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ARABIC MATHEMATICAL MANUSCRIPTS IN MAURETANIA

By Ulrich Rebstock

(PLATES I–IV)

1. *Introduction*

A discussion of the mathematical manuscripts of Mauretania requires an overall introduction to this almost completely unexplored literature.

Its foundations must first be established. Between the years 1978 and 1985 a project financed initially by the Ministry of Foreign Affairs (BRD), then by the Deutsche Forschungsgemeinschaft, was entrusted to Dr. Rainer Oßwald and myself by the Mauretanian Ministry of Cultural Affairs. Its objective was the collection and protection of the selected Arabic manuscript literature of Mauretania.

The term ' Mauretanian literature ' sounds more exotic than it is. Looked at from the physical point of view, its quantity is amazing. While less than two million people live in this country—a quarter of them in the bidonvilles of the capital Nouakchott—dispersed over more than one million square kilometres, close to 100,000 Arabic manuscripts were discovered in countless libraries of varying size, state and ownership.

This ratio is impressive and reflects to a certain extent the uniqueness of Mauretanian culture. Mauretania was out of the mainstream of Islamic history in the Middle Ages, and contacted only peripherally by the conquering Portuguese in the fifteenth and sixteenth centuries. For more than half a century it was hard pressed, unsuccessfully, by French colonialism. The French managed only in 1931 to instal a military garrison in Shinqīṭ, the heartland of the Moors (the *bilād al-Shanāqiṭa* as tradition named the country). It was plunged into independence completely unprepared for the politics and economics of post-colonial times. Not only had the basic political and social structures of the traditionally segmented Mauretanian society survived, but so had the bulk of the cultural patterns of the late Middle Ages.

Mauretania is relatively isolated, its western border being the coastline of the Atlantic ocean, its southern one the Senegal river, the semi-permeable frontier between black and white Africa, its eastern one the fringes of the Saharan high plateau, and its northern one open to the southern foothills of the Atlas mountains. This relative isolation not only permitted the survival of a genuine and vivid Islamic literature produced and reproduced under the archaic circumstances of an untouched bedouin life but also left imprints on its characteristics and content.[1]

In rural areas, for example, education and science remained local privileges until about 15 years ago when the government organized a centrally-controlled school system and offered a university education.

Yet, still in 1978, in Akfallīt, a small tent-settlement just about 100 km. to the south of the capital of Nouakchott, the traditional ' tent-university ' of the well-known Mauretanian scholar M. b. Ḥabībunā al-Tandaghī drew some 40 students to learn Qur'ān, *fiqh*, *naḥw*, etc., under conditions that may well have already prevailed in the learned centres of the Najd and Ḥijāz during the first centuries of Islam.

[1] The most recent and most comprehensive introduction so far to the development of Mauretanian literature, especially with emphasis on the social and economical framework of the intellectual life of the Moors, is the study of Rainer Oßwald: *Die Handelsstädte der Westsahara. Die Entwicklung der arabisch-maurischen Kultur von Šinqīṭ, Wādān, Tīšīt und Walāta* (Berlin, 1986). For further bibliographical sources see the review of H. T. Norris, *BSOAS*, LI, 2 1988, 337–40.

With respect to the limited possibilities of a field-project primarily focused on salvaging the most significant monuments of Mauretania's cultural heritage, the selection of manuscripts to be microfilmed and thereby rescued from the destructive powers of droughts and profound social changes turned out to be rather arbitrary. Nevertheless, I am convinced that the 2239 manuscript units microfilmed in more than 260 private and public libraries represent a reliable cross-section of those parts of Mauretanian literature that physically survived until the nineteen-eighties.[2]

An almost negligible portion of the treatises filmed deals with mathematical subjects. Of course, the largest number (28%) of manuscripts deal with religious and customary law—more precisely, with the interpretation and local adaptation of the classical sources of *mālikī fiqh*. Next come Ṣūfī studies (*taṣawwuf*, 9%), theology (*tauḥīd*, 5%), and Qur'ān (5%), followed by historical treaties (5%) and biographies (4%). Only a minor portion is concerned with natural sciences. Among others, we find six books on geography, three discussing agricultural techniques, at least 13 containing methods of computing the dates of the Islamic calendar (sometimes connected with astrology), and quite a few texts dedicated to the so-called 'Arabic sciences', such as the 'Science of Charms' (*'ilm al-ṭalāṣim*), Astrology (*'ilm al-tanjīm*), the 'Science of Divination' (*'ilm al-sirr*) and the 'Science of Magical Squares' (*'ilm sirr al-jadāwil*).

Most of these texts—in varying degrees—contain splinters of mathematical knowledge and derived techniques. The same holds true, but to a much greater extent, for those which comprise the law of inheritance or the 'science of shares', *'ilm al-farā'iḍ*. Not only do most of these 20 texts explicitly claim the necessity of *ḥisāb* (arithmetic) for the proper calculation of legal shares, but they also stress the undeniable usefulness of *ḥisāb* for those concerned with the distribution of inheritances.

In view of this fact it seems prudent to restrict the description to those texts which offer at least a more or less complete presentation of basic arithmetical operations. A text which fulfils this minimal requirement will be thus designated 'mathematical'.

The whole manuscript corpus does not contain more than 20 mathematical texts. Only half of them are complete and reading some of them is difficult because of the uneven quality of the microfilm.

To be reasonably concise, their characterization will concentrate upon three questions:

(*a*) Where do the texts come from?
(*b*) What are their favoured topics?
(*c*) What historical and social significance can be ascribed to these texts?

2. *Provenance of the texts*

2.1.

According to their provenance, these texts fall into three different groups. The first includes five texts and is associated with the North African mathematician and religious scholar Abū 'Abdallāh M. b. Ghāzī al-'Uthmānī al-Fāsī al-Miknāsī. Manuscript No. 37 (Shinqīṭ), a text of 266 pages called *Idrāk al-*

[2] Short descriptions of the manuscripts are to be found in: Ulrich Rebstock, *Sammlung arabischer Handschriften in Mauretanien: Kurzbeschreibungen von 2 239 Manuskripteinheiten und Indices* (Wiesbaden: Harrassowitz, 1989). The *Katalog der arabischen Handschriften in Mauretanien, bearbeitet von Ulrich Rebstock, Rainer Oßwald und A. Ould 'Abdalqādir* (Beiruter Texte und Studien, 30, Beirut, 1988) provides a fuller insight into the first 100 manuscripts of this collection. The manuscript numbers cited in this text refer to the numbering in the *Sammlung*. Since most of the original texts are not paginated and their number of folios on the microfilms not easily identified, a pagination per pages will be used.

bughya fī ḥall alfāẓ al-munya, confirms on the first page the well-known biographical data of Ibn Ghāzī. Born 841/1437 in Meknes, Ibn Ghāzī moved at an early date in his life to Fez where he studied and later preached at the central mosque towards the end of his life, the year 919/1513.[3] By playing with the so-called '*jummal*-numerals', i.e. the letter-numerals, of Ibn Ghāzī's name, the author of the 'Achievement of the desire to solve the meaning of the words in *al-Munya*', the commentator Aḥmad al-Ṣabbāgh al-'Āqilī who died 1076/1665–6,[4] discovers a divine plan behind the coincidence of the numerical value of *Gh-ā-z-y[an]* (= 900 + 1 + 7 + 10 + 1 = 919) with the latter's death date.[5] Brockelmann mentions a copy of this *Sharḥ* in Rabat (No. 444).

