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THE $M_\bullet$-σ RELATION FOR SUPERMASSIVE BLACK HOLES

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ABSTRACT

We investigate the differences in the $M_\bullet$-σ relation derived recently by Ferrarese & Merritt and by Gebhardt and collaborators. The shallower slope found by the latter authors (3.75 vs. 4.8) is due partly to the use of a regression algorithm that ignores measurement errors and partly to the value of the velocity dispersion adopted for a single galaxy, the Milky Way. A steeper relation is shown to provide a better fit to black hole masses derived from reverberation mapping studies. Combining the stellar dynamical, gasdynamical, and reverberation mapping mass estimates, we derive a best-fit relation $M_\bullet = 1.30(\pm 0.36) \times 10^8\ M_\odot(\sigma_c/200\ \mathrm{km\ s^{-1}})^{4.72(\pm 0.36)}$.

Subject headings: black hole physics — galaxies: kinematics and dynamics — methods: data analysis

1. INTRODUCTION

Ferrarese & Merritt (2000, hereafter FM00) demonstrated a tight correlation between the masses of supermassive black holes (BHs) and the velocity dispersions of their host bulges, $M_\bullet \propto \sigma^\alpha$, $\alpha = 4.8 \pm 0.5$. The scatter in the relation was found to be consistent with that expected on the basis of measurement errors alone; in other words, the underlying correlation between σ and $M_\bullet$ is essentially perfect. The relation is apparently so tight that it surpasses in predictive accuracy what can be achieved from detailed dynamical modeling of stellar kinematical data in most galaxies. As an example, FM00 showed that the BH mass estimates of Magorrian et al. (1998), derived from ground-based optical observations, lie systematically above the $M_\bullet$-σ relation defined by galaxies with secure BH masses, some by as much as 2 orders of magnitude.

The $M_\bullet$-σ relation of FM00 was based on central velocity dispersions σ_c, corrected to an effective aperture of radius $r_e/8$, with r_e the half-light radius. Central velocity dispersions are easily measured and available for a large number of galaxies (Prugniel et al. 1998). An alternative form of the $M_\bullet$-σ relation was investigated by Gebhardt et al. (2000a, hereafter G00) who used σ_e as the independent variable; σ_e was defined as the spatially averaged, rms, line-of-sight stellar velocity within the effective radius r_e. Computing σ_e requires knowledge of the stellar rotation and velocity dispersion profiles at all radii within r_e, as well as information about the inclination of the rotation axis with respect to the line of sight. These data are available for a smaller number of galaxies; on the other hand, σ_e might be expected to reflect the depth of the stellar potential well more accurately than σ_c.

The versions of the $M_\bullet$-σ relation derived by FM00 and by G00 differ in two important ways: the latter authors found a significantly smaller slope ($\alpha = 3.75 \pm 0.3$ vs. 4.8 ± 0.5) as well as a greater vertical scatter—greater both in an absolute sense and relative to measurement errors in $M_\bullet$. G00 estimated that approximately 40% of the scatter in $M_\bullet$ about the mean line was intrinsic and the remainder due to measurement errors. FM00 found no evidence for an intrinsic scatter in $M_\bullet$.

The $M_\bullet$-σ relation is currently our best guide to BH demographics, and it is important to understand the source of these differences. That is the goal of this paper. In addition to using different measures of the velocity dispersion, FM00 and G00 analyzed different galaxy samples and used different algorithms for fitting regression lines to the data. We find that regression algorithms that account correctly for errors in the measured variables always give a steeper slope than that found by G00. We also show that the steeper relation derived by FM00 provides a better fit to galaxies with BH masses computed by reverberation mapping.

2. DATA

Table 1 gives the data used here. The first 12 galaxies (sample 1) are "Sample A" from FM00, consisting of those galaxies with published BH mass estimates that were deemed reliable— roughly speaking, galaxies in which the sphere of influence of the BH has been resolved. Five of these masses are derived from stellar kinematics and seven from gasdynamics. All of these galaxies were included in the G00 sample as well, with the exception of NGC 3115; for this galaxy, we assume $\sigma_e = \sigma_c$. The second part of Table 1 contains the additional 15 galaxies included by G00 (sample 2).[1] Most of the BH mass estimates for these galaxies are based on unpublished Space Telescope Imaging Spectrograph (STIS) data. In addition, G00 included M31 and NGC 1068, which were excluded from FM00 on the grounds that their BH masses were deemed unreliable. We computed distances for the G00 galaxies in Table 1 in the same way as in FM00 and corrected the BH masses accordingly. We also computed aperture-corrected central dispersions σ_c for the G00 galaxies.

