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FRAGMENTATION ORIGIN OF MAJOR SUNGRAZING COMETS C/1970 K1, C/1880 C1, AND C/1843 D1

Zdenek Sekanina and Paul W. Chodas
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109; zs@sek.jpl.nasa.gov, paul.w.chodas@jpl.nasa.gov
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ABSTRACT

Following our recent successful modeling of the common origin of two of the brightest members of the Kreutz system of sungrazing comets, we now examine three other objects: C/1970 K1 (White-Ortiz-Bolelli), the most recent sungrazer discovered from Earth, C/1880 C1 (Great Southern Comet), and C/1843 D1 (Great March Comet). For White-Ortiz-Bolelli, five possible origin and orbit evolution scenarios are explored. We find that its parent was neither C/1965 S1 (Ikeya-Seki), nor C/1882 R1 (Great September Comet), nor the comet of 1106 (the presumed parent of Ikeya-Seki and the 1882 comet). The motion of C/1970 K1 is consistent with a scenario in which the parent was an unknown fragment that separated from the 1106 comet at the same time as, or shortly before, Ikeya-Seki and passed through perihelion in 1970 June–July, shortly after White-Ortiz-Bolelli. The separation of White-Ortiz-Bolelli from this fragment is found to have occurred around the mid-eighteenth century, at a heliocentric distance of about 150 AU, with a relative velocity of 3–5 m s^{-1} in the general direction of the Sun and to the north of the orbital plane. On the other hand, we conclude that the 1880 comet separated directly from C/1843 D1, the second brightest known sungrazer, some 100–150 days after the 1843 comet's previous perihelion passage in the eleventh century, at 2.5–3 AU from the Sun, with a relative velocity of slightly more than 7 m s^{-1} in the generally antisolar direction and to the south of the orbital plane. The pattern of fragmentation of the Kreutz system's members discovered from Earth begins to resemble the evolution of the system's minor fragments detected coronagraphically from aboard the *SOHO* spacecraft, and there is significant qualitative similarity with fragmentation of comet D/1993 F2 (Shoemaker-Levy 9).

Subject headings: comets: general — comets: individual (X/1106 C1, C/1843 D1, C/1880 C1, C/1882 R1, C/1887 B1, C/1945 X1, C/1963 R1, C/1965 S1, C/1970 K1, D/1993 F2) — methods: data analysis

1. INTRODUCTION AND THE OBJECTIVES

It has recently been shown for the first time (Sekanina & Chodas 2002, hereafter Paper I) that the motion of one member of the Kreutz system of sungrazing comets (C/1965 S1, Ikeya-Seki) can very accurately be derived from the motion of another member (C/1882 R1, Great September Comet) on the premise that the two objects are fragments of a common parent comet and that the breakup occurred about 18 days after perihelion with a relative velocity slightly exceeding 7 m s^{-1}. The parent object was most probably the comet observed in 1106 February, but both the breakup's timing relative to perihelion and the separation velocity are fairly insensitive to time (and therefore orbital period) on a scale of tens of years.

Our study of the 1882 and 1965 sungrazers was part of a broader effort aimed at the understanding of the formation and orbital evolution of the Kreutz system in particular and the role of runaway (or cascading) fragmentation in the life cycle of comets in general. A conceptual model for this process extending throughout the orbit about the Sun was introduced by Sekanina (2000) in his study of the pairs (or twins) among the sungrazers discovered[1] in the images taken with the Large Angle and Spectrometric Coronagraph Experiment (LASCO) on board the *Solar and Heliospheric Observatory* (*SOHO*); for the description of this space mission, see Brueckner et al. (1995).

More recent results, based primarily on analysis of the *SOHO* sungrazers (Sekanina 2002, hereafter Paper II), document the merits of modeling the fragmentation process in greater detail and suggest that most of the Kreutz system's mass is still locked in major fragments and that therefore the system is relatively young.

Much work still remains to be done before the conceptual paradigm of runaway fragmentation proposed in Paper II is developed into a comprehensive quantitative model. In an effort to achieve this goal and to enhance the merit of our scenario for the common origin of the 1882 and 1965 sungrazers, it needs to be demonstrated that this kind of relationship also exists among other major Kreutz system members.

The objective of this paper is to address the problem of the origin and orbital evolution of another three sungrazers: C/1970 K1 (White-Ortiz-Bolelli), the most recent member of the system discovered from Earth, C/1843 D1 (Great March Comet), and C/1880 C1 (Great Southern Comet). In comparison with the case of C/1882 R1 and C/1965 S1, the solutions are somewhat less constrained but of sufficient accuracy to provide evidence compelling enough to corroborate the concept of runaway fragmentation.

2. ORBIT OF COMET C/1970 K1

Among the eight sungrazers with a known orbit, which were discovered from the ground between 1843 and 1970, comet White-Ortiz-Bolelli is the sole representative of Marsden's (1989) subgroup IIa. Its orbit is outside the limits for

[1] See an electronic page maintained by D. A. Biesecker at http://sungrazer.nascom.nasa.gov.

TABLE 1

Relativistic Orbital Elements for Comet White-Ortiz-Bolelli, C/1970 K1 (Equinox J2000.0; Epoch 1970 May 14.0 ET)

Orbital Element (1)	General Solution (2)	Forced Solution (3)	Variations $(\partial/\partial e)[T, \omega, \Omega, i, q]$ (4)
Time of perihelion passage T (ET)...........	1970 May 14.48808 ± 0.00129	1970 May 14.48735 ± 0.00133	−15.09
Argument of perihelion ω (deg)..............	61.3122 ± 0.0321	61.3115 ± 0.0332	−17.04
Longitude of ascending node Ω (deg)	337.0916 ± 0.0432	337.0721 ± 0.0449	−407.22
Orbital inclination i (deg).......................	139.0679 ± 0.0202	139.0733 ± 0.0209	111.72
Perihelion distance q (AU)......................	0.0089024 ± 0.0000103	0.0088966 ± 0.0000107	−0.12005
Orbital eccentricity e..............................	0.9998599 ± 0.0000581	0.999908	...
Orbital period P (yr)...............................	507^{+626}_{-206}	951	...

subgroups I and II but deviates less substantially from subgroup II (C/1882 R1, C/1945 X1, and C/1965 S1) than from subgroup I (C/1843 D1, C/1880 C1, C/1887 B1, and C/1963 R1). The discrimination of sungrazers into the two subgroups was advocated by Marsden (1967) on the grounds that their elements display a conspicuous bimodal distribution, but the dynamical significance of this classification has remained unclear because the boundaries of the subgroups are largely washed out among the minor, coronagraphically detected sungrazers (see, e.g., Paper II).

