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JETS IN GAMMA-RAY BURSTS

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ABSTRACT

In the afterglows of several gamma-ray bursts (GRBs), rapid temporal decay, which is inconsistent with spherical (isotropic) blast-wave models, is observed. In particular, GRB 980519 had the most rapidly fading of the well-documented GRB afterglows, with $t^{-2.05 \pm 0.04}$ in optical as well as in X-rays. We show that such temporal decay is more consistent with the evolution of a jet after it slows down and spreads laterally, for which t^{-p} decay is expected (where p is the index of the electron energy distribution). Such a beaming model would relax the energy requirements on some of the more extreme GRBs by a factor of several hundred. It is likely that a large fraction of the weak- (or no-) afterglow observations are also due to the common occurrence of beaming in GRBs and that their jets have already transitioned to the spreading phase before the first afterglow observations were made. With this interpretation, a universal value of $p \cong 2.4$ is consistent with all data.

Subject headings: gamma rays: bursts — hydrodynamics — relativity — shock waves

1. INTRODUCTION

One of the most important open questions in gamma-ray bursts (GRBs) is whether the burst emission is isotropic or strongly beamed in our direction. This question has implications on almost every aspect of the phenomenon, from the energetics of the events to the engineering of the "inner engine" and the statistics and the luminosity function of the sources.

According to the relativistic fireball model, the emission from a spherically expanding shell and a jet would be rather similar to each other as long as we are along the jet's axis and the Lorentz factor γ is large compared to the inverse of the angular width of the jet θ_0 (Piran 1995). When γ drops below θ_0^{-1}, the jet's material begins to spread sideways, and we expect a break in the light curve of the afterglow at this stage. Since we have for spherical adiabatic evolution $\gamma(t) \approx 6(E_{52}/n_1)^{1/8} t_{\rm day}^{-3/8}$, this break should take place at[2]

$$t_{\rm jet} \approx 6.2(E_{52}/n_1)^{1/3}(\theta_0/0.1)^{8/3} \text{ hr}, \quad (1)$$

where E_{52} is the "isotropic" energy of the ejecta in units of 10^{52} ergs, i.e., the inferred energy assuming isotropic expansion, and n_1 is the surrounding interstellar medium (ISM) particle density in cm^{-3}. So far, with the exception of the recent GRB 990123 (Kulkarni et al. 1999), no such break was observed, even for afterglows extending for hundreds of days. More specifically, the well-studied afterglows GRB 970228 and GRB 970508 behave according to a single unbroken power law, as long as the observations continued (Zharikov, Sokolov, & Baryshev 1998; Fruchter et al. 1999), giving a strong indication that those sources were isotropic to a large extent.

We show here that, even without a break in the light curve, one can identify a jet based on the power-law index of the light-curve decline. Since we have a reasonable knowledge of the value of the electrons' energy distribution index $p \sim 2.4$, we expect for high frequencies a spherical decay of $t^{-1.1}$ to $t^{-1.3}$ and a jetlike decay of $t^{-2.4}$. We suggest that at least in one afterglow, GRB 980519, the light curve and spectra are consistent with a spreading jet and inconsistent with a spherical expansion. We suggest that the transition to spreading jet, at $\gamma \sim \theta_0^{-1}$, took place during the few hours between the GRB and the first detection of the afterglow. We conclude that the beaming factor in this burst is at least a few hundred. Together with the appearance of a sharp break in the light curve of the afterglow of GRB 990123, this indicates that jets are common in GRBs. In fact, the rapid decline that corresponds to an expanding jet could also explain the weak or absent optical afterglow seen in some of the other bursts, e.g., GRB 990217 (Piro et al. 1999; Palazzi et al. 1999).

Jets have been discussed extensively in the context of GRBs. First, the similarity between some of the observed features of blazars and AGNs led to the speculation that jets also appear in GRBs (Paczyński 1993; Dermer & Chiang 1998). Second, the regions emitting the GRBs as well as the afterglow must be moving relativistically. The emitted radiation is strongly beamed, and we can observe only a region with an opening angle $1/\gamma$ off the line of sight. Emission outside of this very narrow cone is not observed. These considerations have lead to numerous speculations on the existence of jets and to attempts to search for the observational signature of jets both during the GRB phase (Mao & Yi 1994) and in the context of the afterglow (Rhoads 1997a, 1997b, 1999; Mészáros, Rees, & Wijers 1998; Panaitescu & Mészáros 1998). Finally, jets appear naturally in the context of several leading scenarios for the "inner engine" (Mochkovich et al. 1993; Davies et al. 1994; Katz 1997; Mészáros & Rees 1997; Nakamura 1998).