MS No. 1789, found in Timbedgha in southern Mauretania, runs over 506 beautifully-written pages (see pls. I–III). Although its author and title are nowhere mentioned, its content proves closely similar to MS No. 37. A lengthy introduction of the mathematical terms and numerous examples are responsible for its enormous size. In the introduction, the commentator displays a certain familiarity with the classical structure of sciences. *Ḥisāb*, being part of the first or philosophical sciences, divides into *arithmāṭīqā*, which deals with the peculiarities of numbers, and into the proper *ḥisāb*, which means the use of numbers by adding and apportioning (*jam' wa-tafrīq*). *Al-Jūmaṭīqā* [sic!]—Geometry—comprises Astronomy, *'ilm al-hai'a*, Astrology, *'ilm al-falak wa-buyūtihim*, Geography, and Music, together with Composition.[6]

Numbers 226 and 694 are copies of Ibn Ghāzī's own commentary on his *Munyat al-ḥussāb*, called *Bughyat al-ṭullāb fī sharḥ munyat al-ḥussāb*, 'The desire of the students for an explanation of the calculator's craving'. No. 226, stored in the National Library of Nouakchott, is paginated and its 180 pages are in a much better condition than the incomplete copy, No. 694. Further copies exist in Rabat, Tunis, Tripoli and in Madrid.[7]

No. 695, like 694 photographed in al-'Ārif, a small town to the south-east of Nouakchott, carries the title *Nuzhat dhawī 'l-albāb wa-tuḥfat nujabā' al-anjāb*, 'The stroll of the intellectual and the preciousness of the most noble men'. Its 80 pages are ascribed to a certain M. b. Aḥmad Banīs or Bunais, whose biography remains obscure. According to local informants, the *Nuzha* was written in the year 1200/1785–86; the copy itself was finished some 86 years later. Again we find a lengthy introduction discussing—among other things—the valuation of *ḥisāb* by the traditionalists. The structure of the text and the identity of many examples with those found in *Bughyat al-ṭullāb*[8] justifies a guess before a proper investigation is undertaken: the *Nuzha* might represent a concise local commentary on the *Bughyat al-ṭullāb* by Ibn Ghāzī.

2.2.

The second group of manuscripts is connected with the famous al-Qalaṣādī, the outstanding Spanish mathematician who fled the approaching conquista-

[3] Carl Brockelmann: *Geschichte der arabischen Litteratur*, I–II (Weimar/Berlin, 1898–1902). *Supplementbände* I–III (Leiden, 1937–42) = *GAL*, S II, 337–8; Heinrich Suter: *Die Mathematiker und Astronomen der Araber* (Amsterdam, 1981 [Leipzig, 1900–2]), 186–7; Matvievskaya/Rosenfeld: *Matematiki i Astronomy musulmanskogo srednevekovya i ix trudy* (Moscow, 1983), I–III (= *MM*), II, 533; but all of them give Meknes as the last station of his life.

[4] *GAL*, S II, 338: 1076/1657 (*sic*!) with reference to the remarks of Renaud in *Isis*, XVIII, 1932, 181.

[5] The *Abjad*-numerals, i.e. the 'ABCD-numerals', were used for counting with letters (*ḥisāb al-jummal*) before the introduction of the 'Indian' ciphers and for a long time after. This notation was common in various fields of astronomical practice, see *EI* (1st edition), s.v. *Abdjad*.

[6] MS No. 1789, p. 11/3 f.

[7] *MM*, II, 533. M. Sawīsī's edition of the *Bughya* (Ḥalab 1403/1983) is based on the copies from Tunis and Rabat. Since MS No. 226 differs considerably from this edition, I will cite from the Mauretanian manuscript.

[8] cf. for example MS No. 695, pp. 68, 70 with MS No. 226, pp. 170 f.

dores of Granada and died in Bāja in Tunisia 891/1486.[9] Two of his works dominate this group: *Kashf al-jilbāb 'an 'ilm al-ḥisāb*, which Franz Woepcke rendered as ' Soulèvement du vêtement de la science du calcul ',[10] and *Kashf al-asrār 'an 'ilm ḥurūf al-ghubār*, translated as ' Soulèvement des voiles de la science du Gobār '.[11] Of this last one, five copies are extant, all of them complete: No. 187 from Nouakchott; No. 619 from Abdanghāy and No. 2120 from al-Shārāt in the east; numbers 62 and 1050 from Shinqīṭ carry a slightly different title: *Kashf al-asrār 'an waḍ' ḥurūf al-ghubār*. As the author himself states, he composed this book, which was translated into French in 1859 by Franz Woepcke, as a digest of his earlier *Kashf al-jilbāb*, which again is a commentary on his own composition *al-Tabṣira fī 'ilm al-ḥisāb*.[12]

Since the *Kashf al-jilbāb*, which has survived in quite a few libraries, has not yet been made accessible, I read thoroughly through the 79 pages of the Nouakchott MS No. 101/1, cross-reading the sister-copy No. 101/2 in order to compare original and commentary. In fact, the *Kashf al-jilbāb* and the *Kashf al-asrār* turned out to be almost identical, except that the sequence of chapters is rearranged and fewer examples have been reproduced in *Kashf al-asrār*. This is countered sometimes by more detailed descriptions of the procedures of calculation, obviously for pedagogical purposes.

Both groups of manuscripts, however, have similar roots. They have, for example, at least one common traceable ancestor: the most famous Maghribī mathematician Ibn al-Bannā, teacher of many a remarkable scholar, including al-Maqqarī and Ibn Khaldūn.[13] Both al-Qalaṣādī and his younger contemporary Ibn Ghāzī belong to the numerous North-African scholars who took up the teaching of Ibn al-Bannā' and gained fame by commenting on his *Talkhīṣ a'māl al-ḥisāb* or *Raf' al-ḥijāb*.[14] Consequently there are countless citations of *Raf' al-ḥijāb*, and to a much lesser extent, of the *Talkhīṣ*, to be found in *Bughyat al-ṭullāb*.[15] We also can identify uncited cross-references of *Kashf al-jilbāb* in *Idrāk al-bughya*.[16] Once, Ibn Ghāzī refers to the commentary of Abū 'Uthmān al-'Uqbānī on the *Talkhīṣ* of Ibn al-Bannā', from which we can deduce a further *terminus ante quem* for the biography of al-'Uqbānī,[17] who is elsewhere called the teacher of Ibn Ghāzī.[18]

