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XVI. Properties of the trapezium

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sufficient to communicate motion to the distant parts of the fluid in the direction of the motion of the body. That such increase of resistance as this formula indicates, takes place in great velocities has been ascertained by experiments on the motion of cannon balls.—I hope this paper contains, for those who are acquainted with the subject in theory and experience, sufficient evidence of the accuracy of my views; and if I have any where trodden in the steps of others it is not knowingly, and there is strong presumption that I have not done so, in the fact that I have not the slightest recollection of any theory founded on the same principles.

August 1826. THOMAS TREDGOLD.

XVI. *Properties of the Trapezium.* *By* T. S. DAVIES, *Esq. of Bath**.

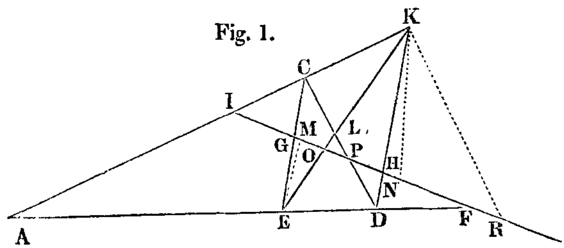
AT a very early period of my mathematical studies the trapezium seemed to offer a far more open field of inquiry, and to yield far more interesting results, than any other figure, the circle excepted, that I examined. The properties of the triangle, numerous as they were, had been so often examined, and their characters stated under almost every possible variety of aspect, that little really novel appeared likely to result from any inquiries, however well directed or vigorously pursued. The trapezium on the contrary had been investigated very little further than its elementary and more obvious properties; and the majority even of these were such as either had some analogy to the triangle, or resulted from dividing it into subordinate triangles. The ground has been occasionally trodden by others; but I think it will be found that every one who has done so has made his discoveries more by accident, or to meet some mathematical exigency, than from any systematic plan of inquiry: and the best proof of this is, that amongst the great men who have noticed any of the properties of the trapezium, not one of them has cared to pursue those results into any of their remoter consequences.

In the course of my researches I have been so fortunate as to meet with several classes of properties which, whether viewed as so many elegant theorems, or in reference to their application to subsequent inquiries, cannot fail to interest the geometrical student; and shall do myself the honour to present to this Society a series of short papers containing a few of the properties that I have noticed, together with a general outline of their demonstrations.

* Read before the Society of Inquirers, Bristol, August 5, 1825, and communicated by the Author.

ECKD a trapezium,—whose opposite sides meet in A and B and whose diagonals intersect each other in L,—is cut by the transversal I, F as in the figure.

Fig. 1.



PROP. I.

FD . CI : FE . KI : : CG . DH : GE . KH *.

Dem.—Draw ME parallel to KD, and KN to CE. Then

EG : EM : : KN : KH,
KN : CG : : IK : IC,
EM : DH : : EF : DF; and compounding

FD . CI : FE . KI : : CG . DH : GE . KH. Q. E. D.

Cor. 1. When F and I coincide with A, we have DA . AC : EA . EA . AK : : CG . DH : GE . KH, and ∴ CG . DH : GE . KH a constant ratio when the transversal passes through A.

Cor. 2. When CG . DH : GE . KH : : DA . AC : GE . KH the transversal passes through A.

Cor. 3. When the transversal passes through A and B
CA . AD : KA . AE : : CB . BD : EB . BK.

PROP. II. (Fig. 1.)

IC . FE : IK . FD : : EO . PC : OK . PD.

Dem.—Draw ME parallel to KD, and KN to CE. Then

FE : FD : : EM : DP,
EM : KN : : OE : OK,
IC : IK : : PC : KN; hence, compounding

IC . FE : IK . FD : : EO . PC : OK . PD. Q. E. D.

* To meet all the varieties which result from taking the trapezium successively *salient, re-entrant,* and *intersectant,* and from interchanging C and K in each of these classes, would require upwards of forty figures; but it is unnecessary to insert them here, as the geometrical reader will easily sketch them for his own use. *Forty-two* is the number I have drawn, but I cannot positively affirm that all possible cases are included in this list.

Cor.

Cor. 1. When F and I coincide with A, we learn that

$$CA.AE : KA.AD :: CP.EO : OK.PD.$$

Cor. 2. When O and P coincide with L, we have

$$IC.FE : IK.FD :: CL.LE : KL.LD.$$

Cor. 3. When the transversal passes through A and L, we find that

$$CA.AE : KA.AD :: CL.LE : KL.LD.$$

Cor. 4. When F and I coincide with A, we have

$$CL.LE : KL.LD :: CP.EO : OK.PD.$$

Cor. 5. Similarly we find when O, P, L coincide

$$IC.FE : IK.FD :: CA.AE : KA.AD.$$

Prop. III. (Fig. 1.)

$$GC.HK : GE.HD :: KO.CP : OE.PD.$$

Dem.—Draw EM, KN as before, and KR parallel to CD. Then

$$CG : KN :: CP : KR,$$
$$KN : GE :: KO : OE,$$
$$KH : K^3R :: H^2D : PD; \text{ hence}$$
$$CG.KH : GE.HD :: KO.CP : OE.PD. \quad \text{Q. E. D.}$$

Cor. 1. When the transversal passes through L, we find

$$CG.KH : GE.HD :: CL.LK : EL.LD.$$
$$:: CB.BK : EB.BD.$$

Prop. IV. (Fig. 1.)

Let the points ECKD lie in the circumference of a circle; then

$$AE : AK :: CL : LK :: EL : LD :: AC : CD.$$

Dem.—By circle $AE.AD : AC.AK :: CL.LD : EL.LK$, and (Cor. 1., Pr. 2.) $AE.AC ; AD.AK :: CL.LE : KL.LD$; hence, &c. Q. E. D.

Prop. V. (Fig. 1.)

The points ECKD being still in the circle we shall have

$$KC.CE : KD.DE :: CL : LD, \text{ and}$$
$$KC.KD : CE.ED :: KL : LE.$$

Dem.—The triangles CLE, KLD are similar, and

$$CE : KD :: EL : LD$$
$$CK : ED :: CL : LE; \text{ hence compounding}$$
$$KC.CE : KD.DE :: CL : LD.$$

In like manner is the second analogy demonstrated. Q. E. D.

Cor. Hence also $CE^2 : KD^2 :: CL.LE : KL.LD,$

$$:: CA.AE : KA.AD,$$

and $CK^2 : ED^2 :: CL.LK : EL.LD$

$$:: BC.BK : EB.BD.$$

Prop.

PROP. VI. (Figs. 2 and 3.)

Fig. 2.

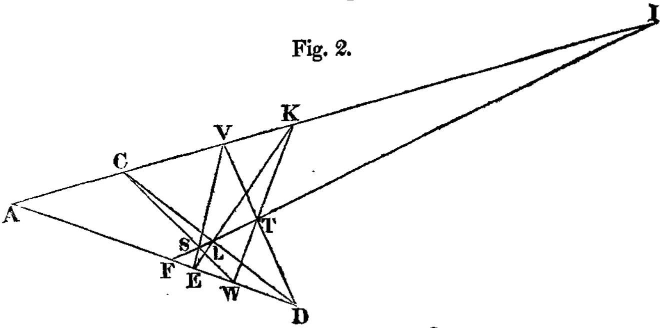
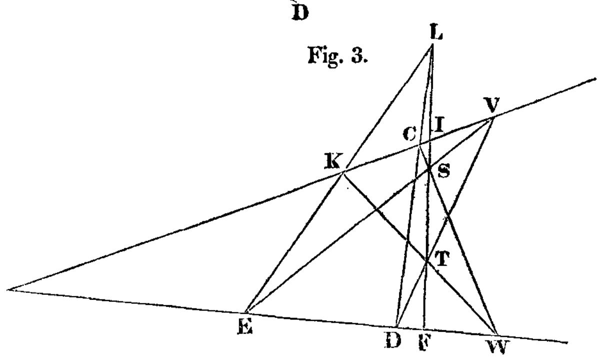


Fig. 3.



