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Abstract

This paper extends the Mirrlees (1971) model of optimal income redistribution with optimal corrective taxes to internalize consumption externalities. It is demonstrated that the optimal second-best tax on an externality-generating good should not be corrected for the marginal cost of public funds. The reason is that the marginal cost of public funds equals unity in the optimal tax system, since marginal distortions of taxation are equal to marginal distributional gains. The Pigouvian tax needs to be modified, however, if polluting commodities or environmental quality are more complementary to leisure than non-polluting commodities are.

JEL-Code: D620, H210, H230.

Keywords: marginal cost of public funds, optimal environmental taxation, optimal redistribution, externalities.

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1 Introduction

Economists have been forceful advocates of the use of economic instruments, such as environmental taxes, to control the emission of greenhouse gasses in the atmosphere, see, for recent, examples Stern (2007), International Monetary Fund (2008) and Fullerton et al. (2010). Pigou (1920) taught us that the optimal tax on externality-generating goods should be equal to the marginal external damage from the consumption of that good. However, the optimality of the Pigouvian tax has been challenged by the analysis of second-best. In particular, setting the corrective tax at the Pigouvian level is no longer guaranteed to be optimal in the presence of distortionary taxes.[1]

The optimal corrective tax should be equal to the marginal external damage *divided* by the marginal cost of public funds – as long as uniform commodity taxes are optimal in the absence of externalities.[2] Providing a better environmental quality is more costly if larger public revenue requirements result in greater labor-tax distortions. The reason is that, it is not optimal, from a revenue-raising perspective, to distort the composition of consumption with pollution taxes, even if tax revenues are recycled in the form of lower tax rates on labor income. By distorting the composition of consumption, environmental taxes exacerbate pre-existing distortions of the labor tax in the labor market. Bovenberg and de Mooij (1994) thus conclude "... In the presence of preexisting distortionary taxes, the optimal pollution tax typically lies below the Pigouvian tax, which fully internalizes the marginal social damage from pollution. Intuitively, the collective good of environmental quality directly competes with other collective goods. Hence, the marginal costs of environmental policy rise with the marginal cost of public funds" (p. 1085).

This paper argues that these second-best results are flawed. The main problem in the literature is that second-best corrective taxes are analyzed in Ramsey-type models assuming a representative agent. Some exceptions are discussed below. Only by artificially ruling out lump-sum taxes does the optimal tax problem become second-best in nature. Often, this assumption is justified by referring to distributional issues that are associated with levying lump-sum taxes. If every agent is identical, however, there is no distributional problem and no economic reason why lump-sum taxes should not be feasible in such settings.

The analysis in this paper explicitly incorporates redistributional concerns by allowing for heterogenous agents. Agents differ in their earnings ability, which is private information. This paper thereby extends the analyses of Mirrlees (1971) and Atkinson and Stiglitz (1976) with externalities in the consumption of certain commodities. Individualized lump-sum taxes are not feasible, as the government can observe neither individual earnings ability nor labor effort. Informational constraints thus impose limits on the redistributive powers of the government.

[1] Research on environmental taxes in second-best settings is discussed and summarized in Goulder (1995), Fullerton and Metcalf (1998), Bovenberg (1999), Sandmo (2000), Goulder and Bovenberg (2002), and Fullerton et al. (2010).

[2] Commodity taxes should optimally be differentiated for non-environmental reasons if some commodities are more complementary to leisure than others. Setting higher taxes on goods that are complementary to leisure alleviates tax distortions on labor supply, cf. Corlett and Hague (1953) and Atkinson and Stiglitz (1976). Sandmo (1975) and Bovenberg and van der Ploeg (1994b) demonstrate that for general utility functions the optimal environmental tax is a weighted average of a Pigouvian term and a Corrlett–Hague term. With weak separability of labor from other commodities in the utility function, and homothetic sub-utility over commodities, optimal commodity taxes should not be differentiated for non-environmental reasons, so that the Corlett–Hague term vanishes, see Bovenberg and van der Ploeg (1994b).