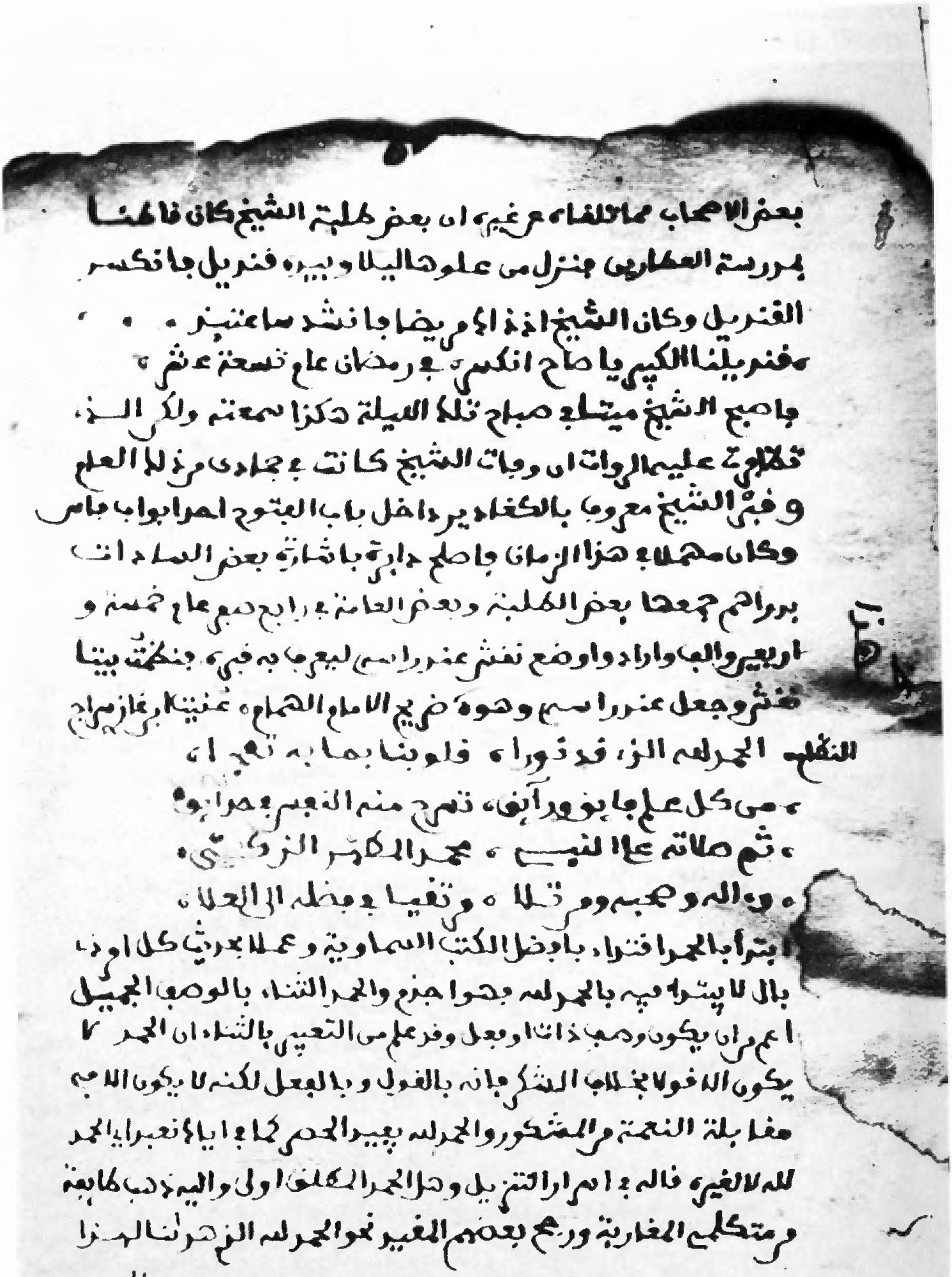
2.3.

The third group, consisting of five texts, could be called Mauretanian. I could not locate the author of one of them: M. al-'Arabī b. Sayyid M. b.

[9] *MM*, II (No. 444), 510–2; Suter, *Mathematiker*, 180–2; *GAL*, II, 266, S II, 378; A. Djebbar, *Enseignement et recherche mathématiques dans le Maghreb des XIIIe-XVIe siècles* (Publications mathématiques d'Orsay, No. 81–2, Orsay, 1981), 144 *passim*. A complete list of his works is listed in *Bīblīughrāfīya al-Qalaṣādī* (Manshūrāt dār al-kutub al-waṭanīya, Bāgha, 1976).
[10] *MM*, II (No. 444), 510–2; Suter, *Mathematiker*, 180–2; *GAL*, II, 266, S II, 378; A. Djebbar, *Enseignement*, 144 *passim*.
[11] Franz Woepcke, *Etudes sur les mathématiques arabo-islamiques* (Frankfurt, 1986), II, 1–2.
[12] *MM*, II, 510.
[13] *MM* II (No. 339), 443–6: died *c.* 721/1320; Suter, *Mathematiker*, pp. 162–4, n. 81: starb gegen 740/1339–40; cf. A. Djebbar, *Enseignement*, 140; Souissi: ' Ibn al-Bannā' ', *Talkhīṣ a'māl al-ḥisāb. Texte établi, annoté et traduit par Mohamed Ṣouissi* (Tunis, 1969), 17–18.
[14] cf. A. Djebbar, *Enseignement*, 11; Souissi, *Talkhīṣ*, 21.
[15] cf. p. 62.
[16] See, for example, the chart for the dissemination of factors, called *jirbāl*, in *Kashf*, 26, reproduced in *Idrāk*, 58.
[17] MS No. 226, p. 128; for al-'Uqbānī see *MM*, II (No. 0192), 38.
[18] MS No. 226, p. 129. al-'Uqbānī is not mentioned among the teachers of Ibn Ghāzī listed in *Bughya* (ed. Sawīsī), pp. w–y.


الى سواء الطريق وليس التوفيق الا اليه، ولا الحول الا عليه
يقول راجي العفو والمغفرة محمد بن احمد بن غازي
الرجاء تعلق القلب بمقصود يحصل في تحصيله مع الاخذ بما يحصله

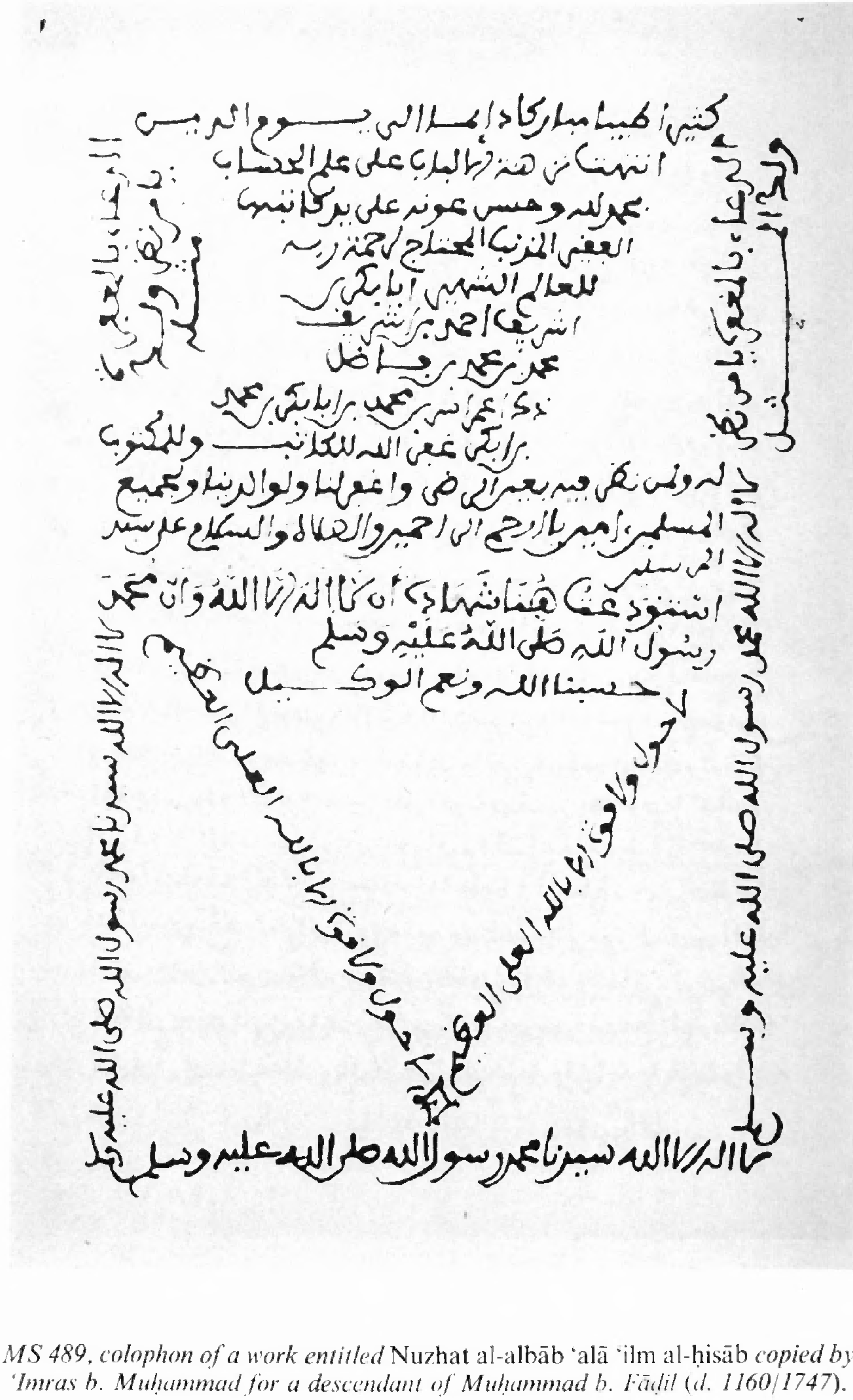
MS 1789. Anonymous compendium of mathematics from Timbedgha, southern Mauretania, folio A.


بعض الاصحاب ممن القاه من غيره ان بعض طلبة الشيخ كان قاطنا
بمدرسة العطارين فنزل من علوها ليلا وبيده قنديل فانكسر
القنديل وكان الشيخ اخذه الام يضا فانشد ساعتئذ
فقنديلنا الكبير يا صاح انكسر ٭ في رمضان عام تسعة عشر
فاصبح الشيخ متوفى صباح تلك الليلة هكذا سمعته ولكن الذي
تظافرت عليه الرواة ان وفاة الشيخ كانت في جمادى من ذلك العام
وقبر الشيخ معروف بالكغاديين داخل باب الفتوح احد ابواب فاس
وكان مهملا في هذا الزمان فاصلح دائرته باشارة بعض السادات
برواه جمعها بعض الطلبة وبعض العامة في رابع عشر عام خمسة و
اربعين والف واراد وضع نقش عند راسه ليعرف به قبره فنظمت بيتا
نقش وجعل عند راسه وهو هذا ضريح الامام الهمام عمدتنا ابن غازي سراج
النظام
الحمد لله الذي قد نورا ٭ قلوبنا بما به تعبدا
من كل علم فايق وراق ٭ تمرح منه العقل في صرايق
ثم صلاته على النبي ٭ محمد المكرم الزكي
وآله وصحبه ومن تلا ٭ وتقفيا في فضله الى العلا
ابتدأ بالحمد اقتداء بافضل الكتب السماوية وعملا بحديث كل امر ذي
بال لا يبتدأ فيه بالحمد لله فهو اجذم والحمد الثناء بالوصف الجميل
اعم من ان يكون وصف ذات او فعل وقد علم من التعبير بالثناء ان الحمد لا
يكون الا قولا بخلاف الشكر فانه بالقول وبالفعل لكنه لا يكون الا في
مقابلة النعمة من المشكور والحمد لله يفيد الحصر كما في اياك نعبد اي الحمد
لله لا لغيره قاله في اسرار التنزيل وهذا الحمد المطلق اولى واليه ذهب طائفة
من متكلمي المغاربة ورجح بعضهم التقييد نحو الحمد لله الذي هدانا لهذا
الحمد

MS 1789, folio B.

الحمد لله الذي اذهب عنا الحزن لما فيه من ذكر المقتضي للحمد ويحكى
كذكر الشيء بدليله وال في الحمد للاستغراق لجميع الانواع او للعهد
كان الله تعالى لما علم عجز خلقه عن كنه حمده حمد نفسه بنفسه في ازله
نيابة عن خلقه قبل ان يحمدوه والحمد مصدر لا يثنى ولا يجمع وحكى ابن
الانباري جمعه على حمود ولام لله للاستحقاق او للملك واصله الاله
حذفت الهمزة وعوض منها حرف التعريف ولذا قيل في النداء يا الله
بالقطع كما يقال يا الاه والالاه من اسماء الاجناس كالرجل والفرس
يقع على كل معبود بحق او باطل ثم غلب على المعبود بالحق واكثر
اهل العلم ان هذا الاسم الكريم هو الاسم الاعظم وذلك لانه دال على
الذات بجميع صفاتها الوجودية والسلبية بخلاف ساير الاسماء
وهذا الاسم الكريم لا يتغير معناه بنقص حرف منه ولان حروفه
ثلاثة فان حذف اوله بقي لله وان حذفت احدى اللامين بقي له
وان حذفت الاخرى بقي هو وكلها ورد في القرآن والقلب
يقع على اللحمة التي هي على الشكل الصنوبري المعلوم عند المهندسين
وعلى المعنى القايم بها وهو العقل عند القايل بانه محله لقوله تعالى
افلم يسيروا في الارض فتكون لهم قلوب يعقلون بها وقيل محله
الدماغ لان الانسان اذا ضرب في راسه اختل عقله وقيل العقل
شجرة اصلها في القلب واغصانها متدلية في الدماغ وسمي قلبا
لتقلبه بين الخواطر الواردة عليه وفي كشف الرموز للمغربي القلب
خلق كامل الوصف له وجهان ظاهر وباطن فظاهره ارضي طبعي
ظلمي جسماني وباطنه سماوي وعلوي نوراني فكثافة ظاهره
وظلمته لمباشرة القوى الطبيعية البشرية ولطافة باطنه لمواجهة

MS 1789, folio C.



انتهت من نزهة الالباب على علم الحساب
بحمد الله وحسن عونه على يد كاتبها
العبد المذنب المحتاج لرحمة ربه
للعالم الشهير ابابكر
الشريف احمد بن شريف
محمد بن فاضل
اشهد ان لا اله الا الله وان محمد
رسول الله صلى الله عليه وسلم
حسبنا الله ونعم الوكيل
لا حول ولا قوة الا بالله العلي العظيم
لا اله الا الله سيدنا محمد رسول الله صلى الله عليه وسلم

MS 489, colophon of a work entitled Nuzhat al-albāb ʿalā ʿilm al-ḥisāb *copied by ʿImras b. Muḥammad for a descendant of Muḥammad b. Fāḍil (d. 1160/1747).*

'Abdarraḥmān al-Qāsim, who composed (No. 438) a commentary on *al-Manẓūma fī 'l-ḥisāb* written by Abū Sālim as-Simlālī, the *Imām al-ḥisāb*, as the commentator calls him.[19] Abū Sālim al-Simlālī, according to the *Matematiki* 'al-Samlāl', lived in the sixteenth century, presumably—as the *nisba* indicates—in Morocco.[20] Two copies of another book by him on the calculation of legal shares (*farā'iḍ*) are preserved in Rabat.[21] The *sharḥ*, which carries the cryptic title, *Rabbānī 'l-muhtadī wa-ṣā' igh al-ḥujja li 'l-muntahī*, runs over 22 pages, but the end is missing. From the content of these pages I judge it to be of Mauretanian origin.

No. 35 and No. 102, the first composed by a certain Sīdī al-Shaikh Muḥyiddīn, the second put into verse by Abū Marwān Bakr b. al-Faqīh 'Uthmān al-Ṣī. . lānī al-Ghallāwī al-Fulanī (sic!), date back to the early nineteenth century and are only reluctantly accepted as mathematical texts.

No. 102 contains hints for addition and subtraction which are difficult to unravel. They mainly explain a rough method for computing the Islamic calendar and the 12 signs of the zodiac.

No. 35 is dedicated to the art of interpreting numbers and deciphering their secrets. It carries the title *Sharḥ risālat kashf al-rān 'an wajh fī 'ilm al-zā'irja al-'adadīya wā-bayān sirr al-ḥurūf al-nūrānīya* (Commentary on the epistle 'The removal of the dust from the face of explanation concerning the numerical divination and the deciphering of the luminous letters'). The epistle offers an introduction to the 'science of letters',[22] which is based on the assumption that the 28 letters of the Arabic alphabet represent four different categories of letters corresponding to the four basic elements: fire, air, water, and earth. Sophisticated combinations of letters, often in the shape of squares, are arranged in order to produce divinatory or magical effects.

No. 401 and No. 489 are copies of the book of a well-known Mauretanian author: Ibn al-Imām M. 'Abdallāh b. al-Imām, called Ambūya, the son of an *amīr* of Walāta who died in 1201/1786.[23]

No. 489 (pl. IV) is complete and the colophon reveals that it was written by a certain 'Imrās b. Muḥammad for a great-grandson of the famous *Sharīf* of Tīshīt, Muḥammad b. Fāḍil, who died in the year 1160/1747.[24] Although the text is entitled *Nuzhat al-albāb 'alā 'ilm al-ḥisāb* it has nothing in common with No. 694 or No. 695. We find rather allusions to *Kashf al-jilbāb* of al-Qalaṣādī. One of his division problems in the chapter on *qismat al-muḥāṣṣāt* is identically reproduced by Anbūya.[25] Nevertheless the author states in the preamble that he had been persuaded to write a commentary on a poem he had composed himself earlier called *Manẓūma fī 'ilm al-ḥisāb al-qalamī.*

2.4.

The last text to be introduced fits into none of these groups: the *Ajniḥat al-rughāb fī ma'rifat al-farā'iḍ wa 'l-ḥisāb* of Aḥmad b. Sulaimān al-Rasmūkī, who lived in Marrākush and died 1721.[26] Al-Rasmūkī announces at the beginning that, having already composed two commentaries on the 34 verses on numbers

[19] MS No. 438, p. 1.

[20] The Simlāla, a sub-tribe of the Guezzoūlā, lived from the tenth century onwards along the southern flank of the Moroccan Atlas. Already al-Ghazzālī knew about their extraordinary contribution to the intellectual life of the Maghrib. 'al-Samlāl' would be a very uncommon *nisba* for a man from a tribe called Simlāla.

[21] *MM*, II (No. 459b), 547.

[22] see *EI* (2nd edition), s.v. *al-ḥurūf.*

[23] Rainer Oßwald, *Handelsstädte*, 501, 503.

[24] ibid., 255, 570.

[25] cf. No. 101, p. 29 with No. 489, p. 65/14.

[26] *GAL*, S II, 709; *MM*, II (No. 583), 637–8.

and the 120 verses on *'Ilm al-farā'iḍ wa 'l-ḥisāb* (by which he means the *Manẓūma fī 'l-ḥisāb* of al-Simlālī), he would like to add a third overlapping one, just for the student's profit. Two copies from al-Ghayra in the Ḥauḍ—i.e. eastern Mauretania—are extant, but unfortunately neither of them is complete. No. 1855 is badly photographed towards the end and No. 1668 breaks off after 28 pages.

This short survey provides a fairly clear picture of the origins of these mathematical texts. A fine network of links connects almost all of them to two outstanding North-African scholars: Ibn Ghāzī and al-Qalaṣādī. The indigenous part remains negligible, at least regarding the sources' background. Since all of them are commentaries, often even sub-commentaries, it would be prudent to withhold such a verdict for now and to look closer at the content of these manuscripts.

3. *Contents*

3.1.

This section is intended to present a general impression of the specific nature of our manuscript corpus. Though many peculiarities may have slipped through the net, I have attempted to sketch an outline of what Mauretanian scholars were eager to read and comment on over the last four centuries.

Little time will be spent on the second group. The studies of Franz Woepcke and recently of Ahmad Djebbar provide a sufficient guide to the mathematical works of al-Qalaṣādī. Two remarkable variations in *Kashf al-jilbāb*, however, should be noted. Like *Kashf al-asrār*, the text comprises an introduction, the four classical chapters on whole numbers, fractions, square and cubic roots and the extraction of the unknown, each composed of eight sections, and finally moves into the epilogue, *al-khātima*.

The last section of chapter i deals with the division of allotments (*qismat al-muḥāṣṣāt*), but in a more precise and voluminous manner than *Kashf al-asrār* does.[27] The knowledge of this technique, al-Qalaṣādī states, is incumbent on the governors (*'ummāl*), the judges (*quḍāt*) and the merchants (*tujjār*).[28] Eight examples demonstrate different cases of the division of profits under varying conditions and numbers of associates.

The second variation takes place in the *khātima*. While *Kashf al-asrār* ends with the summation of cubic progressions, here the summation of arithmetic progressions with an optional first link a_2 is added. Al-Qalaṣādī proceeds to give the formula: [29]

$$s_n = [(a_2' - a_1) \cdot (n - 1) + 2a_1] \cdot n \cdot \tfrac{1}{2}$$

$$\text{which equals } s_n = n \cdot \tfrac{1}{2} \cdot [2a_1 + (n - 1) \cdot d].$$

Before ending he explains the popular secret of the friendly numbers, *al-a'dād al-mutaḥābbūn*, 220 and 284, in the same way as 'Ibn Fallūs', i.e. Abū 'l-Ṭāhir al-Māridīnī, had done in his *Kitāb I'dād al-asrār fī isrār al-a'dād* more than 200 years before.[30]

[27] cf. Franz Woepcke,, 'Traduction du traité d'arithmétique d'Aboúl Hasan Alī Ben Mohammed Alkalsādī', in Franz Woepcke, *Etudes sur les mathématiques arabo-islamiques* (Frankfurt, 1986), II [pp. 1–63], pp. 257–60 with MS No. 101/1, pp. 28/7–33.

[28] MS No. 101/1, p. 28/8.

[29] ibid., 78/3 f.

[30] MS , Berlin No. 5970, p. 13/–1 f.; A. Brentjes, 'Untersuchungen zum Nichomachus Arabicus' (presentation given at Oberwolfach 30.4.1987), paper: p. 8; the expanded version is published under the title 'The first perfect numbers and three types of amicable numbers in a manuscript on elemantary [*sic*] number theory by Ibn Fallūs', in *Erdem*, IV, 11 (May 1988), 467–83.

3.2.

The first group, structured similarly coherently, demonstrates a greater variety of topics than the second one. Since all of the manuscripts display a more or less direct relationship to the *Bughyat al-ṭullāb* of Ibn Ghāzī its description will serve as a starting point.

Ibn Ghāzī chose an unorthodox arrangement for his text. Already one of the first chapters contains the standard example to demonstrate the structure of a geometrical progression:[31] the duplication of the chess squares.[32] The sum is found by way of the formulas:

(1) $a_n = s_{n-1} + 1$

(2) $s_{n-1} = [a_1 \cdot (a_n - a_1)] : (a_2 - a_1)$

from which follows[33]

(3) $s_n = s_{n-1} + a_n$.

Until now Ibn Ghāzī by definition dealt only with 'chess' progressions with $a_1 = 1$ and $q = 2$, which he defines as type (*nauʿ*) 1. Then he proceeds to his second type of 'how'—differentiated progression with $q \neq 2$.

These preliminaries are obviously only meant to introduce a very peculiar problem, since three 'peculiarities' (*khāṣṣīya*) are allocated to a geometrical progression:

The first is the formula (1) mentioned above.

The definition of the second reads:

'The base (*uss*) of each number $[a_m]$ of a geometrical progression starting with one equals the sum of the bases of the two [equal] sides $[a_o, a_p]$ set up – minus one.'[34]

The explanation of the author makes clear what is meant:

if $o - 1 = m - p$
then $m = o + p - 1$
and $a_m = a_o \cdot a_p$

for example: $a_1 = 1, q = 2, m = 8$

$a_8 = a_1 \cdot a_8 = a_2 \cdot a_7 = a_3 \cdot a_6 = \ldots$

so the chess problem can be solved:

$a_{65} = a_{33} \cdot a_{33}$

$\Rightarrow s_{64} = a_{33} \cdot a_{33} - 1.$

[31] The terms 'difference of how' (*tafāḍul fī 'l-kaif*) and 'difference of how much' (*tafāḍul fī 'l-kaif*) are used to describe a geometrical and arithmetical progression respectively.

[32] MS No. 226, p. 18 f.; *Bughya* (ed. Sawīsī), 28 f; this fascinating problem is connected to a legendary Indian ruler who could not keep his promise. A wise man had solved a riddle posed to him at court. But beforehand he had asked the ruler for a reward: the quantity of rice which the doubling of one grain of rice as many times as there are chess fields would yield. The ruler had to realize that this inconspicious demand would swallow up his entire kingdom.

[33] Ibn Ghāzī restricts the validity of formula (2) correctly to all cases where n is an 'even-even' ($\frac{n}{2}$ = even) number. Apart from that, he explicitly states that (2) and (3) may have an optional q. The relevant passage runs:

> This is the second type of 'the difference of how', i.e. of a geometrical proportion (*nisba handasīya*). Its numbers follow each other proportionately, not (necessarily) in the ratio of $\frac{1}{2}$. This is meant by 'halving'. For the sake of practice, I have chosen the following example: Let four successive numbers be in the ratio of $\frac{1}{3}$, in the shape of 1, 3, 9, 27. If we want to sum them up, then we multiply the smallest, i.e. 1, by the difference between 1 and 27. The difference is 26 and the result will be 26. Divide this by the difference between 3 and 1, which is 2. You will get 13. This is the sum of all that is before the 27. Add it to the 27, the largest number. This is 40, the sum of all numbers.

[34] ibid., 19/12; *Bughya* (ed. Sawīsī), 32.

The third 'peculiarity' of which I have not found another example in the Arabic *ḥisāb*-literature, concerns the 'weights' (*auzān*): With the progressive numbers 1, 3, 9, 27 representing the four different weights (*ṣanja*) 1, 3, 9 and 27 *ūqīya* [$1 \bar{u}q = X_1$; $3 \bar{u}q = X_2$; $9 \bar{u}q = X_3$; $27 \bar{u}q = X_4$][35] the weight of anything (*shai'*) between 1 and 40 *ūqīya* can be measured. A handy chart of all 40 combinations gives the list of all solutions.

For example: to weigh a thing of 8 *ūqīya* you get[36]

shai' $+ X_1 \parallel X_3$

to weigh a thing of 20 *ūqīya* you get

shai' $+ X_1 + X_3 \parallel X_4 + X_2$.

Ibn Ghāzī states that this method, which his teacher b. al-Bannā' had already alluded to in *Raf' al-ḥijāb*, results as a 'fruit' from the first two 'peculiarities'. Unfortunately, neither a mathematical proof nor a motive is given for this practical exploitation of the nature of geometrical progression.

Next comes a complete list of the methods to sum up progressions. Starting with the n first natural numbers, where a_1 is always 1, it ends up after the sum of the n first even cubes with the sum of the n first numbers raised four times.[37] Then Ibn Ghāzī proceeds to the *ṭarḥ*-subtraction method to check the multiplication of integers as well as of fractions. The following sections all refer to various simplifications of special multiplications (usually called *nawādir al-ḍarb*), finishing with a chart for the analysis of factors, which, as Ibn Ghāzī states, he owes to the *aṣḥāb al-arithmāṭīqī*.[38] Later he produces the uneven numbers from 1 to 265 where the numbers of the factors of the non-prime numbers are indicated by dots.[39] Examples for the computation of profits then fill several pages. After lengthy chapters on fractions, roots and proportions, where different methods of calculations are demonstrated, including the method of 'the double false attempt', the classical *regula duorum falsorum*, he arrives at the chapter on Algebra, *al-jabr wa 'l-muqābala*. Again, after producing the six classical algebraic equations, he is concerned about the facilitation of problems; for example by a method he calls '*bi 'l-musāwī*':

take the equation $x^2 + 9 = 6x$ $[ax^2 + bx + c = 0]$

then you know that $\left(\frac{6}{2}\right)^2 = 9$ $\left[\left(\frac{b}{2}\right)^2 = ac\right]$

which leads to[40] $x^2 = 9$

At the same time you can use this method to check the solvability of any equation, since $\left(\frac{b}{2}\right)^2 < c$ renders a negative root.

Discussing the calculations of powers Ibn Ghāzī differentiates between the '*ahl al-ta'dīl*', the astronomers who take the 'six' as their base, and the '*ahl al-ḥisāb*', the reckoners who use the 'ten'.[41] Shortly before the end, when solving some complex divisions like:

[35] The canonical ounce, i.e. *ūqīya*, of Arabia had 40 *dirham* and weighed 125 grammes; see Walther Hinz, *Islamische Masse und Gewichte* (Leiden, 1955), 35.
[36] MS No. 226, p. 23.
[37] ibid., 26–31.
[38] ibid., 59.
[39] ibid., 68.
[40] ibid., 143; the negative root, which was regarded as 'absurd', did not concern the author; for the geometrical representation of negative roots, see Adolf P. Youschkevitch, *Les mathématiques arabes* (Paris, 1976), 44, 53–5.
[41] ibid., 128.

$$(12x^3 - 3x^2) : 2x = 6x^2 - 1\tfrac{1}{2}x$$

he cites ' Abū M. b. al-Yāsamīn ' (died 602/1205), most likely from his *Urjūza al-Yāsamīnīya fī 'l-jabr wa 'l-muqābala*.[42]

Now it is most interesting to note that the commentators of Ibn Ghāzī did not lay much stress on the mathematical elements of the *Bughya*. Both MS No. 37 and MS No. 1789 start off with numerous citations referring to the importance of *ḥisāb* for which dicta of the Prophet Muḥammad and his companions are rendered. In Qur'ān and *Ḥadīth* ' reckoning ' (*ḥisāb*) is an eschatological key-word for the accountability of one's acts in this life at the Day of Judgement. Thus the metaphorical meaning of *ḥisāb* was readily used by many Islamic mathematicians to legitimate the study of *ḥisāb*. But the commentator goes beyond this traditional justification. A whole list of fields of application is added to demonstrate the value of mathematical knowledge in everyday life.