The orbital elements of comet White-Ortiz-Bolelli cannot be determined as accurately as the elements for Ikeya-Seki or the 1882 sungrazer. The comet was not discovered until 4 days after perihelion, and only 10 precise astrometric observations were reported, spanning a period of not more than 14 days. The comet's parabolic orbit derived by Marsden (1970) is based on six positions because some of the 10 data points yielded unacceptably large residuals. Perturbations by the planets were ignored, a reasonable approximation given the short orbital arc covered. Effectively, Marsden's elements refer to the epoch of mid-observation, which is 1970 May 31. We have now reexamined the orbit, taking into account both the planetary perturbations and the relativistic effect. There have been two incentives for this effort: (1) with so few observations available, it is imperative to explore the solution's sensitivity to the choice of the data set used for the orbit determination; and (2) because of the correlations among the elements, it is desirable to derive their rates of variation with the eccentricity from a sequence of forced elliptical orbits.

TABLE 2

Residuals of Astrometric Observations of Comet White-Ortiz-Bolelli from General Solution

Time of Observation 1970 (UT)	Residuals[a] (arcsec) R.A.	Decl.	Observer and Location
May 23.957..................	(6.5)	(−1.3)	C. Bolelli (Cerro Tololo)
May 23.964..................	0.3	0.1	C. Bolelli (Cerro Tololo)
May 24.937..................	−0.3	(−1.7)	Z. M. Pereyra (Córdoba)
May 24.942..................	(−8.5)	(4.0)	Z. M. Pereyra (Córdoba)
May 27.438..................	−1.0	0.1	M. P. Candy (Perth)
May 27.951..................	1.0	−0.2	C. Bolelli (Cerro Tololo)
May 27.956..................	0.0	−0.1	C. Bolelli (Cerro Tololo)
June 5.931....................	(7.3)	0.3	Z. M. Pereyra (Córdoba)
June 5.935....................	0.2	(−3.7)	Z. M. Pereyra (Córdoba)
June 6.933....................	−0.2	−0.3	Z. M. Pereyra (Córdoba)

[a] Positional coordinates not used in the solution are parenthesized.

The results are in Table 1, which presents the orbital sets, and Table 2, which lists the residuals left by the astrometric observations. When we choose a tight rejection cutoff of 1″ for the positional residuals, we find that the general solution (col. [2] of Table 1) is elliptical, with the eccentricity departing from unity by 2.4 times its standard deviation. This result corresponds to a $\pm 1\ \sigma$ range of orbital periods from $\sim$300 to $\sim$1130 yr. However, when relaxing the rejection cutoff to 2″, for example, we obtain an eccentricity of $e = 0.9999479 \pm 0.0001345$; i.e., the signal-to-noise ratio for $1 - e$ drops by a factor of 6 to 0.4, and we obtain $\sim$2200 and $\sim$330 yr, respectively, for the most probable orbital period and its $1\ \sigma$ lower limit.

Columns (3) and (4) of Table 1 show an optimized set of orbital elements for a forced eccentricity of 0.999908 (or an orbital period of 951 yr) and the eccentricity dependent rates of variation for the other elements. These rates are essentially constant for orbital periods in the range of at least 500–1500 yr. The standard deviations of such eccentricity-forced orbital elements do not differ significantly from those for the forced solution in column (3) of Table 1, and the residuals left by the astrometric observations are very similar to those listed in Table 2 for the general solution.

Given the severe data constraints, we are confident that Table 1 offers the best possible sets of osculating elements that can serve as critical reference information in our investigation of this comet's most likely origin.

3. CANDIDATE SCENARIOS FOR C/1970 K1

Potential fragmentation scenarios for the origin of comet White-Ortiz-Bolelli have been selected on the basis of two of our previous findings: (1) the nature of the relationship between the sungrazers C/1882 R1 and C/1965 S1, established in Paper I; and (2) the temporal dependence of systematic orbital perturbations brought about by a momentum change that the comet fragment acquires at separation, as presented in Paper II.

Among the known members of the Kreutz system, there are not more than four candidates for the parent object of comet C/1970 K1 because C/1880 C1, C/1887 B1, and C/1945 X1 were not bright enough, not to mention the coronagraphically discovered sungrazers. Of the four that qualify (C/1843 D1, C/1882 R1, C/1963 R1, C/1965 S1), the 1843 comet is the least likely first-generation parent: it is a member of subgroup I and arrived at perihelion too early relative to C/1970 K1. Thus, we do not include the 1843 comet among the potential parents worth thorough investigation.