The government therefore needs to resort to taxing observable labor income and commodity demand in order to achieve its redistributive goals. Tax distortions thus arise endogenously from the redistributive preferences of the government. Nevertheless, the government can always employ non-distortionary, *non*-individualized lump-sum taxes. Intuitively, non-individualized lump-sum taxes are always incentive-compatible, and should therefore be part of the instrument set of the government. Hence, in the absence of any distributional concerns, the government relies exclusively on this non-distortionary source of finance to meet its revenue requirements.

This paper explores whether the optimal environmental tax should be modified in the second-best compared to the first-best Pigouvian tax, which perfectly internalizes all externalities. It also analyzes how the optimal provision of environmental quality deviates from its first-best rule in second-best settings. The main result of this paper is that the rules for the optimal second-best environmental tax and the optimal provision of environmental quality should *not* be corrected for the marginal cost of public funds. Consequently, optimal corrective taxes should not be lowered if public resources are scarcer due to a higher level of labor-income taxation. In the optimal tax system, the marginal cost of public funds equals one also in the second-best. Taxes are distortionary, but in the optimal tax system the deadweight losses are exactly offset by the welfare gains of a more equal welfare distribution. The finding that the marginal cost of public funds equals one is a statement about the optimality of the tax system. At the optimum, the marginal unit of tax revenue should be valued equally by the government and the private sector. Thus, a better environmental quality no longer competes with other public goods. Consequently, models that assume a representative agent produce misleading conclusions regarding the desirability of corrective taxes because they ignore the distributional gains associated with distortionary taxes.

This analysis, furthermore, derives the optimal second-best corrective tax, – what is referred to as the 'modified Pigouvian tax' – which contains some modifications in comparison with the first-best Pigouvian corrective tax. By allowing for general preference structures, which would imply differentiated commodity taxes in the absence of externalities, the analysis demonstrates that if environmental taxes alleviate (exacerbate) labor-market distortions, then optimal environmental taxes should be set higher (lower) for non-environmental concerns. Moreover, when the income tax is restricted to be linear, the tax on polluting consumption should optimally be used for redistributional reasons alongside the income tax. In particular, if the polluting commodity is consumed relatively more by the rich (poor), then the optimal tax on the polluting commodity should be higher (lower). This redistributive role of environmental policy disappears if the government has access to a non-linear income tax.[3]

Optimal non-linear pollution taxes are derived alongside the optimal non-linear income tax so as to explore the conditions under which a flat-rate pollution tax is optimal. It is shown that flat pollution taxes are indeed optimal and equal to the Pigouvian tax under two relatively mild conditions. First, the social marginal damage of consuming one unit of the polluting consumption good should be constant across individuals. And, second, preferences of individuals should be weakly separable between commodities and labor supply.

[3]This result depends on the assumption that preferences are identical – an assumption that is made throughout the paper. Saez (2002) shows that when there is preference heterogeneity in the demand for commodities, higher taxes are levied on commodities for which the skilled have a higher marginal willingness to pay.

Finally, the analysis derives exact conditions on the utility structure that ensure that the optimal second-best environmental tax equals the first-best Pigouvian tax. In particular, when non-linear policy instruments are available, labor and commodities should be weakly separable in utility. However, when only linear tax instruments are available, then not only weak separability is required, but also linear Engel curves in the demand for dirty commodities (homotheticity in utility over dirty and clean consumption goods), and the absence of income effects (quasi-linearity of utility). This latter case most clearly demonstrates the importance of distributional concerns to motivate tax distortions. While satisfying the conditions for the absence of Corlett–Hague motives, the optimal corrective tax no longer contains a correction for the marginal cost of public funds, as in Bovenberg and van der Ploeg (1994b), even though taxes are distortionary.

The rest of this paper is organized as follows. The next section discusses how this paper relates to the literature. Section 3 describes the model. Section 4 derives the optimal tax system under linear policy instruments. Section 5 does the same under non-linear instruments. Section 6 derives the policy implications from the analysis, and Section 7 concludes. The Appendix contains some technical derivations.