2. JET EVOLUTION

The simple fireball model (and the Blandford-McKee 1976 solution) assumes a spherical expansion. However, even if the relativistic ejecta is beamed, as long as the Lorentz factor γ of the relativistic motion satisfies $\gamma > \theta_0^{-1}$, the hydrodynamics of the jet will not be influenced by the fact that it has a finite

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[2] The following numerical factors are different from those given by Rhoads (1999) and Panaitescu & Mészáros (1998). We explain these differences in § 2.

angular size (Piran 1995). The matter does not have enough time (in its own rest frame) to expand sideways. This situation changes drastically when $\gamma \approx \theta_0^{-1}$, when the sideway expansion becomes significant. A full solution of the evolution of a jet at this stage requires two-dimensional relativistic hydrodynamic simulations. However, one can obtain a reasonable idea of what goes on using simple analytical estimates.

Rhoads (1997a, 1997b, 1999) considered the evolution of a relativistic jet that is expanding sideways at the local speed of sound c_s, so that $\theta \sim \theta_0 + c_s t_{\rm proper}/ct \sim \theta_0 + \gamma^{-1}/\sqrt{3}$. In this case, the hydrodynamic transition takes place at $\gamma \sim \theta^{-1}/\sqrt{3}$. However, since the rest mass of the shocked material is negligible compared with its internal energy, the expansion can be ultrarelativistic with a Lorentz factor comparable to the thermal Lorentz factor. This would lead to $\theta \sim \theta_0 + ct_{\rm proper}/ct \sim \theta_0 + \gamma^{-1}$ and to a transition when $\gamma \sim \theta_0^{-1}$. The sideways expansion leads, for an adiabatic evolution, to an exponential slowing down (Rhoads 1997a) as $\gamma \propto \exp[-r/l_{\rm jet}]$, where $l_{\rm jet} \equiv [E_{\rm jet}/(4\pi/3)nm_p c^2]^{1/3}$ is the Sedov length in which a spherical expanding shell with energy $E_{\rm jet}$ acquires mass whose rest mass energy equals to its own energy (n is the ISM density). $E_{\rm jet}$ is the actual energy in the jet. Thus, r is practically a constant during the spreading phase. Therefore, the observer time, which is related to the radius and the Lorentz factor as $t \propto r/\gamma^2$, satisfies simply $t \propto \gamma^{-2}$.

Equation (1) gives the simplest break-time estimate. It is based only on spherical adiabatic expansion. It differs by a factor of 20 in time (corresponding to a factor of ∼3 in the opening angle θ_0) from the expression given by Rhoads (1999). The discrepancy arises from several factors: (1) As discussed above, we assume that the jet expands sideways at the speed of light while Rhoads assumes that jet expands at the sound speed $c/\sqrt{3}$. (2) Rhoads uses $t = R/2\gamma^2 c$. This expression is valid for a point source moving along the line of sight with a constant velocity. We use $t \approx R/4\gamma^2 c$, reflecting the deceleration of the source and its finite angular size (Sari 1997, 1998; Waxman 1997; Panaitescu & Mészáros 1998). (3) We use the simple adiabatic energy condition $E = \gamma^2 mc^2$, where m is the rest mass of the shocked ISM, while Rhoads uses $E = 2\gamma^2 mc^2$. A third possibility is to use the more exact numerical factor derived from the Blandford-McKee (1976) solution: $E = 12\gamma^2 mc^2/17$. (4) We estimated the time in the local frame as $R/\gamma c$. Rhoads noted that the Lorentz factor was higher earlier and hence the effective proper time is shorter by a factor of 2.5, allowing for less spreading. However, far behind the shock, the matter moves with a considerably lower Lorentz factor, allowing it to spread more easily.

Panaitescu & Mészáros (1998) consider similar hydrodynamics as Rhoads (1999), but notice that once $\gamma \sim 1/\theta_0$ the observer is able to see the edge of the jet. They find two transitions, the first one when $\gamma \sim 1/\theta_0$ at around our break-time estimate and the second one around Rhoads's. However, there would be only one transition if the time between the two breaks turns out to be very short. A reliable estimate of the numerical factor clearly requires full two-dimensional simulations. It might also be, as suggested by Rhoads (1999), that the transition takes place over a relatively long time and that most observations, which are conducted in a finite time interval, will show only part of the asymptotic break.

