hep-th/9605033 DAMTP/96-40 EFI-96-13 YCTP-P8-96 RU96-29

I-Brane Inflow and Anomalous Couplings on D-Branes

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Abstract

We show that the anomalous couplings of D-brane gauge and gravitational fields to Ramond-Ramond tensor potentials can be deduced by a simple anomaly inflow argument applied to intersecting D-branes and use this to determine the eight-form gravitational coupling.

May 6, 1996

1. Introduction

Consider a Weyl fermion on a 2*n*-dimensional manifold \mathcal{B} equipped with Yang-Mills and gravitational connections A, ω . If the fermions transform in a representation ρ of a gauge group the anomalous variation of the action $\log Z(A, \omega)$ is given by the famous descent formula [1,2,3,4,5]

$$\delta_{\Lambda} \log Z(A,\omega) = 2\pi i \int_{\mathcal{B}} \left[\operatorname{ch}_{\rho}(F) \hat{A}(R) \right]^{(1)}$$
(1.1)

where

$$\operatorname{ch}_{\rho}(F) = \operatorname{Tr}_{\rho} \exp(\frac{\mathrm{i}F}{2\pi}) = \dim \rho + \operatorname{ch}_{1}(F) + \operatorname{ch}_{2}(F) + \cdots$$
(1.2)

 $(ch_j \text{ is a } 2j\text{-form})$ and

$$\hat{A}(R) = 1 - \frac{p_1}{24} + \frac{7p_1(R)^2 - 4p_2(R)}{5760} + \dots = 1 + \hat{A}_4(R) + \hat{A}_8(R) + \dots$$
(1.3)

is the A-roof genus ¹. In (1.1) we have used the standard notation where for any closed gauge invariant form I we have $I - I_0 = dI^{(0)}$, where I_0 is the leading constant term, and the first order gauge variation is given by $\delta I^{(0)} = dI^{(1)}$. The descent procedure is ambigous, reflecting the ability to add local counterterms to the action. We will comment later on this ambiguity in our context.

The formula (1.1) was given the following simple physical interpretation in [7]. Consider a *p*-brane soliton in a gauge/gravity theory in an s + 1-dimensional spacetime X. If *p* is odd it can happen that the soliton carries chiral fermions transforming in an anomalous representation of the gauge/gravity theory on the soliton with an anomaly determined by (1.1) with \mathcal{B} the *p*-brane world volume. In this case the effective theory on the soliton violates charge conservation and (if $p = 1 \mod 4$) energy-momentum conservation. The apparent charge violation - in a selfconsistent theory - is accounted for by an inflow of charge from the external nonanomalous theory. Equivalently, the gauge variation of the low-energy effective action on the *p*-brane is cancelled by a bulk term whose gauge variation is localized on the *p*-brane.

In this situation the bulk theory has a (p+1)-form coupling to \mathcal{B} , i.e., there is a source term in the equations of motion:

$$d * H_{p+2} = d\tilde{H}_{s-p-1} = \delta_{s-p}(\mathcal{B} \to X) \quad . \tag{1.4}$$

¹ Our conventions for curvatures and connections follows [6]. Thus, ch_2 is *negative* for an ASD connection on a Euclidean 4-fold.

Here $\delta_{s-p}(\mathcal{B} \to X)$ is a delta-function supported Poincare dual form of degree s-p. The nonanomalous theory on X has a corresponding anomalous coupling ²:

$$I_{\text{bulk}}^{\text{anom}} = \int_X \tilde{H} \wedge \left[\text{ch}_{\rho}(F) \hat{A}(R) \right]^{(0)}.$$
 (1.5)

We then see that the anomalous variation of $I_{\text{bulk}}^{\text{anom}}$ cancels the anomalous variation of the effective action for the fermions on \mathcal{B} after using the equation of motion (Bianchi identity) (1.4)

$$\delta_{\Lambda} I_{\text{bulk}}^{\text{anom}} = \int_{\mathcal{B}} \left[\text{ch}_{\rho}(F) \hat{A}(R) \right]^{(1)}.$$
 (1.6)

Note that this construction requires a correlation between the charge carried by the *p*-brane and the number of fermion zero modes. Put another way, given the fermion zero mode structure one can use this argument to determine the quantum of charge carried by the *p*-brane.

This simple argument can be used to deduce the presence of chiral fermion zero modes on the worldvolume given the presence of the correct anomalous couplings, or it can be used, as we do here, to deduce the presence of anomalous couplings given a knowledge of the fermion zero mode structure.

This construction also gives a satisfying picture of the role played by consistent and covariant anomalies [8]. The extension of this mechanism to theories with Green-Schwarz anomaly cancellation was discused in [9] and the relation between this mechanism of anomaly cancellation and the Green-Schwarz mechanism was described in [10]. These couplings have played a crucial role in the study of certain string dualities. For example both Type IIA string theory and M theory have fivebranes with a chiral and anomalous spectrum [11]. Cancelling this anomaly on the fivebrane by a bulk inflow determines a coupling between the potential coupling to the fivebrane and an 8-form polynomial in curvature [12], [13] as can be verified in the IIA theory by a direct string calculation [14].

In this note we show how the same line of argument determines the anomalous couplings of gauge brane fields with bulk fields and verifies Polchinski's calculation of the quantum of Ramond-Ramond charge carried by *D*-branes [15]. For *N* coincident D pbranes on \mathcal{B}_p we find the anomalous couplings

$$I^{CS} = \int_{\mathcal{B}_p} C \wedge \operatorname{Tr}_{N} \exp(\frac{\mathrm{i}F}{2\pi}) \sqrt{\hat{A}(\mathbf{R})}$$
(1.7)

 $^{^2}$ In the literature such terms are sometimes called Chern-Simons couplings and sometimes called WZ terms. We will simply adopt the name "anomalous couplings."

with

$$\sqrt{\hat{A}(R)} = 1 + \frac{1}{2}\hat{A}_4 + \left(\frac{1}{2}\hat{A}_8 - \frac{1}{8}\hat{A}_4^2\right) + \dots$$
(1.8)

where F is the U(N) gauge field strength localized on the brane and R is the 10-dimensional Riemann curvature 2-form pulled back to the brane worldvolume and the RR field strength $H = dC + \cdots$ is a bispinor, equivalent to a sum of even (odd) forms for IIA (IIB) string theory.

The gauge field terms in (1.7) were first found in [16] using the results of [17,18] and have a number of applications to string duality [19,20]. The \hat{A}_4 term was deduced in [21] using a "duality chasing" argument and plays a crucial role in IIA-heterotic duality. We will verify these terms below. The result for the degree eight gravitational coupling is new. In (1.7) the Neveu-Schwarz potential, B, has been set to zero for convenience although non-zero B can easily be included in the following.

2. Derivation

Consider two orthogonally intersecting type II Dirichlet *p*-branes of dimensions p_1, p_2 , multiplicities N_1, N_2 , and filling worldvolumes $\mathcal{B}_1, \mathcal{B}_2$. We will assume they lie along coordinate subspaces and intersect in an I p_{12} -brane $\mathcal{B}_{12} = \mathcal{B}_1 \cap \mathcal{B}_2$ (We use the nomenclature I-brane to indicate the brane occuring at the interesection of two D-branes. The I-brane zero modes differ from those of a D-brane of the same dimension). Accordingly, we may split up the spacetime coordinates into mutually disjoint sets $\{0, \ldots, 9\} = S_{12} \amalg S_1 \amalg S_2 \amalg T$. String endpoints in \mathcal{B}_1 have Neumann (N) boundary conditions in $S_{12} \amalg S_1$ and Dirichlet (D) boundary conditions in $S_2 \amalg T$ while enpoints in \mathcal{B}_2 have Neumann boundary conditions in $S_{12} \amalg S_2$ etc. The I-brane \mathcal{B}_{12} lies along the coordinate plane defined by S_{12} .