At this point, we are already in a position to test the idea, proposed by G00, that the steeper slope of the $M_\bullet$-σ_c relation in FM00 is due to spuriously high values of σ_c for the more nearby galaxies. This idea is rejected based on Figure 1, which shows that there is remarkably little difference on average between σ_c and σ_e. This is presumably due to the flatness of galaxy rotation and velocity dispersion profiles and to the fact that even σ_c is measured on a large enough scale that it is essentially unaffected by the presence of the BH. The mean ratio of σ_e to σ_c is 1.01; the correlation coefficient of log σ_e versus log σ_c is 0.97.

However, we notice that σ_e and σ_c differ significantly for one particular galaxy, the Milky Way. G00 adopted a value

[1] The error bars plotted in Fig. 2 of G00 do not always correspond to the values listed in their Table 1 (e.g., NGC 4291 and NGC 5845). We used the tabulated values.

TABLE 1

Black Hole Mass Estimates and Galaxy Velocity Dispersions

Galaxy	Type	Distance	$M_\bullet$	σ_c	σ_e
		Sample 1 (FM00)			
MW	SbI-II	0.008 ± 0.0009	0.0295 ± 0.0035	100 ± 20	75
I1459	E3	30.3 ± 4.0	4.6 ± 2.8	312 ± 41	323
N221	cE2	0.8 ± 0.1	0.039 ± 0.009	76 ± 10	75
N3115......	S0$^-$	9.8 ± 0.6	9.2 ± 3.0	278 ± 36	...
N3379......	E1	10.8 ± 0.7	1.35 ± 0.73	201 ± 26	206
N4258......	SAB(s)bc	7.2 ± 0.3	0.390 ± 0.034	138 ± 18	120
N4261......	E2	33.0 ± 3.2	$5.4^{+1.2}_{-1.2}$	290 ± 38	315
N4342......	S0$^-$	16.7 ± 1.0	$3.3^{+1.9}_{-1.1}$	261 ± 34	225
N4374......	E1	18.7 ± 1.2	$17^{+12}_{-6.7}$	286 ± 37	296
N4486......	E0pec	16.7 ± 1.0	35.7 ± 10.2	345 ± 45	375
N6251......	E	104 ± 10	5.9 ± 2.0	297 ± 39	290
N7052......	E	66.1 ± 6.4	$3.7^{+2.6}_{-1.5}$	261 ± 34	266
		Sample 2 (G00)			
N821	E6	24.7 ± 2.5	0.51 ± 0.2	196 ± 26	209
N224	Sb	0.77 ± 0.04	0.35 ± 0.25	112 ± 15	160
N1023......	S0	10.7 ± 0.8	$0.39^{+0.09}_{-0.11}$	201 ± 14	205
N1068......	Sb	23.6 ± 3.2	$0.17^{+0.13}_{-0.07}$	149 ± 19	151
N2778......	E	23.3 ± 3.4	$0.20^{+0.16}_{-0.13}$	171 ± 22	175
N3377......	E5+	11.6 ± 0.6	$1.03^{+1.6}_{-0.41}$	131 ± 17	145
N3384......	SB(s)0$^-$	11.9 ± 0.9	$0.185^{+0.072}_{-0.091}$	151 ± 20	143
N3608......	E2	23.6 ± 1.5	$1.13^{+1.44}_{-0.31}$	206 ± 27	182
N4291......	E	26.9 ± 4.1	$1.54^{+3.1}_{-0.68}$	269 ± 35	242
N4473......	E5	16.1 ± 1.1	$1.026^{+0.82}_{-0.71}$	188 ± 25	190
N4564......	E	14.9 ± 1.2	$0.57^{+0.13}_{-0.17}$	153 ± 20	162
N4649......	E2	17.3 ± 1.3	$20.6^{+5.2}_{-10.2}$	331 ± 43	375
N4697......	E6	11.9 ± 0.8	$1.22^{+0.10}_{-0.40}$	163 ± 21	177
N5845......	E*	28.5 ± 4.2	$3.52^{+2.0}_{-0.72}$	275 ± 36	234
N7457......	SA(rs)0$^-$	13.5 ± 1.4	$0.035^{+0.027}_{-0.017}$	73 ± 10	67

Notes.—Type is revised Hubble type. Black hole masses are in units of $10^8\ M_\odot$. Distances are in units of megaparsecs. Velocity dispersions are in units of kilometers per second.