2 Relation to the literature

This paper is related to three strands of literature. First, it is the optimal-tax complement to Kaplow (2006), where the income tax is not necessarily optimized. Kaplow (2006) shows that tax distortions arising from the income tax need not play a role in the second-best policy rule for the optimal corrective tax if the government can design benefit-absorbing changes in the non-linear income tax schedule that perfectly neutralize the distributional impact of environmental taxes. By assuming weakly separable preferences between labor supply and other commodities, such a benefit-absorbing tax change does not generate incentive effects on labor supply, see also Laroque (2005), Gahvari (2006) and Jacobs (2009). Hence, the benefit-absorbing change in the tax schedule perfectly imitates a pure benefit tax, so that neither labor tax distortions nor distributional effects should affect the modified Pigouvian tax.[4] In contrast, this paper derives explicit expressions for optimal environmental and income taxes. If preferences are weakly separable, then the optimal second-best environmental tax is shown to correspond to the Pigouvian tax, even *without* adjusting the tax schedule to neutralize the distributional impact of the environmental tax. In addition, this paper derives the optimal tax structure under linear instruments, which is not explored by Kaplow. Optimal environmental taxes and environmental quality should then take into account also their impacts on the income distribution. Finally, the nature of non-separabilities in preferences for optimal environmental policies in second-best is analytically explored. With non-separable preferences, benefit-absorbing changes in the tax schedule generally affect labor-supply incentives (see also Laroque (2005) and Jacobs (2009)). It is shown that the optimal second-best environmental tax then deviates from the first-best Pigouvian tax.

Second, this paper contributes to the analysis of Pirttilä (2000). By adopting the private

[4] Kaplow (2006) is related to the arguments provided in Kaplow (1996), Kaplow (2004), Laroque (2005) and Gauthier and Laroque (2009), who analyze distortionary taxation and optimal public goods provision (rather than externalities).

marginal value of income as an approximation to the social marginal value of income, Pirttilä (2000) finds that the optimal environmental tax should be corrected for the marginal cost of public funds. Following Jacobs (2010), this paper uses a different and economically more appealing definition for the marginal cost of public funds, which is based on the social marginal value of income of Diamond (1975). It is demonstrated that the marginal cost of public funds should *not* play a role in the optimal second-best policy rules for corrective taxes. Moreover, this paper derives not only clear-cut analytical characterizations for optimal linear income and corrective taxes are derived, but also the conditions are derived under which the optimal linear corrective tax corresponds to the first-best Pigouvian tax in the second-best.

Third, the analysis of this paper generalizes Pirttilä and Tuomala (1997), Cremer et al. (1998), and Micheletto (2008). These authors also extend the analysis of Mirrlees (1971) and Atkinson and Stiglitz (1976) with externalities by exploring the properties of optimal tax rules with respect to the preferences of individuals.[5] However, these authors restrict the pollution tax to be linear, whereas this paper allows for optimal non-linear pollution taxes. The extension to non-linear pollution taxes requires that the government is able to verify individual consumption levels of polluting goods. For many consumption goods this assumption could be difficult to justify. However, for a number of important polluting goods it is, in fact, feasible to levy non-linear taxes. A real-world example can be found in the Netherlands where the government levies a non-linear energy tax based on individual energy consumption (i.e., electricity and gas).[6] It is demonstrated that the optimal non-linear pollution tax is a flat-rate tax as long as preferences are weakly separable between labor and consumption of polluting and non-polluting goods and the marginal social damage of pollution is constant across individuals. These conditions appear to be rather weak. Furthermore, Pirttilä and Tuomala (1997), Cremer et al. (1998), and Micheletto (2008) include all incentive compatibility constraints as separate constraints to the optimal tax problem, which makes it rather difficult to analytically derive general properties of optimal environmental tax schedules. In contrast, this paper obtains explicit and simple second-best non-linear decision rules in terms of behavioral elasticities, which can be estimated from observed behavior.

3 Model

This paper employs a static model that consists of heterogeneous individuals and a government. Individuals maximize utility by supplying labor and consuming non-polluting ('clean') and polluting ('dirty') commodities. The government maximizes social welfare by setting income and commodity taxes on the dirty good. Without loss of generality, a partial equilibrium setting

[5] Cremer et al. (1998) assume that environmental quality is separable from other commodities. This paper allows environmental quality to directly enter utility functions without imposing any form of separability, like in Pirttilä and Tuomala (1997).

[6] In 2010, the Dutch government levies a graduated tax of 0.1629 euro/m^3 for gas use below 5000 m^3 per year until 0.0116 euro/m^3 for non-business users using more than 10,000,000 m^3 of gas per year. Similarly, there is a tax of 0.1114 euro/kWh on electricity use below 10,000 kWh, which declines to 0.0010 euro for non-businesses using more than 10,000,000 kWh per year. Moreover, there a tax credit of 318 euro for electricity, see Dutch Ministry of Finance (2010).

is assumed in which prices are fixed.[7] The model is introduced under the assumption of linear policy instruments. Formally, the informational assumptions for this instrument set are that the government is able to observe aggregate labor incomes and aggregate consumption of dirty goods. Later, this assumption is dropped by allowing for non-linear instruments, which require observability of labor earnings and consumption of dirty goods at the individual level.

3.1 Households

There is a total mass of individuals equal to N. Individuals may differ by a one-dimensional parameter $n \in \mathcal{N} = [\underline{n}, \overline{n}]$. n is used to denote the individual's earning ability ('skill level'). All labor types are assumed to be perfect substitutes in aggregate production. As a result, the wage rate per efficiency unit of skill is constant and normalized to unity. The density of individual types with index n is denoted by $f(n)$ and the cumulative distribution function by $F(n)$. All individual-specific variables are indexed with subscript n.

Each individual n derives utility from non-polluting commodities c_n, polluting commodities q_n, and a better environmental quality E. In addition, the individual derives disutility from supplying labor l_n. The utility function u_n is strictly quasi-concave and identical across individuals:

$$u_n \equiv u\left(c_n, q_n, l_n, E\right), \quad u_c, u_q, -u_l, u_E > 0, \quad u_{cc}, u_{ll}, u_{qq}, u_{EE} < 0, \quad \forall n. \tag{1}$$

The subscripts refer to the argument of differentiation, except where the subscript denotes ability n. All goods are assumed to be non-inferior. It is assumed that the utility function satisfies the single-crossing conditions, which are needed to implement non-linear taxes, see also section 5 and the Appendix. In addition, we assume that the marginal utility of income is decreasing.[8] However, no further structure on the cross derivatives of the utility function is imposed. Hence, this utility specification allows for general cross-substitution patterns between consumption of clean and dirty goods, labor supply and environmental quality.

Households spend their net labor earnings and government transfers on consumption of clean and dirty goods. Gross labor earnings nl_n are subject to tax rate t, polluting goods q_n are subject to tax rate τ, and individuals receive a non-individualized lump-sum transfer T. The individual budget constraint therefore reads as follows:

$$c_n + (1+\tau)q_n = (1-t)nl_n + T, \quad \forall n. \tag{2}$$

Given the presumed absence of non-labor incomes, the tax on the clean commodity is redundant, and, therefore, set to zero.[9] Each individual maximizes utility subject to the budget

[7] Almost all of the papers in the literature fix the marginal rates of transformation between all commodities at one. Hence, all prices are constant, and allowing for general equilibrium provides no additional insights, see e.g., Bovenberg and de Mooij (1994) and Bovenberg and van der Ploeg (1994b). Moreover, the partial equilibrium results fully generalize to general equilibrium settings with non-constant prices, since optimal second-best tax rules in general equilibrium are identical to the ones in partial equilibrium as long as there are constant returns to scale in production and all labor types are perfect substitutes, see also Diamond and Mirrlees (1971).

[8] This requires the utility function to display decreasing returns to scale in all its arguments.

[9] Fullerton (1997) shows that the alternative normalization with a zero labor tax raises the optimal commodity taxes on clean and dirty consumption in a uniform way (which is equivalent to an income tax).

constraint. Households take environmental quality E as given when deciding on their consumption plans. