# Light Curve Analysis of Hipparcos Data for the Massive O-type Eclipsing Binary UW CMa

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#### Abstract

Hipparcos photometric data for the massive O-type binary UW CMa were analysed within the framework of the Roche model. Photometric solutions were obtained for five mass ratios in the  $q = M_2/M_1 = 0.5 - 1.5$  range. The system is found to be in a contact configuration. Independently of q, the best-fitting model solutions correspond to the orbital inclination  $i \sim 71^{\circ}$ and the temperature of the secondary component  $T_2 \sim 33500 \text{ K}$ , at the fixed temperature of the primary  $T_1 = 33750 \text{ K}$ . Considering that the spectrum of the secondary is very weak, photometric solutions corresponding to the contact configuration favor the mass ratio q smaller than unity (in which case the luminosity of the secondary is smaller than that of the primary). The absolute parameters of the system are estimated for different values of the mass ratio.

Keywords:

Preprint submitted to New Astronomy

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# 1. Introduction

The spectroscopic eclipsing binary UW CMa (=29 CMa= HD57060) is an interesting massive O-type close binary system (P=4.4 days). Light curves of UW CMa have been analysed by various authors. Since the spectrum of the secondary component is extremely weak and hardly detected, the spectroscopic mass ratio is uncertain. For this reason, different assumptions on the mass ratio in the system were made in various photometric studies.

Parthasarathy (1978) carried out an analysis of BV observations of Doss (1967) by the method of Russell and Merrill (1952). He determined absolute dimensions of UW CMa by combining derived photometric elements and spectroscopic data by Struve et al. (1958), which suggested that the mass ratio is  $q = M_2/M_1 = 1.20$  (secondary more massive).

Leung and Schneider (1978a) and Bagnuolo et al. (1994) carried out a detailed analysis of spectroscopic and photometric data on UW CMa, but their results are not quite consistent. Leung and Schneider (1978a) analysed the photographic light curve of Seyfert (1941) and BV light curves of Ogata and Hukusaku (1977) using the Wilson and Devinney (1971) algorithm. The authors searched for solutions for five different fixed values of the mass ratio in the q = 0.75 - 1.30 range. As they noticed earlier for several other systems, a unique value for the mass ratio usually could not be determined from a light curve alone (Leung and Schneider, 1978b). Involving some arguments related to the luminosity ratio in the system, the authors came to a conclusion that the most likely value of the mass ratio was  $q \leq 1$  (primary more

massive). The temperature of the O7f star was fixed at  $T_1 = 43000 \text{ K}$ , and the corresponding temperature of the secondary  $T_2 \sim 40000 \text{ K}$  was obtained. The authors reported five sets of photometric parameters and concluded that a contact configuration of this binary was fairly certain.

Bagnuolo et al. (1994) carried out a comprehensive analysis of the UW CMa UV spectra from the IUE archive. The tomography algorithm was used to separate spectra of the two stars. The authors found the mass ratio q = 1.2 using three independent methods: (i) fitting cross-correlation functions; (ii) comparing radial velocity semi-amplitude  $K_1$  and  $V \sin i$  (Gies and Bolton, 1986); (iii) measuring the goodness-of-fit of shifted secondary spectra produced by the tomography algorithm for an assumed grid of mass ratios. They also obtained a new spectral classification for the primary (O7.5-8 Iabf) and the secondary (O9.7 Ib). The intensity flux ratio of the stars in the UV was found to be  $r = 0.36 \pm 0.07$  (primary brighter). By using spectral type calibration of Howart and Prinja (1989) new temperatures of stars were estimated:  $T_1 = 33750K$  and  $T_2 = 29000K$ .

Then Bagnuolo et al. fitted the V-band light curve (van Genderen et al., 1988) and the UV light curve (Eaton, 1978) using GENSYN code (Mochnacki and Doughty, 1972; Gies and Bolton, 1986). It was shown that a good fit could be obtained for a semi-contact configuration, the orbital inclination  $i = 74^{\circ} \pm 2^{\circ}$  and a reasonable intensity ratio r < 0.5. The photometric model implies that the primary fills its inner Roche lobe (fill-out ratio  $f_1 = 1.0$ ), while the fill-out ratio of the secondary is  $f_2 = 0.7$ . The authors conclude that the radius of the secondary is about 70 - 80% as large as that of the primary. This result is inconsistent with the model of Leung and Schneider (1978), which suggested a large radius of the secondary.