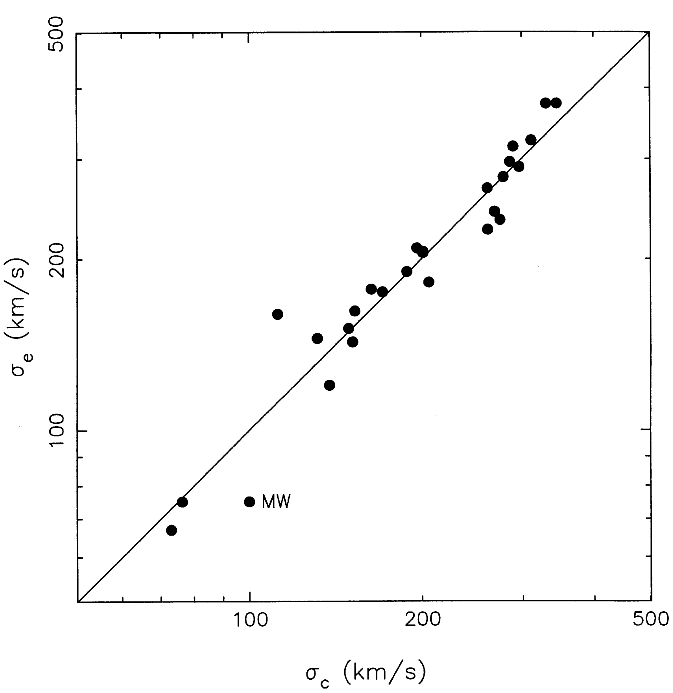

Fig. 1.—Comparison of σ_c (the central velocity dispersion) and σ_e (the rms velocity within 1 effective radius) for the galaxies in Table 1. The solid line has a slope of 1.

of $\sigma_e = 75$ km s^{-1} for the Galaxy based on the velocity dispersion between 50″ and 500″ (Kent 1992; Genzel et al. 2000). They apparently neglected to account for the contribution of the rotational velocity, which in the same region is 103 ± 15 km s^{-1} (Kent 1992). More importantly, 500″ corresponds to a projected radius of 20 pc at the Galactic center, more than 2 orders of magnitude smaller than the effective radius of the Galactic bulge ($\sim$2.7 kpc; see Gilmore, King, & van der Kruit 1990). The bulge velocity dispersion has been measured by several authors at various Galactocentric distances within 4 kpc, all giving values between 75 and 110 km s^{-1}, with a tendency for σ to increase slowly toward the center (e.g., Kent 1992; Tiede & Terndrup 1997, 1999; Minniti 1996; Côté 1999; Zhao, Rich, & Biello 1996). The rotational velocity in the inner 1.5 kpc is well approximated by a solid-body curve with $v \sim 65$–87 km s^{-1} kpc^{-1} (e.g., Tiede & Terndrup 1997, 1999; Morrison & Harding 1993; Menzies 1990; Kinman, Feast, & Lasker 1988). In view of these results, we question the choice of $\sigma_e = 75$ km s^{-1} for the Milky Way. FM00 adopted $\sigma_c = 100$ km s^{-1}; in what follows we will perform regression analyses assuming values of both 75 km s^{-1} and 100 km s^{-1} for σ_e. We will show that the slope of the $M_\bullet$-σ_e relation depends significantly on which value is used.

G00 assumed constant errors on log $M_\bullet$ and zero measurement errors in σ_e when carrying out their least-squares fits. However, ignoring measurement errors in the independent variable is well known to bias the slope downward (e.g., Jefferys 1980). Even when high signal-to-noise ratio data are used, measurement errors on the velocity dispersions are easily at the 10% level (e.g., van der Marel et al. 1994) and cannot be neglected. Unfortunately, the data used by G00 to compute σ_e are mostly unpublished, and the authors do not give error estimates in their paper. Therefore, in the regression analyses described below, we will make various assumptions about the measurement uncertainties in σ_e.

To understand how the different galaxy samples used by FM00 and by G00 may have affected their respective conclusions about the $M_\bullet$-σ relation, we analyze sample 1 (the 12 galaxies from FM00) and sample 2 (the additional 15 galaxies from G00) separately. The BH masses in sample 2 are significantly less accurate than those in sample 1, with an rms uncertainty of 0.28 dex, compared to 0.18 dex for sample 1. We also present results from the analysis of the entire set of 27 galaxies, called the "combined sample" below.