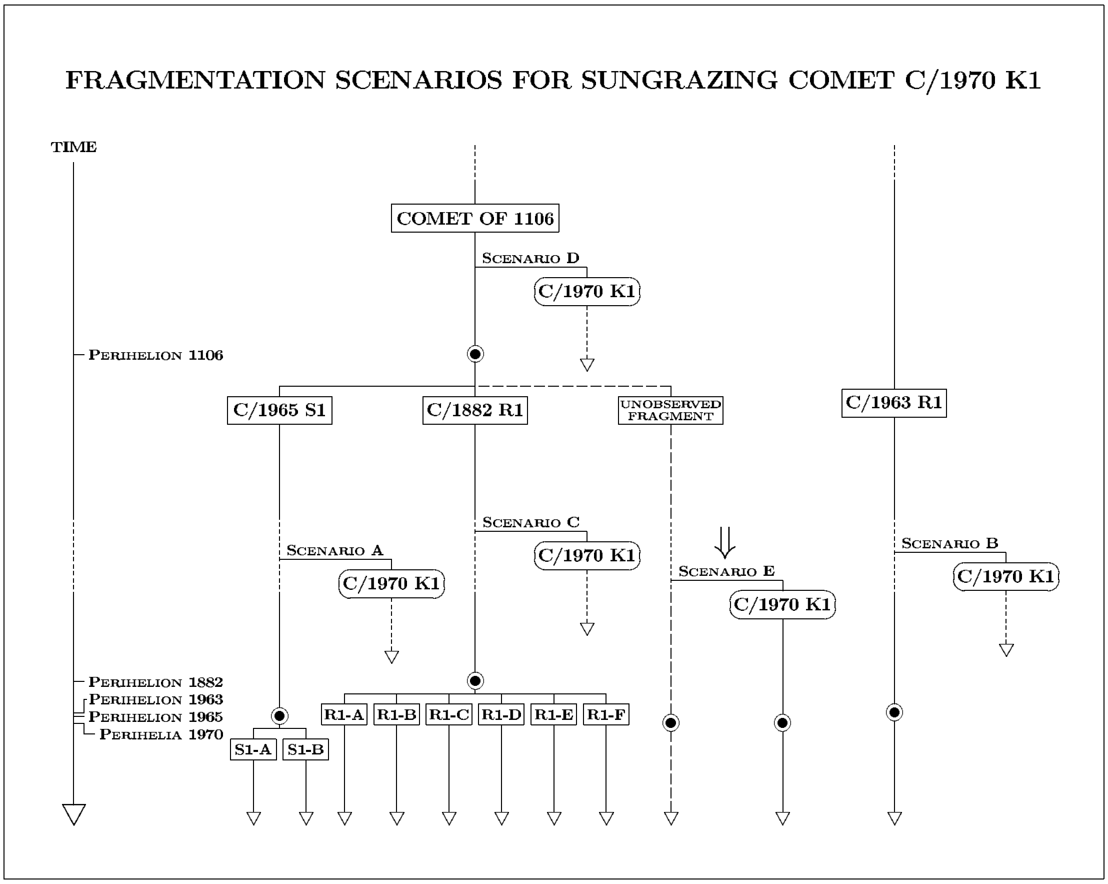

FIG. 1.—Schematic overview of the five fragmentation scenarios investigated in order to explain the origin and identify the source of sungrazer C/1970 K1 (White-Ortiz-Bolelli). The scenarios are described in § 3. To present the problem in a broader context, the evolutionary tracks of other sungrazing comets, notably 1106, C/1882 R1, C/1963 R1, and C/1965 S1, are also shown, including the observed fragmentation of the 1882 and 1965 objects. The arrow highlights scenario E, the only acceptable evolutionary track we were able to find for C/1970 K1. Time increases from top to bottom.

However, we might consider it in the future, if we are successful in identifying a specific fragmentation hierarchy that links subgroups I and II together.

In the following, we describe in some detail five fragmentation scenarios for the origin of comet White-Ortiz-Bolelli:

A. Separation from Ikeya-Seki after its separation from C/1882 R1.

B. Separation from C/1963 R1 (Pereyra) during its pre-1963 revolution about the Sun.

C. Separation from C/1882 R1 after its separation from Ikeya-Seki.

D. Separation from the comet of 1106 before its splitting into C/1882 R1 and Ikeya-Seki.

E. Separation from an unobserved fragment of the 1106 comet after its separation at or about the same time as C/1882 R1 and Ikeya-Seki.

The five scenarios are schematically presented in Figure 1. It shows the fragmentation events involving the members of the Kreutz system, which we consider as potential sources for comet White-Ortiz-Bolelli, and the evolution of these sungrazers during the past millennium.

The 1970 osculating orbits of the three known candidate parents are listed in Table 3. It is noted that especially for

TABLE 3

ORBITAL ELEMENTS FOR C/1965 S1 (IKEYA-SEKI), C/1963 R1 (PEREYRA), AND C/1882 R1 (GREAT SEPTEMBER COMET) AS POTENTIAL PARENT COMETS FOR C/1970 K1 (EQUINOX J2000.0; EPOCH 1970 MAY 14.0 ET)

Orbital Element	C/1965 S1	C/1963 R1	C/1882 R1
T (ET)	1965 Oct 20.472699 $\pm$ 0.000087	1963 Aug 22.14971 $\pm$ 0.00491	1882 Jul 27.447130 $\pm$ 0.000037
ω (deg)	70.47194 $\pm$ 0.00084	83.7574 $\pm$ 0.0335	66.36954 $\pm$ 0.00179
Ω (deg)	348.69982 $\pm$ 0.00107	4.8576 $\pm$ 0.0489	343.39500 $\pm$ 0.00223
i (deg)	142.34212 $\pm$ 0.00023	144.3486 $\pm$ 0.0055	141.04888 $\pm$ 0.00054
q (AU)	0.00829681 $\pm$ 0.00000034	0.0057611 $\pm$ 0.0000146	0.00979408 $\pm$ 0.00000073
e	0.99990405 $\pm$ 0.00000024	0.9999324 $\pm$ 0.0000005	0.99987863 $\pm$ 0.00000016
P (yr)	804.1 $\pm$ 3.4	786.7 $\pm$ 9.2	724.9 $\pm$ 2.0

C/1882 R1 the 1970 elements differ quite significantly from the near-perihelion elements. At first sight, a separation of C/1970 K1 from Ikeya-Seki appears to be the most probable event because the two comets passed through perihelion within 5 yr of each other and Ikeya-Seki is a member of subgroup II. Since comet Pereyra may have a common parentage with C/1843 D1 (Marsden 1989), scenario B is worth examining as a representative of the scenarios in which White-Ortiz-Bolelli would not derive from subgroup II. Scenario C is included because, in terms of the 1970 osculating angular elements, C/1882 R1 is closer to C/1970 K1 than the other two candidate comets. In scenario D White-Ortiz-Bolelli is assumed to be a first-generation product of the 1106 comet, while in scenarios A, C, and E it is its second-generation product.