#### 2. Hipparcos light curve of UW CMa

New photometric data on UW CMa (HIP 35412) were obtained during the Hipparcos mission. For our light curve analysis we used photometric observations in the broad-band Hp system (effective wavelength  $\lambda_{eff} \sim 4500$ Å). The light curve includes 217 data points obtained between 1990 March 31 and 1993 March 3. Orbital phases were calculated with the following ephemeris for the primary minimum (Herczeg et al., 1981):

# $JD(hel) MinI = 2440877.563 + 4^{d}.39336 \cdot E$

The primary minimum is due to the eclipse of the more luminous star by the less luminous companion. The asymmetries in the light curve noticed by the earlier observers are also present in the Hipparcos light curve. The observed light curve (Figs.1-5) shows almost symmetrical primary minimum while the secondary minimum is asymmetrical. Both branches of the secondary minimum are not smooth, notably the ascending branch is more distorted. These distortions are likely due to a mass flow, gas streams and/or stellar wind in the binary.

#### 3. Analysis

In the current study we adopted an approach similar to that of Leung and Schneider (1978). We have analysed the Hippparcos photometric light curve in two fashions: (i) including all observations; (ii) omitting orbital phases 0.5 - 0.7 on the ascending branch of the secondary minimum (the most distorted part of the light curve). It turned out that in both cases the obtained solutions were nearly identical. For this reason, significance levels of the derived model parameters were estimated for the second case only.

The Hipparcos photometric light curve was analysed within the framework of the Roche model in eccentric orbit, similar to Wilson's (1979) model. The algorithm is described in detail by Antokhina (1988, 1996). Here we describe its main features only briefly. The computer code allows one to calculate a radial velocity curves, monochromatic light curves and absorption line profiles of the stars simultaneously, either for a circular or an eccentric orbit. Axial rotation of the components may be non-synchronized with the orbital revolution. Tidal distortion of the components as well as their mutual radiative heating are taken into account. The intensity of the radiation coming from an elementary area of the stellar surface and its angular dependence are determined by the temperature of the star, gravitational darkening, limb darkening, and heating by radiation from the companion. Input parameters of the model are summarized in Table 1.

# 3.1. Input Parameters

We fixed some parameters which values were defined in previous studies of the system or can be assumed from global stellar properties. A light curve solution is only sensitive to the temperature difference between the stars, thus the temperature of one star has to be fixed. Usually it is the temperature of the primary, which can be determined more reliably. As we mentioned earlier, Bagnuolo et al. (1994) obtained new spectral classification of the primary (O7.5-8 Iabf) and the secondary (O9.7 Ib) and their temperatures  $T_1 = 33750K$  and  $T_2 = 29000K$  using the spectral type calibration from

Table 1: Input Parameters of the synthesis program

Parameters	Description
$q = M_2/M_1 \ .$	Mass ratio
<i>e</i>	Eccentricity
ω	Longitude of periastron, star No.1
<i>i</i>	Orbital inclination
$\mu_1, \mu_2 \ldots \ldots$	Roche lobe filling coefficients, $\mu = R/R^*$ , where R is the
	polar radius of a star and $R^*$ is the polar radius of the
	corresponding inner critical Roche lobe at periastron position
$T_1, T_2 \ldots$	Average effective temperatures of the components
$\beta_1,\beta_2$	Gravity darkening coefficients (the temperature of an
	elementary surface area $T = T_{1,2} \times \left(\frac{g}{\langle g \rangle_{1,2}}\right)^{\beta_{1,2}}$ , where
	$g$ and $\langle g \rangle$ are the local and mean gravity acceleration)
$A_1, A_2 \ldots$	Bolometric albedos (coefficients of reprocessing
	of the emission of a companion by "reflection")
$F_1, F_2 \ldots \ldots$	Ratio of surface rotation rate to synchronous rate
$x_{1,2}, y_{1,2} \ldots$	Limb darkening coefficients (see the text)
$l_3$	Third light
$\lambda(n)$	Effective wavelengths of monochromatic light curves

Howart and Prinja (1989). Using these data we fixed the average effective temperature of the primary star at  $T_1 = 33750$  K.

We fixed the gravity-darkening coefficients  $\beta_1 = \beta_2 = 0.25$  and albedos  $A_1 = A_2 = 1$  to values typical for early type stars. A non-linear "square-

root" limb darkening law (Diaz-Cordoves and Gimenez, 1992; Diaz-Cordoves et al., 1995; Van Hamme, 1993) was used:

$$I(\cos\gamma) = I(1)[1 - x(1 - \cos\gamma) - y(1 - \sqrt{\cos\gamma})],$$

where  $\gamma$  is the angle between the line of sight and the normal to the surface, I(1) is the intensity at  $\gamma = 0$ , and x, y are limb darkening coefficients. As shown by Van Hamme (1993), this is the most appropriate limb-darkening law at optical wavelengths for  $T \geq 10000$  K.

The Hipparcos light curve indicates circular orbit, so we fixed eccentricity at e = 0. The rotation of both stars was assumed to be synchronous with the orbital revolution  $F_1 = F_2 = 1$ .

#### 3.2. Adjustable Parameters

Thus the adjustable parameters of the models were (i) the Roche lobe filling coefficients for the primary and the secondary  $\mu_1, \mu_2$  (these parameters define the dimensionless surface potentials  $\Omega_1, \Omega_2$ ), (ii) the average effective temperature of the secondary star  $T_2$ , and (iii) the orbital inclination *i*. We fitted the model light curves for five fixed values of mass ratios (see below).

The search for the adjustable parameters was done with the well-known Simplex algorithm (Nelder and Mead's method) (Himmelblau, 1971; Kallrath and Linnell, 1987). In the vicinity of the minima found, additional calculations were done on a fine grid, to explore the details of the shape of the residuals surface and to determine the confidence intervals for the free parameters of the model. The confidence intervals for the parameters were estimated using  $\chi^2$  test at a confidence level of 1%.



Figure 1: Observed and model light curves for q = 0.5. For this mass ratio minimal deviation exceeds the critical value  $\chi^2$  at the confidence level of 1%

Since the mass ratio is unknown we searched for solutions at its five different values, q = 0.5, 0.75, 1.0, 1.25, 1.5. The resulting parameters for the five solutions are presented in Table 2. The corresponding model light curves along with the observed light curve are shown in Figs.1-5. The sky plane view of UW CMa for one of the solutions from Table 2 is shown in Fig 6.



Figure 2: Observed and model light curves at q = 0.75



Figure 3: Observed and model light curves at q = 1.0



Figure 4: Observed and model light curves at q = 1.25



Figure 5: Observed and model light curves at q = 1.5



Figure 6: The sky plane view of UW CMa at different orbital phases. The mass ratio q = 0.75

# 3.3. Absolute Dimensions

From Table 2 it can be concluded that for all five mass ratios the obtained solutions are rather close. Indeed, the  $\chi^2$  values are rather similar except the model for q = 0.5 where  $\chi^2$  is somewhat larger. The values of the adjustable parameters  $\mu_1, \mu_2, T_2, i$  are also rather close for all models with q > 0.5. The absolute dimensions of UW CMa for different values of q are given in Table 3. While computing the absolute values we used the semi-amplitude of the radial velocity curve of the primary component  $K_1 = 224.5 \ km/s$  (Stickland, 1989).

#### 4. Results and Discussion

Current analysis of the Hipparcos light curve of UW CMa allows us to definitely state that the system is in a contact configuration. This result confirms a similar conclusion made by Leung and Schneider (1978a) from their analysis of the light curves by Seyfert (1941) and Ogata & Hukusaku (1977). Another conclusion similar to that of Leung and Schneider (1978a), is that in a contact configuration the mass ratio q should be smaller than unity. Indeed, the spectrum of the secondary is very weak which implies that  $L_2/L_1$  (visual) is less than unity.

These conclusions are in disagreement with those reached by Bagnuolo et al. (1994) from the analysis of V (van Genderen et al., 1988) and UV (Eaton, 1978) light curves. The authors argue for a semi contact configuration (the secondary underfills its Roche lobe) to explain the low luminosity of the secondary. However, from the appearance of their light curves (Bagnuolo et al., 1994) it seems they could be better fitted in a contact model than in a semi contact one.

A more serious problem concerns the mass ratio in the system. Indeed, the contact configuration favors a small mass ratio, q < 1. However, Bagnuolo et al. (1994) obtained an estimate q = 1.2 from the UV data. Their estimate was consistent for three different methods they used. Presently, it is difficult to resolve the issue and to make a final conclusion on the mass ratio in the binary. Detailed spectroscopic study could probably allow one to derive the mass ratio. However such a study is beyond the scope of the present paper.

