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ON THE LOCATION OF THE SNOW LINE IN A PROTOPLANETARY DISK

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ABSTRACT

In a protoplanetary disk, the inner edge of the region where the temperature falls below the condensation temperature of water is referred to as the snow line. Outside the snow line, water ice increases the surface density of solids by a factor of 4. The mass of the fastest growing planetesimal (the isolation mass) scales as the surface density to the 3/2 power. It is thought that ice-enhanced surface densities are required to make the cores of the gas giants (Jupiter and Saturn) before the disk gas dissipates. Observations of our solar system's asteroid belt suggest that the snow line occurred near 2.7 AU. In this paper we revisit the theoretical determination of the snow line. In a minimum-mass disk characterized by conventional opacities and a mass accretion rate of $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, the snow line lies at 1.6–1.8 AU, just past the orbit of Mars. The minimum-mass disk, with a mass of $0.02\ M_\odot$, has a lifetime of 2 million years with the assumed accretion rate. Moving the snow line past 2.7 AU requires that we increase the disk opacity, accretion rate, and/or disk mass by factors ranging up to an order of magnitude above our assumed baseline values.

Subject headings: planetary systems: formation — planetary systems: protoplanetary disks

1. INTRODUCTION

Most of the extrasolar planets that have been detected so far are Jupiter-like gas giants. The most widely accepted theory for the formation of gas giants is the core accretion model (e.g., Pollack et al. 1996; Rafikov 2004), which requires a core of 5–15 $M_\oplus$ (Guillot 2005). Some of the extrasolar planets seem to have masses in this range (McArthur et al. 2004; Santos et al. 2004); they would serve as cores if there were gas for them to accrete.

In the minimum-mass solar nebula (MMSN), the surface density of refractory materials is about 0.64 g cm^{-2} at 5 AU. The surface density of all condensable material increases to 2.7 g cm^{-2} once the volatiles (ices) freeze out. The isolation masses of the early planetary embryos, after they have swept up all the material in their annular feeding zones in the parent disk, are proportional to the 3/2 power of the surface density. Taking the radial width of the feeding zone to be $2\sqrt{3}$ Hill radii (Gladman 1993), and using the ice-enhanced surface density, we find that the isolation mass at Jupiter's distance is about the mass of the Earth. These embryos then merge to form the 5–15 $M_\oplus$ cores of the gas giants. It has been traditionally believed that the surface density needs to be enhanced by ices to form the cores of the giant planets before disk gas dissipates.

Ice forms at (and beyond) the snow line, where the temperature falls below 145–170 K, depending on the partial pressure of nebular water vapor. Previous work (Hayashi 1981; Sasselov & Lecar 2000) neglected the dependence of the sublimation temperature on the gas density. Podolak & Zucker (2004) showed that for the densities in the MMSN, the sublimation temperature can be as low as 145 K.

In an earlier paper (Sasselov & Lecar 2000), we found that the midplane temperature dropped to 170 K at a distance of 1.5 AU (the heliocentric distance of Mars) for a disk heated purely by incident starlight (a "passive" disk that does not accrete). The intent of that paper was to see if close-in extrasolar planets could be formed in situ, i.e., if cores weighing a few Earth masses could be formed at the distance of Mercury. We were content to show that they could not. However, in our solar system, the snow line was definitely outside the orbit of Mars. The evidence points to about 2.7 AU, in the outer asteroid belt where icy C-class asteroids abound (Abe et al. 2000; Morbidelli et al. 2000; Rivkin et al. 2002). Comets are more water-rich by about a factor of 4, while the inner asteroid belt is largely devoid of water. While this evidence has been questioned and alternatives proposed (e.g., Grimm & McSween 1993), it appears that the solar nebula at the time of planetesimal formation was hotter than the models discussed by Sasselov & Lecar (2000).

In this paper we revisit the issue of the snow line. We aim to find out how global disk parameters (surface density, mass accretion rate, opacity) affect the location where the snow transition occurs. We are concerned with large-scale disk properties and ignore here local perturbations due to protoplanets discussed by Jang-Condell & Sasselov (2004). In § 2 we describe our model for protoplanetary disks, and in § 3 we calculate the temperature and density-dependent rates of ice sublimation and condensation. We present our results and conclusions in § 4.