Upon the opposite sides ED, CK of a trapezium are constituted the triangles EVD, CWK, each having its vertex in the other's base. If S be the point of intersection of CW, EV, and T that of VD, WK; then will ST always pass through L, the intersection of the diagonals CD, KE of the trapezium*.

Dem.—Let A be the intersection of KC, ED, and ST cut CK, ED in I and F. Then

$$CS \,.\, SE : VS \,.\, SW :: CA \,.\, AE : AV \,.\, AW,$$
$$VT \,.\, TW : KT \,.\, TD :: AV \,.\, AW : KA \,.\, AD \ (ib.); \text{ hence}$$
$$\frac{CS \,.\, SE}{VS \,.\, SW} : \frac{KT \,.\, TD}{VT \,.\, TW} :: CA \,.\, AE : KA \,.\, AD$$
$$CL \,.\, LE : KL \,.\, LD \ \ldots\ldots (a)$$

Again,
$$IC \,.\, FE : IV \,.\, FW :: SC \,.\, SE : VS \,.\, SW \ (ib.)$$
$$IV \,.\, FW : IK \,.\, FD :: VT \,.\, TW : KT \,.\, TD \ (ib.) \text{ hence}$$
$$\frac{CS \,.\, SE}{VS \,.\, SW} : \frac{KT \,.\, TD}{VT \,.\, TW} :: IC \,.\, FE : IK \,.\, FD \ \ldots\ldots (b)$$

* This theorem also includes a considerable number of figures, which however it is unnecessary to insert here.

Hence

Hence from (*a*) and (*b*) we find

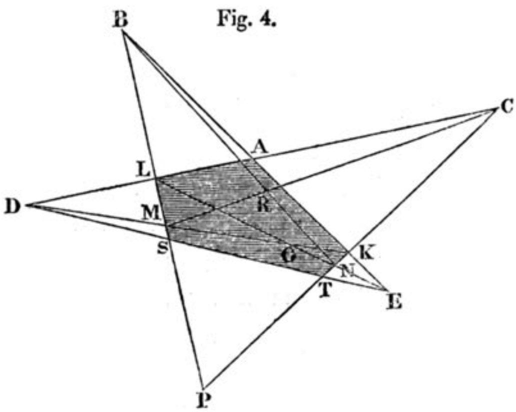
IC . FE : IK . FD : : CL . LE : KL . LD; and thence (Converse of cor. 2. Pr. 3.) that IF or ST passes through L.

Q. E. D.

The geometrical reader will readily perceive that C and K may interchange their places, so as to throw L without the trapezium whilst a similar mode of proof obtains. This will be rendered obvious from constructing the figure.

Schol. From this curious property a great number of beautiful porisms may be deduced; and it is probable that very many *linear loci* may be brought within the reach of actual demonstration by the efficiency of this single theorem. I gave a demonstration of it from other principles in the Monthly Magazine for July 1825. This property is not, however, yet stated in its most general or its most interesting form; for instead of being posited in the sides of the angle KAD, the points V and W may range *in the periphery of any Conic section whatever*, the points ECKD being in the same periphery. The demonstration depends upon a system of investigation something different from that employed in this paper: I shall therefore defer the proof till I have completed a paper upon which I am now employed, in which I trace a number of related properties of that class of curves. I shall, however, here set down two or three corollaries from Pr. 6.; for such they may be called, as they flow from it without the slightest reducing analysis.

Fig. 4.



Cor. 1. Let BDPEC (fig. 4) be a *pentalpha**, formed by pro-

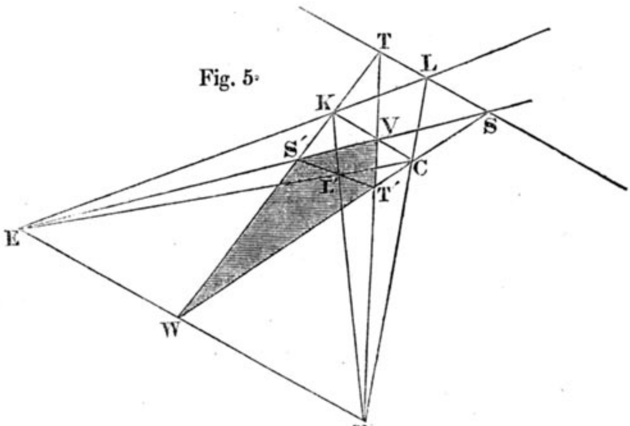
* This very appropriate name was given to the figure, I believe, by J. C. Hobhouse, Esq. M.P.; at least I do not recollect to have met with it except in his "Illustrations of the Fourth Canto of Childe Harold." This figure,

prolonging the sides of the pentagon ALSTK till they meet. Draw DK cutting EL in O and BP in M, and draw EL cutting PC in N. Then if R be the intersection of NB, MC, the line OR will tend to A, the summit of the pentagon.

Cor. 2. BE, CD (fig. 4) are two straight lines intersecting in A; let two points B, E and C, D be taken one pair in each of these lines, and P a point between those lines. From P draw PB, PC cutting DC in L and BE in K; draw DK cutting BP in M and LE in O, and let LE cut PC in N. Lastly, draw NB, MC intersecting each other in R. Then OR always tends to the same point, the intersection of the lines BE, CD.

This corollary has fifteen separate cases, all of which are referable to the different cases of Prop. 6.

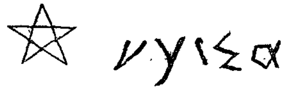
Fig. 5.



Cor. 3. Let S′VT′W (fig. 5) be any trapezium whatever. Through

figure, however, had been long celebrated amongst the Greeks, Romans, Etruscans, and other nations, before it was introduced into this country. Whether it was a Druidical figure I have not been able to learn, though I should think it highly probable that it was. The *medial section*, or *extreme and mean ratio*, probably obtained the name of the "*divine section*," from its necessity in constructing the pentagonal base of this figure, and the figure itself having been considered emblematical of the Deity.

Bishop Wilkins, speaking concerning characters that express words, goes on to say: "Of this nature was that angular figure so much used by the Grecians of old, which might be resolved into the letters υγιεα.



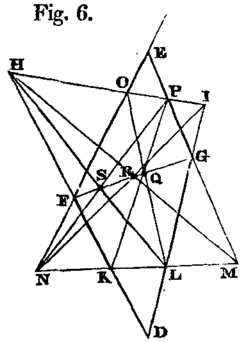
"This mark was esteemed so sacred amongst the ancients, that Antiochus Soter, a perpetual conqueror, did always instamp it on his coin, and in-

 scribe

Through either pair of opposite angles, as V and W, draw any lines KC, ED limited by the sides. The lines DK, EC will intersect in the diagonal S'T', and the lines EK, DC will intersect in the diagonal ST of the intersecting trapezium TS'ST'.

Prop. VII.

Fig. 6.



Let EFDG be any trapezium whatever, and through any point R in either of the diagonals, as in FG, draw lines, each to cut an opposite pair of sides in H, M and N, I; join also HI cutting the pair of sides EF, EG in O and P and NM cutting the pair FD, DG in K and L. Then OL, PK being drawn, their intersection Q is in the diagonal FG.

Dem.—Draw PN, HL intersecting in S; then since HLI, NPM are two triangles situated as in Pr. 6, the line joining their intersections S, G will pass through R the intersection of the diagonals HM, NI; and F, S, R, G, are in a straight line.

Again, because NOL, HKP, are triangles also situated as in Prop. 6. and intersecting in F, Q, the line FQ passes through S the intersection of the diagonals HL, NP; or Q is in the line FG. Q. E. D.

