

The Principle of Locality. Effectiveness, fate and challenges

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Abstract

The Special Theory of Relativity and Quantum Mechanics merge in the key principle of Quantum Field Theory, the Principle of Locality. We review some examples of its “unreasonable effectiveness” in giving rise to most of the conceptual and structural frame of Quantum Field Theory, especially in absence of massless particles. This effectiveness shows up best in the formulation of Quantum Field Theory in terms of operator algebras of local observables; this formulation is successful in digging out the roots of Global Gauge Invariance, through the analysis of Superselection Structure and Statistics, in the structure of the local observable quantities alone, at least for purely massive theories; but so far it seems unfit to cope with the Principle of Local Gauge Invariance.

This problem emerges also if one attempts to figure out the fate of the Principle of Locality in theories describing the gravitational forces between elementary particles as well. An approach based on the need to keep an operational meaning, in terms of localisation of events, of the notion of Spacetime, shows that, in the small, the latter must lose any meaning as a classical pseudoRiemannian manifold, locally based on Minkowski space, but should acquire a quantum structure at the Planck scale.

We review the Geometry of a basic model of Quantum Spacetime and some attempts to formulate interaction of quantum fields on Quantum Spacetime. The Principle of Locality is necessarily lost at the Planck scale, and it is a crucial open problem to unravel a replacement in such theories which is equally mathematically sharp, namely a Principle where the General Theory of Relativity and Quantum Mechanics merge, which reduces to the Principle of Locality at larger scales.

Besides exploring its fate, many challenges for the Principle of Locality remain; among them, the analysis of Superselection Structure and Statistics also in presence of massless particles, and to give a precise mathematical formulation to the Measurement Process in local and relativistic terms; for which we outline a qualitative scenario which avoids the EPR Paradox.

1 Local Quantum Physics and Field Theory

Special relativity requires that no physical effect can propagate faster than light. Quantum Mechanics states that two observables are compatible if their measurement operations do not perturb each other, and this is the case if and only if the associated operators commute. Brought together, these principles lead to *Locality*.

In Quantum Mechanics the observables are given as bounded operators on a fixed Hilbert space; in Quantum Field Theory we may take the Hilbert space \mathcal{H}_0 , describing a single superselection sector, *the vacuum sector*. Their collection is therefore irreducible. The main postulate is that this is the collection of *(quasi) local observables* [1, 2], i.e. we have an inclusion preserving map from nice regions (say the set \mathcal{K} of double cones - the intersections of open forward and backward light cones with a common interior point) in Minkowski space to $*$ subalgebras of operators

$$\mathcal{O} \mapsto \mathfrak{A}(\mathcal{O}) \subset B(\mathcal{H}_0) \quad (1)$$

whose selfadjoint elements are the observables which can be measured in the spacetime region \mathcal{O} , and such that *local commutativity* holds, i.e. the measurements of two spacelike separated observables must be compatible, so that they commute with each other:

$$\mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)' \quad \text{if} \quad \mathcal{O}_1 \subset \mathcal{O}_2' \quad (2)$$

where the prime on a set of operators denotes its commutant (the set of all bounded operators commuting with all the operators in the given set) and on a set in Minkowski space denotes the spacelike complement. Thus each $\mathfrak{A}(\mathcal{O})$ is included in the intersection of the commutants of all $\mathfrak{A}(\mathcal{O}_n)$, as \mathcal{O}_n runs through all the double cones spacelike to \mathcal{O} .

This is the principle of *Locality*; in its strongest form, *Duality*, it also requires that each $\mathfrak{A}(\mathcal{O})$ is maximal with the above property: more precisely, the mentioned inclusion is actually an equality:

$$\mathfrak{A}(\mathcal{O}) = \mathfrak{A}(\mathcal{O}')', \quad (3)$$

where, here and in the following, $\mathfrak{A}(\mathcal{O}')$ denotes the norm closed $*$ subalgebra generated by all the local algebras associated to the various double cones which are spacelike separated from \mathcal{O} , i.e. included in \mathcal{O}' .

A weaker form of this assumption is “essential duality”, requiring only that the *dual net* defined by

$$\mathfrak{A}^d(\mathcal{O}) = \mathfrak{A}(\mathcal{O}')',$$

is its own dual, that is

$$\mathfrak{A}^{dd}(\mathcal{O}) = \mathfrak{A}^d(\mathcal{O}),$$

or, equivalently, is again a local net.

If the theory is suitably described by Wightman fields, essential duality can be proved to hold [3] ; the weakening of duality to essential duality indicates the presence of spontaneously broken global gauge symmetries, which, at the end of the day, can actually be recovered within the group of all automorphisms, leaving \mathfrak{A} pointwise fixed, of the unique, canonically constructed, field algebra associated to \mathfrak{A} (see below), where the unbroken global gauge symmetries are given by the subgroup leaving the vacuum state invariant [4].

In physical terms, characteristic of the vacuum sector is the presence of the *vacuum state* ω_0 , induced by a unit vector Ω_0 in \mathcal{H}_0 ; the distinguished property is *stability*: the joint spectrum of the energy-momentum operators on \mathcal{H}_0 must be included in the forward light cone and the vacuum corresponds to the zero joint eigenvalue, i.e. its energy is minimal in all Lorentz frames. Here the energy-momentum operators are the generators of the unitary, continuous representation of the spacetime translation group, which implements its action on the quasilocal observables, expressing *covariance*. That action might be a restriction of an action of the whole Poincaré group if the theory is also Lorentz invariant; in both cases, or in the case of other spacetime symmetry groups, the group acts geometrically on the collection \mathcal{K} of regions, and covariance is expressed by an action of the group as automorphisms of the quasilocal algebra \mathfrak{A} such that the map \mathfrak{A} in (1) intertwines the two actions.

Translation and Lorentz covariance of the net (1), Covariance and Spectrum Condition in the vacuum sector, play no role in a large part of the analysis we survey here, except for a mild technical consequence, proven long ago by Borchers, that we called *the Property B*¹, which can just be as-

¹ *Property B*: If \mathcal{O}_1 and \mathcal{O}_2 are double cones and the second includes the closure of the first, then any selfadjoint projection E localised in the first is of the form $E = WW^*$,

sumed as an additional axiom besides duality. Most of the analysis requires nothing more.

The collection \mathfrak{A} of quasilocal observables will be the operator norm closure of the union of all the $\mathfrak{A}(\mathcal{O})$, that is, due to (1) and (2), their norm closed inductive limit. Thus \mathfrak{A} is a norm closed $*$ subalgebra of $B(\mathcal{H})$ (i.e. a *C* Algebra of operators* on \mathcal{H}) which is irreducible. The physical states of the theory are described by normalised positive linear functionals (in short: *states*) of \mathfrak{A} , i.e. are identified with the corresponding expectation functionals.

Unit vectors in \mathcal{H}_0 induce pure states all belonging to the same superselection sector, identified with the vacuum superselection sector; among these pure states a reference vector state ω_0 , induced by the unit vector Ω_0 , will be called the *Vacuum State* (resp. the Vacuum State Vector).

In general, there will be a maze of other pure states, all appearing, by the GNS construction, as vector states of other inequivalent irreducible representations of the algebra \mathfrak{A} . In order to describe the superselection sectors, we ought to consider representations describing, in an appropriate precise mathematical sense, *elementary perturbations of the vacuum*.

Such a criterion will select a collection of representations, among which we have to study irreducibility, equivalence, containment, computing their intertwiners; the unitary equivalence classes of its irreducible elements will be the *superselection sectors*. Their collection is thus determined by a category, whose objects are the representations of \mathfrak{A} fulfilling the selection criterion, and whose arrows are the intertwining operators.

Crucial is the choice of the selection criterion; for the sake of clarity of the exposition, we will concentrate in this survey on a rather restrictive choice, adopted early; the core of the results extend however to most general choice that is physically admissible in massive theories on the Minkowski space (or in not less than three space dimensions). Both these choices restrict states by their *localisability* properties; the other possibility, to restrict states by properties of *covariance and positivity of the energy* has been proposed still earlier by Borchers, and might be more crucial in the discussion of theories with massless particles [6].

The restrictive choice describes charges that can be localized exactly in any bounded region of spacetime; more precisely, one adopts the following *selection criterion*: the representations π of \mathfrak{A} describing elementary

where $W^*W = I$ and W is localised in the second.

