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OBLIQUITY TIDES ON HOT JUPITERS

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ABSTRACT

Obliquity tides are a potentially important source of heat for extrasolar planets on close-in orbits. Although tidal dissipation will usually reduce the obliquity to zero, a nonzero obliquity can persist if the planet is in a Cassini state, a resonance between spin precession and orbital precession. Obliquity tides might be the cause of the anomalously large size of the transiting planet HD 209458b.

Subject headings: celestial mechanics — planetary systems: formation — planets and satellites: formation — stars: individual (HD 209458)

1. INTRODUCTION

A wonderful surprise of the last decade was the discovery of "hot Jupiters": giant planets that orbit their parent stars at a distance smaller than 10 stellar radii (Mayor & Queloz 1995; Butler et al. 1997). The proximity of the star and planet raises the possibility of tidal interactions between them. Of these interactions, the most frequently discussed are the synchronization of the planetary rotation period and orbital period, which happens over $\sim 10^6$ yr, and orbital circularization, over 10^8–10^9 yr (see, e.g., Rasio et al. 1996 and Lin & Gu 2004). Less frequently discussed, but often implicit, is the tidal evolution of the obliquity (the angle between the planetary spin axis and the orbit normal), which occurs on the same timescale as synchronization (Peale 1999).

We wish to point out that in some circumstances, a close-in planet can maintain a nonzero obliquity and that the consequent heat from obliquity tides can be large enough to affect the internal structure of the planet. We present the heating calculation first, in § 1, since it does not depend on our particular scheme for maintaining the obliquity. Our proposal, given in § 2, is that hot Jupiters occupy Cassini states, in which spin precession resonates with orbital precession. In § 3, we ask whether or not obliquity tides can explain the famously small density of the extrasolar planet HD 209458b. In § 4, we discuss the implications, strengths, weaknesses, and possible tests of this theory.

2. THE POWER OF OBLIQUITY TIDES

Tidal torques synchronize the planetary spin frequency (ω) and orbital mean motion (n), and often reduce the obliquity (θ) to zero (Goldreich & Peale 1970). Henceforth, if the orbit is circular, the tidal bulge is motionless in the reference frame of the planet, and energy dissipation ceases. However, dissipation continues if the planet somehow maintains a nonzero eccentricity or obliquity. For a star of mass $M_\star$ and a planet of mass M_p and radius R_p in an orbit with semimajor axis a, the rate of energy loss through tidal friction is

$$\frac{dE}{dt} = \left[\frac{9}{10}\frac{h}{Q}\, n \left(\frac{GM_p^2}{R_p}\right)\left(\frac{M_\star}{M_p}\right)^2\left(\frac{R_p}{a}\right)^6\right](7e^2 + \sin^2\theta). \quad (1)$$

This formula, derived in a different context by Wisdom (2004), employs the customary model of friction within astronomical bodies: Q is the "quality factor" of tidal oscillations (the inverse of the fractional energy dissipated per cycle). The factor h is the displacement Love number, parameterizing our ignorance of the planet's deformability. The factor in square brackets may be written

$$2 \times 10^{27}\ \mathrm{ergs\ s^{-1}} \left(\frac{Q/h}{10^6}\right)^{-1}\left(\frac{P}{3\ \mathrm{days}}\right)^{-5}\left(\frac{R_p}{R_{\mathrm{Jup}}}\right)^5, \quad (2)$$

where P is the orbital period.

For this heat source to play a significant role in determining the structure of the planet, it must be comparable to the intrinsic luminosity L_0 of other processes (e.g., gravitational contraction, radioactivity, and nuclear reactions). For Jupiter, $L_0 = 3 \times 10^{24}$ ergs s^{-1}. For hot Jupiters, theoretical estimates of L_0 are difficult to summarize (depending as they do on the planet's age, temperature, composition, atmosphere, and interior structure), but for billion-year-old planets of Jupiter's radius and mass, one expects $L_0 \approx 2 \times 10^{25}$ ergs s^{-1} (see, e.g., Guillot & Showman 2002 and Baraffe et al. 2003). Tidal heating is dominant when $e \gtrsim 0.04$ or $\theta \gtrsim 0.1$. The importance of eccentricity tides is well known (Bodenheimer et al. 2001), but the equivalent importance of obliquity tides does not seem to have been appreciated.