3. ANALYSIS

We assume a relation of the form

$$Y_i = \alpha X_i + \beta + \epsilon_i \tag{1}$$

between the measured variables, where Y is log $M_\bullet$ and X is either log σ_c or log σ_e. The units are solar mass for $M_\bullet$ and kilometers per second for σ. The ϵ_i describe measurement errors as well as intrinsic scatter in the relation, if any. A large number of regression algorithms are available for recovering estimates $\hat{\alpha}$ and $\hat{\beta}$ of the slope and intercept and their uncertainties, given (X_i, Y_i) and their estimated errors $(\sigma_{Xi}, \sigma_{Yi})$. These algorithms differ in the degree of generality of the model that is assumed to underlie the data. The following four algorithms were used here.

Ordinary least-squares (OLS).—All of the error is assumed to lie in the dependent variable (i.e., log $M_\bullet$), and the amplitude of the error is assumed to be the same from measurement to measurement. This is the algorithm adopted by G00. We use the implementation G02CAF from the NAG subroutine library. The OLS estimator is biased if there are measurement errors in the independent variable or if the errors in the dependent variable vary from data point to data point (e.g., Jefferys 1980).

General least-squares (GLS).—All of the error is still assumed to reside in the dependent variable, but the amplitude of the error may vary from point to point. Press et al. (1989) implement this model in their routine FIT, which we use here.

Orthogonal distance regression (ODR).—The underlying variables are assumed to lie exactly on a straight line, i.e., to have no intrinsic scatter, but the observed quantities are allowed to have measurement errors in both X and Y, which may differ from point to point. This model is incorporated in the routines FITEXY of Press et al. (1989) and FV of Fasano & Vio (1988). We use the former routine here; the latter was found to give essentially identical results. The ODR estimator may be biased if the true variables exhibit intrinsic scatter about the linear relation, in addition to measurement errors (e.g., Feigelson & Babu 1992).

Regression with bivariate errors and intrinsic scatter (BRS).—As in ODR, the data are permitted to have measurement errors in both X and Y that differ from point to point. In addition, the underlying variables are allowed to have an intrinsic scatter about the regression line. We use the routine BCES($Y|X$) of Akritas & Bershady (1996), the same routine used in FM00.

Tables 2 and 3 give estimates of the slope and intercept, $\hat{\alpha}$ and $\hat{\beta}$, and their uncertainties as computed by each of the four algorithms, using σ_c and σ_e as independent variables. Values in parentheses correspond to setting $\sigma_e = 100$ km s^{-1} for the Milky Way, as discussed above. The results are summarized below and in Figure 2.

1. Accounting for errors in one or both variables increases the slope of the relation, whether expressed in terms of σ_c or σ_e. Ignoring measurement errors biases the slope too low, for two reasons. The BH masses in sample 2 are significantly more uncertain than those in sample 1 and, as a group, exhibit a shallower slope (particularly when expressed in terms of σ_c); routines like OLS that weight all data equally therefore underestimate the true slope. Second, ignoring measurement errors in the independent variable (log σ) *always* yields spuriously low slopes (e.g. Jefferys 1980). We find that the shallowest slope for every sample is returned by OLS, the routine used by G00. All other algorithms give slopes in the range $4 \lesssim \hat{\alpha} \lesssim 5$ for the combined sample.
2. The slope inferred for the $M_\bullet$-σ_e relation using ODR and BRS depends somewhat on the assumed errors in σ_e. Increasing the assumed error from 5% to 20% increases the BRS slope of the combined sample from 3.9 to 4.8.
3. Even when the appropriate fitting routines are used, the $M_\bullet$-σ_c relation tends to have a steeper slope than the $M_\bullet$-σ_e relation. This difference, however, is driven by one galaxy only: when σ_e for the Milky Way is increased from 75 km s^{-1} (used by G00) to a more appropriate—in our opinion —value of 100 km s^{-1}, both relations have a best-fit slope of $\sim 4.5 \pm 0.5$ for the combined sample (assuming a plausible 10%–15% error on σ_e).
4. Adding the galaxies of sample 2 (from G00) has little impact on the results, as long as measurement errors are taken into account by the fitting routine and $\sigma_e = \sigma_c = 100$ km s^{-1} is used for the Galaxy: the regression lines for

TABLE 2

RESULTS OF THE LINEAR REGRESSION FITS: log $M_\bullet = \alpha$ log $\sigma_c + \beta$

	Combined Sample				Sample 1				Sample 2			
Fit	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$
OLS	4.00	0.35	−1.10	0.80	4.39	0.38	−1.94	0.89	3.28	0.61	0.47	1.37
GLS	4.49	0.13	−2.18	0.28	4.71	0.14	−2.64	0.31	3.32	0.38	0.40	0.86
ODR......	4.52	0.36	−2.31	0.83	4.46	0.43	−2.06	1.03	4.20	0.60	−1.66	1.37
BRS.......	4.43	0.39	−2.08	0.92	4.81	0.55	−2.92	1.30	3.75	0.59	−0.59	1.32