We consider now synchrotron emission from a power-law distribution of accelerated electrons produced by shocks in an expanding jet. The instantaneous spectrum is given by the four broken power laws discussed in Sari, Piran, & Narayan (1998). However, the time dependence of the break frequency and the overall normalization depend strongly on the hydrodynamic evolution. Therefore, the light curve from a jet differs strongly from the light curve of a spherical evolution. Surprisingly, it is possible to obtain general expressions, appropriate to both spherical ($\gamma > \theta^{-1}$) and jet ($\gamma \leq \theta^{-1}$) evolution. We write these generalized expressions and specialize to jet and sphere only at the very end. We begin with the typical frequency ν_m, at the observer frame:

$$\nu_m = \frac{eB}{m_e c}\gamma_e^2\gamma \propto \gamma^4 \propto \begin{cases} t^{-3/2}, & \text{spherical,} \\ t^{-2}, & \text{jet.} \end{cases} \quad (2)$$

The cooling frequency is given by

$$\nu_c = \frac{36\pi^2 e m_e c}{\gamma B^3 t^2} \propto \gamma^{-4} t^{-2} \propto \begin{cases} t^{-1/2}, & \text{spherical,} \\ \text{const.,} & \text{jet.} \end{cases} \quad (3)$$

The peak flux is obtained at the lowest of the two frequencies ν_m and ν_c. Let $\underline{N}_e$ be the total number of electrons radiating toward the observer, i.e., those located in a cone of opening angle γ^{-1}. ($\underline{N}_e$ is different from N_e [Sari et al. 1998; Sari & Piran 1999], which is the total number of radiating electrons, including those that are not radiating toward the observer.) $\underline{N}_e$ can be approximated by $\underline{N}_e = 4\pi\gamma^{-2}R^3 n/3$. The total energy per unit time per unit frequency emitted by these electrons, $\sigma_T m_e c^2 \underline{N}_e B\gamma/6\pi e$, is distributed over an area of $\pi\gamma^{-2}d^2$ at a distance d from the source. The observed peak flux density is therefore

$$F_{\nu,\max} = \frac{2\sigma_T m_e c^2}{\pi e}\frac{R^3 nB\gamma}{d^2} \propto R^3\gamma^2 \propto \begin{cases} \text{const.,} & \text{spherical,} \\ t^{-1}, & \text{jet.} \end{cases} \quad (4)$$

It seems to hold quite generally at late times (except perhaps the first few hours; see Sari & Piran 1999) that $\nu_c \gg \nu_m$. The electrons responsible for low-energy emission are therefore those with ν_m. In this case, the self-absorption frequency can be estimated as

$$\nu_a \propto R^{3/5}\gamma^{2/5} \propto \begin{cases} \text{const.,} & \text{spherical,} \\ t^{-1/5}, & \text{jet.} \end{cases} \quad (5)$$

We now turn to calculate the light curves for several frequency ranges. The flux at low frequencies, which is self-absorbed, evolves as

$$F_{\nu<\nu_a} \propto \left(\frac{\nu}{\nu_a}\right)^2 \left(\frac{\nu_a}{\nu_m}\right)^{1/3} F_{\nu,\max} \propto R^2 \propto \begin{cases} t^{1/2}, & \text{spherical,} \\ \text{const.,} & \text{jet.} \end{cases} \quad (6)$$

The flux above the self-absorption frequency but below the typical frequency ν_m evolves as

$$F_{\nu_a<\nu<\nu_m} = \left(\frac{\nu}{\nu_m}\right)^{1/3} F_{\nu,\max} \propto R^3\gamma^{2/3} \propto \begin{cases} t^{1/2}, & \text{spherical,} \\ t^{-1/3}, & \text{jet.} \end{cases} \quad (7)$$

The flux at low frequencies ($\nu < \nu_m$) would, therefore, rise like $t^{1/2}$ as long as the evolution is spherical. Then, once γ drops below θ_0^{-1} and the jet begins to spread, the flux at frequencies above the self-absorption would decrease as $t^{-1/3}$. At lower frequencies which are in the self-absorbed regime, the flux will be a constant until the self-absorption frequency is reached. These predictions are different from those derived by Rhoads, who considered the case when $\nu_m < \nu_a$, for which the flux rises linearly with time. However, based on GRB 970508, it seems that this regime of $\nu_m < \nu_a$ is relevant only after 100 days.

At high frequencies two light curves are possible, depending on whether the radiating electrons are cooling ($\nu > \nu_c$) or not ($\nu < \nu_c$). The slope itself also depends on the electron power-law distribution index p. Below the cooling frequency, we obtain

$$F_{\nu_m<\nu<\nu_c} = F_{\nu_m}(\nu/\nu_m)^{-(p-1)/2} \propto R^3\gamma^{2p} \propto \begin{cases} t^{-3(p-1)/4}, & \text{spherical,} \\ t^{-p}, & \text{jet.} \end{cases} \quad (8)$$

Above the cooling frequency, we have

$$F_{\nu_m<\nu_c<\nu} = F_{\nu_m}(\nu_c/\nu_m)^{-(p-1)/2}(\nu/\nu_c)^{-p/2}$$
$$\propto R^3\gamma^{2p-2}t^{-1} \propto \begin{cases} t^{-3p/4+1/2} & \text{spherical,} \\ t^{-p} & \text{jet.} \end{cases} \quad (9)$$

Note that for a spreading jet, the light-curve decay index (but not the spectrum) is independent of whether $\nu > \nu_c$ or $\nu < \nu_c$. This is because of the fact that ν_c is constant in time in the case of a spreading jet. Since p determines both the light curve and spectrum, a parameter-free relation between the temporal decay index α and the spectral index β [defined by $F_\nu(t) \propto t^{-\alpha}\nu^{-\beta}$] can be given. The relations are summarized in Table 1.

TABLE 1

THE SPECTRAL INDEX β AND THE LIGHT-CURVE INDEX α AS FUNCTION OF p

Frequency	Spectral Index β, Sphere and Jet ($F_\nu \propto \nu^{-\beta}$)	Light-Curve Index α ($F_\nu \propto t^{-\alpha}$)	
		Sphere	Jet
$\nu < \nu_c$	$(p-1)/2 \cong 0.7$	$3(p-1)/4 \cong 1.05$	$p \cong 2.4$
		$3\beta/2$	$2\beta+1$
$\nu > \nu_c$	$p/2 \cong 1.2$	$(3p-2)/4 \cong 1.3$	$p \cong 2.4$
		$3\beta/2 - 1/2$	2β

NOTE.—Typical values are quoted using $p = 2.4$. The parameter-free relation between α and β is given for each case (eliminating p).

3. OBSERVATIONS

3.1. *GRB 980519*

GRB 980519 was one of the brightest of the bursts detected in the *BeppoSAX* WFC (Muller et al. 1998; in 't Zand et al. 1999), second only to the recent GRB 990123 (Feroci et al. 1998). GRB 980519 had the most rapid fading of the well-documented GRB afterglows, consistent with $t^{-2.05\pm0.04}$ in *BVRI* (Halpern et al. 1999). The power-law decay index of the X-ray afterglow, $\alpha_X = 2.07 \pm 0.11$ as reported by Owens et al. (1998), is consistent with the optical. The X-ray temporal decay of GRB 980519 is the fastest of the seven afterglows that were well measured by *BeppoSAX* (Owens et al. 1998). The optical spectrum alone is well fitted by a power law of the form $\nu^{-1.20\pm0.25}$, while the optical and X-ray spectra together are adequately fitted by a single power law, $\nu^{-1.05\pm0.10}$.

The relation between the spectral slope and the temporal decay is inconsistent with the simple spherical fireball model that predicts $\alpha = 3\beta/2$ or $\alpha = 3\beta/2 - 1/2$. This inconsistency is independent of the value of p. These are consistent with each other if we assume an expanding jet phase. It is difficult to determine the exact value of p from these observations. However, we note that they are consistent with a value of $p \sim 2.4$ that arises in other bursts. This fits the optical and X-ray power-law decay if the full asymptotic spreading behavior $t^{-2.4}$ was not been reached yet. It also fits the optical spectral index, which has a large uncertainty. The optical–to–X-rays slope is intermediate between the value obtained for slow cooling (~ -0.8) and that obtained for fast cooling (~ -1.25). The cooling frequency is, therefore, between the optical and X-rays.

The interpretation that this transition from a spherical-like behavior to a jetlike behavior took place less than 8.5 hr after the burst implies that the opening angle of the jet was rather small: $\theta < 0.1$, leading to a beaming factor of 300 or larger! We note that the two strongest GRBs detected by the *BeppoSAX* WFC are inferred to have a large beaming factor. This may indicate that a significant fraction of the spread in luminosities is contributed by the beaming effect.

3.2. *GRB 990123*

GRB 990123 was a remarkable burst with a very high GRB fluence and with a prompt optical emission. We interpret this emission as resulting from early reverse shock. The reverse shock has also produced the early radio flare (Sari & Piran 1999). This reverse-shock emission decayed like t^{-2} and disappeared quickly. The forward shock produced the prompt and late X-ray as well as the later optical emission. The optical afterglow, from about 3.5 hr to about 2.04 ± 0.46 days, showed a power-law decay with $t^{-1.1\pm0.03}$. Then, the optical emission began to decline faster (Kulkarni et al. 1999). The simplest explanation is that we have observed the transition from a spherical-like phase to an expanding jet phase. The transition took place at ~2 days, corresponding to $\theta_0 \sim 0.1$. This implies a beaming factor of about 200, reducing the energy of the burst to 3×10^{52} ergs. This is the only burst in which such a break has been detected. The decay before the break is well measured and fits an electron distribution with $p \cong 2.4$.

3.3. *GRB 980326*

GRB 980326 was another burst with a rapid decline. Groot et al. (1998) derived a temporal decay slope of $\alpha = 2.1 \pm 0.13$ and a spectral slope of $\beta = 0.66 \pm 0.7$ in the optical band. Such rapid temporal decay suggests a jetlike evolution. As Groot et al. (1998) note, the large uncertainty in the spectral index allows in this case also a spherical expansion interpretation (with somewhat unusual values $p = 4.2$ or $p = 5.2$). However, this measured temporal decay was dependent upon a report of a host galaxy detection at $R = 25.5 \pm 0.5$, which was included as a constant term. The detection of a host has since been determined to be spurious; better data show no constant component to a limiting magnitude of $R = 27.3$ (Bloom & Kulkarni 1998). When the previously assumed constant component is removed, the overall light curve is concave, in disagreement with a jet interpretation. If the last detection is interpreted as a different phenomenon (Bloom et al. 1999), then the remaining points show a rapid decline—in agreement with a jet.

3.4. *GRB 970228 and GRB 970508*

GRB 970228 and GRB 970508 had no observed break in the light curve as long as the afterglow could be observed. GRB 970228 was observed by *Hubble Space Telescope*

6 months later, at which point it was still following a power-law decay as $t^{-1.14\pm0.05}$ (Fruchter et al. 1999). GRB 970508 was observed for 9 months to decline as $t^{-1.23\pm0.04}$ (Zharikov et al. 1998), at which point it became as faint as its host galaxy. This sets a limit on the beaming in these events of $\theta_0 \geq 1$. The beaming factor is therefore less than an order of magnitude.

4. CONCLUSIONS

Under the standard assumptions that the magnetic field as well as the electron energy are a constant fraction of equipartition, the temporal decay rate and its relation with spectral index depend strongly on the hydrodynamic evolution. Therefore, jets can be identified based on the temporal decay rate and its relation with the spectral index, even without detecting a break in the afterglow light curve.

We have seen indication of a jetlike behavior in three bursts. Two other bursts did not show any break in their optical light curves, which have been observed for a long time. In several other bursts the situation is inconclusive, and their short afterglow is consistent with rather narrow jets. We suggest that jetlike behavior is the common one in GRBs. Moreover, the range of possible beaming angles, from $\theta_0 \leq 0.1$ for GRB 980519 to $\theta \sim 0.1$ for GRB 990123 and $\theta \geq 1$ for GRB 970228 and GRB 970508, is quite large. These beaming angles are consistent with the limits set by searches for "orphan" radio (Perna & Loeb 1998) and X-ray (Grindlay 1999) afterglows.

The suggestion that GRBs are beamed has several implications. First, this implies that the GRB inner engines must include a collimation mechanism in addition to the required acceleration mechanism. This makes the similarity between GRBs and some AGNs, more specifically blazars, even greater. Second, the beaming reduces the energy budget of this phenomenon. Beaming of $\theta_0 \sim 0.1$ reduces the required energy by a factor of 200. Interestingly, the evidence for jets arises most clearly in the two strongest bursts detected by *BeppoSAX* so far. It may provide a hint on the energy budget and on the effect of beaming on the luminosity function. GRB models based on "regular" compact objects become more appealing once more.

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Note added in proof.—The afterglow following GRB 990510 was recently found to have a transition from a shallow slope to a steep one, indicating a transition from spherical evolution to spreading jet (J. S. Bloom, S. R. Kulkarni, S. Djorgovski, D. A. Frail, T. S. Axelrod, J. R. Mould, & B. P. Schmidt, GCN Circ. 323 [http://gcn.gsfc.nasa.gov/gcn/gcn3/323.gcn3] [1999]). This burst was again very bright, fitting the discussion in the last paragraph.