An overview table of perturbations of the orbital elements due to nucleus splitting, presented in Paper II, provides us with an excellent diagnostic tool for estimating the location of a fragmentation event along the orbit; for the reader's benefit, this information is reproduced in Table 4. It is apparent from comparison of the orbital sets in Tables 1 and 3 that the differences, between C/1970 K1 on the one hand and C/1882 R1 or C/1965 S1 on the other hand, of $\sim 5^\circ$–10° in ω and Ω, $\sim 2^\circ$–3° in i, and $\sim$0.1–0.2 $R_\odot$ in q ($1\ R_\odot = 0.0046524$ AU is the Sun's radius) are diagnostic, according to Table 4, of a fragmentation event at a very large heliocentric distance, which requires that the fragments arrive at their next perihelion within several years if the event occurred before aphelion, but nearly simultaneously if after aphelion. Thus, with the perihelion time separation of less than 5 yr between C/1965 S1 and C/1970 K1, the criteria in Table 4 indicate that scenario A is clearly preferable to scenario C because the perihelion times of C/1882 R1 and C/1970 K1 differ too much. Scenario B appears to be the least likely of the three because the very large differences between C/1963 R1 and C/1970 K1 in the angular elements and perihelion distance ($>20^\circ$ in ω and Ω, $>5^\circ$ in i, and $\sim\frac{2}{3}\ R_\odot$ in q) point to a fragmentation event near aphelion, which in turn requires that the perihelion be reached by the fragments within $\sim$1 yr, in contradiction with the actual difference of almost 7 yr between the arrivals of the two comets.

The approach employed to search for the solutions in each scenario has been described in detail in § 2 of Paper I. Here we only remark that it consists of (1) an orbit integration code, which computes the motion between two arbitrary epochs within the interval from 3000 B.C.E. to A.D. 3000 using a variable integration step and a prescribed tolerance threshold for error accumulation; and (2) an iterative least-squares differential correction procedure, which provides an efficient technique for deriving the optimized values of up to five parameters introduced in Sekanina's (1978, 1982) model for the split comets. For a pair of fragments, the full-scale version of this model allows the user to determine (1) the time of separation t_s, (2) the components of the separation velocity V_s in three cardinal directions, and (3) the differential deceleration γ. The directions defined by the heliocentric orbit of the parent comet are the radial (away from the Sun), transverse, and normal directions of the right-handed RTN coordinate system. The respective components of the separation velocity are V_R, V_T, and V_N. The iterative search procedure offers an option to solve for any combination of fewer than the five parameters, which proves extremely convenient when the convergence is slow.

In each proposed scenario, the orbital elements of the presumed parent comet serve as the initial conditions for the search procedure, whereas the orbital elements of comet White-Ortiz-Bolelli represent proxy observations. Because of the uncertainty in the orbital eccentricity of C/1970 K1 (Table 1), only the remaining five elements are used in our effort to determine the separation parameters for each of the five candidate fragmentation scenarios. To make the comparison among the scenarios more straightforward, we use what in Paper I was called system II, in which the individual elements enter the search procedure with their weights based on the formal errors of the secondary fragment, in this case C/1970 K1. Thus, regardless of the identity of the parent comet, the weighting system remains the same for all the candidate scenarios. As in Paper I, the set of separation parameters is obtained when the solution converges. The

TABLE 4

Overview of Perturbations of Orbital Elements of Fragments due to Separation Velocity of $\sim$5 m s^{-1}

Fragments that are products of a breakup...	...Reach next perihelion...	...In orbits that are...
At or shortly after perihelion	At considerably different times (up to two millennia apart)	Identical in ω, Ω, i, and q, but very different in P
Near 1 AU after perihelion	Less than one century apart	Slightly different in ω, Ω, i ($\ll 1^\circ$), and q ($<0.01\ R_\odot$), but moderately different in P
At heliocentric distance of tens of AU after perihelion	Tens of years apart	Somewhat different in ω, Ω (both up to a few degrees), i (up to $\sim 1^\circ$), and q (up to $\sim 0.1\ R_\odot$), but fairly similar in P
At very large heliocentric distance preaphelion	Several years apart	Significantly different in ω, Ω (both $\sim 10^\circ$), i (a few degrees), and q (up to $\sim 0.5\ R_\odot$), but very similar in P
In general proximity of aphelion	1 yr or less apart	Very different in ω, Ω, i, and q, but virtually identical in P
After aphelion	Almost simultaneously, not more than weeks apart	Determined by very approximate symmetry of perturbations relative to aphelion, except for those of perihelion time

quality of the fit is expressed in terms of $[\mathscr{P}_0]$, the sum of squares of the residuals between the normalized observed elements of comet C/1970 K1 (i.e., the elements divided by their standard deviations) and its normalized elements derived from the fragmentation model. Since the eccentricity is not employed in optimizing the fragmentation solutions, the acceptable values for the sum of squares of the residuals must satisfy the condition $[\mathscr{P}_0] \leqslant 5$ (see Paper I for details). In addition, all solutions with separation velocities exceeding 10 m s^{-1} are rejected because such velocities are deemed unrealistic. Together, these two conditions provide us with a powerful tool for constraining the genuine fragmentation scenarios.

To make the problem tractable, we introduced two assumptions. First, we neglect the effect of differential deceleration between the fragments. Because of the nature of the sungrazer orbits, the deceleration has practically no effect on the normal component of the separation velocity and, except when the separation occurs very close to perihelion, also no effect on the transverse component. The deceleration is highly correlated with the velocity's radial component, but dynamically a relative deceleration of $\sim 10^{-4}$ the solar gravitational acceleration, appropriate for a sizable fragment like C/1970 K1, is equivalent to only a very small fraction of 1 m s^{-1} in V_R, an insignificant amount considering that the separation velocity is on the order of several meters per second. Ignoring the deceleration, only four separation parameters—t_s, V_R, V_T, and V_N—remain to be determined. Unlike in Paper I, three-parameter solutions are now found to converge rapidly (usually in not more than four iterations), so that the variations in $[\mathscr{P}_0]$ with the separation time t_s can readily be obtained and the optimum solution determined from the minimum value of $[\mathscr{P}_0]$.

Second, we postulate that because of the presumed considerable nuclear mass difference between C/1882 R1 and C/1965 S1 (cf. Papers I and II), the entire momentum change during their 1106 breakup, corresponding to the separation velocity of ~ 7 m s^{-1}, was acquired by Ikeya-Seki. The orbital motion of the center of mass of the 1882 comet after the breakup, as derived by us in Paper I, can on this assumption be used to describe the orbital motion of the 1106 comet before its splitting, so that scenarios C and D effectively merge into one.