3. CASSINI STATES FOR EXTRASOLAR PLANETS

The reason is probably a mistaken intuition that $\theta = 0$ is the only possible endpoint of tidal evolution. In general, the planet's spin axis and its orbit both precess, in response to additional planets, satellites, the stellar quadrupole, or other torquing agents. With spin and orbital precession, the outcomes of tidal evolution are Cassini states (Colombo 1966; Peale 1969; Ward 1975), in which the orbit normal $\hat{\boldsymbol{n}}$ and spin axis $\hat{\boldsymbol{s}}$ precess at the same rate about the same axis $\hat{\boldsymbol{k}}$ (see Fig. 1). For a given body, there are at most two stable Cassini states, differing in whether $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{n}}$ are on the same side of $\hat{\boldsymbol{k}}$ (state 1) or opposite sides (state 2).[2] The obliquity $\theta = \cos^{-1}(\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{n}})$ of a Cassini state is not generally zero nor is it necessarily close to zero. Although many satellites in the solar system are

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[2] Formally, there are two other equilibria: state 3 is linearly stable but is unstable to tidal evolution (Goldreich & Peale 1970), and state 4 is linearly unstable.

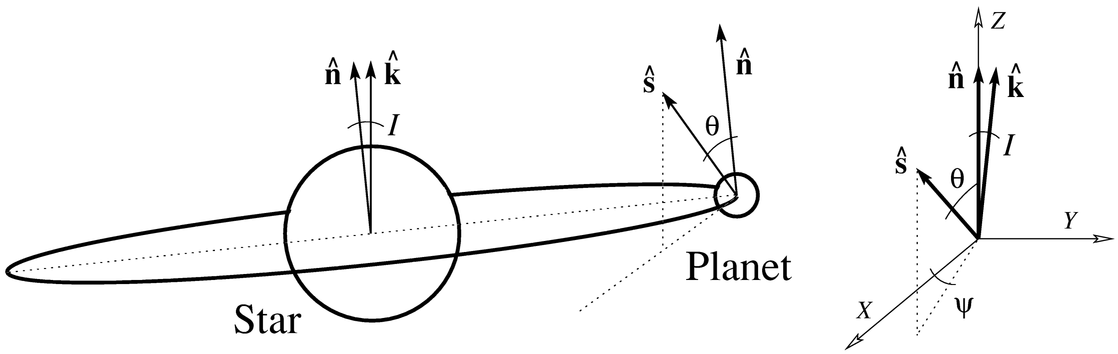

FIG. 1.—*Left*: Illustration of the spin axis $\hat{\boldsymbol{s}}$, orbit normal $\hat{\boldsymbol{n}}$, precession axis $\hat{\boldsymbol{k}}$, orbital inclination I, and obliquity θ. A general orientation for $\hat{\boldsymbol{s}}$ is depicted. In a Cassini state, $\hat{\boldsymbol{s}}$ is coplanar with $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{n}}$. *Right*: A convenient set of Cartesian axes, in which $\hat{\boldsymbol{n}}$ is along the Z-axis, $\hat{\boldsymbol{k}}$ is in the Y-Z plane, and (θ, ψ) are traditional polar angles describing $\hat{\boldsymbol{s}}$. In a Cassini state, $\psi = \pm 90°$.

in state 1 with vanishingly small obliquity, the Moon is in state 2 with $\theta = 6.5°$.[3]

Idealizing the planet as an oblate rigid body on a circular and uniformly precessing orbit of inclination $I = \cos^{-1}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})$, the Cassini obliquities θ_i obey

$$\cos\theta_i \sin\theta_i - \epsilon \sin(\theta_i - I) = 0. \tag{3}$$

Here we have defined $\epsilon = -g/\alpha$, where g is the nodal precession frequency and α is the spin precessional constant for a fixed orbit. The latter can be written

$$\alpha = \frac{3}{2}\left(\frac{C-A}{C}\right)\left(\frac{n^2}{\omega}\right), \tag{4}$$

where $C > B = A$ are the planet's principal moments of inertia and ω is its spin frequency. During synchronization, $\omega \to n$. When $\epsilon < \epsilon_{\rm crit} \equiv (\sin^{2/3} I + \cos^{2/3} I)^{-3/2}$, equation (3) has four roots, corresponding to the two stable states, 1 and 2, and two unstable states, 3 and 4. For $\epsilon > \epsilon_{\rm crit}$, there are only two roots, corresponding to states 2 and 3. Equation (3) can be derived from the governing Hamiltonian under the assumption of principal-axis rotation (see, e.g., Ward 1975),

$$\mathcal{H} = \mathcal{H}_0 - \frac{\alpha}{2}(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{s}})^2 - g(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{s}}), \tag{5}$$

where $\mathcal{H}_0$ is the $\hat{\boldsymbol{s}}$-independent portion. Figure 2 shows contours of $\mathcal{H}(\hat{\boldsymbol{s}})$ for an illustrative case.