TABLE 3

RESULTS OF THE LINEAR REGRESSION FITS: $\log M_\bullet = \alpha \log \sigma_e + \beta$

		Combined Sample				Sample 1				Sample 2			
Fit	%[1]	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$	$\hat{\alpha}$	$\hat{\sigma}(\alpha)$	$\hat{\beta}$	$\hat{\sigma}(\beta)$
OLS		3.85	0.30	−0.75	0.68	3.86	0.29	−0.65	0.67	3.53	0.55	−0.13	1.25
		(4.00)[2]	(0.32)	(−1.10)	(0.74)	(4.12)	(0.38)	(−1.30)	(0.90)				
GLS		3.90	0.11	−0.73	0.23	3.93	0.12	−0.74	0.25	3.58	0.039	−0.21	0.089
		(4.47)	(0.13)	(−2.14)	(0.28)	(4.71)	(0.14)	(−2.64)	(0.31)				
ODR	5	3.90	0.15	−0.81	0.35	3.95	0.16	−0.83	0.37	3.96	0.43	−1.08	0.99
		(4.33)	(0.17)	(−1.83)	(0.39)	(4.42)	(0.19)	(−1.96)	(0.43)				
	10	4.07	0.24	−1.20	0.55	3.98	0.25	−0.91	0.59	4.18	0.51	−1.58	1.17
		(4.40)	(0.27)	(−2.01)	(0.60)	(4.42)	(0.30)	(−1.97)	(0.70)				
	15	4.17	0.33	−1.44	0.75	4.00	0.36	−0.96	0.84	4.27	0.65	−1.77	1.49
		(4.45)	(0.36)	(−2.13)	(0.83)	(4.42)	(0.43)	(−2.00)	(1.01)				
	20	4.23	0.43	−1.59	0.99	4.02	0.47	−1.00	1.11	4.33	0.82	−1.92	1.91
		(4.49)	(0.47)	(−2.23)	(1.10)	(4.44)	(0.56)	(−2.04)	(1.35)				
	25	4.27	0.53	−1.70	1.24	4.03	0.58	−1.04	1.40	4.40	1.01	−2.08	2.40
		(4.53)	(0.58)	(−2.31)	(1.37)	(4.45)	(0.71)	(−2.06)	(1.70)				
BRS	5	3.90	0.24	−0.86	0.55	3.89	0.22	−0.73	0.47	3.60	0.49	−0.29	1.10
		(4.05)	(0.32)	(−1.22)	(0.74)	(4.16)	(0.41)	(−1.40)	(0.96)				
	10	4.05	0.27	−1.20	0.62	4.00	0.22	−0.97	0.47	3.84	0.57	−0.83	1.30
		(4.22)	(0.36)	(−1.61)	(0.83)	(4.29)	(0.43)	(−1.71)	(1.01)				
	15	4.33	0.34	−1.85	0.79	4.19	0.24	−1.41	0.53	4.32	0.83	−1.91	1.91
		(4.54)	(0.45)	(−2.36)	(1.04)	(4.54)	(0.48)	(−2.28)	(1.15)				
	20	4.81	0.50	−2.95	1.18	4.49	0.32	−2.13	0.74	5.27	1.65	−4.04	3.77
		(5.11)	(0.65)	(−3.64)	(1.53)	(4.94)	(0.62)	(−3.22)	(1.48)				
	25	5.64	0.89	−4.84	2.08	4.98	0.53	−3.25	1.26	7.42	4.62	−8.87	10.5
		(6.11)	(1.15)	(−5.93)	(2.70)	(5.60)	(0.93)	(−4.75)	(2.25)				

[1] Percentages refer to assumed measurement errors in σ_e.
[2] Values in parentheses used $\sigma_e = 100$ km s^{-1} for the Milky Way.

sample 1 (from FM00) and for the combined sample are essentially the same. In other words, the BH masses added by G00 are too uncertain to significantly alter the fit determined by the galaxies from sample 1 alone.