3.1. *Scenario A*

Table 5 shows that our analysis fails to support the hypothesis that comet White-Ortiz-Bolelli is a fragment of Ikeya-Seki. The best match between the orbital elements for C/1970 K1 derived from the observations on the one hand and from this scenario on the other hand ($[\mathscr{P}_0]_{\rm min} = 2.69$) occurs some 630 yr after aphelion (highlighted with an asterisk in Table 5). One needs a totally unacceptable separation velocity exceeding 70 m s^{-1} (mostly in the radial direction) to account for the nearly 5 yr difference between the perihelion times of the two objects. In addition, whereas for fragmentation events during the first 100 yr or so after the 1106 perihelion the calculated separation velocity is in a reasonable range, the residuals for any such event are, especially in the angular elements, much too high. For a 1206 breakup, they still reach as much as $0\overset{\circ}{.}13$ in the argument of perihelion and $0\overset{\circ}{.}08$ in the inclination, i.e., almost 4 times the standard deviation in either case, with $[\mathscr{P}_0] \simeq 31$.

An introduction of a moderate differential acceleration of 10^{-4} the solar attraction changed the radial component by about 0.1 m s^{-1}, the transverse component by 0.001 m s^{-1}, and the normal component by 0.0004 m s^{-1}, hence marginally at most, as expected.

3.2. *Scenario B*

We encountered a much more severe case of the same problem when testing the hypothesis of White-Ortiz-Bolelli having separated from comet Pereyra. The sum of squares of the residuals decreased from $[\mathscr{P}_0] \simeq 230$ for a separation time at the beginning of the year 1300 down to ~ 43 at the beginning of 1800, with the corresponding separation velocity increasing from ~ 11 to ~ 180 m s^{-1}, again mainly in the radial direction.

The primary contributors to the large values of $[\mathscr{P}_0]$ were the residuals of the argument of perihelion, which varied between $-0\overset{\circ}{.}38$ and $-0\overset{\circ}{.}14$, and of the longitude of the ascending node, which were near $0\overset{\circ}{.}2$ throughout the five

TABLE 5

SEPARATION TIME–DEPENDENT SOLUTIONS FOR FRAGMENTATION SCENARIO A (C/1970 K1 RELATIVE TO C/1965 S1)

Separation Time[a] (yr)	Distance at Separation (AU)		Sum of Squares of Residuals[c]	Separation Velocity (m s^{-1})			
	From Sun	From Ecliptic[b]		Total	V_R	V_T	V_N
100	103.9	−59.5	31.01	7.78	6.20	2.87	3.73
200	146.8	−84.3	16.46	10.67	10.14	2.68	1.96
300	170.5	−98.0	10.06	15.50	15.23	0.95	2.70
400	180.4	−103.8	6.32	22.95	22.79	2.58	0.82
500	177.9	−102.5	3.96	35.53	35.43	1.37	2.36
600	162.8	−93.9	2.78	60.43	60.36	2.79	1.10
620	158.1	−91.2	2.70	68.47	68.40	1.27	2.81
630.9*	155.3	−89.6	2.69*	73.56	73.49	1.45	2.79
640	152.8	−88.1	2.70	78.27	78.20	2.37	2.14
660	146.7	−84.7	2.79	90.41	90.35	3.08	1.25
700	132.4	−76.5	3.31	125.83	125.78	2.54	2.66

[a] Reckoned from the time of perihelion passage on 1106 January 26.5 ET; the positive values indicate postperihelion times.

[b] The negative values indicate distances measured in the direction of the south pole of the ecliptic.

[c] Dimensionless sum of squares of the residuals $[\mathscr{P}_0]$ in units of the standard deviation for all five elements used; if the residual in each element should equal the standard deviation, then $[\mathscr{P}_0] = 5$.

centuries. The optimization process was then terminated, with the conclusion that White-Ortiz-Bolelli not only could not separate from Pereyra but also could be neither a first-generation nor, very probably, a second-generation product of any known member of subgroup I.

3.3. *Scenarios C and D*

A tidal, near-perihelion fragmentation event is ruled out by the differences between C/1882 R1 and C/1970 K1 in the angular elements and perihelion distance. We explored the possibilities that White-Ortiz-Bolelli separated (1) from C/1882 R1 some 10–100 yr after its parent's perihelion in 1106 (Scenario C) or (2) from the 1106 comet between the years 600 and 1080 (scenario D).

All scenario C results were found to be unacceptable. In the investigated time period of 90 yr, the sum of squares of the residuals was reduced from $\sim$156 for a separation time 10 yr after perihelion to 37 for 100 yr after perihelion, but the required separation velocity increased during this time from 50 to $\sim$130 m s^{-1}. In some runs it was not only the radial but also the transverse and normal components that were too high. Like for scenario A, the poorest match was in the argument of perihelion and the inclination.

The scenario D results were marginally better but still unacceptable. We experienced severe convergence problems for all separation times before the year 900. However, we are confident that we did not miss any acceptable solution because the variations in the sum of squares of the residuals displayed a broad minimum of $\sim$23 between the years 1000 and 1050. The required separation velocity reached at that time about 73 m s^{-1}. All three angular elements yielded residuals that exceeded their standard deviations.

3.4. *Scenario E*

Whereas the number of options we have in this case is obviously infinite, we focus on finding a first-generation parent of comet White-Ortiz-Bolelli that separated from the 1106 comet either (1) together with C/1882 R1 and C/1965 S1, but with a different separation velocity in the same direction as the latter (scenario E$_1$); or (2) somewhat earlier, with the separation velocity identical with that of Ikeya-Seki (scenario E$_2$). The parent is unlikely to have separated after the breakup of the 1882 and 1965 comets because it would have to have a velocity higher than 7 m s^{-1}, thus being at best on the verge of the adopted velocity cutoff.

Before describing the two search efforts, we note that the solutions have to satisfy a further condition. The parent to C/1970 K1 must have been a bright comet that could not escape attention of the observers, unless it reached perihelion between late May and mid-August. The comet's orbit would then be approximately aligned with the subsolar meridian both before and after perihelion, and the object could only be seen in daylight, which would require it to be truly spectacular. Even if this parent should be somewhat brighter than Ikeya-Seki, for example, the comet would hardly be detected in daylight with no advance notice. The line of apsides of the Kreutz system dictates that the most unfavorable observing conditions occur annually for objects with perihelia in early July (0.51 of a year from its beginning), and the full width of the window is conservatively estimated at 0.24 yr.