A hot Jupiter settles into a Cassini state on the same $\sim 10^6$ yr timescale as spin-orbit synchronization. Whether it ends up in state 1 or state 2 depends on its initial obliquity and orbital inclination, and on ϵ, which may be a function of time. Reasons for variations in ϵ include the disappearance of the protoplanetary disk, migration of the planet or its fellow planets or satellites, contraction of the planet, and spin alteration of the planet or star. If ϵ varies slowly compared to g^{-1} and α^{-1} ("adiabatically"), and slowly compared to the Cassini state settling time, then the obliquity tracks the evolving Cassini obliquity. Abrupt changes in ϵ or θ may cause the planet to leave a Cassini state and ultimately to switch states if it lands in the basin of attraction of the other stable state.

Figure 2 suggests three ways in which a hot Jupiter can maintain a significant obliquity. First, if $\epsilon > \epsilon_{\rm crit}$, the only stable state is 2, for which the obliquity is nonzero ($\theta_2 \to I$ as $\epsilon \to$

[3] The observation that the Moon's $\hat{\boldsymbol{s}}$, $\hat{\boldsymbol{n}}$, and $\hat{\boldsymbol{k}}$ are coplanar was the basis of G. D. Cassini's third law of lunar motion, formulated in 1693.

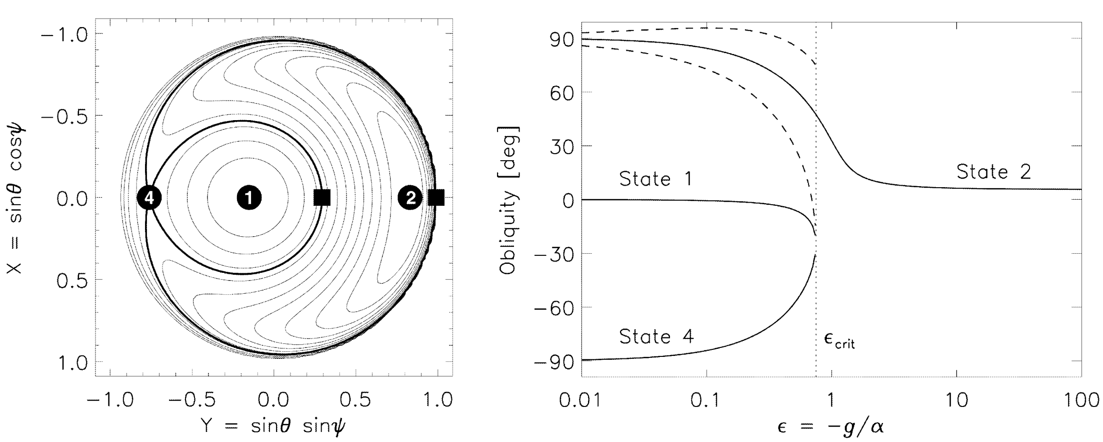

FIG. 2.—*Left*: Contours of $\mathcal{H}(\hat{\boldsymbol{s}})$, for $\epsilon = 0.6$ and $I = 0.1$. The unit sphere is projected onto the X-Y plane. The thick contour is the separatrix, which divides the basins of attraction of states 1 and 2 under tidal evolution. The numbered circles show states 1, 2, and 4. The retrograde state 3 is not shown. The separatrix angles θ_s are marked by squares. *Right*: Cassini state obliquities θ_i (*solid lines*) and separatrix angles θ_s (*dashed lines*) as a function of ϵ, for $I = 0.1$.

∞). This is the case for the Moon. Second, it could be in state 1 or state 2 with $\epsilon \sim \epsilon_{\rm crit}$, but this requires a special coincidence, since ϵ involves both planet-specific properties (namely, its moments of inertia) and seemingly unrelated quantities specific to other bodies (which determine the precessional torques). Third, the planet could be in state 2 with $\epsilon < \epsilon_{\rm crit}$. When ϵ is small, the basin of attraction of state 2 is also small, but it is possible to be captured into state 2 when ϵ is large and then tipped over to large obliquity as ϵ is gradually reduced. Ward & Hamilton (2004) proposed this scenario to explain why Saturn has a much larger obliquity than Jupiter. In the next section, we propose a similar scenario for a particular extrasolar planet.

4. THE CASE OF HD 209458b

The planet HD 209458b transits its parent star (Charbonneau et al. 2000; Henry et al. 2000), a fortuitous circumstance that enables one to make many interesting measurements, including that of the planet's mean density, which is 0.33 g $\rm cm^{-3}$. This is 27% of the Jovian value and is the smallest mean density of all seven known transiting extrasolar planets (see, e.g., Alonso et al. 2004, Konacki et al. 2005, and Pont et al. 2004). Theorists have struggled to explain this anomaly, usually by attempting to identify an overlooked internal heat source, although Burrows et al. (2003) argued that the density is not so terribly anomalous. We present a new hypothesis: HD 209458b resides in Cassini state 2 with a large obliquity, whereas most hot Jupiters reside in state 1 with small obliquities.

We are led to imagine the following sequence of events: (1) The planet forms at a large orbital distance, with a nonzero (but not necessarily large) obliquity. (2) The planet migrates inward to its current position. (3) As the spin and orbit are synchronized over $\sim 10^6$ yr, the planet falls into Cassini state 2, whether by chance or because state 2 is the only possibility. (4) As the disk disappears over $\sim 10^7$ yr, the orbital precession rate (and ϵ) decreases, forcing the obliquity to grow. (5) The planet remains in state 2 for billions of years.

Is this scenario compatible with order-of-magnitude estimates of the relevant quantities? We assume $I = 0.1$, because Winn et al. (2005) found an angle of $\approx 4°$ between the sky projections of the stellar spin axis and orbit normal, and it seems reasonable that I is of the same order of magnitude. We estimate $(C - A)/C$ by scaling Jupiter's observed value (de Pater & Lissauer 2001, p. 221) by $(\omega/\omega_{\rm Jup})^2$. Application of equation (4) gives $\alpha/n \sim 10^{-3}$ after synchronization. We assume that g was initially determined by the torque of the protoplanetary disk, which we idealize as a minimum-mass solar nebula. The precession rate is sensitive to the (unknown) inner radius of the disk: for $r_{\rm min} = 1.6a$, $|g/n| = 5 \times 10^{-5}$, rising to 10^{-4} for $1.3a$. These estimates ignore resonance effects, which we find through trial numerical integrations to be capable of enhancing g by an order of magnitude. As the disk disappears, g falls until other torques dominate, such as those from the stellar quadrupole or additional orbiting bodies. In the former case, assuming $J_2 = 10^{-6}$, the precession slows to $|g/n| = 4 \times 10^{-8}$. There is no evidence of additional bodies in current radial velocity data (Laughlin et al. 2005), but even a body too small or too distant to have been detected could nevertheless cause faster precession than the stellar quadrupole alone. For example, a 1 $M_\oplus$ planet in a 1.2 day orbit would cause $|g/n| = 2 \times 10^{-5}$ while producing undetectable radial velocity variations of $\sim$0.5 m $\rm s^{-1}$. We note that g is measurable, in principle, through time variations of the transit duration (Miralda-Escudé 2002).

With these considerations, it seems plausible that ϵ had an initial value ranging from 0.02 to near unity, and a final value between 10^{-2} and 10^{-5}. The initial value of ϵ could have been large enough that capture into Cassini state 2 occurred with reasonable probability, or even 100% probability if $\epsilon > \epsilon_{\rm crit}$. The final value of ϵ is small enough to force $\theta_2 \rightarrow 90°$. Thus, the current obliquity must be quite large.

The energy dissipated through obliquity tides comes at the expense of the orbital energy. We must examine whether or not it is possible for the dissipation rate to be large enough to inflate the planet, yet small enough to allow the orbit to survive for 5×10^9 yr, the approximate main-sequence age of the star (Cody & Sasselov 2002). To inflate the planet, Bodenheimer et al. (2003) found the required power to be 4×10^{27} ergs $\rm s^{-1}$ if the planet has a dense core and an order of magnitude smaller if the planet is coreless. With reference to equation (1), this corresponds to an upper limit on Q/h of 10^6 for a cored planet and 10^7 for a coreless planet. The condition

$$\frac{a}{|da/dt|} = \frac{GM_\star M_p/2a}{dE/dt} > 5 \times 10^9 \text{ yr} \tag{6}$$

yields a lower limit on Q/h of 5×10^6. In reality, equation (6) is probably too restrictive by a factor of a few; the dissipation rate was smaller in the past, when the planet had a larger radius, a larger orbital distance, and possibly a larger mass (Vidal-Madjar et al. 2003). Thus, Q/h should be $\sim 10^6$ if the planet has a core and 10^6–10^7 if it does not. In comparison, for Jupiter it is thought that Q is between 10^5 and 10^6 (Goldreich & Soter 1966), and $h \gtrsim 0.6$ (Gavrilov & Zharkov 1977). The required Q/h for HD 209458b is comparable to, or somewhat larger than, the nominal Jovian value.

The last chapter of the story is that the planet remains in Cassini state 2 for billions of years. Here the difficulty is that as ϵ decreases, the width of the resonance also decreases, reducing the robustness of state 2 to perturbations. For $\epsilon = 10^{-5}$, the angle between the separatrix angles (see Fig. 2) is only $0\overset{\circ}{.}2$. For $\epsilon = 10^{-2}$, it is increased to 7°. The impact of a body of mass m at escape velocity would produce a maximum obliquity shift of

$$\frac{m v_{\rm esc} R_p}{L_\omega} = \frac{m\sqrt{2GM_pR_p}}{Cn} = 20° \left(\frac{m}{M_\oplus}\right), \tag{7}$$

where the maximum is achieved for grazing incidence at the pole. Hence, we must also suppose HD 209458b suffered no major collisions after the disappearance of the protoplanetary disk.

5. DISCUSSION

Hot Jupiters should be in Cassini states. Since the obliquity of a Cassini state is not necessarily small, obliquity tides are a potentially important internal heat source. Whether or not a given planet can maintain a significant obliquity depends on its initial obliquity as well as its precessional and collisional histories. Obliquity tides could be the "missing" heat source that bloats the transiting planet HD 209458b. The implications of this hypothesis are that the planet's obliquity is nearly 90°, that its tidal dissipation factor Q and displacement Love number h obey $10^6 < Q/h < 10^7$, and that it had a quiescent history of no major collisions after its orbital precession rate declined from an initially large value.

This hypothesis has certain strengths and weaknesses relative to the two leading prior hypotheses. Guillot & Showman (2002) proposed that atmospheric circulation patterns convert a small fraction of the stellar radiation into heat deep within the planet. An advantage of this hypothesis over ours is that no fine-tuning of Q/h or the collision history is required. A disadvantage is that it is not obvious why the other hot Jupiters should not experience the same phenomenon. In contrast, Cassini state 2 is naturally a minority state for hot Jupiters. After landing in state 2 (possibly by chance), a planet must avoid being jostled into state 1 by collisions or the synchronization process.

In the scenario of Bodenheimer et al. (2001), tidal heating from orbital circularization is extended indefinitely because of a periodic eccentricity exchange with another planet. The extra heat is the fault of a well-placed third body, which is naturally expected to be a rare occurrence. Unfortunately, no such body has yet been detected, and the current eccentricity is small (Laughlin et al. 2005; Winn et al. 2005). These problems have made the hypothesis less appealing than it once was, although they might be overcome through tuning of Q/h or other embellishments. A relative strength of our hypothesis that no third body is required, although a third body could increase the robustness of the high-obliquity state.

Further work is needed to improve on our order-of-magnitude calculations. The synchronization process must be followed in detail because α changes on the same timescale as obliquity evolution and is therefore nonadiabatic (Peale 1974). We treated the planet as an oblate spheroid, but in reality it is triaxial due to tidal distortion, a complexity that alters the Cassini obliquities and dynamics (Peale 1969). We ignored any evolution of e or I and any nonuniform precession. Finally, a realistic description of perturbations is needed to estimate the capture probability and lifetime of Cassini state 2.

We can think of two possible tests of the theory that HD 209458b has a large obliquity, neither of which is easy. First, the obliquity can be measured or bounded through high-precision photometry of the transit (Seager & Hui 2002; Barnes & Fortney 2003), but the expected signal is smaller than 10^{-5} in relative flux. Second, a hot Jupiter with $\theta \approx 90°$ has no permanent day side or night side and should have a smaller day-night temperature difference than a planet with $\theta = 0°$. To quantify the expected temperature difference, atmospheric models of hot Jupiters (Showman & Guillot 2002; Cho et al. 2003; Menou et al. 2003; Burkert et al. 2005) should be generalized to cases of large obliquity.

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