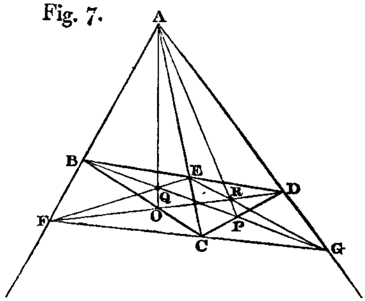
Cor. When HI passes through E, the points O, P coalesce with E and K, L with D; in which case, the theorem becomes the same with Cor. 3. Pr. 6.

scribe it on his ensigns; unto which he did pretend to be admonished in a dream by an apparition of Alexander the Great. And there are many superstitious women in these times who believe this to be so lucky a character, that they always work it upon the swaddling clothes of their young children, thinking thereby to make them healthful and prosperous in their lives. Unto this kind also, some refer the characters that are used in magic, which are maintained to have not only a secret signification, but likewise a natural efficacy."—Works, vol. ii. p. 49.

Mr. Hobhouse thinks the figure found its way into the Northern superstitions from its similarity to the hammer of Thor. He remarks that "the English shepherd who never heard of Antiochus, nor saw his coin, still cuts it in the grass": and perhaps some analogy might be traced by those who are curious in matters of this kind, between the signification of the heraldic mullet, the cinquefoil, and the rose, when compared with the signification of the figure in the older superstitions.

Prop. VIII.

Fig. 7.



Through either angle as C of any quadrilateral figure ABCD draw a line FG limited by the other sides AB, AD in F and G. Join GB, FD cutting the sides BC, CD in O and P, and let AO, AP cut BG, FD in Q and R. Then FQ and GR will pass through E, the intersection of the diagonals AC, BD.

Dem.—The triangles FCA, PBD being constituted in the trapezium FPDA according to the conditions of Pr. 6. show that E, R, G, are three points in the same straight line. Also, the triangles GCA, BDO in the trapezium GOBA, show in like manner that E, Q, F, are points in the same straight line.

Q. E. D.

Prop. IX.

Fig. 8.

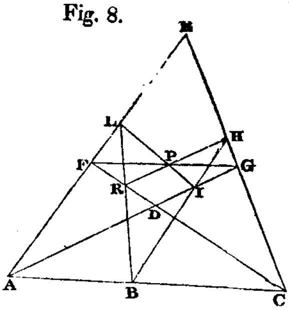
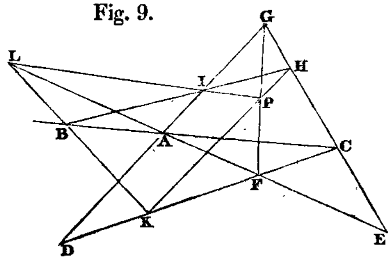


Fig. 9.



Let EFDG be a trapezium, whose opposite pairs of sides EF, GD and EG, FD*, meet in A and C. From any point B in the line AC, draw lines to cut the adjacent pairs of sides of the trapezium, viz. BIH, to cut DG in I and EG in H, and BKL to cut FD in K and FE in L. Then LI, KH will always intersect in the diagonal FG.

Dem.—1. The transversal AC cutting the triangles KEL and IGH give

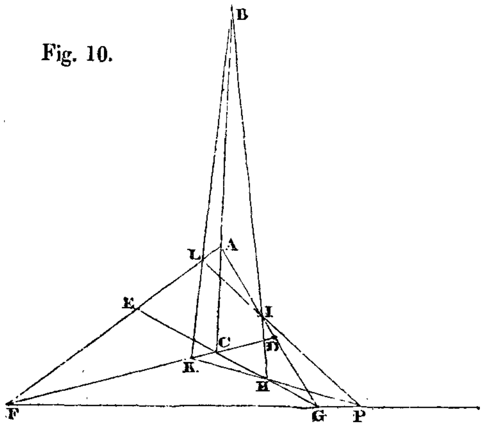
BK : BL : : AF . CK : AL . FC, (Bland's Geom. Prob. Pr. 47. Sect. 4.) and

BI : BH : : CG . AK : CH . AG. Hence, compounding

* It is to be taken for granted, that all lines given by position may be prolonged when necessary, as well as that lines joining given points, are also drawn.

