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X. *On a new fundamental Law of Electrodynamics.* *By* Professor R. CLAUSIUS *.

IN order to explain electrodynamic phenomena, W. Weber, as is well known, advanced a law on the force exerted on each other by two moving particles of electricity. Let e and e' be the two particles, each of which may be either positive or negative, and let r be their distance from each other, which is to be regarded as a function of the time t; according to Weber these particles exert a repulsion upon one another which is represented by the formula

$$\frac{ee'}{r^2}\left[1-\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2+\frac{2}{c^2}r\frac{d^2r}{dt^2}\right],$$

in which c is a constant.

Against this formula objections have been raised by Helmholtz; and from reasons quite independent of Helmholtz's, the conviction has forced itself upon me that it does not correspond to the reality. On the other hand, my considerations have led me to another dynamic law, which I take leave to communicate provisionally, reserving for a subsequent occasion the complete exposition of the reasons which have induced me to advance it. Two remarks only, serving for elucidation, I must premise before introducing the formulæ.

If we start from the conception that the electrodynamic action upon each other of two moving particles of electricity takes place through an intervening substance, we need not assume that it is dependent only on the relative motion of the particles, but we can also ascribe an influence upon it to the absolute motions of the two individual particles. If, for instance, two electric particles are moving in parallel directions with equal velocity, and consequently are, relatively to one another, at rest, they may yet exert a reciprocal electrodynamic action; for to the medium existing between them they behave differently from particles actually at rest. Further, with this conception, we need not assume that the direction of the electrodynamic force coincides with the line connecting the two particles, but may also admit forces from other directions as possible.

Let, now, x, y, z and x', y', z' be the rectangular coordinates of the two electric particles e and e' (concentrated in points) at the time t; and for the relative coordinates of the particle e to the particle e' let us introduce the following symbols—

$$\xi=x-x',\ \eta=y-y',\ \zeta=z-z'.$$

* Translated from a separate impression communicated by the Author, having been read before the Niederrheinische Gesellschaft für Natur- und Heilkunde, December 6th, 1875.

Call the distance between the two particles r. Further, let ds and ds' be two path-elements simultaneously passed through by the particles, ϵ the angle between them, and v and v' the velocities. If then the components (falling in the coordinate-directions) of the total electrostatic and electrodynamic force suffered by the particle e from the particle e' are represented by Xee', Yee', and Zee', the following equations, written first in the most general form, hold good, in which k is a positive constant referred to the quantitative ratio between the electrodynamic and electrostatic portions of the force, and n another constant, of which we shall speak further on:—

$$X = \frac{\xi}{r^3} - k\left(\frac{\xi}{r^3}\cos\epsilon + n\frac{d^2\frac{\xi}{r}}{ds\,ds'}\right)vv' + k\frac{d}{dt}\left(\frac{1}{r}\frac{d\xi}{dt}\right);$$

$$Y = \frac{\eta}{r^3} - k\left(\frac{\eta}{r^3}\cos\epsilon + n\frac{d^2\frac{\eta}{r}}{ds\,ds'}\right)vv' + k\frac{d}{dt}\left(\frac{1}{r}\frac{d\eta}{dt}\right);$$

$$Z = \frac{\zeta}{r^3} - k\left(\frac{\zeta}{r^3}\cos\epsilon + n\frac{d^2\frac{\zeta}{r}}{ds\,ds'}\right)vv' + k\frac{d}{dt}\left(\frac{1}{r}\frac{d\zeta}{dt}\right).$$

From these equations all the forces and induction-actions exerted on one another by galvanic currents can be deduced.

The three components of the force exerted on a current-element ds by a current-element ds' are represented generally by the following expressions—

$$cii'\,ds\,ds'\left(-\frac{\xi}{r^3}\cos\epsilon + \frac{d\frac{1}{r}}{ds'}\frac{d\xi}{ds} + \frac{d\frac{1}{r}}{ds'}\frac{d\xi}{ds'} - n\frac{d^2\frac{\xi}{r}}{ds\,ds'}\right),$$

$$cii'\,ds\,ds'\left(-\frac{\eta}{r^3}\cos\epsilon + \frac{d\frac{1}{r}}{ds'}\frac{d\eta}{ds} + \frac{d\frac{1}{r}}{ds}\frac{d\eta}{ds'} - n\frac{d^2\frac{\eta}{r}}{ds\,ds'}\right),$$

$$cii'\,ds\,ds'\left(-\frac{\zeta}{r^3}\cos\epsilon + \frac{d\frac{1}{r}}{ds'}\frac{d\zeta}{ds} + \frac{d\frac{1}{r}}{ds}\frac{d\zeta}{ds'} - n\frac{d^2\frac{\zeta}{r}}{ds\,ds'}\right),$$

in which i and i' signify current-intensities, and c is a positive constant dependent on the constant k and also on the unit selected for the measure of the current-intensity.

The question now is, what value is to be given to the constant n? If the value 1 be chosen, the preceding expressions will represent the components of the same force that was deduced by Ampère, namely an attraction of the strength

$$cii'\,ds\,ds'\left(\frac{\cos\epsilon}{r^2}+r\,\frac{d^2\frac{1}{r}}{ds\,ds'}\right).$$

This Ampèrian formula, however, cannot be proved by experiment for two single elements of current, but the experimental proof always refers to cases in which at least one of the currents is closed. Hence other forces may be regarded as admissible between two current-elements, if, for the case in which one current is closed, they only give the same result that was given by Ampère's calculation. Such forces are obtained when various values are given to the constant n in the above expressions; for the term affected by the factor n, occurring in each of the three expressions, which is a differential coefficient of the second order according to s and s', becomes in the integration over a closed current zero, and consequently cannot have any influence on the force which a closed current exerts on a current-element.

Hence, if we admit as certain only that which has been confirmed by experiment, we can provisionally consider n a constant yet to be determined. Theoretically, however, that value is the most probable which makes the fundamental equations simplest, viz. the value 0, by which those equations are transformed into

$$\mathrm{X}=\frac{\xi}{r^3}(1-k\,vv'\cos\epsilon)+k\,\frac{d}{dt}\left(\frac{1}{r}\,\frac{d\xi}{dt}\right),$$

$$\mathrm{Y}=\frac{\eta}{r^3}(1-k\,vv'\cos\epsilon)+k\,\frac{d}{dt}\left(\frac{1}{r}\,\frac{d\eta}{dt}\right),$$

$$\mathrm{Z}=\frac{\zeta}{r^3}(1-k\,vv'\cos\epsilon)+k\,\frac{d}{dt}\left(\frac{1}{r}\,\frac{d\zeta}{dt}\right).$$

XI. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

June 17, 1875.—Joseph Dalton Hooker, C.B., President, in the Chair.

THE following Papers were read:—

"On a new Form of Dynamo-Magneto-Electric Machine." By S. C. Tisley.

In the first machines constructed by Siemens and Wheatstone in