2. THE MODEL

Our model is that of a disk that is heated not only by steady mass accretion at rate $\dot{M}$, but also by absorption of light emitted by the central star. We work with a disk whose surface density is that of the MMSN: $\Sigma(r) = \Sigma_0(r/\mathrm{AU})^{-3/2}$, where r is the disk radius, Σ is the total surface density in gas and condensables, and $\Sigma_0 = 1700$ g cm^{-2}. We avoid explicitly accounting for the usual dimensionless viscosity parameter α (e.g., Frank et al. 1992) by fixing the value of $\dot{M}$ and using our prescribed surface density law. These choices define an α that is not constant with radius.

To estimate the midplane temperature, we first neglect absorption of starlight and consider accretional heating only. The flux

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emitted by a disk that steadily accretes mass at rate $\dot{M}$ in a potential due to a central star of mass $M_\star$ and radius $R_\star$ is (Lynden-Bell & Pringle 1974)

$$F_{\rm acc}(r) \equiv \sigma T_{\rm eff}^4 = \frac{3}{8\pi}\frac{GM_\star \dot{M}}{r^3}\left(1-\sqrt{\frac{R_\star}{r}}\right). \quad (1)$$

Here $T_{\rm eff}(r)$ is the effective temperature corresponding to the total flux released by accretional heating. We use this effective temperature to evaluate the midplane temperature of the disk under the assumption that the accretional energy is transported radiatively from the midplane to the surface. Treating radiative diffusion in an optically thick medium, we can safely adopt the Eddington approximation. We employ the Rosseland optical depth,

$$\tau_{\rm R} = \int_0^\infty \kappa\rho(r,\ z)\, dz,$$

to derive the midplane temperature due to accretional heating only,

$$T_{\rm mid,acc}^4 = \frac{3}{4}\left(\tau_{\rm R}+\frac{2}{3}\right)T_{\rm eff}^4. \quad (2)$$

Here $\rho(r, z)$ is the total mass density at radius r and vertical height z above the midplane, and κ is the Rosseland opacity. The latter quantity is taken from D'Alessio et al. (2001); it is dominated by particle condensates and is a function of temperature. It is uncertain insofar as the properties of the condensates—their mineral composition, allotropic state, and distribution with size—are uncertain. We make use of the dependence of the opacity on whether the temperature is above (300 K) or below (100 K) ice sublimation as given by D'Alessio et al. (2001), but point out that the effects due to that dependence on the temperature structure of the disk are small and continuous, as discussed by Jang-Condell & Sasselov (2004).

Next, we restore irradiation from the central star. The true midplane temperature is

$$T_{\rm mid}^4 = T_{\rm mid,acc}^4 + T_{\rm irr}^4. \quad (3)$$

For details on computing $T_{\rm irr}$, see Sasselov & Lecar (2000) and Jang-Condell & Sasselov (2004). For the central star, we used model parameters for stars of 1 $M_\odot$ with ages of 1 and 2 Myr from the models of Siess et al. (2000). The models are with a mild overshoot parameter. We note that models of such young stellar objects are notoriously uncertain. The span of ages that we consider provides a wide range of stellar irradiation fluxes and, we hope, covers some of this uncertainty.

3. THE ICE CONDENSATION/ SUBLIMATION TEMPERATURE

Although the commonly followed rule of thumb for computing the position of the snow line is simply to take it where the gas temperature drops to 170 K (see, e.g., Sasselov & Lecar 2000), this procedure is too naive. As pointed out by Podolak & Zucker (2004), ice grains are unstable whenever the grain temperature is high enough that the rate of water vapor sublimation from the grain exceeds the rate of water vapor condensation from the surrounding gas. The grain temperature, in turn, is determined by balancing the relevant heating and cooling processes. For the case of ice grains in a gas disk, the grain is heated by the ambient radiation field and by the release of latent heat when water vapor condenses on the surface. The grain is cooled by reradiation and by the removal of latent heat when ice sublimates. Gas and grains also exchange energy by gas-grain collisions. The details of the model have been presented elsewhere (Mekler & Podolak 1994; Podolak & Mekler 1997). In all the calculations presented in this section, we assume a fixed gas temperature, $T_{\rm gas}$, and calculate the resulting grain temperature, $T_{\rm grain}$.