(We could even choose W in the same algebra $\mathfrak{A}(\mathcal{O}_1)$ if the latter were a so called type III factor, which is most often the case by general theorems [5]).

perturbations of the vacuum are those whose restriction to $\mathfrak{A}(\mathcal{O}')$, for each double cone \mathcal{O} , is unitarily equivalent to the restriction to $\mathfrak{A}(\mathcal{O}')$ of the vacuum representation.

This means that any such representation will have, among its vector states, sufficiently many strictly *localised states*, with all possible double cone localisations. A state ω is strictly localised in a double cone \mathcal{O} if the expectation value in ω of any local observable which can be measured in the spacelike complement of \mathcal{O} coincides with the expectation value in the vacuum. Note that an electric charge cannot be localised in this sense, as a result of Gauss theorem [7, 8].

If the unitary operator U implements the equivalence of π and the vacuum representation when both are restricted to $\mathfrak{A}(\mathcal{O}')$ for a chosen double cone \mathcal{O} , we can realize the representation π in question on the same Hilbert space as the vacuum representation, carrying it back with U^{-1} . The representation ρ we obtain this way is now the identity map on $\mathfrak{A}(\mathcal{O}')$:

$$\rho(A) = A \quad \text{if} \quad A \in \mathfrak{A}(\mathcal{O}') \quad (4)$$

and the duality postulate implies that it must map $\mathfrak{A}(\mathcal{O})$ into itself; if \mathcal{O} is replaced by any larger double cone, $\mathfrak{A}(\mathcal{O}')$ is replaced by a smaller algebra, hence the foregoing applies, showing that any larger local algebra is mapped into itself; hence ρ is an *endomorphism* of \mathfrak{A} .

Since the choice of \mathcal{O} was arbitrary up to unitary equivalence, our localised morphisms are endomorphisms of \mathfrak{A} which, up to unitary equivalence, can be localised in the sense of (4) in any double cone.

Unitary equivalence, inclusion or reduction of representations are decided studying their *intertwining operators* $T : T\pi(A) = \pi'(A)T, A \in \mathfrak{A}$. Duality implies that the intertwining operators between two localised morphisms must be *local observables*, in particular they belong to \mathfrak{A} . Hence localised morphisms *act* on their intertwiners.

More generally, let \mathfrak{A} be a C^* Algebra with identity I and whose centre are the complex multiples of I ; the foregoing comments suggest to consider the category $\text{End}(\mathfrak{A})$ whose objects are the unital endomorphisms and whose arrows are their intertwiners *in* \mathfrak{A} .

We can define a *product* of objects as the composition of morphisms and on arrows, say $R \in (\rho, \rho'), S \in (\sigma, \sigma')$, by:

$$R \times S \equiv R\rho(S) \in (\rho\sigma, \rho'\sigma') \quad (5)$$

so that $\text{End}(\mathfrak{A})$ becomes a *strict associative C^* tensor category*, with a tensor unit, the identity automorphism, which is irreducible, since its self arrows are in the centre, hence are the complex multiples of I .

The category describing superselection structure is thus equivalent to the full tensor subcategory of $\text{End}(\mathfrak{A})$ whose objects are the *transportable localized morphisms*, that is endomorphisms of \mathfrak{A} which, up to unitary equivalence, can be localised in the sense of (4) in any double cone.

The property of transportability has two aspects. On one side, it is a very weak replacement of translation covariance: the unitary equivalence class of the considered representations does not change if we change the localisation region of a representative by any spacetime translation. On the other side, it carries the requirement that there is no *minimal size* for the region where a given superselection charge can be localised. Note, however, that only the first of these conditions is really essential for the analysis exposed here below.

A subrepresentation of a ρ corresponds to a selfadjoint projection E in (ρ, ρ) , and by Property B there is a local isometry W such that $E = WW^*$ so that composing $\text{Ad } W^*$ with ρ gives a corresponding *subobject* of ρ in our category; similarly we can use local isometries with range projections summing up to I to construct finite direct sums of objects; thus our category has *subobjects and direct sums*.

The Locality principle by itself implies that this category has a surprisingly rich structure. For the sake of the smoothness of the exposition we avoid here the full details in the definitions and results, which can be found in the literature we refer to. That structure, described in more detail below, can be summarized saying that it is a *strict symmetric tensor C^* category* with irreducible tensor unit, endowed with an integer (or infinite) valued intrinsic dimension function. The finite dimensional objects form a full tensor subcategory, whose objects are all finite direct sums of irreducible objects.

We will call this subcategory the *superselection category* and denote it by \mathcal{T} ; it possesses automatically a further important piece of structure: it is a *rigid* strict symmetric tensor C^* category with irreducible tensor unit.

Rigidity means that to any object we can assign a *conjugate* object, such that their tensor product contains the tensor identity (the identity morphism) as a component, with some minimality conditions which make its class unique [7, 9, 10].

The structure of the superselection category we just mentioned corresponds exactly to the structure of the category $\text{Rep } G$ of finite dimensional, continuous unitary representations of a compact group G , with the linear intertwining operators as arrows, equipped with the ordinary tensor product, the symmetry given by the flip of tensor products, the rigidity given by complex conjugation of representations. In other words, $\text{Rep } G$ is a symmetric tensor subcategory of $\text{Vect}_{\mathbb{C}}$, the category of finite dimensional complex

vector spaces.

The analogy is complete if we look at $\text{Rep } G$ as an abstract C^* tensor category, forgetting that actually the objects are finite dimensional vector spaces, and the arrows are linear operators between those spaces (to be more precise, the analogy is complete if we replace $\text{Rep } G$ by an equivalent rigid symmetric *strict* associative tensor category).

The analogy is limited precisely by the fact that, unlike $\text{Rep } G$, \mathcal{T} is *not* given, nor a priori represented, as a symmetric tensor *subcategory* of $\text{Vect}_{\mathbb{C}}$: in other words there is no a priori given faithful symmetric tensor functor \mathcal{F} of \mathcal{T} into $\text{Vect}_{\mathbb{C}}$.

The crucial importance of F lies in the fact that, if it exists, the classical theorems of Tannaka and Krein imply that the analogy we mentioned is actually described by a functor \mathcal{F} , for a unique compact group G .

However, by a completely different strategy, it has been possible to prove [10] that, for any rigid strict symmetric tensor C^* category with irreducible tensor unit, say \mathcal{T} , there exists a unique compact group G and faithful symmetric tensor functors F of \mathcal{T} into $\text{Vect}_{\mathbb{C}}$, respectively \mathcal{F} of \mathcal{T} into an equivalent subcategory of $\text{Rep } G$, such that the following diagram is commutative:

$$\begin{array}{ccc} \mathcal{T} & \xrightarrow{\mathcal{F}} & \text{Rep } G \\ & \searrow F & \downarrow f \\ & & \text{Vect}_{\mathbb{C}} \end{array}$$

where f denotes the forgetful functor.

The proof is obtained reducing to the crucial case where \mathcal{T} is a full tensor subcategory of $\text{End}(\mathfrak{A})$, where \mathfrak{A} is a unital C^* algebra whose centre reduces to the complex multiples of the identity [11]. In this case, G arises as a dual action: dual to the action of \mathcal{T} on \mathfrak{A} , on a crossed product $\mathfrak{B} = \mathfrak{A} \rtimes \mathcal{T}$. More precisely, G is the set of all automorphisms of \mathfrak{B} leaving \mathfrak{A} pointwise fixed.

The crossed product will in particular contain \mathfrak{A} as a subalgebra with trivial relative commutant; as a consequence, for each endomorphism ρ of \mathfrak{A} , the subspace of all intertwiners in \mathfrak{B} between the actions on \mathfrak{A} of the identity map and of ρ is a *Hilbert space in \mathfrak{B}* [12], which naturally defines a tensor functor \mathcal{G} of $\text{End}(\mathfrak{A})$ into the category of Hilbert spaces in \mathfrak{B} , (where the arrows are the continuous linear operators ²), the tensor product of two Hilbert spaces in \mathfrak{B} being given by the operator product in \mathfrak{B} . The

²if the Hilbert spaces in question are finite dimensional; otherwise we must add: which are defined by the product with an element of \mathfrak{B}

finite dimensional Hilbert spaces in \mathfrak{B} with trivial left annihilator (i.e. with support I) form a *symmetric tensor C^* category $\mathcal{H}(\mathfrak{B})$* , with a symmetry defined by the flip operators of the tensor product (that, as mentioned, coincides with the operator product).