Although scenarios A–D all failed, they provide us with very compelling information on the source of the failure. The angular elements and the perihelion distance of C/1970 K1 differ from those of the other sungrazers considered as the candidate parents (cf. Table 3 with Table 1) to such a degree that the difference can only be explained by a breakup at large heliocentric distance, as demanded by the constraints in Table 4. Since it is always the radial component of the separation velocity that is unacceptably high and since in extremely elongated orbits the radial component determines almost exclusively the time of next perihelion (Paper II), it appears that the genuine parent of White-Ortiz-Bolelli should be at perihelion at approximately the same time as the fragment. In the following we consider the years 1969, 1970, and 1971. In conformity with the seasonal constraint above, the search for appropriate perihelion times of the parent is therefore limited to the intervals 1969.39–1969.63, 1970.39–1970.63, and 1971.39–1971.63.

3.4.1. *Scenario E$_1$*

If the parent of C/1970 K1 splits off from the comet of 1106 at the same time as Ikeya-Seki, its separation velocity should somewhat exceed the latter fragment's 7.08 m s^{-1}. We searched for the optimized solutions in five selected scenarios, which we call, respectively, E$_{11}$, . . ., E$_{15}$: one in 1969, three in 1970, and one in 1971. The principal parameters for both the parent comet and White-Ortiz-Bolelli are listed in columns (2)–(6) of Table 6, while the parent's orbital elements at their 1969–1971 epochs are presented in Table 7.

While each scenario yields a sum of squares of the residuals $[\mathscr{P}_0]_{\min}$ that is entirely satisfactory, only scenarios E$_{12}$–E$_{14}$ offer an acceptably low separation velocity. Indeed, the correlation between the parent's perihelion time and the separation velocity limits acceptable perihelion times ($V_s \leq 10$ m s^{-1}) to an interval from 1969.8 to 1970.9, approximately centered on the perihelion time of C/1970 K1. Combined with the seasonal constraint, this condition restricts the parent's perihelion time in the E$_1$ scenarios to a period of 1970.39–1970.63.

In the E$_1$ scenarios, the parent's velocity of separation from the 1106 comet was only $\sim$0.4 m s^{-1} higher than Ikeya-Seki's. Comet C/1970 K1 broke off from its parent almost six and a half centuries later, long after aphelion (but still $\sim$150 AU from the Sun), corresponding to nearly exactly the mid-eighteenth century. Within these scenarios, the uncertainty in the separation time is only about $\pm$1 month. The relative velocity acquired by White-Ortiz-Bolelli at separation is some 3.2–5.7 m s^{-1}, depending on the event's timing.

3.4.2. *Scenario E$_2$*

We now searched for the optimized solutions in three selected scenarios, one each in 1969, 1970, and 1971. We call them, respectively, E$_{21}$, E$_{22}$, and E$_{23}$. The parameters for the parent and White-Ortiz-Bolelli are listed in columns (7)–(9) of Table 6, while the parent's orbital elements at their 1969–1971 epochs are in Table 8.

Table 6 indicates that taking Ikeya-Seki's separation velocity for the release of C/1970 K1's parent from the 1106 comet requires that the breakup occur 15–16 days after perihelion instead of 18 days. The table also shows that scenarios E$_1$ and E$_2$ offer remarkably similar results: the sum of squares of the residuals is equally satisfactory; the middle (E$_{22}$) entry is the only one yielding an acceptably low separation velocity for C/1970 K1, so that the parent's perihelion

TABLE 6

PARAMETERS FOR SELECTED FRAGMENTATION SCENARIOS E_{11}–E_{15} AND E_{21}–E_{23}

PARAMETER (1)	SCENARIO E_{11} (2)	E_{12} (3)	E_{13} (4)	E_{14} (5)	E_{15} (6)	E_{21} (7)	E_{22} (8)	E_{23} (9)
Parent of C/1970 K1 (relative to 1106)								
Time of separation:								
Days after 1106 perihelion	18.0	18.0	18.0	18.0	18.0	16.0	15.5	15.0
Date 1106 Feb (ET, old style)	13.5	13.5	13.5	13.5	13.5	11.5	11.0	10.5
Heliocentric distance (AU)	0.747	0.747	0.747	0.747	0.747	0.690	0.675	0.661
Distance from ecliptic (AU)	−0.385	−0.385	−0.385	−0.385	−0.385	−0.354	−0.346	−0.338
Separation velocity (m s^{-1}):								
Radial component, V_R	7.33	7.40_5	7.41	7.41_5	7.49	7.04	7.04	7.04
Transverse component, V_T	0.75	0.76	0.76	0.76	0.77	0.72	0.72	0.72
Normal component, V_N	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total velocity	7.37	7.44	7.45	7.45	7.53	7.08	7.08	7.08
Next perihelion passage[a]	1969.51	1970.48	1970.54	1970.61	1971.58	1969.45	1970.47	1971.55
Comet C/1970 K1 (relative to Parent)								
Time of separation:								
Years after 1106 perihelion	643.20	643.21	643.18	643.12	642.02	642.94	642.89	641.68
Year and decimal of year (ET)	1749.30	1749.31	1749.29	1749.23	1748.13	1749.05	1749.00	1747.79
Heliocentric distance (AU)	153.1	153.4	153.4	153.4	154.1	153.1	153.5	154.1
Distance from ecliptic (AU)	−88.3	−88.5	−88.5	−88.5	−88.9	−88.3	−88.5	−88.9
Separation velocity (m s^{-1})								
Radial component, V_R	14.89	−2.02	−3.13	−4.24	−20.93	15.81	−1.93	−20.22
Transverse component, V_T	1.56	1.56	1.56	1.57	1.81	1.62	1.62	1.90
Normal component, V_N	2.77	2.77	2.76	2.76	2.59	2.74	2.73	2.53
Total velocity	15.23	3.77	4.46	5.30	21.17	16.13	3.72	20.47
Sum of squares of residuals, $[\mathscr{P}_0]$	1.863	1.670	1.659	1.646	1.469	1.864	1.659	1.462

[a] For full-precision information, see Tables 7 and 8.

