The Inner Structure of Black Holes

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Abstract

We study the gravitational collapse of a self-gravitating *charged* scalar-field. Starting with a *regular* spacetime, we follow the evolution through the formation of an apparent horizon, a Cauchy horizon and a final central singularity. We find a null, *weak*, mass-inflation singularity along the Cauchy horizon, which is a precursor of a strong, *spacelike* singularity along the $r = 0$ hypersurface. The inner black hole region is bounded (in the future) by singularities. This resembles the classical inner structure of a Schwarzschild black hole and it is remarkably different from the inner structure of a charged static Reissner-Nordström or a stationary rotating Kerr black holes.

The simple picture describing the exterior of a black-hole [1] is in dramatic contrast with its *interior*. The singularity theorems of Penrose and Hawking [2] predicts the occurrence of inevitable spacetime singularities as a result of a gravitational collapse in which a black-hole forms. According to the *weak cosmic censorship* conjecture [3], these spacetime singularities are hidden beneath the black-hole's event-horizon. However, these theorems tell us nothing about the nature of these spacetime singularities. In particular, the *final* outcome of a generic gravitational collapse is still an open question.

Our physical intuition regarding the nature of these inner singularities and the outcome of gravitational collapse is largely based on the spherical Schwarzschild black hole solution and the idealized Oppenheimer-Snyder collapse model [4]. The Schwarzschild black hole contains a strong *spacelike* central singularity. All the matter that falls into the black hole

crashes into this singularity within a finite proper time. The Schwarzschild singularity is unavoidable. This behaviour is manifested in the Penrose diagram describing the conformal structure of a spacetime in which a Schwarzschild black hole forms (see Fig. 1).

However, spherical collapse is not generic. We expect some angular momentum and this might change this picture drastically. The inner structure of a stationary rotating, Kerr, black hole contains a strong inner *timelike* singularity, which is separated from external observes by both an apparent horizon and a Cauchy horizon (CH). A free-falling test particle cannot reach this singularity. Instead it will cross a second Cauchy horizon and emerge form a white hole into another asymptotically flat region. A remarkably similar structure exists in a charged Reissner-Nordström black hole (see Fig. 1) . We do not expect to find charged collapse in nature. However, this similarity motivates us to study spherically symmetric charged gravitational collapse as a simple toy model for a realistic generic rotating collapse (which is at best axisymmetric).

Does the inner structure of a Reissner-Nordström black hole describe the generic outcome of gravitational collapse? Novikov [5] studied the collapse of a charged shell and found that the shell will reach a minimal radius and bounce back, emerging into another asymptotically flat region - a different universe. The idea of reaching other universes via a black hole's interior is rather attractive. It immediately captured the imagination of the popular audience and SciFi authors coined the "technical" term "Stargate" for this phenomenon. However as predictability is lost at the CH this leads to serious conceptual problems.

We are faced with two gravitational collapse models. The "traumatic" collapse to Schwarzschild in which nothing can escape the central singularity and the "fascinating" collapse to Kerr or Reissner-Nordström in which a generic infalling observer might escape unharmed to another Universe. Which of the two possibilities is the generic one?

Penrose, [6] who was the first to address this issue pointed out that small perturbations, which are remnants of the gravitational collapse outside the collapsing object are infinitely blueshifted as they propagate in the black-hole's interior parallel to the Cauchy horizon. The resulting infinite energy leads to a curvature singularity. Matzner et. al [7] have shown

that the CH is indeed unstable to linear perturbations. This indicates that the CH might be singular - "Stargate" might be closed. A detailed modeling of this phenomena suggests that the CH inside charged or spinning black-holes is transformed into a *null, weak* singularity [8–10]. The CH singularity is weak in the sense that an infalling observer which hits this null singularity experiences only a finite tidal deformation [10]. Nevertheless, curvature scalars (namely, the Newman-Penrose Weyl scalar Ψ_2) diverge along the CH, a phenomena known as *mass-inflation* [9].

Despite this remarkable progress the physical picture is not complete yet. The evidence for the existence of a null, weak CH singularity is mostly based on perturbative analysis. The pioneering work of Gnedin and Gnedin [11] was a first step towards a full non-linear analysis. They have demonstrated the appearance of a central *spacelike* singularity deep inside a charged black-hole coupled to a (neutral) scalar-field. Much insight was gained from the numerical work of Brady and Smith [12] who studied the same problem. These authors established the existence of a *null* mass-inflation singularity along the CH. Furthermore, they showed that the singular CH contracts to meet the central $r = 0$ *spacelike* singularity. More recently, Burko [13] demonstrated that there is a good agreement between the numerical results and the predictions of the perturbative approach.