Now we can formulate the crucial conditions, which define uniquely the crossed product:

1. \mathfrak{A} is unittally embedded in \mathfrak{B} with trivial relative commutant;
2. the restriction \mathcal{G}_0 of \mathcal{G} to \mathcal{T} is a faithful symmetric tensor functor into $\mathcal{H}(\mathfrak{B})$;
3. the objects in the range of \mathcal{G}_0 generate \mathfrak{B} as a C^* algebra.

To appreciate condition 2., note that in general \mathcal{G} would map most objects to the Hilbert space consisting only of the 0 element, or whose support is not I ; or, sometimes, to a Hilbert space which is infinite dimensional.

The condition on the relative commutant in 1., and condition 3., express the minimality of the crossed product.

Defining G as the set of all automorphisms of \mathfrak{B} which leave \mathfrak{A} pointwise fixed, we have that its elements must leave each object in the range of \mathcal{G}_0 stable, hence must induce on each such object (a finite dimensional Hilbert space inducing a given localised morphism on \mathfrak{A}) a finite dimensional unitary representation; the strong (point - norm) topology on G coincides, by 3., with the (Tychonov) topology defined these representations, and thus makes of G a compact group. Composing \mathcal{G}_0 with the map of its range to representations of G we just mentioned, we obtained the desired functor \mathcal{F} .

It is worth noting that the need for an abstract duality theory for compact groups, a problem which arose in Algebraic Quantum Field Theory at the end of the 60's and was solved at the end of the 80's, emerged meanwhile in similar terms (for Algebraic Groups) in Mathematics, in the context of Grothendieck Theory of Motives; an independent solution, just slightly later and with slightly different assumptions, was given by Deligne [13]. In recent years, Mueger gave an alternative proof of the Abstract Duality Theorem for Compact Groups, following the line of the Deligne approach [14].

As previously stressed, the detailed, crucial properties of \mathcal{T} (namely to be a *rigid, strict symmetric, tensor C^* category with irreducible tensor unit*), are all important consequences of *Locality*.

The *tensor* structure is just inherited from that of $\text{End}(\mathfrak{A})$, where \mathcal{T} can be embedded thanks to *Duality*; and the irreducibility of the tensor unit accordingly amounts to the triviality of the centre of \mathfrak{A} .

The *symmetry* arises from the fact that *Locality* of \mathfrak{A} propagates to the category of transportable localised morphisms: if the localised morphisms ρ, σ are localised in mutually spacelike double cones, they commute:

$$\rho\sigma = \sigma\rho;$$

moreover,

$$T \times S = S \times T$$

if the sources of the arrows T and S are mutually spacelike localised morphisms, and the same is true for the targets.

It is worth noting that while the first relation (local commutativity of localised morphisms) holds unconditionally, the second one (local commutativity of arrows) holds only if we deal with theories on a spacetime with more than one space dimensions (that is spacetime dimension is at least three). This is apparent from its derivation: it is evident if there are two spacelike separated double cones, the source and target of the first arrow being localized in one of them, and those of the second arrow in the other one. The general case is reduced to this one by a sequence of small moves, which is possible if there is enough room, but not in two spacetime dimensions.

By a similar argument, this property extends to a larger class, of morphisms localised in spacelike cones (see below), only if there are at least three space dimensions.

Thus in low dimensional theories the superselection category might fail to be symmetric, it would be a rigid *braided* tensor C^* category [15][16].

Any two arrows T, S can be made spacelike separated in the above sense, if we compose them with suitable unitary intertwiners, between the considered morphisms and suitably localised ones. By simple algebra, this relation provides a unitary

$$\epsilon(\rho, \sigma) \in (\rho\sigma, \sigma\rho)$$

which is easily seen to depend only upon ρ and σ , such that

$$\epsilon(\rho, \sigma) \circ T \times S = S \times T \circ \epsilon(\rho, \sigma)$$

which, together with compatibility with the tensor product, and the symmetry property

$$\epsilon(\rho, \sigma) = \epsilon(\sigma, \rho)^{-1}$$

are the defining properties of a *symmetry*. By its very construction, it is *the unique symmetry for the category of transportable localized morphisms which reduces to the identity on spacelike separated objects*.

These properties of being a symmetry imply that, associating to the exchange of j with $j + 1$ the intertwiner

$$\epsilon_\rho^{(n)}(j, j + 1) = I_{\rho^{j-1}} \times \epsilon(\rho, \rho)$$

one defines a unitary representation of the permutation group of n elements, for each n larger than j , with values in the selfintertwiners of ρ^n .

The importance of these pieces of structure for the physical content can now be seen, if we associate to each localised morphism the localised state obtained composing the vacuum state with that morphism.

Local commutativity of morphisms show that the product of morphisms is transported by that map to a commutative product of mutually spacelike strictly localised states,

$$\omega_1 \times \omega_2 \times \cdots \times \omega_n \equiv \omega_0 \circ \rho_1 \rho_2 \cdots \rho_n \quad (6)$$

if $\omega_j = \omega_0 \circ \rho_j$, $j = 1, 2, \dots, n$, where the product state restricts to each factor when tested with observables localized spacelike to the remaining ones. Thus this product has the meaning of “composition of states”, and it factors to classes giving a product of unitary equivalence classes of localised morphisms, which is commutative owing to the locality of morphisms, and thus has the meaning of “composition of charges”.

Accordingly, a pair of objects are conjugate if the composition of their charges can lead, among the possible channels, to the charges of the vacuum sector; namely rigidity means particle–antiparticle symmetry of superselection quantum numbers.

If the morphisms in (6) are all equivalent to a given ρ , and say U_j in \mathfrak{A} are associated (local) unitary intertwiners, then the product $\rho_1 \rho_2 \cdots \rho_n$ is equivalent to ρ^n and

$$U_1 \times U_2 \times \cdots \times U_n \in (\rho^n, \rho_1 \rho_2 \cdots \rho_n).$$

Now obviously our states ω_j are vector states in the representation ρ induced by the state vectors

$$\Psi_j = U_j^* \Omega_0$$

and we can define a *product state vector* $\Psi_1 \times \Psi_2 \times \cdots \times \Psi_n$ which induces the state $\omega_1 \times \omega_2 \times \cdots \times \omega_n$ in the representation ρ^n by setting:

$$\Psi_1 \times \Psi_2 \times \cdots \times \Psi_n \equiv (U_1 \times U_2 \times \cdots \times U_n)^* \Omega_0.$$

If we change the order $(1, 2, \dots, n)$ by a permutation p , the product state will not change but the product state vector changes to

$$\begin{aligned} \Psi_{p^{-1}(1)} \times \Psi_{p^{-1}(2)} \times \dots \times \Psi_{p^{-1}(n)} &= \\ &= (U_{p^{-1}(1)} \times U_{p^{-1}(2)} \times \dots \times U_{p^{-1}(n)})^* (U_1 \times U_2 \times \dots \times U_n) \Psi_1 \times \Psi_2 \times \dots \times \Psi_n \\ &= \epsilon_\rho^{(n)}(p) \Psi_1 \times \Psi_2 \times \dots \times \Psi_n, \end{aligned}$$

where $\epsilon_\rho^{(n)}(p)$ is precisely the *representation of the permutation group* canonically associated to ρ , with values in the commutant of ρ^n .

If ρ is changed to another localised morphism ρ' by a unitary equivalence, say U in (ρ, ρ') , $\epsilon_\rho^{(n)}$ is changed to $\epsilon_{\rho'}^{(n)}$ by a unitary equivalence, implemented by $U \times U \times \dots \times U$; thus the hierarchy of unitary equivalence classes of the representations $\epsilon_\rho^{(n)}$, $n = 2, 3, \dots$ depends only upon the unitary equivalence class of ρ , a *superselection sector* if ρ was irreducible.

This hierarchy is then the *statistics* of that superselection sector.

The main result on statistics says that (as a consequence solely of our assumptions, that is essentially as a consequence of Locality alone) the statistics of a superselection sector is uniquely characterised by a “statistics parameter” associated to that sector, which takes values $\pm 1/d$, or 0, where d is a positive integer. The integer d will be *the order of parastatistics*, and $+$ or $-$ will be its *Bose or Fermi character* (no distinction for infinite order, when the parameter vanishes).

More explicitly, let \mathcal{K} be a fixed Hilbert space of dimension d , and let $\theta_n^{(d)}$ denote the representation of the permutation group of n objects which acts on the n th tensor power of \mathcal{K} permuting the factors; our theorem says that, given a superselection sector, if its statistics parameter λ is $+1/d$ then, for each n , $\epsilon_\rho^{(n)}$ is unitarily equivalent to the sum of infinitely many copies of θ_n^d ; if λ is $-1/d$, the same is true provided we further multiply with the sign of the permutation; the latter being irrelevant if $d = \infty$, i.e. if $\lambda = 0$.