We consider grains in the optically thick midplane of the disk. The radiative heating flux (energy absorbed per unit area of the grain) is given by

$$E_{{\rm rad},h} = \pi\int_0^\infty Q_{\rm abs}B_\lambda\left(T_{\rm gas}\right)d\lambda, \quad (4)$$

while the radiative cooling flux is given by

$$E_{{\rm rad},c} = \pi\int_0^\infty Q_{\rm emis}B_\lambda\left(T_{\rm grain}\right)d\lambda. \quad (5)$$

Here $B_\lambda(T)$ is the Planck function, and $Q_{\rm abs}$ and $Q_{\rm emis}$ are the efficiency factors for absorption and emission of radiation. We compute $Q_{\rm abs} = Q_{\rm emis}$ from Mie theory; values depend on grain size and the complex refractive index of the constituent material. In this model we considered mixtures of ice and some generic absorbing material. The complex refractive index for ice was taken from the work of Warren (1984). Since ice is essentially transparent in the visible, where there is a peak in the solar spectrum, the temperatures of pure ice grains exposed to solar heating can be substantially different from grains with a small admixture of material that absorbs in the visible. As shown in Podolak & Mekler (1997), the results are not sensitive to the details or amount of absorbing material, provided it produces some absorption in the visible. For grains in the midplane, where the optical depth to the sun is high, the difference in temperature between pure and dirty ice grains is negligible.

To compute the heating by water vapor condensation, we assume that every molecule of water vapor that hits the grain condenses and releases a latent heat of q. If $n_{\rm H_2O}$ is the number density of water molecules and $m_{\rm H_2O}$ is the mass of a water molecule, the energy flux into the grain due to water condensation is

$$E_{{\rm cond},h} = \frac{n_{\rm H_2O}}{2q}\sqrt{\frac{2kT_{\rm gas}}{\pi m_{\rm H_2O}}}, \quad (6)$$

where k is Boltzmann's constant. We assume that the number density of water molecules never exceeds the number density for saturation at the ambient gas temperature or the solar ratio to H_2, whichever is lower.

The evaporative cooling is given by

$$E_{{\rm evap},c} = q\frac{P_{\rm vap}\left(T_{\rm grain}\right)}{\sqrt{\pi m_{\rm H_2O}kT_{\rm grain}}}, \quad (7)$$

where $P_{\rm vap}$ is the vapor pressure over ice at the grain temperature.

Finally, the heat flux into the grain from the ambient gas is given by

$$E_{{\rm gas},h} = \frac{n_{\rm H_2}}{2}\sqrt{\frac{2kT_{\rm gas}}{\pi m_{\rm H_2}}}\frac{jk\left(T_{\rm gas}-T_{\rm grain}\right)}{2}, \quad (8)$$

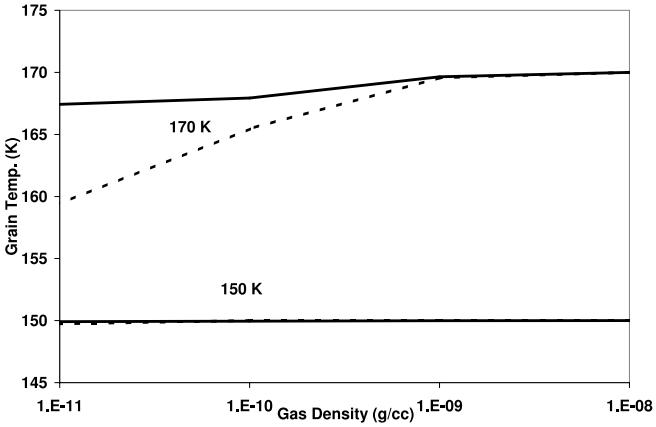

FIG. 1.—Grain temperature as a function of gas density for gas temperatures of 150 and 170 K. Solid curves are for 10 μm pure ice grains; dotted curves are for 0.1 μm pure ice grains.

where n_{H_2} is the number density of hydrogen molecules, and j is the number of molecular degrees of freedom ($j = 5$ for H_2). We assume a value for T_{gas}, equate the total heating and cooling rates, and solve for T_{grain}. The condition that the grain be stable against evaporation is that $E_{cond,h} \geq E_{evap,c}$.

Figure 1 shows the temperature of pure ice grains as a function of the ambient gas density for $T_{gas} = 150$ and 170 K. The solid curves are for grains of 10 μm radius, and the dashed curves are for grains of 0.1 μm radius. While grains in the 150 K gas are all at nearly the same temperature, independent of the gas density, the grains in the 170 K gas have a temperature that varies both with gas density and grain size. To explain these results, we first note that if $E_{cond,h} = E_{gas,h} = 0$, $T_{grain} < T_{gas}$ due to $E_{evap,c}$. At $T_{gas} = 170$ K, the rise in T_{grain} with gas density reflects the increasing importance of $E_{cond,h}$ and $E_{gas,h}$. The rise is even more pronounced for 0.1 μm grains than for 10 μm grains, since the optical absorption and emission efficiencies of the former are lower than those of the latter by 2 orders of magnitude; nonradiative terms are especially important for small grains. At $T_{gas} = 150$ K, the vapor pressure of water is so low that evaporative cooling and condensation heating are never important compared to radiation heating and cooling, and T_{grain} equilibrates to T_{gas}.