We are interested in the case when the I-brane possesses chiral, anomalous zero modes. It is not hard to see that this happens when there are chiral unbroken supersymmetries on the I-brane. Unbroken supersymmetries will exist if there exist SO(1,9) MW spinors $\epsilon, \tilde{\epsilon}$ such that [22]:

$$\epsilon = \Gamma^{S_{12}} \Gamma^{S_1} \tilde{\epsilon}$$

$$\epsilon = \pm \Gamma^{S_{12}} \Gamma^{S_2} \tilde{\epsilon}$$
(2.1)

Each equation in (2.1) has solutions iff p_1, p_2 are both even (odd) in the IIA (IIB) theory. Moreover, each linearly independent solution of $\epsilon = \eta \Gamma^{12} \Gamma^T \epsilon$ (where η is a sign depending on S_{12}, S_1, S_2) gives a linearly independent supersymmetry. Solutions exist iff $|S_1| + |S_2| =$ 0 mod 4. This may be proved by squaring the Γ-matrix or by noting that $|S_1| + |S_2|$ is invariant under *T*-duality transformations along all coordinate axes. Therefore we may map one configuration to a D-instanton and apply the result of [22]. The supersymmetry can only be chiral for $p_{12} = 1 \mod 4$ and $T = \emptyset$. Up to *T*-duality there are exactly two distinct cases with chiral supersymmetry. We can have two 5-branes intersecting on a string: $S_{12} = \{0,1\}, S_1 = \{2,3,4,5\}, S_2 = \{6,7,8,9\}$ or we can have two 7-branes intersecting on a 5-brane: $S_{12} = \{0,1,2,3,4,5\}, S_1 = \{6,7\}, S_2 = \{8,9\}.$

The excitation spectrum of the brane theory is easily derived using the techniques explained in [22]. There are four sectors with boundary conditions:

$$S_{12} S_{1} S_{2} T \qquad S_{12} S_{1} S_{2} T$$

$$\sigma = 0 N N D D \qquad \sigma = 0 N D N D$$

$$\sigma = \pi N N D D \qquad \sigma = \pi N D N D$$

$$S_{12} S_{1} S_{2} T \qquad S_{12} S_{1} S_{2} T$$

$$\sigma = 0 N D N D \qquad \sigma = 0 N N D D$$

$$\sigma = \pi N N D D \qquad \sigma = \pi N D N D$$
(2.2)

The first two sectors lead to $U(N_i)$ SYM on \mathcal{B}_i . The second two sectors provide supermultiplets in the (N_1, \bar{N}_2) and (\bar{N}_1, N_2) of the gauge group $U(N_1) \times U(N_2)$. These fields, corresponding to the two orientations of DN strings, give fields related by complex conjugation. Evidentally, these "mixed sector" fields only have zeromodes along \mathcal{B}_{12} . There are always massless states in the Ramond sector and, by the GSO projection, they will be chiral fermions if and only if p_{12} is odd (regardless of whether we work in IIA or IIB theory). Since the states confined to \mathcal{B}_{12} only come from open strings we never encounter chiral bosons or gravitini. In short, the massless spectrum on the I-brane consists of one Weyl fermion in the (N_1, \bar{N}_2) and one in the (\bar{N}_1, N_2) .

In four or eight world-volume dimensions this spectrum is not chiral since complex conjugation flips the chirality and there is thus no anomaly on the I-brane. In two or six world-volume dimensions there is an anomaly determined by descent from the four-form or eight-form part of

$$I = (\mathrm{ch}_{(\bar{N}_1, N_2)}(F) + \mathrm{ch}_{(N_1, \bar{N}_2)}(F))\hat{A}(R) \cong 2\mathrm{ch}_{N_1}(F_1)\mathrm{ch}_{N_2}(F_2)\hat{A}(R)$$
(2.3)

where we have used the fact that in two or six dimensions only even powers of the gauge field strength appear so that traces in the N and \overline{N} are equal.