Furthermore, $d(\rho) = 1$ iff ρ is an *automorphism*; the quotient of the group of all *localised automorphisms* modulo its normal subgroup of inner elements is a commutative group, naturally equipped with its discrete topology. Its Pontryagin - van Kampen dual is a compact abelian group, the quotient of the previously described compact group G , dual to \mathcal{T} , modulo its closed normal subgroup generated by commutators.

In absence of parastatistics the duality problem is then solved by the classical theorems; moreover, if the superselection group has independent generators, it acts on \mathfrak{A} via a section, and the crossed product is just a covariance algebra, i.e. an ordinary crossed product by that action. An

induction procedure solves the general (commutative) case [17].

The inverse of a localised automorphism provides a *conjugate*; if ρ has dimension d , the subobject of its d th power corresponding to the image through $\epsilon_\rho^{(n)}$ of the totally antisymmetric projection - the *determinant* of ρ - may be shown to be one dimensional, hence an automorphism; if we compose its inverse with the $(d - 1)$ th power of ρ and take an appropriate subobject we construct a conjugate of ρ .

A physically more illuminating picture of the conjugate emerges if we take a sequence of local unitaries U_n intertwining ρ and morphisms whose localisation double cones run to spacelike infinity when n tends to infinity.

Then $\text{Ad}U_n^*$ are inner (bi-)localised morphisms, readily seen to converge to ρ in the point - norm topology. Limiting points of $\text{Ad} U_n$ in the point - weak operator topology provide “left inverses” of ρ ; but if the dimension of ρ is finite, that sequence is actually convergent to a unique left inverse, from which the conjugate may be constructed. If we assume furthermore translation covariance and the spectrum condition, a weak converse showed at the same time that if there is a left inverse which, when composed with the vacuum state, gives a vector state in a positive energy representation, then the statistics of ρ is finite [9].

We may interpret the states obtained composing the vacuum state with $\text{Ad} U_n^*$ or $\text{Ad} U_n$ as bilocalised states; the first sequence will contain the charges of ρ in a fixed region, and some compensating charge (so that the state as a whole lies in the vacuum sector) in a region running to spacelike infinity; the second one will contain the charges of ρ in the region running to spacelike infinity, so that they disappear in the limit, and the compensating charge in the fixed double cone; thus, if the statistics of ρ is finite, that compensating charge may be caught in the limit as the conjugate sector.

Can infinite statistics actually occur? The answer is yes in low dimension [18, 19, 20], where anyway the theory above does not apply; in $3 + 1$ dimensions, however, a key result of Fredenhagen shows that, in theories with purely massive particles, statistics is automatically finite [21].

We have seen that to each superselection sector is associated (an integer, the order of parastatistics, and) a sign, $+1$ for paraBose and -1 for paraFermi. In relativistic theories, to each sector another sign is intrinsically attached, $+1$ for sectors with integer and -1 for those with half integer spin values, describing univalence.

Then the *Spin Statistics Theorem*, based solely on First Principles, holds: for sectors with an isolated point in mass spectrum with finite particle multiplicity, those signs must agree.

This theorem, first proved for the class of sectors described here [9], was then extended to sectors localisable only in spacelike cones, covering all massive particle theories (cf below) [22]. More recent variants replaced the assumptions of covariance and finite mass degeneracy by that of *modular covariance* [23]. It has been generalised even to QFT on some appropriate kinds of curved spacetimes [24].

It has been established in low dimensional theories as well, as a relation between the phase of the statistics parameter and a phase generalising univalence [23, 25].

Existence of conjugates in the category language is called rigidity; this term refers to the specific properties of conjugation in a tensor category of finite dimensional unitary representations of a group. The tensor product of a representation on H with its complex conjugate acting on the complex conjugate Hilbert space \bar{H} always contain the identity representation on the one dimensional Hilbert space of complex numbers. Special intertwiners are defined as the maps changing the number λ to $\lambda \cdot \sum_{j=1}^d e_j \otimes \bar{e}_j$ or to $\lambda \cdot \sum_{j=1}^d \bar{e}_j \otimes e_j$, for any orthonormal basis in the d dimensional Hilbert space H . These intertwiners are related to one another by the flip operator, defining the standard symmetry of the category. They obey precise *conjugate relations*.

The same conjugate relations are fulfilled in our category \mathcal{T} if we equip it with another symmetry; to specify it, it suffices to tell its values on irreducible pairs of objects: in that case it will be the opposite of the previously described canonical choice if both objects are parafermi, and agree with it in the other cases.

It is this new symmetry that corresponds functorially to the flip in the crossed product construction. Recall that the flip changes the order in the operator product in \mathfrak{B} of two Hilbert spaces; thus, if ψ and ψ' are elements of \mathfrak{B} which respectively implement on \mathfrak{A} two *irreducible* spacelike separated morphisms ρ, ρ' , we will have

$$\psi\psi' = \theta\psi'\psi = \pm\epsilon(\rho, \rho')\psi'\psi = \pm\psi'\psi$$

where the minus sign occurs only if both the chosen morphisms are parafermi, and the plus sign occurs otherwise. We used of course the characterizing property of our original symmetry, namely its being the identity on spacelike separated objects.

Composing the vacuum state of \mathfrak{A} with the G invariant conditional expectation of \mathfrak{B} onto \mathfrak{A} given by the integration over the action of G , we extend it to the *vacuum state* of \mathfrak{B} , a pure G invariant state. The associated GNS representation will be the *vacuum representation* of \mathfrak{B} .

We can define the local field algebras associating to each double cone \mathcal{O} the von Neumann algebra $\mathfrak{F}(\mathcal{O})$ generated, in that representation, by the Hilbert spaces in \mathfrak{B} which implement on \mathfrak{A} morphisms localized in the given double cone. The C^* algebra generated is the quasilocal field algebra \mathfrak{F} , in its (irreducible) defining representation. Restricting this representation to the subalgebra of observables \mathfrak{A} we get a *reducible* representation, which is the direct sum of irreducible objects of \mathcal{T} , each class being represented, and with a multiplicity equal to the order of the parastatistics, which agrees with the dimension of the associated irreducible continuous unitary representation of G ; all such representations must occur [26].

The properties we listed, including notably completeness (all superselection sectors must be described by \mathfrak{F}), normal commutation relations at spacelike separations, imply also the *uniqueness* of our field net.

If only *essential duality* is fulfilled in the vacuum representation, we may still apply this theory to the *dual net* $\mathcal{O} \mapsto \mathfrak{A}^d(\mathcal{O})$, a local net fulfilling duality.

The representations of the given net fulfilling the selection criterion can be seen to be exactly the restrictions of those of the dual net fulfilling the selection criterion, with the same intertwiners. Consequently we obtain the inclusions

$$\mathfrak{A} \subset \mathfrak{A}^d \subset \mathfrak{F}$$

and a compact group defined by the second inclusion as the group G of all automorphisms of \mathfrak{F} leaving \mathfrak{A}^d pointwise fixed; this is the group of all *unbroken* global gauge symmetries, whose dual is given by the superselection structure. The (possibly larger and not necessarily compact) group \mathcal{G} of all automorphisms of \mathfrak{F} leaving \mathfrak{A} pointwise fixed can be shown to leave automatically each local field algebra globally stable, to commute with translations if the theory is translation covariant, and to include G precisely as the stabilizer of the vacuum state [4].

Thus the global gauge group G exists by the virtue of Locality, and is entirely encoded in the *local observable* quantities; *any* compact metrizable group must appear this way [27].

The Goldstone Theorem, its variants, and limits of its validity, can be thoroughly discussed in this frame [4].

As mentioned at the beginning, the selection criterion of the representations which form the superselection structure does not apply to theories with massless particles like QED. But does it cover the most general theory *without* massless particles?

Characteristic of such a theory would be existence (and abundance)

of representations which are translation covariant, with energy momentum spectrum included in the forward light cone (and hence necessarily Lorentz invariant [8]), and mass spectrum starting with a positive *isolated* point.

Adopting this as the definition of *particle representation*, Buchholz and Fredenhagen were able to prove that each irreducible (or factorial) particle representation is necessarily localisable in all *spacelike cones*; namely it obeys the weaker but similar selection criterion where the (*bounded*) double cones \mathcal{O} are replaced by the *unbounded* regions \mathcal{C} , defined each as the cone with vertex in a generic point in spacetime spanned by all half lines joining the vertex to a chosen double cone in the spacelike complement of that point [8].