Still, the mass-inflation scenario has never been demonstrated explicitly in a collapsing situation beginning from a regular spacetime. All previous numerical studies began with a singular Reissner-Nordström spacetime with an additional infalling scalar field. We demonstrate here explicitly that mass-inflation takes place during a dynamical charged gravitational collapse. We show that the generic black hole that forms in a charged collapse is engulfed by singularities in all future directions.

We consider the gravitational collapse of a self-gravitating *charged* scalar-field. The physical model is described by the coupled Einstein-Maxwell-charged scalar equations. We solve the coupled equations using a characteristic method. Our scheme is based on *double null* coordinates: a retarded null coordinate u and an advanced null coordinate v. The axis, $r = 0$, is along $u = v$. For $v \gg M$ our null ingoing coordinate v is proportional to the

Eddington-Finkelstein null ingoing coordinate v_e. These coordinates allow us to begin with a regular initial spacetime (at approximately past null infinity), calculate the formation of the black-hole's event horizon, and follow the evolution inside the black-hole all the way to the central and the CH singularities.

Fig. 2 describes the numerical spacetime that we find. The upper panel (Fig. 2a) displays the radius $r(u,v)$ as a function of the ingoing null coordinate v along a sequence of outgoing ($u = const$) null rays that originate from the *non*-singular axis $r = 0$. One can distinguish between *three* types of outgoing null rays: (i) The outer-most (small-u) rays, which *escape* to infinity. (ii) The intermediate outgoing null rays which approach a fixed radius $r_{CH}(u)$ at late-times $v \to \infty$ indicating the existence of a CH. (iii) The inner-most (large-u) rays, which terminate at the *singular* section of the $r = 0$ hypersurface. These outgoing rays reach the $r = 0$ singularity in a *finite* v, *without* intersecting the CH. This structure is drastically different from the Reissner-Nordström spacetime, in which all outgoing null rays which originate inside the black-hole intersect the CH. Moreover while in a Reissner-Nordström spacetime the CH is a *stationary* null hypersurface, here $r_{CH}(u)$ depends on the outgoing null coordinate u. The CH *contracts* and reaches the inner $r = 0$ singularity. The CH is smaller if the charge is smaller, and if the charge is sufficiently small it is difficult (numerically) to notice the existence of a CH in the solution.

Fig. 2b. depicts the $r(u,v)$ contour lines. The outermost contour line corresponds to $r = 0$; its left section (a straight line $u = v$) is the *non*-singular axis and its right section corresponds to the central singularity at $r = 0$. Since $r_v < 0$ along this section, the central singularity is *spacelike*. Previously $r_v = 0$ indicated the existence of an apparent horizon (which is first formed at $u \approx 1$ for this specific solution). The CH is a *null* hypersurface located at $v \to \infty$. This follows because the intermediate outgoing null rays (in the range $1 \lesssim u \lesssim 2.1$ for this specific solution) terminate at a finite (u-dependent) radius $r_{CH}(u)$. The singular CH contracts to meet the central ($r = 0$) spacelike singularity (along the $u \simeq 2.1$ outgoing null ray). Thus, the *null* CH singularity is a precursor of the final *spacelike* singularity along the $r = 0$ hypersurface.

As expected from the *Mass Inflation* scenario the mass function $m(u, v)$ (and the curvature) diverge exponentially along the outgoing null rays (see Fig. 3a). The mass function increases not only along the outgoing (u=const) null rays (as v increases) but also along ingoing (v=const) null rays (as u increases). The *weakness* of the singularity is demonstrated here by the metric function g_{uV} (see Fig. 3b) which approaches a finite value at the CH. This confirms the analytical analysis of Ori [10], according to which a suitable coordinate transformation can produce a *non*-singular metric.