BK . BI : BL . BH : : AF . CK . CG . AK : AL . CF . CH . GA.

Fig. 10.



2. Again, the triangles ALI, CKH, cut by the tranversal FG in the points P′ and P″, give

LP′ : P′I : : GA . FL : IG . AF (Bland. ib.) and
HP″ : P″K : : FC . GH : KF . CG. Whence, comp.

LP′ . P″H : P′I . P″K : : GA . FL . FC . GH : IG . AF . KF . CG.

3. Further, the triangles AFC, ACG give

LF : LA : : BC . KF : AB . CK. (Bl. ib.) and
GH : CH : : GI . AB : BC . IA. (ib.) Hence

$\frac{LF . GH}{CH . AL} = \frac{IG . KF}{CK . AI}$.

4. Dividing the terms of the first proportion by the corresponding terms of the second, we have

$\frac{LP' . HP''}{LB . HB} : \frac{P'I . P''K}{BI . BK} : : \frac{FL . GH}{CH . AL} : \frac{IG . KF}{CK . AI}$. But by (3) we learn that the second of these is a ratio of equality; hence

$$\frac{LP' . HP''}{LB . HB} = \frac{P'I . P''K}{BI . BK}, \text{ or}$$

LP′ . HP″ : P′I . P″K : : LB . BH : KB . BI,
: : LP . PH : IP . PK (Cor. 3. Pr. 2),

and hence (converse of Cor. 2. Pr. 3.) the lines LI, KH, and FG intersect in the same point P. Q. E. D.

Cor. 1. Conversely we find, *ex absurdo*, that when LI, KH, situated as in the theorem, intersect in FG, the lines LK, HI, will intersect in AC.

Schol. The three figures above given are sufficient for the purpose

purpose of showing the nature of the mutations which the theorem undergoes when we take the trapezium successively *salient* (fig. 8), *re-entrant* (fig. 9), and *intersectant* (fig. 10). The number of figures, however, which a complete exhibition of all the cases of the theorem would require, is not less than a hundred and twenty; perhaps even more, which have escaped my notice. These three figures are sufficient to show, also, that had we defined AC, FG, DE, the three diagonals of the trapezium in fig. 8, an elegant enunciation might be formed for the combination of them in pairs in reference to this theorem.

It is easy by a similar train of investigation to discover several analogous and equally beautiful properties of the trapezium: but as this paper has already exceeded the limits which I originally proposed to myself, I shall defer them to a future but not distant period. In conclusion, I shall just remark, that from the properties already given a series of interesting properties of the conic-sections are immediately deducible.

August 5, 1825.

XVII. Decas sexta novarum Plantarum Succulentarum; *Autore* A. H. Haworth, *Soc. Linn. Lond.—Soc. Horticult. Lond.—necnon Soc. Cæs. Nat. Curios. Mosc. Socio, &c. &c.*

To the Editor of the Philosophical Magazine and Journal.

Sir,

HAVING just finished my sixth Decade of new Succulent Plants, I lose no time in sending it for insertion in the next Number of your Magazine.

All the plants included in this decade, appear to me, after the most diligent research, to be entirely nondescript, and belong to the vast genus *Mesembryanthemum*; to which they form a conspicuous addition: for some of them are not only new as *species*, but constitute *new* and unrecorded *sections*. They are all flourishing and flowering in the royal and unrivalled gardens at Kew; and were sent thither by our most assiduous friend Bowie, whose adventurous and discriminating eye detected them growing spontaneously in the remote and arid regions of Southern Africa, whither this enterprising traveller is again about to repair. And he proposes to go not only with the intention of collecting plants, seeds and roots, but also, as heretofore, in almost all the branches of animated nature.

And now, sir, most heartily wishing this extensive discoverer of succulent plants every possible success in his laborious undertakings,—