in fundamental physics.[13]

[12] In his section 7, Gabriele (part II) claims to disagree with me on this point, but I think the first two sentences of his section 8 indicate that we are actually in agreement. If, for example, the alien tells us that he observes the decay $p \to H + e^+$, then we can be sure that his α is different from ours. Choosing to attribute this effect (or any other effect) to a difference in c rather than $\hbar$ or e, however, is entirely a matter of convention, just as the difference between left and right would be a matter of convention in a world with no CP violation. So c fails the Feynman test.

[13] Indeed, participants of the Strings 2000 conference placed it in the top ten [24].

6. What about theories with time-varying constants?

Suppose that our "alien" came not from a different universe but from a different epoch in our own universe and we stumbled across his historical records. In this way of thinking, the issue of whether $\hbar$, c and G are fundamental devolves upon the issue of whether the results of any experiments could require the unambiguous conclusion that $\hbar$, c and G are changing in time. According to our criterion above, any such time-dependence would be merely convention, without physical significance.

On the other hand, many notable physicists, starting with Dirac [25], have nevertheless entertained the notion that G or c are changing in time. (For some reason, time-varying $\hbar$ is not as popular.) Indeed, papers on time-varying c are currently in vogue as as an alternative to inflation. I believe that these ideas, while not necessarily wrong, are frequently presented in a misleading way and that the time-variation in the physical laws is best described in terms of time-varying dimensionless ratios, rather than dimensionful constants.[14] So, in my opinion, one should talk about time variations in the dimensionless parameters of the standard model but not about time variations in $\hbar$, c and G. For example, any observed change in the strength of the gravitational force over cosmological times should be attributed to changing mass ratios rather than changing G. For example, the proton is approximately 10^{19} times lighter than the black hole discussed in section 3, whose Compton wavelength equals its Schwarzschild radius. It is then sensible to ask whether this dimensionless ratio could change over time.[15]

Unfortunately, this point was made insufficiently clear in the recent paper presenting astrophysical data suggesting a time-varying fine structure constant [34]. As a result, a front page article in the New York Times [35] announced that the speed of light might be changing over cosmic history.[16]

In the context of M-theory which starts out with no parameters at all, these standard model parameters would appear as vacuum expectation values of scalar fields.[17] Indeed, replacing parameters by scalar fields is the only sensible way I know to implement time varying constants of Nature. The role of scalar fields in determining the fundamental constants in a TOE was also emphasized by Gabriele [14, 15, 16].

7. Conclusions

The number and values of fundamental dimensionless constants appearing in a Theory of Everything is a legitimate subject of physical enquiry. By contrast, the number and values of dimensionful constants, such as

[14] This point of view is also taken in [33].

[15] One could then sensibly discuss a change in the number of protons required before a star reaches its Chandrasekar limit for gravitational collapse. I am grateful to Fred Adams for this example.

[16] I am reminded of the old lady who, when questioned by the TV interviewer on whether she believed in global warming, responded: "If you ask me, it's all this changing from Fahrenheit to Centigrade that causing it!".

[17] The only other possibility compatible with maximal four-dimensional spacetime symmetry is the vacuum expectation value of a 4-index field strength. For example, the cosmological constant can receive a contribution from the vev of the M-theory 4-form [31].

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$\hbar, c, G, \ldots$ is a quite arbitrary human construct, differing from one choice of units to the next. There is nothing magic about the choice of two, three or seven. The most economical choice is zero. Consequently, none of these dimensionful constants is fundamental.

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Note added. Warren Siegel (private communication) makes the following interesting points:

1. Planck was actually a member of the Four Constants Party, since his original paper introduced not only a basic length, mass and time but also a temperature.[18]

2. In 1983, the Conference Generale des Poids et Mesures declared c to have the value 299,792,458 metres/second exactly, by definition, thus emphasizing its role as a nothing but a conversion factor.[19]

3. Sailors use the perverse units of section 3, when they measure intervals along the x and y axes in nautical miles and intervals along the z axis in fathoms. The same observation was made independently by Steve Weinberg (private communication).

[18] By analyzing Planck's papers [3] Lev came to the conclusion that by adding k to c, $\hbar$ and G, Planck contradicts his definition of natural units [36].

[19] So asking whether the value of c has changed over cosmic history is like asking whether the number of litres to the gallon has changed.

References

[1] S. Weinberg, Overview of theoretical prospects for understanding the values of fundamental constants, in The constants of physics, W.H. McCrea and M.J. Rees eds., Phil. Trans. R. Soc. London A 310 (1983) 249.

[2] G.J. Stoney, The philosophical magazine and journal of science, 11 (1881) 381.

[3] M. Planck, Uber irrevesible Strahlungsvorgange, S.-B. Preuss Akad. Wiss. (1899) 440–480; Ann. d. Phys. 1 (1900) 69 reprinted in Max Planck, Physikalische Abhandlungen und Vortrage, Band I. Friedr. Vieweg. 1958, pp. 560–600, pp. 614–667.

[4] G. Gamov, D. Ivanenko and L. Landau, Zh. Russ. Fiz. Khim. Obstva. Chast' Fiz. 60 (1928) 13 (in Russian).