Such representations cannot be described any longer by endomorphisms of the quasilocal C^* Algebra itself, but still suitable variants of the above construction apply with the same results [8, 26].

We dealt in some detail with but one of the lines where the Principle of Locality has shown its “unreasonable effectiveness”; it is worth at least to mention other important lines, we will not dwell on here.

One of them is the weak form of the Quantum Noether Theorem, based on the *Split Property* [28]. This is an enhancement of Locality, requiring that, if the double cone \mathcal{O}_2 contains the closure of the double cone \mathcal{O}_1 , there is a type I factor N such that

$$\mathfrak{F}(\mathcal{O}_1) \subset N \subset \mathfrak{F}(\mathcal{O}_2).$$

It implies the analog for the observables, which is equivalent to a principle of local preparation of states [32]. It excludes physically unreasonable models as most generalized free fields, and would follow from growth conditions at high energies of the densities of localized states; the required *nuclearity* conditions would guarantee the existence of thermal equilibrium states [29].

The split property provides an exact form of local current algebras associated to exact global symmetries, and local charge operators associated to the superselection quantum numbers [30, 31, 32].

While the local observables in general have no specific individual characterisation, all that matters being their localisation region, the weak form of the Quantum Noether Theorem provides specific local observables with precise physical meaning. This opens the way to an intrinsic definition of local observables [33].

These local aspects of superselection rules suggested from the very beginning a possible approach to a full Quantum Noether Theorem; this idea has been so far successfully tested on some free field models (cf [34] and references therein).

The nuclearity conditions have been studied in various forms, global and local [35].

Important and successful lines are, furthermore, the study of the scale limit, the analysis of the phenomenon of confinement and of renormalisation in terms of local algebras [36, 37, 38, 39].

But the Algebraic Approach proved quite fruitful also in the formulation of Quantum Field Theory on curved Spacetimes [40] and in the perturbative approach [41].

In a world with only one (or, for sectors which are only localisable in spacelike cones, with only two) space dimensions, as mentioned above, the statistics might be described by a braiding, not necessarily by a symmetry [15, 16] ; the problem of extending the abstract duality theory to this case is still open (see, however, [42, 43, 44]).

A very rich and successful field of reseaches grew up in recent years, on the Algebraic Approach to Conformal Quantum Field Theory; a review can be found in [45]; for recent relations to Noncommutative Geometry, see [46].

There one finds deep relations to the Theory of Subfactors; here, we limit ourselves to quote the general relation, established by Longo [16], between the square modulus of the statistics parameter of a localised morphism ρ and the Jones index of the inclusion of local algebras determined by ρ , more precisely

$$d(\rho)^2 = \text{ind}(\rho(\mathfrak{A}(\mathcal{O})), \mathfrak{A}(\mathcal{O}))$$

where the Jones index is meant to be extended to infinite factors [16].

This relations confirms an early view that the statistics parameter of a localised morphism ought to be related to a noncommutative version of the analytical index of a Fredholm operator (coinciding with the defect for injective operators, as in the previous relation); remarkable developments may be found in [47].

In the physical Minkowski space, the study of possible extensions of superselection theory and statistics to theories with massless particles like QED, is still a fundamental open problem. The electrically charged states will not be captured by the selection criterion described above (not even by its more general form in terms of spacelike cones). While the laws of Nature are believed (and indirectly checked, down to the scale of 10^{-17} cm) to be local, those states will not be localised, due to the slow decay of Coulomb fields [48, 49]. The relevant family of representations describing superselection sectors will have only asymptotic localisation properties; it might still be, however, described by a tensor category of morphisms of our algebra of quasilocal observables; this category can at most be expected to

be asymptotically Abelian in an appropriate sense; but this might well be enough to derive again a symmetry [50].

More generally the algebraic meaning of quantum gauge theories in terms local observables is missing.

This is disappointing since the success of the Standard Model showed the key role of the Gauge Principle in the description of the physical world; and because the validity of the Principle of Locality itself might be thought to have a dynamical origin in the *local nature* of the fundamental interactions, which is dictated by the Gauge Principle combined with the principle of minimal coupling.

In view of the last comment, a deep understanding of these principles in local algebraic terms might be of extreme importance as a guide to understand their variants on Quantum Spacetime, where, as discussed in Section 3, locality is lost; but it might well lift to an equally stringent and precise physical principle, having its dynamical origin in the Gauge Principle, but taking a different form due to the noncommutativity of the Spacetime manifold, while reducing to Locality at distances which are large compared to the Planck length.

2 Local Quantum Physics and the Measurement Process

The quantum process of measurement of an observable A in a given state of the observed system \mathcal{S} can be thought, following von Neumann [51], as the result of the time evolution of the composed system, where we added to \mathcal{S} the *measuring apparatus* of A , \mathcal{A} ; of the latter, we distinguish here its *microscopic part* $\mu\mathcal{A}$, which interacts with \mathcal{S} and is left, after the measurement, in mutually orthogonal states distinguishing the different values of A in \mathcal{S} , and its *macroscopic part* $M\mathcal{A}$, which does not interact appreciably with the system, but is coupled to $\mu\mathcal{A}$ in such a way that it is left in different states, which amplify to a macroscopically accessible level the different final states of $\mu\mathcal{A}$, and thus render accessible to the observer the value of A in the given state of the system.

If that value was not sharp, of course, the resulting state of the composed system \mathcal{S} plus $\mu\mathcal{A}$ plus $M\mathcal{A}$ will be an entangled state, resulting from the superposition of the different product states.

But if, as it is the case in practice, the amplifying part $M\mathcal{A}$ is composed by an enormous number N of particles, its different final states will be associated to *disjoint representations* [52] in the limit $N \rightarrow \infty$. The coherence

between the different outcomes, in principle still accessible with the measurement of the nearly vanishing interference terms (vanishing exactly only in the limit $N \rightarrow \infty$), will be totally inaccessible in practice as soon as N is sufficiently large, as the number of molecules in a bubble from the trace of a charged particle in a bubble chamber.

This (personal) summary of the ideas of von Neumann combined with those of Ludwig, Daneri-Loinger-Prosperi [53], Hepp [54], Zurek [55], Sewell [56] Castagnino [57], and many others, takes a more specific form if we require that A is a *local* observable and that the whole theory (describing, with the same dynamics, the time evolution of our system \mathcal{S} and its interactions with $\mu\mathcal{A}$, as well as those of $\mu\mathcal{A}$ with $M\mathcal{A}$), is *local* in the sense of the previous Section.

In this case, the operation of adding the measuring apparatus \mathcal{A} to \mathcal{S} is not described, as it was in the picture by von Neumann, by taking the tensor product of the state of \mathcal{S} before the measurement with the state of \mathcal{A} “ready to measure A ” (that is by an isometry of the Hilbert space of state vectors of the system into its tensor product with that of state vectors of the measuring apparatus), but rather by an isometry which maps *into itself* the Hilbert space of state vectors of the Field Theory describing both the system with or without the apparatus (namely, the vacuum Hilbert space \mathcal{H} of the fields, including all superselection sectors as discussed in the previous Section).

Now, if we consider a local observable A , say in $\mathfrak{A}(\mathcal{O})$, the operation of adding to the system the *microscopic part* of the apparatus which measures A should be described by an isometry W which is *localised in the same region*, that is W is in $\mathfrak{F}(\mathcal{O})$.

Since the time duration T of the measurement is supposed to be very short compared to the independent evolution of our system, the measurement process will be described by the change of W under its time evolution, in the Heisenberg picture, to $\alpha_T(W)$, localised in $\mathcal{O} + T$; thus, changing \mathcal{O} to a slightly larger double cone, we may say that both W and $\alpha_T(W)$ are isometries in $\mathfrak{F}(\mathcal{O})$.

As in the picture given by von Neumann, if A has finite spectrum with spectral projections E_j , the effect of the evolution will be

$$\alpha_T(W) = \sum_j W_j E_j, \quad W_j^* W_k = \delta_{j,k}, \quad (7)$$

where the W_j are isometries with mutually orthogonal ranges, which commute with the E_j , and, if the further addition to the system of the

macroscopic part $M\mathcal{A}$ of the measurement apparatus is taken into account, will trigger the further evolution of the composed state into macroscopically accessible states of $M\mathcal{A}$, orthogonal to each other for different j 's, and belonging to disjoint representations (and hence with vanishing off diagonal interference terms) in the limit $N \mapsto \infty$ [52]. However, if we keep N very large but finite, as is the case in practice, the same statement will be approximately true with an extremely high precision, and the operation of adding $M\mathcal{A}$ will be again described by an isometry W' of \mathcal{H} into itself, still localized in some larger region, which will be of macroscopic size, as the time T' needed for the amplification process.