Figure 2 shows the energy fluxes due to condensation and sublimation for 0.1 μm (*dotted curve*) and 10 μm (*dashed curve*) ice grains

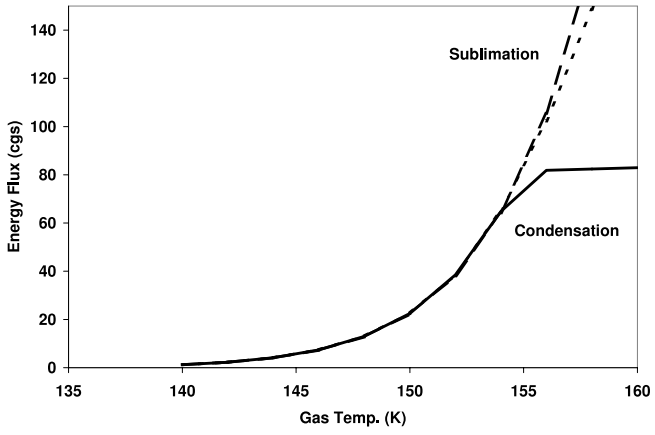

FIG. 2.—Energy fluxes due to condensation (*solid line*) and sublimation for 10 μm (*dashed line*) and for 0.1 μm (*dotted line*) pure ice grains. The background gas density is fixed at 5×10^{-11} g cm^{-3}.

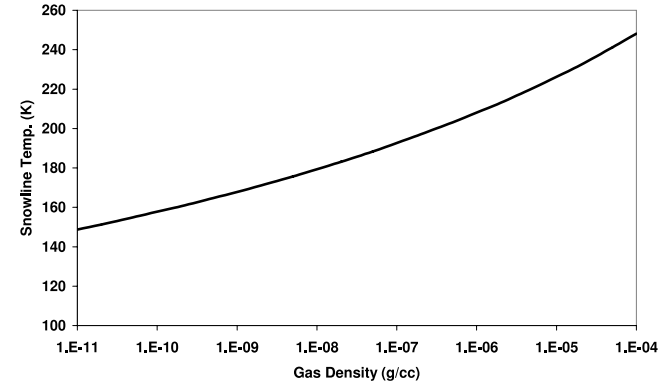

FIG. 3.—Gas temperature at the snow line as a function of gas density. The result is insensitive to grain size and composition.

for a background gas density of $\rho_{gas} = 5\times10^{-11}$ g cm^{-3}. Grains become unstable when the temperature goes above 154 K. Note how this sublimation temperature is insensitive to grain size.

For lower gas densities, the snow line appears at even lower temperatures, dropping to 150 K at $\rho_{gas} = 2\times10^{-11}$ and to 145 K at $\rho_{gas} = 1\times10^{-11}$ g cm^{-3}. This is shown in Figure 3, where the gas temperature at the snow line is shown for different values of the gas density. These values are insensitive to the size of the grain and its composition (e.g., pure water ice or ice with an admixture of some other absorber). In fact, at the snow line, where the condensation heating of a grain is almost exactly balanced by the evaporative cooling, a much simpler model is possible if the optical depth to the star is high. In this case, the radiative heating and radiative cooling of the grain also balance, and the temperature is given simply by the condition that the saturation vapor pressure equals the local partial pressure of the water vapor. For computing the snow-line temperature in the optically thick midplane, the difference between this simple model and the detailed model is too small to be discernible in the figure.

4. RESULTS AND CONCLUSIONS

By adding a modest amount of accretion, $10^{-8}\ M_\odot\ \mathrm{yr}^{-1} \approx 10^{-5} M_{Jup}\ \mathrm{yr}^{-1}$, to our standard model of the MMSN at age