[5] M. Bronshtein, K voprosu o vozmozhnoy teorii mira kak tselogo, (On a possible theory of the world as a whole), in Osnovnye problemy kosmicheskoy fiziki (Basic problems of cosmic physics). Kiev, ONTI (1934), pp. 186–218, in particular p. 210 (in Russian).

[6] M. Bronshtein, Physikalische Zeitscrift der Sowjetunion 9 (1936) 140.

[7] A. Zelmanov, Kosmologia (Cosmology), in Razvitie astronomii v SSSR (Development of astronomy in USSR), Nauka, Moscow, 1967, pp. 320–390, in particular p. 323 (in Russian).

[8] A. Zelmanov, Mnogoobrazie materialnogo mira i problema beskonechnosti Vselennoi (Diversity of the material world and the problem of infinity of the Universe), in Beskonechnost' i Vselennaya (Infinity and Universe), Mysl', Moscow, 1969, pp. 274–324, in particular p. 302 (in Russsian).

[9] G. Gorelik, Razmernost' prostranstva (Dimension of space), MGU, Moscow, 1983, chapter 5 (in Russian).

[10] L.B. Okun, The fundamental constants of physics, Sov. Phys. Usp. 34 (1991) 818.

[11] Symbols, units and nomenclature in physics: document U.I.P. 20 (1978), International Union of Pure and Applied Physics, S.U.N. Commission.

[12] Review of Particle Properties, Phys. Rev. D 45 (1992) Part II; page III.4.

[13] L.B. Okun, Particle physics: the quest for the substance of substance, Harwood academic publishers, Chur, New York 1985, Appendix 1. On systems of physical units.

[14] G. Veneziano, A stringy nature needs just two constants, Europhys. Lett. 2 (1986) 199.

[15] G. Veneziano, Quantum strings and the constants of nature, in The challenging questions, proceedings of the 27th Course of International School held July 26 – August 3, 1989, Erice, Italy, A. Zichichi ed., Plenum Press, New York 1990, p. 199.

[16] G. Veneziano, Fundamental constants in field and string theory, CERN-TH.6725/92, talk given at the 6th session of the International Workshop of Theoretical Physics, in String quantum gravity and physics at the Planck scale, Erice, Sicily, 21–28 June 1992, N. Sanchez and A. Zichichi eds., World Scientific 1993, p. 552.

[17] O.A. Volberg, Zanimatel'naya progulka v strane Einshteina (An entertaining trip into Einstein's country), in Zanimatel'naya Mekhanika (Entertaining Mechanics), Ya.I. Perel'man ed., GRN-PYuL, Leningrad – Moscow 1935, ch. 11, p. 188 (in Russian).

[18] G. Gamow, Mr. Tompkins in paperback, containing Mr. Tompkins in wonderland and Mr. Tompkins explores the atom, Cambridge University Press, Cambridge, England 1988.

[19] Ya.B. Zeldovich and I.D. Novikov, The structure and evolution of the universe, University of Chicago press 1982.

[20] G. Veneziano, Physics with a fundamental length, in Physics and mathematics of strings, Vadim Knizhnik Memorial Volume, L. Brink, D. Friedan and A.M. Polyakov eds., WSPC, 1990, p. 509.

[21] N. Feather, Mass, length and time, Edinburgh University Press, 1959.

[22] M.J. Duff, The world in eleven dimensions: supergravity, supermembranes and M-theory, I.O.P. Publishing 1999, http://bookmark.iop.org/bookpge.htm/book=815p.

[23] M.J. Duff, J.T. Liu and R. Minasian, Eleven-dimensional origin of string/string duality: a one loop test, Nucl. Phys. B 452 (1995) 261.

[24] M.J. Duff, The top ten questions in fundamental physics, in Strings 2000, proceedings of the International superstrings conference, University of Michigan, Ann Arbor, M.J. Duff, J.T. Liu and Lu eds., World Scientific, 2001, http://feynman.physics.lsa.umich.edu/strings2000/millennium.html.

[25] P.A.M. Dirac, The cosmological constants, Nature 139 (1937).

[26] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, D-branes and short distances in string theory, Nucl. Phys. B 485 (1997) 85 [hep-th/9608024].

[27] The NIST reference on constants, units and uncertainty, http://physics.nist.gov/cuu/Units/international.html.

[28] Particle Data Group, Review of particle properties, Phys. Rev. D 45 (1992) S1, part II; page III.4., erratum ibid. D 46 (1992) 5210.

[29] J.-M. Levy-Leblond, The importance of being (a) constant, in Problems in the foundations of physics, Enrico Fermi School LXXII, G. Torraldi ed., (North Holland 1979), p. 237.

[30] R.P. Feynman, The character of physical law, Penguin, 1992, p. 101.

[31] M.J. Duff and P. van Nieuwenhuizen, Quantum inequivalence of different field representations, Phys. Lett. B 94 (1980) 179.

[32] J. Gribbin and P. Wesson, Fickle constants of physics, New Scientist, 4 July 1992, p. 30.

[33] J.D. Barrow and J. Magueijo, Varying-α theories and solutions to the cosmological problems, astro-ph/9811072.

[34] J.K. Webb et al., Further evidence for cosmological evolution of the fine structure constant, astro-ph/0012539.

[35] J. Glanz and D. Overbye, Anything can change, it seems, even an immutable law of nature, New York Times, August 15, 2001.

[36] L. Okun, Cube or hypercube of natural units?, hep-ph/0112339.