We conclude that the different slopes found by G00 and FM00 (3.75 vs. 4.8) are due partly to the neglect of measurement errors by the former authors and partly to the difference between σ_e and σ_c for a single data point, the Milky Way. If we use the more appropriate value of $\sigma_e = 100$ km s^{-1} for the Milky Way and a plausible 10%–15% error on σ_e, the $M_\bullet$-σ_c and $M_\bullet$-σ_e relations have essentially the same slope, $\sim$4.5. The data points added by G00, based mostly on unpublished modeling of stellar kinematical data from STIS, appear to contain little information about the $M_\bullet$-σ relation that was not already contained in the more accurate masses from FM00.

We next address the scatter in the $M_\bullet$-σ relation. The χ^2 merit function for a linear fit to data with errors in both variables is

$$\tilde{\chi}^2 = \frac{1}{N-2} \sum_{i=1}^{N} \frac{(Y_i - \hat{\alpha} X_i - \hat{\beta}_i)^2}{\sigma_{Y,i}^2 + \hat{\alpha}^2 \sigma_{X,i}^2} \tag{2}$$

(e.g., Press et al. 1989), where, in our case, $Y = \log M_\bullet$ and $X = \log \sigma$. A good fit has $\tilde{\chi}^2 \lesssim 1$. Since measurement uncertainties $\sigma_{X,i}$ are not available for the σ_e, we computed $\tilde{\chi}^2$ only for the $M_\bullet$-σ_c relation. We are also interested in the absolute scatter in $\log M_\bullet$, which we define as

$$\Delta_\bullet = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Y_i - \hat{\alpha} X_i - \hat{\beta}_i)^2} \ . \tag{3}$$

We computed $\tilde{\chi}^2$ and $\Delta_\bullet$ using the fits given by the BRS regression algorithm (Table 2). The results are

$$\text{Sample 1: } \tilde{\chi}^2 = 0.74 \ \Delta_\bullet = 0.26 \ ,$$

$$\text{Sample 2: } \tilde{\chi}^2 = 1.67 \ \Delta_\bullet = 0.35 \ ,$$

$$\text{Combined Sample: } \tilde{\chi}^2 = 1.20 \ \Delta_\bullet = 0.34 \ .$$

The sample 2 galaxies exhibit a larger scatter in $\log M_\bullet$ than the galaxies in sample 1 (0.35 dex vs. 0.26 dex), consistent with their greater measurement uncertainties (Table 1). Furthermore, the 12 galaxies from FM00 define a significantly tighter correlation, as measured by $\tilde{\chi}^2$, than the 15 galaxies added by G00 ($\tilde{\chi}^2 = 0.74$ vs. $\tilde{\chi}^2 = 1.67$) or than the combined sample. Thus, we confirm the conclusion of G00 that the scatter in their data about the best-fit linear relation exceeds that expected on the basis of measurement error alone. The large $\tilde{\chi}^2$ for sample 2 may indicate that the measurement uncertainties quoted by G00 are too small.

4. REVERBERATION MAPPING MASSES

A long-standing discrepancy exists between BH masses determined from stellar kinematics and from reverberation mapping; the latter technique uses emission lines in active galactic nuclei (AGNs) to probe the virial mass within the broad-line region (Netzer & Peterson 1997). Since there are currently no galaxies with BH masses determined independently by the two techniques, any comparison must be statistical. The standard approach (e.g., Wandel 1999) has been to compare the average BH mass at a given bulge luminosity as computed from reverberation mapping with the mass predicted by the Magorrian et al. (1998) relation; the latter is based on stellar kinematical data, mostly of low

