ELEMENTS OF TITANIA.

FROM LETTERS OF MR. FERGUSON TO THE EDITOR.

I have computed the accompanying Elements of the Sixtieth Asteroid from observations of September 15, 22, and 29:—

$$\begin{aligned}
T &= 1860, \text{ Oct. } 1, \text{ Washington M. T.} \\
M &= 197^\circ\ 28'\ 55''.9. \\
\pi &= 158\ \ 5\ \ 38.6 \\
\Omega &= 187\ \ 12\ \ 10.3
\end{aligned}\Bigg\}\ \text{Mean Equinox of Epoch.}$$

$$\begin{aligned}
i &= \ \ 4\ \ 41\ \ \ 4.4 \\
\varphi &= 11\ \ 27\ \ 27.0 \\
\log a &= 0.359763 \\
\log \mu &= 3.010362
\end{aligned}$$

1860, *October* 28.

I send the annexed improved elements, which, though not yet quite correct, are nearer than those sent before. I have not been able to find time to make an Ephemeris, but will do so soon.

$$\begin{aligned}
T &= 1860, \text{ Oct. } 1, \text{ Washington M. T.} \\
M &= 199^\circ\ 48'\ 13''.8 \\
\pi &= 158\ \ 6\ \ 45.7 \\
\Omega &= 187\ \ 15\ \ 15.8
\end{aligned}\Bigg\}\ \text{Mean Equinox of Epoch.}$$

$$\begin{aligned}
i &= \ \ 4\ \ 40\ \ 18.8 \\
\varphi &= 11\ \ 27\ \ 10.2 \\
\log a &= 0.359725 \\
\mu &= 1024''.3
\end{aligned}$$

JAMES FERGUSON.

ON THE SUPPOSED INTRA-MERCURIAL PLANETS.

By SIMON NEWCOMB.

A very simple computation will show that a planet of the same reflective power with *Saturn*, and of $\frac{1}{400}$ part his diameter, placed at the distance 0.15 from the Sun, would at quadrature glow with about the same brilliancy as *Saturn* does at opposition. The bulk of such a planet would be $\frac{1}{64\,000\,000}$ that of *Saturn;* and if its density were 120 times as great, it would still have only $\frac{1}{500\,000}$ of his mass. A planet at this distance would require $\frac{1}{650}$ the mass of *Saturn* to produce the motion in the perihelion of *Mercury* indicated by the researches of Leverrier. The further the planet should be from the Sun, the less the mass which would be necessary, and the more easily it would be found.

It is incredible that a group of planets each as bright as *Saturn* at opposition, surrounding the Sun, should not have been observed during total eclipses, if they existed. I think, therefore, that we are justified in concluding that, if the observed motion in the perihelion of *Mercury* is caused by intra-Mercurial planets, *they must be several hundred in number.*

From what has been said it would seem that five hundred would be a very moderate estimate of the number of the intra-Mercurial planets; and if so numerous, it is evident from the following table, that, unless their inclinations be enormously great, there must be a transit *nearly every day.* They may well be so minute as to escape observation entirely with ordinary instruments, and that mode of looking for them which gives most hope of success is to examine the sun carefully with very high powers.

To estimate as well as possible the number of these occurrences, I have prepared the following table, giving the number of transits in ten years of planets of different mean distances and inclinations. It is founded on the fact that a planet of any given mean distance and inclination will, on the whole, transit the disk of the Sun with a certain frequency, depending on the values of those elements. Strictly speaking, this frequency would be slightly affected by the eccentricity, but as the change would only be of the second order with respect to that element, it is neglected.

Table showing the number of Transits in ten Years of Planets having different Mean Distances and Inclinations.

i	Mean Distance.				
	.10	.15	.20	.25	.30
0°	306	162	102	70	51
1°	306	162	102	41	21
2	306	88	36	18	10
3	181	53	23	12	7
4	126	37	18	8	5
5	98	31	14	7	5
6	79	24	12	6	4
7	67	21	10	6	3
8	50	19	9	5	3
9	55	18	8	4	2
10	48	17	8	4	2
11	42	16	7	4	2
12	38	14	6	3	2
13	36	12	6	3	1
14	34	11	5	3	1
15	32	10	5	2	1
16	30	10	4	2	1

Whatever motion of the perihelion of *Mercury* might be produced by the action of a numerous group of asteroids, it is easily deducible that nearly the same amount of motion would be produced in its node, supposing that each asteroid is very small compared with the sum of all the others. This results from the circumstance that the invariable plane of the planetary system will always be very near the probable mean position of the orbits of a numerous group of planets at any time. This may be shown as follows.