So al-'Āqilī mentions the legal problems connected with the *zakāt* and *khums*, the *farā'iḍ* (inheritance laws), the *musāqāt* (irrigated land), *ijāra* (rent), *taflīs* (money-changing), *shuf'a* (pre-emption) and the *ayyām al-ḥaiḍ wa 'l-ṭuhr* (days of menstruation and cleanness).[43] The author's list of MS No. 1789 runs similarly: *masā'il fiqhīya* (juridical problems), *'ibādāt* (religious performances), *mu'ashsharāt* (tithe) and *khums* (fifth), *ghanā'im* (booty), *farā'iḍ*, *musāqāt*, *ijāra*, *shuf'a* and *ayyām al-ḥaiḍ wa l-ṭuhr*.[44] Consequently we find, apart from the major chapters on the technique of *ḥisāb*, quite a few allusions to the application of arithmetic. Both MSS, just like *al-Bughya*, extensively treat the calculation of profits. Inheritance cases are set and the correct shares allotted. MS No. 1789 (pls. I–III) further on contains the conversion of weights and measures, the fixation of wages with respect to the labour executed and another example of the widespread ' ladder-problem ', the historical diffusion of which has recently been thoroughly studied by Jacques Sesiano.[45]

Then al-'Āqilī constructs a hypothetical case which nevertheless carries the vivid imprint of his economic surroundings: a farmer intends to lay out a plantation of date-palms on an area 20 *dhirā'* long (l) and 10 *dhirā'* wide (w). But the slips must be planted at a distance (d) of 2 *dhirā'*. Al-'Āqilī then delivers the formula for both cases, the corners being used or not:[46]

$$\left(\frac{l}{d} + 1\right) \cdot \left(\frac{w}{d} + 1\right) = (11 \cdot 6) = 66 \text{ slips.}$$

On the other hand we have the *homo ludens* at work, or possibly his teaching fantasy: [47]

> [let us assume] a donkey-rider comes across a group of people and greets them: ' Peace upon you, O one hundred and one men '; then one of them

[42] ibid., 170; for Muḥammad b. al-Yāsamīn see *MM*, II (No. 320), 358; A. Djebbar, *L'analyse combinatoire au Maghreb: L'example d'Ibn Mun'im* (XII-XIII *s.*), (Publications mathématiques d'Orsay, No. 85–01, Orsay, 1980), 142: mort 1204.

[43] MS No. 37, p. 8/12 f.

[44] MS No. 1789, p. 11/–6 f.

[45] J. Sesiano, ' Survivance médiévale en Hispanie d'un problème né en Mésopotamie ', *Centaurus*, 30, 1987, 18–61; see pp. 20 ff.

[46] MS No. 37, p. 108/6 f.

[47] The Arabic text (ibid., 109) runs as follows: *wa-lau marra rākib ḥimar bi-unās, fa-sallama 'alaihim; ' wa-qāla: al-salām 'alaikum yā mi'at rajul wa-rajul! ' fa-qāla aḥaduhum: ' lau anna ma'anā mithlunā wa-mithl niṣfinā wa-rub' inā wa-inta wa-ḥimāruka la-kunnā mi'ata wa-wāḥid.'*

In the oldest known version of this problem, Metrodoros ' the Egyptian ' (first half of the sixth century) counts dancing girls instead of bedouins; see Herbert Hunger and Kurt Vogel, *Ein byzantinisches Rechenbuch des 15. Jahrhunderts. 100 Aufgaben aus dem Codex Vindobonensis Phil. Gr. 65. Text, Übersetzung und Kommentar* (Wien, 1963), 39–40.

answers him: 'If we were double and a half and a quarter as many as we are, including yourself and your donkey—then we would be one hundred and one.'

Which he solves:

$$x + x + \tfrac{1}{2}x + \tfrac{1}{4}x + 2 = 101 \Rightarrow x = 36.$$

To the same type belongs a poem cited by M. b. Aḥmad Banīs in *Nuzhat dhawī 'l-albāb* where mathematics and erotics combine in a curious unity:

1 two thirds of my heart belong to him
2 and the third of the rest of it
3 and again a third
4 but the last third of the rest to the cupbearer
5 six parts do remain
6 to be divided among (my other) lovers!

transformed into a cooler notation:[48]

$$|\,1\,|\; x - \tfrac{2}{3}x \;|\,2\,|\; - \tfrac{1}{3}(x - \tfrac{2}{3}x) \;|\,3\,|\; - \tfrac{1}{3}\{\tfrac{1}{3}(x - \tfrac{2}{3}x)\}$$
$$|\,4\,|\; - \tfrac{1}{3}[\tfrac{1}{3}\{\tfrac{1}{3}(x - \tfrac{2}{3}x)\} = |\,5\,|\; 6$$
$$\Rightarrow \tfrac{2}{81}x = 6$$
$$\Rightarrow x = 243$$

resulting in the love-rating for 'him' of $\frac{79}{81}$.

The same poem but with changed sexes had been popular among the population of Baghdād in the late tenth century according to al-Ṣūlī in his *Adab al-kuttāb*.[49] Thus the commentators dress, whenever possible, their rather dry subject in a colourful gown without neglecting their initial aim. They could serve themselves from a rich fund of this type of exercise which had accumulated over centuries between India and the Atlantic.

3.3.

The 'Mauretanian' group displays a similar tendency although on a lower level. In MS No. 438 we find again the practical usefulness ascribed to the art of *ḥisāb*, especially in connexion with the application of the inheritance laws, the financial administration and company trading.[50] But the main concern is explaining the basic principles of arithmetic: addition, substraction, multiplication, division, carried out with integers as well as fractions, isolating the factors of large numbers and proving the results by way of the method of *ṭarḥ*.

These explanations are performed with great verbal precision and in full length. Anbūya b. al-Imām, for example, covers several pages with the demonstration of three different methods of multiplication.

The first of them is called *al-mujannaḥ*, the 'winged-one'. It is the same as al-Qalaṣādī, from whom—as already mentioned—Anbūya borrowed at least one *muqāsama* problem, put forth in *Kashf al-jilbāb*.[51]

[48] MS No. 695, p. 68/-5 f.
[49] *Adab al-kuttāb*, ed. M. Bahjat al-Athrī (al-Qāhira 1341/1922), p. 243/3 f.
[50] p. 1; MS No. 489, p. 65: *qismat al-muḥāṣṣāt*; MS No. 1855, p. 15/4: *qismat ar-ribā'*; MS No. 401, p. 45: *qismat al-muḥāṣṣāt*.
[51] MS No. 101/1, p. 15/-1 f.