Now let us compare the anomalous charge violation (2.3) on the I-brane with the variation coming from the bulk terms on the two intersecting branes. We will follow the convention of [15] with two parameters μ , α appearing in the normalization of the RR kinetic terms and the coupling of the RR potential to D-branes, and we also take $4\pi^2 \alpha' = 1$. We provisionally assume a coupling on each brane of the form

$$\mu \int_{\mathcal{B}} C \wedge Y(F, R) \tag{2.4}$$

where Y(F, R) is a gauge invariant polynomial of mixed degree with $Y(F, R) = N + \cdots$. Strictly speaking, the coupling (2.4) is not well-defined in the presence of branes since the RR potentials C are not single valued or mutually local. This can be remedied by integrating by parts all terms except for the top RR potential. Thus (2.4) is more properly written as

$$\mu \int_{\mathcal{B}} NC + Y^{(0)}H \quad . \tag{2.5}$$

Since H and dC differ in the presence of branes (H is gauge invariant) (2.5) in fact differs from (2.4). We consider (2.5) as the correct expression of the brane anomalous coupling.

The equation of motion (Bianchi identity) for the RR field strength is: ³

$$dH = \frac{\mu}{\alpha} \sum_{\text{branes}} \delta(\mathcal{B}_p \to M_{10}) Y(F, R).$$
(2.6)

The RR potential C thus has an anomalous variation in the presence of branes given by

$$\delta C = -\frac{\mu}{\alpha} \sum_{\text{branes}} \delta(\mathcal{B}_p \to M_{10}) Y^{(1)}(A, \omega)$$
(2.7)

where A is the brane gauge field.

We can now compute the variation of (2.5) for two intersecting branes using (2.6) and (2.7) and find a variation on the I-brane of

$$-\frac{\mu^2}{\alpha} \int_{\mathcal{B}_{12}} \left[Y(1)Y^{(1)}(2) + Y(2)Y^{(1)}(1) + N_1 Y^{(1)}(2) + N_2 Y^{(1)}(1) \right]$$
(2.8)

where Y(i) indicates that it is a function of ω and the gauge fields on the i^{th} brane. The anomalous variation (2.8) follows from descent from the polynomial

$$-\frac{\mu^2}{2\pi\alpha}2Y(1)Y(2).$$
 (2.9)

 $^{^3}$ In order to avoid factors of $\sqrt{-1}$ we work in Minkowski space for IIB and Euclidean space for IIA.

Symmetry between the two branes fixes the local counterterm ambiguity to give:

$$(2Y(1)Y(2))^{(0)} = Y(1)^{(0)}Y(2) + Y(1)Y^{(0)}(2) + N_1Y^{(0)}(2) + N_2Y^{(0)}(1).$$
(2.10)

Comparing this to the anomaly on the I-brane (2.3) we see that the anomaly is cancelled provided that

$$Y(F,R) = \operatorname{ch}_N(F)\sqrt{\hat{A}(R)},$$

$$\mu^2/\alpha = 2\pi,$$
(2.11)

thus verifying (1.7) as well as the quantum of RR charge found in [15].

Quantization of μ constrains, in particular, the nine-form potential whose source is the eight-brane, and the Romans mass of type IIA ten-dimensional supergravity. However, when this mass is non-zero there are extra terms in (2.11) that should be easy to incorporate following [23] and [24]. Likewise it is easy to include a non-zero Neveu–Schwarz potential which was set equal to zero in the above.

It is natural to wonder about the inflow mechanism for multiply intersecting branes. However, since open strings have only two ends all charge violation is already accounted for by considering pairs of branes.

Acknowledgements

We would like to thank the Rutgers department of physics for hospitality during the course of this work. In particular we would like to thank T. Banks, M. Douglas, A. Losev, E. Martinec, N. Nekrasov, A. Sen, and S. Shatashvili for useful discussions. The research of GM was supported by DOE grant DE-FG02-92ER40704, and by a Presidential Young Investigator Award. The work of JH was supported in part by NSF Grant No. PHY 91-23780.

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