Our numerical solution has put together all the different ingredients found in the previous analyses into a single coherent picture. The inner structure of a black hole that forms in a gravitational collapse of a charged scalar-field is remarkably different from the inner structure of a Reissner-Nordström (or Kerr) black hole (see Fig. 1). The inner region is bounded by singularities in all future directions: a *spacelike* singularity forms on $r = 0$ and a *null* singularity forms along the CH, which contracts and meets the *spacelike* singularity at $r = 0$. This structure is much closer to the "traditional" Schwarzschild inner structure than to the seemingly more generic Reissner-Nordström or Kerr structures. However, while the *spacelike* singularity is *strong*, the null singularity along the CH is *weak*. Matter is able to cross this singularity without being crushed by tidal forces. Thus, in spite of this "singular" picture, "Stargate" might not be completely closed after all (provided that the travelers are willing to suffer a strong, yet finite distortion). These travelers will not have, of course, the slightest idea what is expected for them beyond the CH. The weakness of the CH singularity leaves open the question of predictability beyond the CH.

ACKNOWLEDGMENTS

REFERENCES

[1] R. Ruffini and J. A. Wheeler, Physics Today **24**, 30 (1971).

[2] S. W. Hawking and R. Penrose, Proc. R. Soc. Lond. **A314**, 529 (1970).

[3] R. Penrose, Riv. Nuovo Cimento I **1**, 252 (1969).

[4] J. R., Oppenheimer and H. Snyder, Phys. Rev, **56**, 455 (1939).

[5] I., D., Novikov, Soviet Phys. - JEPT Lett., **3**, 142 (1966).

[6] R. Penrose, in Battelle Rencontres, edited by C. de Witt and J. Wheeler (W. A. Benjamin, New York, 1968), p. 222.

[7] R. A. Matzner, N. Zamorano, and V. D. Sandberg, Phys. Rev. **D19**, 2821, (1979).

[8] W. A. Hiscock, Phys. Lett. **A83**, 110 (1981).

[9] E. Poisson and W. Israel, Phys. Rev. **D41**, 1796 (1990).

[10] A. Ori, Phys. Rev. Lett. **67**, 789 (1991); A. Ori, Phys. Rev. Lett. **68**, 2117 (1992).

[11] M. L .Gnedin and N. Y. Gnedin, Class. Quant. Grav. **10**, 1083 (1993).

[12] P. R. Brady and J. D. Smith, Phys. Rev. Lett. **75**, 1256 (1995).

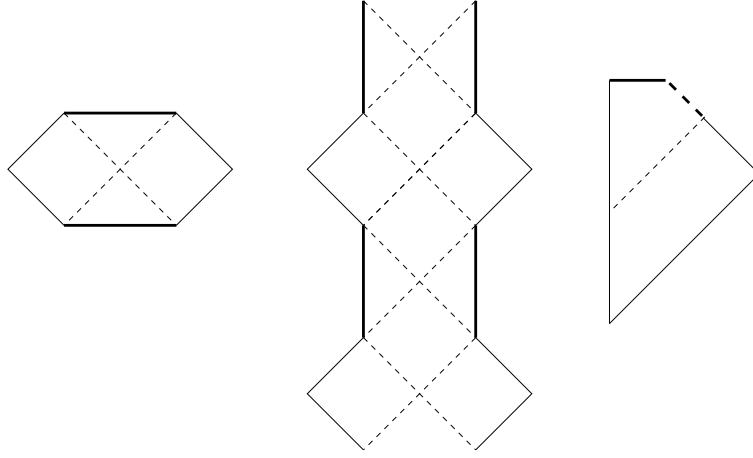[13] L. M. Burko, Phys. Rev. Lett. **79**, 4958 (1997).

FIGURES

FIG. 1. Penrose Diagrams of Schwarzschild (a) Reissner-Nordström (b) and Charged Collapse (c) spacetimes. Thick solid lines denote strong singularities, thick dashed lines denote the weak null CH singularity, thin dashed lines denote the various horizons.
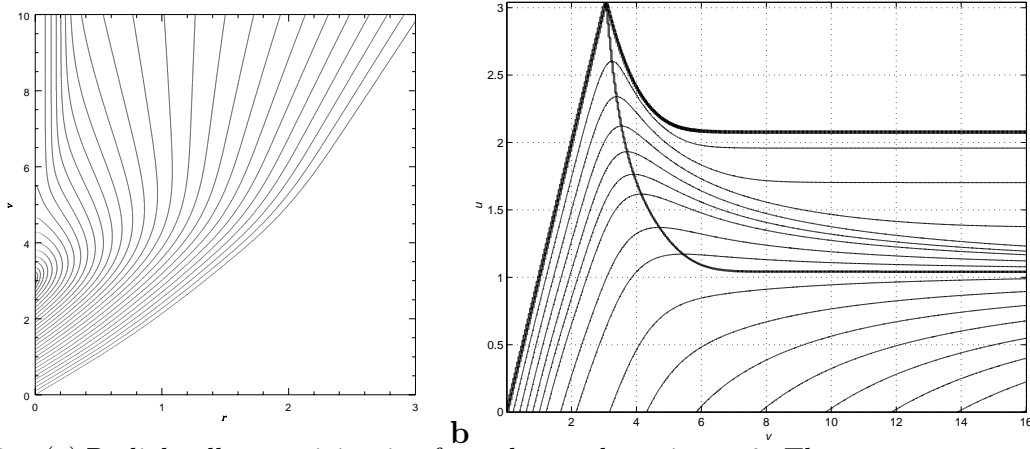