1. *al-mujannaḥ*:

52 · 73 =

<table>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>1</td><td>4</td><td>6</td><td></td></tr>
<tr><td></td><td>1</td><td>5</td><td></td><td></td><td></td><td>1</td><td>5</td><td></td><td></td></tr>
<tr><td>3</td><td>5 |</td><td>5̣</td><td>2</td><td>→</td><td>3</td><td>5 |</td><td>5̣</td><td>2̣</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>=</td><td>3</td><td>7</td><td>9</td><td>6</td><td>| [52]</td></tr>
</table>

The second one is named *al-ḍarb bi 'l-usūs*, the 'multiplication with the bases':

2. *al-ḍarb bi 'l-usūs*:

432 · 321 =

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<tr><td>(III)</td><td>1</td><td>2</td><td>8</td><td>4</td><td></td><td></td><td></td></tr>
<tr><td>(II)</td><td></td><td></td><td>9</td><td>6</td><td>3</td><td></td><td></td></tr>
<tr><td>(I)</td><td></td><td></td><td></td><td>6</td><td>4</td><td>2</td><td>↑</td></tr>
<tr><td></td><td></td><td></td><td></td><td>4</td><td>3̇</td><td>2̇</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>3</td><td>2</td><td>1</td><td></td></tr>
<tr><td></td><td>1</td><td>3</td><td>8</td><td>6</td><td>7</td><td>2</td><td></td></tr>
</table>

Begin with the first position of the multiplicand = '2'. Multiply it with the first position of the multiplicator; put the result above the line on position $(1 + 1) - 1$ = *the first*; $(1 + 2) - 1$ = *the second*... After having finished with the first position of the multiplicand, make a dot on top of it; then add the results.

The characteristic advantages of this method is the immediate knowledge of the single position while progressing with the different multiplications.[53]

The third method Anbūya calls *al-ḍarb bi-niṣf al-tanjīl*, that is, 'multiplication by halving the moving of the positions to the right'. He expressly states that the *ḥussāb*, the arithmeticians, elaborated this method when dealing with squares. The following example illuminates this procedure which works exactly opposite to the way al-Qalaṣādī proceeded in *Kashf al-jilbāb* when extracting whole-numbered roots.[54]

3. *al-ḍarb bi-niṣf al-tanjīl*:

438 · 438 =

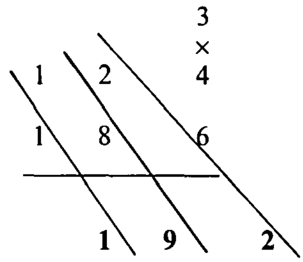
<table>
<tr><td>(V)</td><td></td><td></td><td></td><td></td><td>6</td><td>4</td></tr>
<tr><td>(IV)</td><td></td><td></td><td></td><td>4</td><td>8(6·8)</td><td></td></tr>
<tr><td>(III)</td><td></td><td></td><td>6</td><td>4(8·8)</td><td></td><td></td></tr>
<tr><td>(II)</td><td></td><td>2</td><td>4(3·8)</td><td>9(3·3)</td><td></td><td></td></tr>
<tr><td>(I) ↑</td><td>1</td><td>6(4·4)</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>(2·4)8</td><td></td><td>(2·3)6</td><td></td><td></td></tr>
<tr><td></td><td>1</td><td>9</td><td>1</td><td>8</td><td>4</td><td>4</td></tr>
</table>

Thereafter a rather well-known method, also to be found in MS No. 489, the 'diagonal one', is demonstrated by Anbūya:

[52] ibid., 20; the dots show the successive substitution of places.
[53] ibid., p. 20.
[54] MS No. 101/1, p. 47 ff.

4. '*diagonal*':

$$64 \cdot 3 =$$



All of these methods have at least one common feature: they cannot be handled *without* the *takhṭ*, the dust-board, which entered the Islamic lands together with the Hindī numerals,[55] or any other comparable medium to note down the numbers. And, indeed, this seems to be the main object of the author: to introduce *ḥisāb*-methods that accommodate dealing with '*ṭirs wa-qalam*' paper and pencil. In MS No. 401 he reminds the reader of an earlier poem he had composed on *'ilm al-ḥisāb al-qalamī*, the 'science of pencil arithmetics'.[56] At the same time the nine Hindī numerals are presented, in their 'western' version, and the advantage of their use is emphasized. Apparently this Mauretanian scholar of the closing eighteenth century had reasons enough to wage a literary battle against the antiquated use of the *jummal* numerals or the *ha-wā'ī*-method, finger-reckoning, and for the propagation of the more effective and elegant way with paper and pencil.

3. *Conclusions: historical and social significance*

Let us now turn to the attempt to evaluate this scattered information with a view to a general assessment of the mathematical priorities in the Mauretanian literature.

First of all, the homogeneity of our sources, i.e. the cross-sectional character of the manuscript corpus, can be considered as a relatively sound base for this attempt. If it should be argued that the few texts on *ḥisāb* do not allow generalizations, one should keep in mind that these texts are truly all we have. The situation calls for making a virtue out of necessity. With respect to their quantitative representation, the mathematical manuscripts occupy the leading position within the field of natural sciences. In this context it is worth noting that no text on geometry could be identified. Although only 19 MSS fitted the self-defined criteria of mathematical texts, it should not be forgotten that a respectable number cover adjacent fields like *falak*, astronomical and astrological studies, or *'ilm al-farā'iḍ*, the science of fixing inheritance shares, which both frequently make use of arithmetical techniques. Their existence underlines the interdisciplinary role arithmetic played in various sections of socially relevant sciences.

Evidently the scientific level of this art was completely and unilaterally dependent on the North-African achievements. No texts other than *maghribī* could have been used as teaching material, which adds a further brick to the more general asumption that Mauretanian literature not only started as an offshoot of the *maghribī* one, but that it also remained relatively isolated from

[55] See A. Saidan, *The arithmetic of al-Uqlīdisī, The story of Hindu-Arabic arithmetic as told in Kitāb Fuṣūl fī al-Ḥisāb al-Hindī by Abū al-Ḥasan Aḥmed ibn Ibrāhīm al-Uqlīdisī. Translated and annotated by A. S. Saidan* (Dordrecht, 1978), p. 35, n. 1.6.

[56] p. 2.

developments in the eastern Islamic centres.[57] The genuine local accomplishments, on the other hand, disclose some specific features. up to the end of the eighteenth century the Mauretanian student of *ḥisāb* had evidently to be persuaded of the advantages of the Hindī numerals, let alone the more refined methods of calculating with paper and pencil. This is certainly no evidence of any kind of backwardness for this region. It may reflect rather the specific needs of a society which lacked big urban centres and their international economic network, and which was strictly stratified into a small class of learned people, the *zawāyā*-tribes, and a majority of a mainly nomadic population who did not seem to need this kind of science to cope with their daily affairs. The absence, for example, of chapters on the rates of money-exchange and their equivalence to different grain-weights, almost ubiquitous in most of the Arabic works on arithmetic, may strengthen this argument. The stubborn persistence of traditional Arabic reckoning methods sheds light onto the two-class system of knowledge in this society.

But conclusions of this kind should be drawn cautiously and be reserved for more detailed and comprehensive studies of this fascinating culture, where the path to the *yaum al-ḥisāb*[58] remained narrow and dangerous: settling the accounts of one's life before God—*ḥisāb*—did not leave much room for philosophical leisure.

[57] cf. Oßwald, *Handelsstädte*, 280 ff.

[58] Qur'ān (Rudi Paret: *Der Koran. Übersetzung von Rudi Paret*, Stuttgart, 1979), 40:27, 38:16, 26, 53.