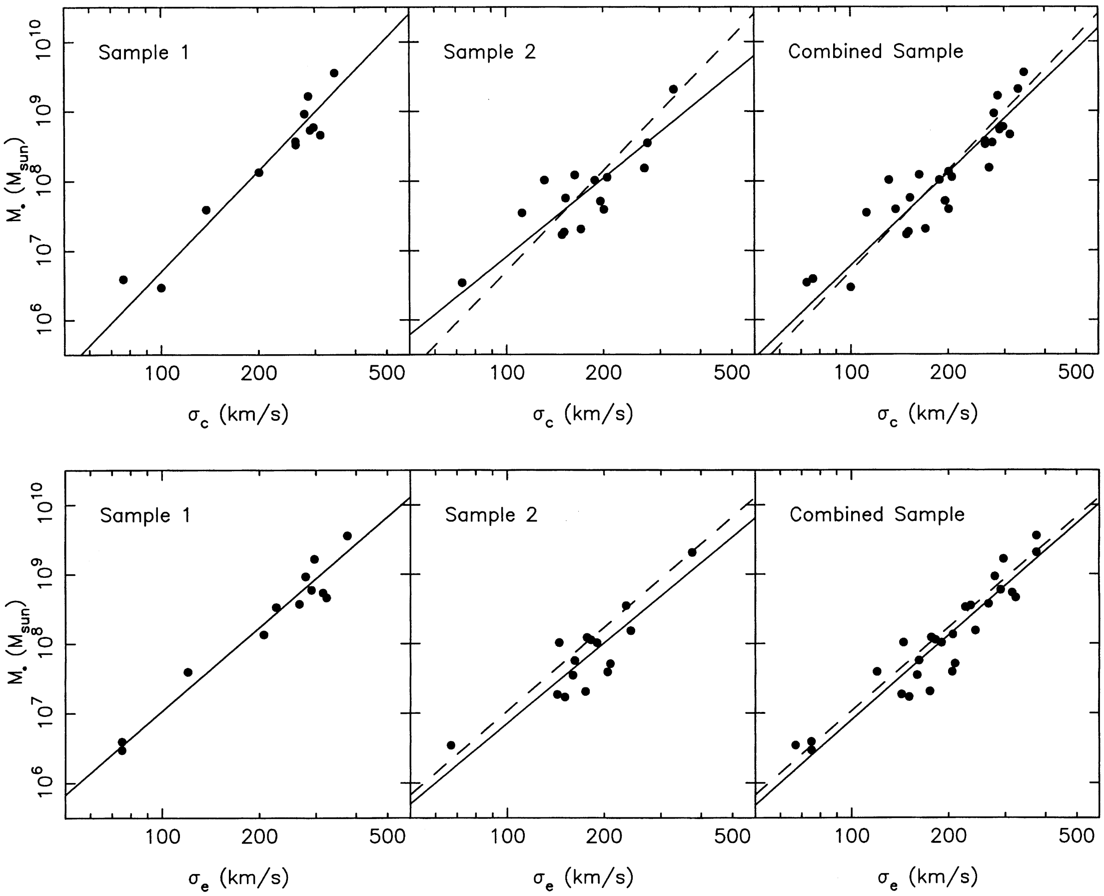

FIG. 2.—Regression fits to the three galaxy samples, with σ_c or σ_e as independent variable. Sample 1: Galaxies from FM00. Sample 2: Additional galaxies from G00 (also Table 1). Regression lines were computed with the BRS algorithm, assuming 10% errors in σ_e. The fit to sample 1 is repeated in the right two panels as a dashed line.

spatial resolution. The discrepancy is a factor of ~ 20 in the sense that the reverberation mapping masses are too low (Wandel 1999). This discrepancy has most often been attributed to some systematic error in the reverberation mapping masses (e.g., Richstone et al. 1998; Faber 1999; Ho 1999).

FM00 showed that the Magorrian et al. masses fall systematically above the $M_\bullet$-σ_c relation defined by galaxies with more secure BH mass estimates, some by as much as 2 orders of magnitude. The offset is strongly correlated with distance, suggesting a systematic, resolution-dependent error in the Magorrian et al. modeling. Much or all of the discrepancy with the reverberation mapping masses might therefore be due to systematic errors in the Magorrian et al. masses, contrary to the usual assumption. Gebhardt et al. (2000b) tested this idea by plotting seven AGN BH masses against their $M_\bullet$-σ_e relation.[2] We reproduce that plot here as Figure 3. The fit is reasonable, although the points tend to scatter below the line. We also plot in Figure 3 the steeper $M_\bullet$-σ_c relation derived in FM00 (given here, in Table 2, as the BRS regression fit on σ_c for sample 1). The steeper relation of FM00 is clearly a better fit.

We stress that several of the AGN data points lie at the low-mass end of the distribution where the $M_\bullet$-σ relation is