TABLE 7

ORBITAL ELEMENTS FOR POTENTIAL PARENT FRAGMENTS OF C/1970 K1 IN SCENARIOS E_{11}–E_{15} (EQUINOX J2000.0)

Orbital Element	Scenario E_{11}	Scenario E_{12}	Scenario E_{13}	Scenario E_{14}	Scenario E_{15}
T (ET)	1969 Jul 4.43906	1970 Jun 25.05619	1970 Jul 18.46212	1970 Aug 10.87231	1971 Aug 2.25853
ω (deg)	67.3587	66.8601	66.8421	66.8246	66.7650
Ω (deg)	344.7593	344.1822	344.1624	344.1451	344.1237
i (deg)	141.4170	141.2773	141.2720	141.2672	141.2497
q (AU)	0.00725665	0.00741408	0.00742659	0.00743918	0.00763576
e	0.99992606	0.99992356	0.99992333	0.99992308	0.99991960
P (yr)	972.3	955.2	953.3	951.1	925.5
Epoch (ET)	1969 Jun 28.0	1970 Jun 23.0	1970 Aug 2.0	1970 Aug 2.0	1971 Jul 28.0

TABLE 8

ORBITAL ELEMENTS FOR POTENTIAL PARENT FRAGMENTS OF C/1970 K1 IN SCENARIOS E_{21}–E_{23} (EQUINOX J2000.0)

Orbital Element	Scenario E_{21}	Scenario E_{22}	Scenario E_{23}
T (ET)	1969 Jun 14.68504	1970 Jun 3.16371	1971 Jul 18.93797
ω (deg)	67.3988	66.8615	66.7609
Ω (deg)	344.8066	344.1838	344.1160
i (deg)	141.4280	141.2777	141.2485
q (AU)	0.00724926	0.00741282	0.00762747
e	0.99992622	0.99992264	0.99991974
P (yr)	973.9	938.0	926.5
Epoch (ET)	1969 Jun 28.0	1970 Jun 23.0	1971 Jul 28.0

TABLE 9
ORBITAL ELEMENTS FOR COMETS C/1843 D1 (GREAT MARCH COMET) AND C/1880 C1 (GREAT SOUTHERN COMET) (EQUINOX J2000.0)

Orbital Element (1)	Comet C/1843 D1 (2)	Comet C/1880 C1 (3)
Time of perihelion passage T (ET)	1843 Feb. 27.91434 ± 0.00120	1880 Jan 28.09679
Argument of perihelion ω (deg)	82.8063 ± 0.0600	85.1285
Longitude of ascending node Ω (deg)	3.7283 ± 0.0735	6.4762
Orbital inclination i (deg)	144.3893 ± 0.0091	144.5226
Perihelion distance q (AU)	0.0054897 ± 0.0000161	0.0055347
Orbital eccentricity e	0.9999363[a]	0.9999358[a]
Orbital period P (yr)	800[a]	800[a]
Epoch (ET)	1843 Mar 21.0	1880 Feb 15.0

[a] Forced element.

time is once again limited to the 1970 window; and, most astonishingly, the derived time of separation of C/1970 K1 from its parent differs from the time found from E_{12}–E_{14} by just a small fraction of a year! (Note also that this separation time agrees to within 12 yr or so with the time of the best orbital match between White-Ortiz-Bolelli and Ikeya-Seki in scenario A.) We interpolate that the separation velocity now ranges from 3.2 to 5.5 m s^{-1}, again depending on the timing of the breakup.

4. ORBITS AND RELATIONSHIP OF SUNGRAZERS C/1843 D1 AND C/1880 C1

This is another pair of sungrazers with similar orbits, even though not as similar as C/1882 R1 and C/1965 S1. Both C/1843 D1 and C/1880 C1 belong to subgroup I. Even though already addressed in the past, the problem of their common origin still presents a major challenge.

Several orbits for C/1843 D1 and C/1880 C1 were calculated by Kreutz (1901). Some of them are of historical value only, but his "definitive" elements for C/1843 D1 and his "most probable" parabola for C/1880 C1 are still listed in the latest edition of the comet catalog by Marsden & Williams (2001). Kreutz's other useful sets of elements are those derived for either comet on the assumption that the orbital period was 800 yr.

In a major effort to understand the orbital evolution of the Kreutz system, Marsden (1989) examined the relationship between C/1843 D1 and C/1880 C1. He assumed that all sungrazers continue to split at perihelion and believed that all orbital differences among the sungrazers can be accounted for by the indirect planetary perturbations acting over sufficiently long periods of time. He was able to solve the puzzle of the 1843 and 1880 comets only by postulating that they both had orbital periods shorter than 400 yr. He could not match Kreutz orbits for these objects derived for an assumed orbital period of 800 yr. Since no other studies of this kind have been published, we decided to search for a scenario to show that the relationship of the two objects is independent of the constraint on the orbital period.

We began this part of our investigation by precessing Kreutz's elements for the two comets (for an orbital period of 800 yr) to the equinox of J2000.0. The results are in Table 9. Since the planetary perturbations were unaccounted for by Kreutz, we assumed that the elements refer in either case to a standard epoch nearest the middle of the observed arc.

The integration of the orbit of C/1843 D1 back in time gave for the previous perihelion the date of 1048 November 30, which is as uncertain as the orbital period. Although Kreutz (1901) claims that the standard deviation of his most probable orbital period of 512 yr is ±71 yr, the actual error must be at least $\pm 3\ \sigma$. While in 1048 there are no candidates for possible Kreutz sungrazers found in Hasegawa & Nakano's (2001) list based on Chinese, Japanese, Korean, and European historical records, there is one in 1041 and another one in 1034. Our experience with the pair of C/1882 R1 and C/1965 S1 shows, however, that the results are insensitive to the exact choice of the perihelion time and that the tolerance is at least tens of years. Furthermore, while the orbits calculated for different orbital periods would by no means be identical, in the particular case of the year 1048 the calculated perihelion point is located merely 0.00491 AU from the Sun's center, that is, less than 40,000 km above the photosphere.

The orbital elements for C/1880 C1 are of somewhat lower accuracy than those for C/1843 D1, and they were not integrated back in time. They served as the proxy observations in our search technique, except, again, for the forced eccentricity. Since Kreutz published the standard deviations of the orbital elements only for C/1843 D1, we used those to weight the derived elements of C/1880 C1 in our search for an optimum orbital solution, thus applying what in Paper I was called weighting system I. As with C/1970 K1, we applied a four-parameter model and found that it was again possible to solve for up to three parameters at the same time (albeit the solutions were converging more slowly), so the fragmentation parameters were obtained in a manner identical with that for comet White-Ortiz-Bolelli (see § 3).

Table 10 presents the derived separation parameters for C/1880 C1 and C/1843 D1. We find that this fragmentation

TABLE 10
PARAMETERS OF FRAGMENTATION EVENT FOR C/1880 C1 RELATIVE TO C/1843 D1

Fragmentation Parameter	Value
Time of fragmentation:	
Days after 1048 perihelion, $t_s - T$	127 ± 46
Date (ET, old style)	1049 Apr 6
Heliocentric distance (AU)	2.76 ± 0.66
Distance from ecliptic (AU)	−1.58 ± 0.38
Separation velocity (m s^{-1}):	
Radial component, V_R	6.04 ± 0.79
Transverse component, V_T	2.55 ± 0.59
Normal component, V_N	−3.37 ± 0.80
Total velocity	7.37 ± 0.77

event occurred longer after perihelion than the breakup of C/1882 R1 and C/1965 S1, but with a similar separation velocity. Thus, it is possible to account for C/1843 D1 and C/1880 C1 as fragments of the same parent object even if their orbital period is ~800 yr.

5. SUMMARY AND CONCLUSIONS

Table 11 summarizes information on the match to the orbital elements for both C/1970 K1 and C/1880 C1 employed as the proxy observations to search for the fragmentation events. C/1970 K1 is represented by scenarios E_{14} and E_{22}, which, in part because of the difference in the parent's perihelion time (close to the boundaries of the window), nearly bracket all scenario E solutions.

Although the eccentricity was not used in the search procedure for either C/1970 K1 or C/1880 C1, the other orbital elements of White-Ortiz-Bolelli employed in the iterations were repeatedly adjusted by applying the eccentricity-dependent rates of variation from Table 1, as dictated by the eccentricity derived from the solutions. For this reason the (parenthesized) residuals of the eccentricity in Table 11 are necessarily zero. For C/1880 C1, the orbital elements used were those from Table 9.

The derived timing of the fragmentation event involving C/1843 D1 and C/1880 C1 indicates a postperihelion delay ~7 times longer than the delay of the separation of C/1882 R1 and C/1965 S1, thus confirming our previous conclusion (e.g., Paper II) that fragmentation of a sungrazer does not terminate with tidal splitting in the immediate proximity of the Sun but tapers off in a quasi-stochastic fashion in a long sequence of nontidal (or post-tidal) events. The results for this pair, in which C/1843 D1 was intrinsically much brighter, remind us of the physical developments observed in comet D/1993 F2 (Shoemaker-Levy 9). On the other hand, the birth of White-Ortiz-Bolelli resembles fragmentation events experienced by minor sungrazers of the *SOHO* type.

Remarkably, the separation velocities are consistently near, or below, 7 m s^{-1}. It is increasingly likely that this effect is a product of the distribution of angular momentum of the Kreutz system among fragments.

Given that sufficiently accurate orbits are known for only one member of subgroup IIa, two members of subgroup II, and three members of subgroup I, we find, among these sungrazers, no direct, first-generation relationship between subgroups II and IIa, and certainly no such relationship between subgroups I and IIa. However, we suggest a very reasonable indirect relationship between the known members of subgroups II and IIa: the proposed unknown parent of C/1970 K1, if born together with C/1882 R1 and Ikeya-Seki, needed to acquire at its separation a velocity only a fraction of 1 m s^{-1} greater than Ikeya-Seki.

TABLE 11

ORBITAL ELEMENTS FOR COMETS C/1970 K1 AND C/1880 C1 DERIVED FROM FRAGMENTATION SOLUTIONS (EQUINOX J2000.0)

	COMET C/1970 K1		
ORBITAL ELEMENT	Scenario E_{14}	Scenario E_{22}	COMET C/1880 C1
	Derived Orbital Elements		
T (ET)	1970 May 14.48735	1970 May 14.48735	1880 Jan 28.09679
ω (deg)	61.3335	61.3338	85.1086
Ω (deg)	337.0267	337.0265	6.4995
i (deg)	139.0824	139.0823	144.5228
q (AU)	0.0088966	0.0088966	0.0055354
e	0.9999084	0.9999083	0.9999371
P (yr)	957.2	955.6	825.6
Epoch (ET)	1970 May 14.0	1970 May 14.0	1880 Feb 15.0
	Residuals: Observed Minus Derived Orbital Elements[a]		
T (days)	0.00000	0.00000	0.00000
ω (deg)	−0.0220	−0.0223	0.0199
Ω (deg)	0.0452	0.0454	−0.0233
i (deg)	−0.0091	−0.0090	−0.0002
q (AU)	0.0000000	0.0000000	−0.0000007
e	(0.0000000)	(0.0000000)	(−0.0000013)
	Dimensionless Normalized Residuals		
T	0.00	0.00	0.00
ω (deg)	−0.66	−0.67	0.33
Ω (deg)	1.01	1.01	−0.32
i (deg)	−0.44	−0.43	−0.02
q (AU)	0.00	0.00	−0.04
	Sum of Squares of Dimensionless Residuals		
$[\mathscr{P}_0]$	1.646	1.659	0.212

[a] Residuals left by subtracting the derived elements in this table from the elements found from information in cols. (3) and (4) of Table 1 for C/1970 K1 and in col. (3) of Table 9 for C/1880 C1. For the eccentricity, the residuals are in parentheses to show that this element was not used in the solutions.

It is interesting to compare our results with Marsden's (1989) ideas about the evolution of the Kreutz system. In his Figure 7, he speculates that subgroup IIa evolved entirely independently of subgroup II, starting with an ancient ancestor, presumably the famous Aristotle comet of 371 B.C.E. Thus, in Marsden's scenario, White-Ortiz-Bolelli has not evolved from the 1106 comet and may be as much as 2.4 millenia old. By contrast, our results suggest that, as a separate object, this comet has existed for only 250 yr!

All evidence from this study supports the conceptual model of runaway fragmentation. However, the critical issue that eventually will need to be addressed is the evolution of the Kreutz system long before the year 1106. The problem is difficult not only because its solution is computer time intensive, but primarily because of the increasingly complex fragmentation hierarchy that one confronts when proceeding further into the past. The availability of "check points," like the comet of 1106, is critical for this research. Searches in historical records, like that by Hasegawa & Nakano (2001), are clearly helpful, but no other "beacon" as obvious as the 1106 comet has so far been recognized. Because of these difficulties, it seems prudent to develop the model for the fragmentation hierarchy gradually from the present to the past.

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