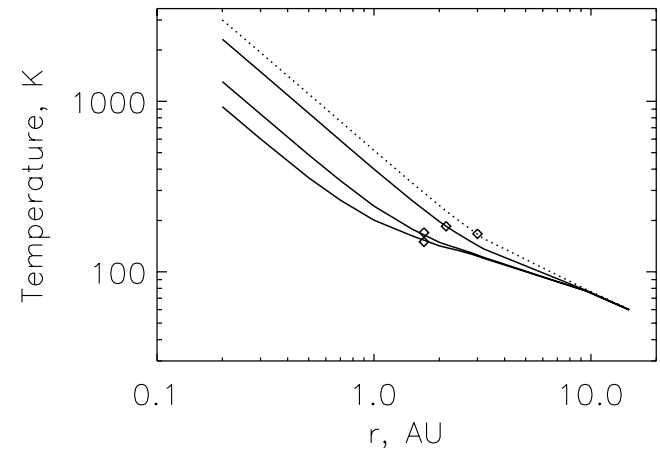

FIG. 4.—Midplane temperatures for disks with masses of 0.1, 1, and 10 times the MMSN, and the snow-line locations indicated for each of them (*solid lines*). The disks have a steady $\dot{M} = 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, $\Sigma(r) \propto r^{-3/2}$, and their central stars are 1 Myr old. Also shown is a MMSN disk model in which the opacity has been boosted fivefold and $\dot{M} = 4\times10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ (*dotted line*). The temperature gradient becomes shallow where the snow transition occurs, because viscous and irradiation heating exchange dominance at those radii for disk models considered here.

TABLE 1
SNOW-LINE LOCATIONS AND DISK MIDPLANE TEMPERATURES AT 1 Myr AGE FOR FOUR MASS ACCRETION RATES

$\dot{M}$	0.1 × MMSN r(AU)	T (K)	MMSN r(AU)	T (K)	MMSN (high-κ) r(AU)	T (K)	10 × MMSN r(AU)	T (K)
1×10^{-8}.....	1.6	151	1.7	174	2.20	172	2.1	185
2×10^{-8}.....	1.8	150	2.0	172	2.55	170	2.5	172
4×10^{-8}.....	2.0	150	2.3	170	3.00	167	3.1	168
8×10^{-8}.....	2.2	149	2.7	166	3.45	162	3.5	174

NOTE.—High-κ stands for a fivefold opacity increase.

1 Myr, we move the snow transition outward to 1.6–1.8 AU, beyond the orbit of Mars, as can be seen in Figure 4. Observations suggest, however, that the snow line in our solar system was located even farther out, near the outer asteroid belt at 2.7 AU, where C-class asteroids contain some water, albeit a factor of 4 less than comets at 5 AU (see § 1). In Table 1, we document the ways in which we can further increase disk midplane temperatures so as to push the snow line outward.

We can increase the temperature of the disk by increasing the accretion rate, but the disk temperature varies only as the fourth root of $\dot{M}$, while the lifetime of the disk varies as $1/\dot{M}$. In other words, increasing the temperature by 10% comes at the cost of decreasing the lifetime of the disk by 40%. The mass of the MMSN is about 2% of a solar mass. Therefore, the lifetime of the MMSN disk with an accretion rate of $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ is $t_{\mathrm{disk}} = M_{\mathrm{disk}}/\dot{M} = 0.02\ M_\odot/10^{-8}\ M_\odot\ \mathrm{yr}^{-1} = 2\times10^6$ yr.

Increasing the surface density of the disk is another possibility, although more problematic. The optical depth increases linearly with the surface density, but the midplane temperature scales only as the fourth root of the optical depth. And higher densities are accompanied by higher pressures, which demand higher sublimation temperatures. For a fixed accretion rate ($10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$), varying the disk surface density from 0.1 to 10 times that of the MMSN moves the snow line from 1.6 to only 2.1 AU (see Fig. 4). Also, one gains only slightly from using a flatter density profile (say $\Sigma \propto r^{-1}$). It is worth noting that Kuchner (2004) derived a $\propto r^{-2}$ disk for the minimum-mass extrasolar nebula.

Perhaps the most natural resolution to the problem is to boost the opacity in the disk. The Rosseland mean opacities increase 10-fold for a 10-fold decrease in the maximum grain radius; the same effect is accomplished by increasing the power-law exponent of the dust size distribution from 3.0 to 4.0 (e.g., Table 1 of D'Alessio et al. 2001). However, such change of grain size properties might be difficult to justify given recent observations of 1–2 Myr old disks (Rodmann et al. 2006). The range of possible values of κ should be explored further.

In summary, accounting for an accretion rate of $\dot{M} = 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ in our standard MMSN disk succeeds in moving the snow line past Mars. However, moving it out past 3 AU requires, for example, that we simultaneously increase $\dot{M}$, Σ_0, and κ by factors of 2–5 above our assumed baseline values.

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