Still, the effect of the measurement and of the macroscopic detection of the result, will be described by the evolution of the isometry $W'W$ to the isometry $\alpha_{T'}(W'W)$, both localised in some large but finite region $M\mathcal{O}$.

Now the state vector Ψ will be changed, by the presence of the measurement apparatus of A , into a state vector $W\Psi$ before the measurement or $\alpha_T(W)\Psi$ immediately after, but, in both cases, the effect of the microscopic part of the measurement apparatus will not be detectable with observables B localized in a double cone \mathcal{O}_0 which is *spacelike separated* from \mathcal{O} (if we do not neglect the interaction of the amplifying part of the measurement apparatus with the system, the same would be true only in the spacelike complement of $M\mathcal{O}$). For, the local commutativity of B with the isometries $W, \alpha_T(W)$ implies

$$(W\Psi, BW\Psi) = (\alpha_T(W)\Psi, B\alpha_T(W)\Psi) = (\Psi, B\Psi).$$

The conventional picture of the measurement process in Quantum Mechanics, as an instantaneous jump from a pure state to a mixture, which affects the state all over space at a fixed time in a preferred Lorentz frame, appears, in the scenario we outlined, as the result of several limits:

1. the time duration T of the interaction giving rise to the measurement (which, in an exact mathematical treatment, would involve the whole interval from minus infinity to plus infinity, as all scattering processes) is set equal to zero;
2. the number of microconstituents of the amplifying part of the measurement apparatus is set equal to infinity, thus allowing *exact* decoherence;
3. the volume involved by the measurement apparatus in its interaction with the system (thus occupied by the microscopic part of the apparatus) tends to the whole space, allowing the reduction of wave packets to take place *everywhere*;

In the conventional picture, some form of *nonlocality* is unavoidable, al-

beit insufficient for transmission of perturbations (hence not contradicting local commutativity) or even of information [58]: for a given observer, a *coherent superposition* of two possibilities might be changed, instantaneously in some preferred Lorentz frame, to a state where only one possibility survives, by the measurement performed by another observer in a very far spacelike separated region.

(One should be aware, however, that vector states in the vacuum representation never restrict to pure states of the local algebras, since those are type III von Neumann algebras).

This possibility of one observer of “steering” (as Schroedinger termed it) the findings of the other observer seems however incompatible with locality if we insist that the measurement process should be in the end reconciled with the description of time evolution which governs all interactions, including those between the observed quantum system and the measurement apparatus.

In our picture there is no contradiction with Lorentz covariance, and the Einstein Podolski Rosen paradox [59] does not arise.

Similar conclusions have been proposed long ago by Hellwig and Kraus [60]; further denials of the reality of the EPR paradox keep emerging from time to time in the literature (for recent ones, cf [61, 62]).

The existence of entangled states in Local Quantum Field Theory is of course out of question [52]; most of the remarkable experiments checking the violations of the Bell inequalities, notably by Aspect [63], do confirm the existence of entanglement.

Do they also contradict the *local* picture of the measurement process we outlined? This is not entirely clear; maybe we still need a clear cut experimental check of whether entanglement of eigenstates of mutually *spacelike separated* observables can, or, as we anticipated, cannot, be revealed also by means of *equally spacelike separated* observations.

3 The Quantum nature of Spacetime and the fate of locality in presence of gravitational interactions

While a deep understanding of electric charge and local gauge theories is a challenge for Locality, its fate is to breakdown if gravitational forces are taken into account. We turn now to this point.

At large scales spacetime is a pseudo Riemannian manifold locally modelled on Minkowski space. But the concurrence of the principles of Quantum

Mechanics and of Classical General Relativity points at difficulties at the small scales, which make that picture untenable.

If we do give an operational meaning to the localisation of an event in a neighborhood of a point, specified with the accuracy described by uncertainties in the coordinates, we see that, according to Heisenberg principle, an uncontrollable energy has to be transferred, which is the larger the smaller is the infimum of the spacetime uncertainties.

This energy will generate a gravitational field which, if all the space uncertainties are very small, will be so strong to prevent the event to be seen by a distant observer. However, if we measure one of the space coordinates of our event with great precision but allow large uncertainties L in the knowledge of at least one of the other space coordinates, the energy generated may spread in such a way that the gravitational potential it generates would vanish everywhere as $L \rightarrow \infty$.

One has therefore to expect *Space Time Uncertainty Relations* emerging from first principles, already at a semiclassical level. Carrying through such an analysis [64, 65] one finds indeed that at least the following minimal restrictions must hold

$$\Delta q_0 \cdot \sum_{j=1}^3 \Delta q_j \gtrsim \lambda_P^2; \quad \sum_{1 \leq j < k \leq 3} \Delta q_j \Delta q_k \gtrsim \lambda_P^2, \quad (8)$$

where λ_P denotes the Planck length

$$\lambda_P = \left(\frac{G\hbar}{c^3} \right)^{1/2} \simeq 1.6 \times 10^{-33} \text{ cm}. \quad (9)$$

Thus points become fuzzy and *locality loses any precise meaning*. We believe it should be replaced at the Planck scale by an equally sharp and compelling principle, yet unknown, which reduces to locality at larger distances.

The Space Time Uncertainty Relations strongly suggest that spacetime has a *Quantum Structure* at small scales, expressed, in generic units, by

$$[q_\mu, q_\nu] = i\lambda_P^2 Q_{\mu\nu} \quad (10)$$

where Q has to be chosen not as a random toy mathematical model, but in such a way that (8) follows from (10).

To achieve this in the simplest way, it suffices to select the model where the $Q_{\mu\nu}$ are central, and impose the “Quantum Conditions” on the two invariants

$$Q_{\mu\nu}Q^{\mu\nu}; \quad (11)$$

$$\begin{aligned} [q_0, \dots, q_3] &\equiv \det \begin{pmatrix} q_0 & \cdots & q_3 \\ \vdots & \ddots & \vdots \\ q_0 & \cdots & q_3 \end{pmatrix} \\ &\equiv \varepsilon^{\mu\nu\lambda\rho} q_\mu q_\nu q_\lambda q_\rho = \\ &= -(1/2)Q_{\mu\nu}(*Q)^{\mu\nu}, \end{aligned} \quad (12)$$

(we adopt here and henceforth Planck units, where $\hbar = c = G = 1$) whereby the first one must be zero and the square of the second is of order 1; we must take the square since it is a *pseudoscalar* and not a scalar; so that, more precisely, our Quantum Conditions read

$$(1/4)[q_0, \dots, q_3]^2 = I, \quad (13)$$

$$[q_\mu, q_\nu][q^\mu, q^\nu] = 0, \quad (14)$$

$$[[q_\mu, q_\nu], q_\lambda] = 0. \quad (15)$$

One obtains in this way [64, 65] a model of Quantum Spacetime (for brevity, the basic model) which implements *exactly* our Space Time Uncertainty Relations and is fully Poincaré invariant.

In any Lorentz frame, however, the *Euclidean* distance between two independent events can be shown to have a lower bound of order one in Planck units. Two distinct points can never merge to a point. However, of course, the state where the minimum is achieved will depend upon the reference frame where the requirement is formulated. (The structure of length, area and volume operators on QST has been studied in full detail [66]).

Here we will limit ourselves to motivate the statement on distances. First note that a classical locally compact manifold is fully described, by the Gelfand-Naimark Theorem, by the *commutative C* Algebra* of the complex continuous functions vanishing at infinity on that manifold; the Basic Model replaces the algebra of continuous functions vanishing at infinity on Minkowsky Space by a *noncommutative C* Algebra* \mathcal{E} , the enveloping C* - Algebra of the Weyl form of the commutation relations between the coordinates:

$$e^{i\alpha_\mu q^\mu} e^{i\beta_\nu q^\nu} = e^{-(i/2)\alpha_\mu Q^{\mu\nu}\beta_\nu} e^{i(\alpha+\beta)_\mu q^\mu}; \quad \alpha, \beta \in \mathbb{R}^4.$$

The unbounded operators q_μ are *affiliated* to the C*Algebra \mathcal{E} and fulfill the desired commutation relations. Poincaré covariance is expressed by an

action τ of the full Poincaré group by automorphisms of \mathcal{E} , determined by the property that its canonical extension to the q_μ 's fulfill

$$\tau_L(q) = L^{-1}(q).$$

The C*-Algebra \mathcal{E} turns out to be the C* - Algebra of continuous functions vanishing at infinity from a manifold Σ to the C* - Algebra of compact operators on the separable infinite dimensional Hilbert space.

Here Σ is the (maximal) *joint spectrum of the commutators*, which is the manifold of the real antisymmetric two-tensors fulfilling constraints imposed by the above *quantum conditions*; namely, specifying such a tensor by its electric and magnetic components \vec{e}, \vec{m} , $\vec{e}^2 = \vec{m}^2$, $\vec{e} \cdot \vec{m} = \pm 1$. Thus Σ can be identified with the full Lorentz orbit of the standard symplectic form in four dimensions, that is Σ is the union of two connected components, each omeomorphic to $SL(2, \mathbb{C})/\mathbb{C}_*$, or to the tangent manifold TS^2 to the unit sphere in three dimensions. If $\vec{e} = \pm \vec{m}$ they must be of length one, and span the *base* $\Sigma^{(1)}$ of Σ . Thus Σ can be viewed as $T\Sigma^{(1)}$.

The bounded continuous functions on Σ span the centre \mathcal{Z} of the *multiplier algebra* of \mathcal{E} , and the commutator $Q_{\mu\nu}$ of the q 's is affiliated to \mathcal{Z} , and of course is the function taking the point σ of Σ to $\sigma_{\mu\nu}$.

The manifold Σ does survive the large scale limit; thus, *QST predicts extra-dimensions*, which indeed manifest themselves in the *compact* manifold $\Sigma^{(1)} = S^2 \times \{\pm 1\}$ if QST is probed with *optimally localised states*.

The discrete two-point space which thus appears here as a factor reminds of the one postulated in the Connes-Lott theory of the Standard Model.

In this light QST looks similar to the phase space of a 2 - *dimensional* Schroedinger particle; and thus naturally divides into cells (of volume governed by the 4-th power of the Planck length); so that, though being continuous and covariant, QST is effectively discretised by its Quantum nature. (Compare the earlier discussion of the “fuzzy sphere” by John Madore).

To motivate these conclusions note first that irreducible representations of \mathcal{E} are in one-to-one correspondence with irreducible representations of the Weyl relations, i.e. with *regular* irreducible representations of (10), where the commutators become multiples of the identity, hence described by a point σ_0 in Σ .

Picking a suitable Lorentz frame, that point may be chosen as the standard symplectic form, where the only nonvanishing above-diagonal entries are $\sigma_{0,1} = \sigma_{2,3} = 1$; thus (10) becomes, in that irreducible representation, just the Heisenberg commutation relations for momentum and position operators in two degrees of freedom; since the representation is regular, it is

unitarily equivalent to the corresponding Schroedinger representation.

Thus in a pure state associated to that representation the expectation value of the sum of squares of the q_μ 's (the Euclidean square length of the four-vector), is twice an expectation value of the Hamiltonian of the two-dimensional harmonic oscillator, and is consequently bounded below by 2.

Therefore the sum of the squares of the uncertainties of the q_μ 's is bounded below the same way.

This is easily seen to happen also in any other irreducible representation obtained by a (possibly improper) rotation, and for their (discrete or continuous) convex combinations, where 2 is an actual minimum for those sums; which can be shown to take necessarily a larger value in any other representation.

Thus \mathcal{E} possesses states of *optimal localisation* of a single event.

If we consider n independent events, Quantum Mechanics tells us that we must describe them by tensor products of n copies of \mathcal{E} . However it appears immediately that it makes more sense not to take the tensor product over the complex numbers, but rather the “module tensor product over the center \mathcal{Z} of the multiplier algebra” of \mathcal{E} .

This simply means to stipulate that $Q_{\mu\nu}$, or better their functional calculus with bounded continuous functions on Σ , which span \mathcal{Z} , can be moved through in different spots of the tensor product like complex numbers, giving a result independent of their position. Thus the commutator between the coordinate μ of the j th event with the coordinate ν of the k th event is zero if j is different from k , and equal to $Q_{\mu\nu}$ independent of j if $j = k$.

As a consequence, the difference variables scaled by $2^{-1/2}$ obey *the same commutation relations* (10) between the μ and ν components.

Accordingly, by the above discussion it follows that, for distinct j, k ,

$$\sum_{\mu=0}^4 (q_\mu^j - q_\mu^k)^2 \gtrsim 4, \quad (16)$$

that is, the *Euclidean distance* between two independent events is bounded below by order of the Planck length *in every Lorentz frame*.

Thus the existence of a minimal length is not at all in contradiction with the Lorentz covariance of the model (nor with the possibility of measuring *a single* coordinate with arbitrary precision: this is not contradicted by the famous Amati Ciafaloni Veneziano relation, which implicitly presupposes a joint precision in the measurement of all space coordinates).

Similarly, the difference $q_\mu^j - q_\mu^k$ *commutes strongly* with the weighted

barycenter coordinates $n^{-1/2}(\sum_{j=1}^n q_\mu^j)$ and the latter obey *the same commutation relations* (10) between the μ and ν components.

Now two commuting representations of the algebra of all compact operators act necessarily on the distinct factors of a tensor product decomposition of the representation Hilbert space.

This implies that there is a *-isomorphism η of the n th \mathcal{Z} - module tensor power of \mathcal{E} into the $(n+1)$ th which, if extended canonically to the affiliated unbounded selfadjoint operators, maps the weighted barycenter coordinates $n^{-1/2}(\sum_{j=1}^n q_\mu^j)$ to $q_\mu \otimes I$, (the identity in the n th tensor power of the multiplier algebra $M\mathcal{E}$ of \mathcal{E}), and $q_\mu^j - q_\mu^k$ to $I \otimes (q_\mu^j - q_\mu^k)$ (where I is the identity of $M\mathcal{E}$).

One can therefore define a “Quantum Diagonal Map” [67] $E^{(n)}$ which takes the Euclidean length of the difference variables to the (nonzero) minimal allowed value, by composing the previously defined map η of $\mathcal{E}^{\otimes n}$ into $\mathcal{E}^{\otimes n+1}$, with the evaluation, on each factor in the places successive to the first in that $(n+1)$ fold tensor product, of the “universal optimally localised map” of \mathcal{E} , which, composed with any probability measure on the base $\Sigma^{(1)}$ of Σ , produces the most general optimally localised state localized around the origin (cf [67] for details).

The Quantum Diagonal Map obviously depends upon a chosen Lorentz frame.

The models where the commutators of the coordinates take fixed numerical values θ , which appear so often in the literature, arise as irreducible representations of our model; such models, taken for a fixed choice of θ rather than for its full Lorentz orbit, necessarily break Lorentz covariance. To restore it as a twisted symmetry is essentially equivalent to going back to the model where the commutators are operators. This point has been recently clarified in great depth ([68]; see also [69]).

On the other side, a theory with a fixed, numerical commutator (*a θ in the sky*) can hardly be realistic.

The geometry of Quantum Spacetime and the free field theories on it are *fully Poincaré covariant*.

Considering for simplicity a neutral scalar free field $\phi(x)$, its evaluation on q_μ gives [65]

$$\phi(q) = \frac{1}{(2\pi)^{3/2}} \int (e^{iq_\mu k^\mu} \otimes a(\vec{k}) + e^{-iq_\mu k^\mu} \otimes a(\vec{k})^*) d\Omega_m^+(\vec{k})$$

where $d\Omega_m^+(\vec{k}) = \frac{d^3\vec{k}}{2\sqrt{\vec{k}^2+m^2}}$ is the usual invariant measure over the positive energy hyperboloid of mass m : $\Omega_m^+ = \{k \in \mathbb{R}^4 / k_\mu k^\mu = m^2, \quad k_0 > 0\}$.

In order to give a precise mathematical meaning to this expression, we may think of a quantum field over QST acting on a Hilbert space \mathcal{H} as a linear map, continuous in the appropriate topology, assigning to test functions f linear operators affiliated to the C^* -tensor product $\mathcal{E} \otimes \mathcal{B}(\mathcal{H})$ and formally denoted by

$$f \rightarrow \int \phi(q + aI) f(a) d^4a .$$

The free field so constructed defines a map from states $\omega \in \mathcal{S}(\mathcal{E})$ to operators on \mathcal{H} by

$$\phi(\omega) \equiv \langle \omega \otimes id, \phi(q) \rangle, \quad \omega \in \mathcal{S}(\mathcal{E}).$$

If we choose for ω an optimally localised state and compute the commutator of the $\phi(\omega)$ with its space translate by a , in the case of the massless free field we find a simple explicit expression, which vanishes for large a as a Gaussian, but does not vanish exactly at any spacelike separation (cf [65] for details).

Thus locality is lost already for the free fields. But Lorentz covariance survives, and it is easily seen to be summarised in a simple form by the relation

$$\tau_L \otimes \alpha_L(\phi(q)) = \phi(q),$$

for each Poincaré transformation L , where α and τ denote the actions of the Poincaré group on the algebras of field operators and of Quantum Spacetime respectively.

We can still define a net of “local von Neumann algebras of Fields” associated to the free field, indexed no longer by subsets in Minkowski space, but rather by their noncommutative analogs: the selfadjoint projections E in the Borel completion $\tilde{\mathcal{E}}$ of \mathcal{E} .

Namely we have a map

$$E \rightarrow \mathfrak{F}(E) = \{e^{i(\phi(\omega)+\phi(\omega)^*)}, \tilde{\omega}(E) = 1\}''$$

where $\tilde{\omega}$ denotes the normal extension of the state ω of \mathcal{E} to $\tilde{\mathcal{E}}$, and the double prime on a set of bounded operators denotes the double commutant (that is, the von Neumann algebra they generate).

This net obeys isotony in an obvious sense, and is Poincaré covariant,

$$\alpha_L(\mathfrak{F}(E)) = \mathfrak{F}(\tau_L E)$$

But locality is lost. There is no meaning to “ E_1 and E_2 are spacelike separated”, unless we pick a point σ in Σ , and limit ourselves to a special wedge W associated to σ and its spacelike complement $-W$. In this special case locality survives for free fields, but is bound to be destroyed by interactions on QST.

That remnant of locality has been exploited to construct *deformations* of local nets for which the two particle S matrix is nontrivial [70, 71], at the price of loosing locality in terms of fields localised in bounded regions.

The various formulation of interaction between fields, all equivalent on ordinary Minkowski space, provide inequivalent approaches on QST; but all of them, sooner or later, meet problems with *Lorentz covariance*, apparently due to the nontrivial action of the Lorentz group on the *centre* of the algebra of Quantum Spacetime. On this point in our opinion a deeper understanding is needed.

The earliest form of interaction, proposed in [65], led to an ansatz for the Scattering Matrix S given by the Gell-Mann - Low formula for the interaction Hamiltonian (for the interaction given by the product of n basic fields)

$$H_I(t) \equiv \int_{\Sigma(1)} d\sigma \int_{q_0=t} d^3q \lambda : \psi_1(q) \dots \psi_n(q) : .$$

which gives rise to a perturbative expansion of S coinciding with that defined by a suitable nonlocal *effective* interaction on ordinary Minkowski space, where

$$\begin{aligned} H_I(t) &= \int_{x_0=t} d^4x H_{eff}^I(x), \\ H_{eff}^I(x) &= \int G_n(x - x_1, \dots, x - x_n) \lambda : \psi_1(x_1) \dots \psi_n(x_n) : d^4x_1 \dots d^4x_n , \end{aligned}$$

and the nonlocal kernels G_n can be explicitly computed [65].

The time ordering in the Dyson expansion of the Scattering Matrix has to be defined for the t - variables appearing as arguments of the $H_I(t)$, (*not for the time variables of the field operators themselves*: such a choice

is suggested if one regards the nonlocal interaction as if it *were* local, and leads to violation of unitarity, unlike the proposal we describe here [65, 72]).

As a consequence, the usual Feynman rules cannot be applied; the necessary modifications involve the *Denk-Schweda propagators* rather than Feynman propagators, and have been precisely formulated in [73].

The ansatz involves the integral over $\Sigma^{(1)}$, which breaks Lorentz invariance. This choice is dictated by the fact that there is no finite invariant measure (or mean) on Σ . This ansatz does not fully regularise the theory in the ultraviolet, except the special case of the ϕ^3 interaction [74].

One can however introduce interactions in different ways, all preserving spacetime translation and space rotation covariance; among these it is just worth mentioning here one of them, where one takes into account, in the very definition of Wick products, the fact that in our Quantum Spacetime two distinct points can never merge to a point.

It seems therefore more legitimate to apply to the ordinary Wick product of field operators evaluated at independent events the *quantum diagonal map*, which is associated, as explained above, with the minimum of the Euclidean length of the difference of the independent coordinates (in a given Lorentz frame!).

The “Quantum Wick Product” obtained by this procedure leads to a Dyson perturbation expansion of the S matrix which is, as above, again coinciding with that determined by a nonlocal interaction Hamiltonian on ordinary Minkowski space, where now the nonlocal kernels G_n have the explicit form [67]

$$G_n(x_1, \dots, x_n) = c_n \delta^{(4)}\left(\sum_{j=1}^n x_j\right) \cdot e^{-\frac{1}{2} \sum_{j=1}^n \sum_{\mu=0}^3 (x_j^\mu)^2},$$

namely, the nonlocal regularizing kernel is now Gaussian in all variables, except for the presence of a Dirac measure of the sum, which expresses translation invariance.

It follows that the Gell-Mann - Low formula for S matrix, where the vacuum - vacuum diagrams are divided out, is *free of ultraviolet divergences* at each order of the perturbation expansion.

The Gaussian kernel forces the cross sections to vanish as a polynomial multiple of a Gaussian at transplanckian energies and momentum transfers.

Note that the nonlocal kernel is now independent of the points on the base of Σ , so no ad hoc integration is needed.

However, while no UV problems are left, a hard IR problem shows up: it is necessary to introduce an adiabatic time cutoff of the interaction, which

is difficult to remove [67] .

Note that UV finiteness does not mean that renormalisation is not needed at all: a *finite* renormalisation, with renormalisation constants depending on the Planck length, is needed, in order to subtract physically meaningless contributions; it should be possible to choose that dependence so that, applying this procedure to the usual renormalized interaction derived on the classical Minkowski space, the resulting perturbation expansion reproduces, in the limit $\lambda_P \rightarrow 0$, the usual renormalised perturbation expansion; however the interplay with the adiabatic limit and with the renormalised one particle states have to be considered; for progress on this line, see [75, 76, 77].

The common feature of all approaches is that, due to the quantum nature of spacetime at the Planck scale, locality is broken, even at the level of free fields, and more dramatically by interactions. Which, as far as our present knowledges go, lead to a breakdown of Lorentz invariance as well. Note however that the invariance under translation and space rotations is preserved by the previous prescription.

In the approach to interacting fields on Quantum Spacetime based on the Yang - Feldman equations, Lorentz invariance is preserved at the level of field equations, but covariance problems arise at the level of one particle states and of asymptotic scattering states [72, 76, 78].

One might expect that a complete theory ought to be covariant under general coordinate transformations as well. This principle, however, is grounded on the conceptual experiment of the falling lift, which, in the classical theory, can be thought of as occupying an infinitesimal neighbourhood of a point. In a quantum theory the size of a “laboratory” must be large compared with the Planck length, and this might pose limitations on general covariance. One might argue that such limitations ought to be taken care of by the quantum nature of Spacetime at the Planck scale.

On the other side elementary particle theory deals with collisions which take place in narrow space regions, studied irrespectively of the surrounding large scale mass distributions, which we might well think of as described by the vacuum, and worry only about the short scale effects of gravitational forces.

We are thus led to consider Quantum Minkowski Space as a more realistic geometric background for Elementary Particle Physics.

But the energy distribution in a generic quantum state will affect the Spacetime Uncertainty Relations, suggesting that the commutator between the coordinates ought to depend in turn on the metric field. This scenario could be related to the large scale thermal equilibrium of the cosmic microwave background, and to the non vanishing of the Cosmological Constant

[79, 80].

This might well be the clue to restore Lorentz covariance in the interactions between fields on Quantum Spacetime.

In the course of the last dozen of years Quantum Field Theory on Non-commutative Spacetime became quite popular, mostly adopting coordinates with commutators which are multiples of the identity, and under the influence of string theory; we refrain from giving references, which would necessarily be very numerous. Much work has been dedicated to renormalisability of Euclidean theories. Rather seldom, however, the necessary depart from Feynmann rules has been taken into account. We limit ourselves to mention that, in the noncommutative case, there is no analog of the Osterwalder and Schrader theorem, and the ultraviolet behaviour in the Euclidean might be unrelated to that in the Minkowskian. For recent interesting results on this problem see [81].

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