The following definitive conclusions can be drawn from our analysis:

1. The system is in contact configuration.

- 2. Independently of a particular value of q, the best-fitting model solutions correspond to the following values of the free parameters:  $i = 71^{\circ}.0 - 71^{\circ}.6$  and  $T_2 = 33300 - 33700 K$ .
- 3. It is impossible to reliably determine the value of q from the light curve solution alone. This conclusion was predictable and has been already discussed by Leung and Schneider (1978a,b).

One possibile way to explain the discrepancy in the determination of the mass ratio is a possibility that the geometry of UW CMa is different from the standard Roche geometry. The secondary component of a contact system could be a star surrounded by an optically and geometrically thick envelope (disk). Such a model was used by Antokhina and Cherepashchuk (1987) and Antokhina and Kumsiashvili (1999) in the light curve analysis of the massive interactive binary system RY Sct. In this model a binary consists of a primary component treated as a normal star in a Roche model and a disk-shaped secondary (oblate spheroid). This model was first suggested by Wilson (1974) for the analysis of  $\beta$  Lyrae.

Another model where the secondary star is surrounded by a disk was used by Djurašević et al. for light curve analysis of RY Sct (Djurašević et al., 2008) and V448 Cyg (Djurašević et al., 2009). RY Sct is an active masstransferring system. It contains the O9.5-B0 primary filling its Roche lobe which transfers mass to the more massive and hotter (although apparently fainter) secondary component, hidden within a dense accretion disk (Giuricin and Mardirossian, 1981; Antokhina and Cherepashchuk, 1987, Djurašević et al., 2008; Grindstrom et al., 2007; and references therein). Giuricin and Mardirossian (1981) presented a list of OB-binaries, which could presumably be at the same evolutionary stage as RY Sct. They included UW CMa in this list. This again could suggest existence of a dense accretion disk in the system.

In the current paper we do not apply a disk model to light curve analysis of UW CMa. The considerations above are just speculations on one possible way to resolve the mass ratio problem in the system. They are not arguments for the disk existence in UW CMa. To apply a disk model, we need some observational evidences for the presence of a disk. Such arguments (if any) could possibly be obtained from spectroscopy.

UW CMa is a massive early type contact binary. Spectroscopic observations in UV, optical, and X-ray domains reveal colliding winds. The system also shows active mass transfer and mass loss. The evolutionary time scale of massive early type contact systems is relatively short. UW CMa appears to be close to the common envelope phase of its evolution. The common envelope evolution in massive close binary stars leads to various degrees of stripping the envelope of the more massive star (Nomoto et al. 1995). This can turn UW CMa into a Type II-L, IIn, IIb, Ib, or Ic of Supernova (Nomoto et al. 1995). The future evolution of UW CMa is governed by large scale mass transfer and mass loss. It may rapidly evolve into a luminous blue variable (LBV) and then evolve into a Type II Supernova similar to LMC SN 1987A (Parthasarathy et al. 2006). From the LBV phase it may alternatively evolve into a Wolf-Rayet binary and end up again as a Type II Supernova.

		Parameter				
Parameters	$0.50^{\mathrm{b}}$	0.75	1.00	1.25	1.50	status
<i>i</i> (°)	72.6	$71.3\pm0.6$	$71.0 \pm 0.5$	$71.2 \pm 06$	$71.6\pm0.4$	adjusted
$\Omega_1=\Omega_2\ \ldots\ldots\ldots$	2.876	$3.331 \pm 0.056$	$3.750\pm0.059$	$4.209\pm0.063$	$4.526\pm0.066$	adjusted
$\mu_1$	0.999	$1.000\pm0.018$	$0.994\pm0.022$	$0.998\pm0.023$	$0.997\pm0.024$	adjusted
$\mu_2$	0.997	$0.993 \pm 0.021$	$0.999\pm0.020$	$0.999 \pm 0.018$	$1.000\pm0.019$	adjusted
$T_1(K)$	33750	33750	33750	33750	33750	adopted
$T_2(K)$	32800	$33300\pm700$	$33400\pm900$	$33600\pm700$	$33700\pm800$	adjusted
$L_1/(L_1+L_2)^{\rm a}$ .	0.662	0.570	0.501	0.451	0.409	computed
$L_2/(L_1+L_2)^{\rm a}$ .	0.338	0.430	0.499	0.549	0.591	computed
$F_1$	1.0	1.0	1.0	1.0	1.0	adopted
$F_2$	1.0	1.0	1.0	1.0	1.0	adopted
$\beta_1$	0.25	0.25	0.25	0.25	0.25	adopted
$\beta_2$	0.25	0.25	0.25	0.25	0.25	adopted
$A_1$	1.0	1.0	1.0	1.0	1.0	adopted
$A_2$	1.0	1.0	1.0	1.0	1.0	adopted
<i>x</i> <sub>1</sub>	-0.188	-0.188	-0.188	-0.188	-0.188	adopted
$y_1$	0.719	0.719	0.719	0.719	- 0.719	adopted
$x_2$	-0.141	-0.141	-0.141	-0.141	0.141	adopted
$y_2$	0.746	0.746	0.746	0.746	0.746	adopted
e	0.	0.	0.	0.	0.	adopted
ω	0.	0.	0.	0.	0.	adopted
<i>l</i> <sub>3</sub>	0.	0.	0.	0.	0.	adopted
$\chi^2$	250	235	230	235	232	computed
Relative radii $(R/a)$						
$r_1(pole)$	0.4143	$0.3802 \pm 0.0076$	$0.3561 \pm 0.0071$	$0.3376 \pm 0.0067$	$0.3227 \pm 0.0064$	
$r_1(point)$	0.5707	$0.5295 \pm 0.0061$	$0.5000 \pm 0.0587$	$0.4770 \pm 0.0568$	$0.4583 \pm 0.0551$	
$r_1(side)$	0.4399	$0.4009 \pm 0.0093$	$0.3740 \pm 0.0086$	$0.3537 \pm 0.0081$	$0.3376 \pm 0.0077$	
$r_1(back)$	0.4679	$0.4308 \pm 0.0125$	$0.4050 \pm 0.0119$	$0.3853 \pm 0.0114$	$0.3696 \pm 0.0111$	
$r_2(pole)$	0.2998	$0.3323 \pm 0.0066$	$0.3561 \pm 0.0071$	$0.3748 \pm 0.0075$	$0.3902 \pm 0.0078$	
$r_2(point)$	0.4292	$0.4705 \pm 0.0563$	$0.5000 \pm 0.0587$	$0.5229 \pm 0.0605$	$0.5415 \pm 0.0619$	
$r_2(side)$	0.3129	$0.3480 \pm 0.0080$	$0.3740 \pm 0.0086$	$0.3949 \pm 0.0092$	$0.4122 \pm 0.0097$	
$r_2(back)$	0.3454	$0.3797 \pm 0.0113$	$0.4050 \pm 0.0119$	$0.4250 \pm 0.0124$	$0.4415 \pm 0.0128$	

Table 2: Photometric Solutions for Assumed Mass Ratios

<sup>a</sup>  $L_1, L_2$  - relative monochromatic luminosities of the stars

<sup>b</sup> For q=0.5 the confidence intervals of the adjustable parameters are not listed as the minimal deviations exceed the critical value  $\chi 2$  at the confidence level of 1%

	$q = M_2/M_1$					
Parameters	0.50	0.75	1.00	1.25	1.50	
$M_1(M_{\odot})$	106.9	44.0	24.4	15.8	11.2	
$M_2(M_{\odot}) \ldots$	53.5	33.0	24.4	19.7	16.8	
$a(R_{\odot})$	61.4	48.1	41.3	37.1	34.3	
$R_1(R_{\odot})$	27.2	19.5	15.7	13.4	11.8	
$R_2(R_{\odot})$	19.7	17.1	15.7	14.8	14.3	
$T_1(K)$	33750	33750	33750	33750	33750	
$T_2(K)$	32800	33300	33400	33600	33700	
$L_1(10^5 L_{\odot})$	8.7	4.5	2.9	2.1	1.7	
$L_2(10^5L_\odot)$	4.1	3.3	2.9	2.6	2.4	

Table 3: Absolute Parameters of UW CMa

# Acknowledgments

The authors thank Prof N. Samus and Dr I. Antokhin for useful discussions. We also thank Prof S. Conti and the anonymous referee for the constructive review of our paper. E. A. acknowledges support by the Russian Foundation for Basic Research through the grant 08-02-01220.

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