FIG. 2. (a) Radial null rays originating from the regular axis $r = 0$. The outer-most rays escape to infinity, the inner-most rays terminate at the singular section of the $r = 0$ and the intermediate outgoing null rays reach a (u-dependent) finite radius. (b) Contour lines of the coordinate r in the vu-plane. The $r = 0$ contour line is indicated by a thicker curve. Its left section ($u = v$) is the *non*-singular axis, while its right section corresponds the the central *spacelike* singularity. The apparent horizon is indicated by $r_v = 0$. The (singular) CH is a null hypersurface located at $v \to \infty$. It contracts to meet the central spacelike singularity (in a finite proper time).
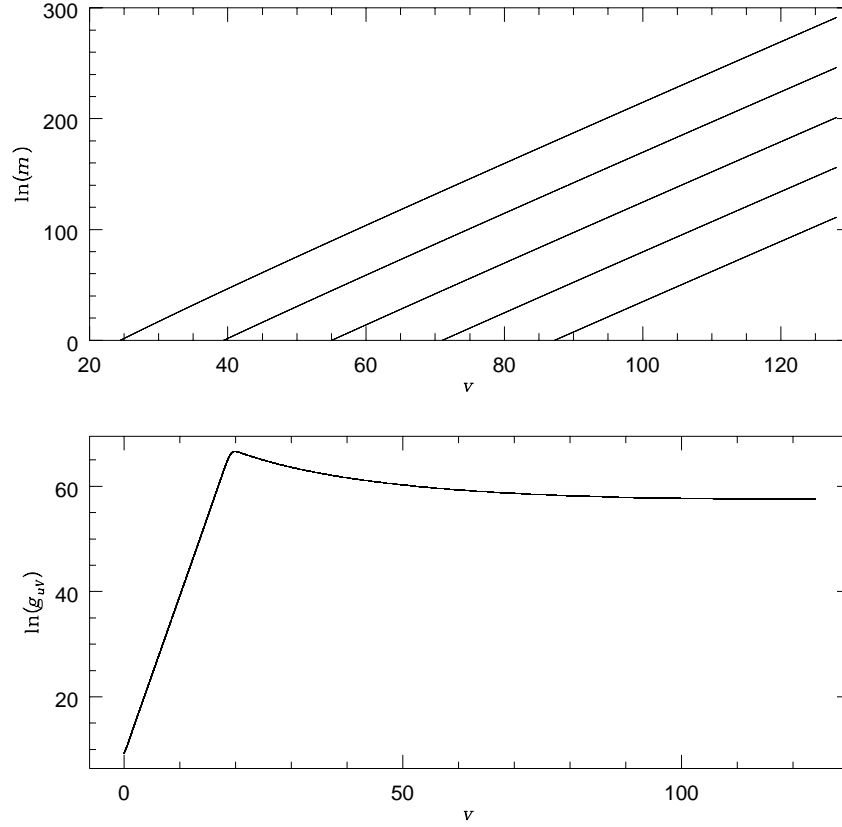
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FIG. 3. The CH singularity. The top panel displays $\ln(m)$ vs. advanced time v, along a sequence of outgoing null rays. The exponential growth of the mass-function demonstrates the appearance of the mass-inflation scenario [9]. The bottom panel displays the metric function g_{uV} along an outgoing null ray. V is a Kruskal-like ingoing null coordinate. The CH is at $V \to 0$. g_{uV} approaches a finite value as $V \to 0$ in agreement with the simplified model of Ori [10]. This demonstrates the *weakness* of the null mass-inflation singularity.