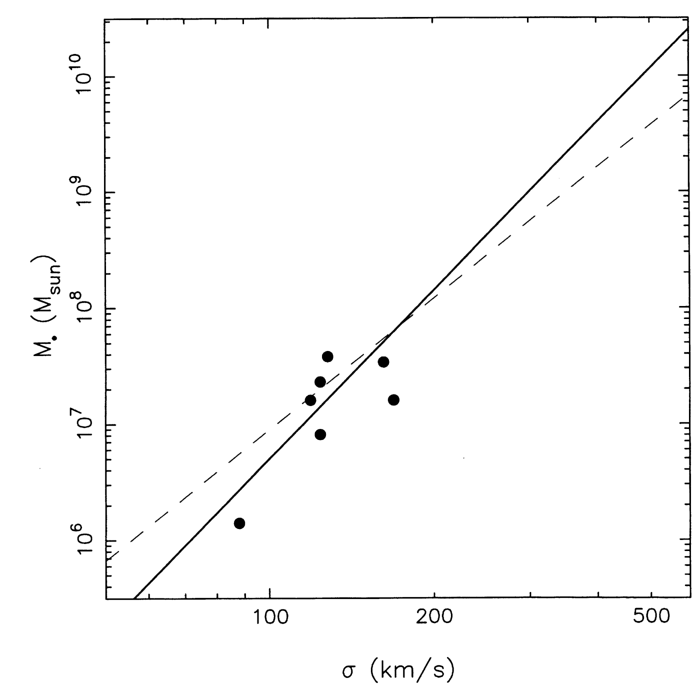

FIG. 3.—Reverberation mapping masses for seven galaxies. Solid line is the $M_\bullet$-σ_c relation from Ferrarese & Merritt (2000); dashed line is the $M_\bullet$-σ_e relation from Gebhardt et al. (2000a).

[2] The velocity dispersions plotted by Gebhardt et al. (2000b) are labeled σ_e even though they are central values.

strongly affected by uncertainties in the slope. Nevertheless, there would no longer appear to be any prima facie reason for believing that the reverberation mapping masses are systematically in error. Furthermore, the scatter in these masses about the $M_\bullet$-σ relation appears to be comparable to that of the sample 2 data from G00. We therefore carried out regression fits combining the reverberation mapping masses with sample 1 and sample 2, for a total of 34 galaxies. We assumed 50% measurement errors in the AGN $M_\bullet$ and 15% errors in the σ_c. The results (using the BRS regression routine) were

$$\hat{\alpha} = 4.72 \pm 0.36, \hat{\beta} = -2.75 \pm 0.82 , \quad (4)$$

very close to the parameters derived in FM00 using sample 1 alone. This fit has $\tilde{\chi}^2 = 1.11$ and $\Delta_\bullet = 0.35$, about as good as obtained using the galaxies in Table 1.

5. SUMMARY

We investigated the differences in the $M_\bullet$-σ relation as derived by Ferrarese & Merritt (2000) and by Gebhardt et al. (2000a). The latter authors found a shallower slope (3.75 vs. 4.8) and a greater vertical scatter, larger than expected on the basis of measurement errors alone. Three possible explanations for the differences were explored: different galaxy samples; different definitions of the velocity dispersion, central (σ_c) versus integrated (σ_e); and different routines for carrying out the regression. The shallower slope of the G00 relation was found to be due partly to the use of a regression algorithm that does not account properly for measurement errors and partly to the adoption of a value of σ_e for the Milky Way which is, in our opinion, implausibly low. The greater scatter seen by G00 is due to larger uncertainties associated with the additional BH masses included by them, mostly from unpublished STIS data. When measurement uncertainties are properly accounted for, the parameters of the best-fit relation derived from the combined samples of FM00 and G00 are essentially identical to those derived from the sample of FM00 alone. The steeper relation derived by FM00 also provides a better fit to BH masses obtained from reverberation mapping. A regression fit to the combined sample of 34 galaxies, including stellar dynamical, gasdynamical, and reverberation mapping masses, yields

$$M_\bullet = 1.30(\pm 0.36) \times 10^8 \, M_\odot \left(\frac{\sigma_c}{200 \text{ km s}^{-1}}\right)^{4.72(\pm 0.36)} . \quad (5)$$

The scientific implications of equation (4) are discussed briefly by FM00 and extensively in Merritt & Ferrarese (2000). This relation is essentially identical to the one derived in FM00. We suggest that there is no longer any reason to assume, as a number of authors (Richstone et al. 1998; Faber 1999; Ho 1999) have done, that the reverberation mapping masses are less accurate than masses derived from stellar kinematics (Magorrian et al. 1998).

We stress that the current sample of galaxies with reliable BH mass estimates is likely affected by severe selection biases, which are very difficult to quantify. Our results highlight the need for accurate BH masses if the $M_\bullet$-σ relation is to be further refined. Only a handful of galaxies observed with STIS are likely to yield mass estimates as accurate as those already available for the galaxies in FM00. Uncertainties in the reverberation mapping masses are probably comparable to those obtained from *Hubble Space Telescope* data in most galaxies; however, the number of galaxies with reverberation mapping masses is large (~ 35) and growing. Furthermore, many of these galaxies are in the critical, low-mass range, $10^6 \, M_\odot \lesssim M_\bullet \lesssim 10^8 \, M_\odot$. An aggressive campaign to measure stellar velocity dispersions in AGNs might be the best route toward refining the $M_\bullet$-σ relation.

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