If we obtain the general expressions for the inclination and node of an asteroid during all time by the method of LAGRANGE developed in the *Mémoires de Berlin*, 1781–2, and since explained by LAPLACE, PONTECOULANT, LEVERRIER, and JACOBI, they will be of the form

$$\begin{aligned}
\sin i \, \sin \Omega &= \varepsilon \sin (k t + \beta) + \varepsilon_1 \sin (k_1 t + \beta_1) + \ldots + E \sin (K t + B) \\
\sin i \, \cos \Omega &= \varepsilon \cos (k t + \beta) + \varepsilon_1 \cos (k_1 t + \beta_1) + \ldots + E \cos (K t + B) \\
\sin i' \, \sin \Omega' &= \varepsilon' \sin (k t + \beta) + \varepsilon'_1 \sin (k_1 t + \beta_1) + \ldots + E' \sin (K' t + B') \\
\sin i' \, \cos \Omega' &= \varepsilon' \cos (k t + \beta) + \varepsilon'_1 \cos (k_1 t + \beta_1) + \ldots + E \cos (K' t + B').
\end{aligned}$$

In these expressions the quantities ε, ε_1, &c. are all very small, and if E, E', &c. also be small, it is evident that the inclinations will always be included within narrow limits. But, in order that the asteroids should have no influence in changing the node of *Mercury*, it is requisite that the mean positions of the planes of their orbits should not differ from the plane of the orbit of *Mercury*, and the latter planet having a large inclination, this condition cannot be fulfilled if the inclinations of the asteroids are small.

Let us now take the expression given in the *Mécanique Céleste*, Liv. II., No. 60, for the secular variation of the node of a planet.

$$\frac{d\theta}{dt} = -\{(0.1) + (0.2) + (0.3) + \ldots\} + (0.1)\frac{\tan i'}{\tan i}\cos(\theta - \theta') + (0.2)\frac{\tan i''}{\tan i}\cos(\theta - \theta'') + \ldots \quad (2)$$

If now the inclinations of the orbits of the disturbing bodies are considerable, E, E', &c. must be considerable in comparison with the εs. If the bodies are very numerous, several hundred in number, it will follow from the theory of probabilities that we may expect to find the angles $Kt + B$, $K't + B'$, &c. pretty evenly distributed throughout the circle. It is true that there is no physical necessity that they should be thus distributed; still, the probability against it is so small, that it would be entirely neglected in the ordinary concerns of life. But it is easy to see that, in the case I have supposed, these angles would represent approximately the longitudes of the nodes; therefore we are to expect that on the invariable plane the nodes will also be nearly evenly distributed in longitude. Again, there being no relation between the nodes, the inclinations, and the masses, and the angles $\theta - \theta'$, $\theta - \theta''$, &c. being nearly evenly distributed, it is in the highest degree probable that the second part of (2) will be very small in comparison with the first part, which will, therefore, very nearly represent the motion of the node of *Mercury*, produced by the intra-Mercurial planets. But this part is also the expression for the motion of the longitude of the perihelion, when the eccentricities are neglected, their effect being supposed to cancel each other, like those of the inclinations. The motion of the node on the ecliptic will be about $\frac{1}{8}$ less than on the invariable plane; we are, therefore, justified in concluding, that, if the motion of 38 seconds per century in the perihelion of *Mercury* is produced by a group of intra-Mercurial planets, the planets must also produce a retrograde motion of about 34 seconds in the node of *Mercury*.

It will, of course, be remembered, that this conclusion is founded on the hypothesis, reasons for which have been given, that each separate planet of the group is very small in comparison with the sum of the others.

THE SOLAR ECLIPSE IN WASHINGTON TERRITORY.

FROM THE REPORT OF LIEUT. GILLISS.

[Communicated by Prof. A. D. BACHE, Supt. U. S. Coast Survey.]

THE latitude of the station was determined from circummeridian altitudes of the *Sun*, *Polaris*, and *Altair*, measured with a sextant and artificial horizon, as follows: —

1860, July 13,	5 observations	sun's lower limb,	$+47^\circ\ 2'\ 26''.2$
" " 16,	5 "	Polaris	3 11.2
" " 18,	8 "	Altair	3 4.4
		Mean by weight,	47 2 54.1

The camp knoll was 47 feet north of that on which the eclipse was observed, and the latitude adopted is $+47^\circ\ 2'\ 54''$.

Neither of the chronometers was uniform in its rate. No. 2113 left New York with a daily gaining rate of $0^s.2$. From July 13th to July 18th, the observed rate was losing $3^s.31$; and during the whole time it was absent from New York it lost $0^s.95$ per day.

No. $\frac{2}{739}$ on leaving New York gained $0^s.5$ per day; its rate in camp was losing $0^s.62$, and for the whole absence losing $2^s.09$ per day.

The chronometric results for longitude are therefore: —

Chronometer.	From the rate of May 21.	From rate of July 13-18.	From rate of May 21 - Aug. 24.
	h m s	h m s	h m s
No. 2113,	8 11 16.44	8 9 8.33	8 10 33.89
No. $\frac{2}{739}$,	8 12 2.24	8 11 22.56	8 10 26.41
Means,	8 11 39.34	8 10 15.44	8 10 30.15

The correction applicable to observations at camp to reduce them to the telescope knoll being $-0^s.56$, if we adopt the determination from the rates between 21st May and 24th August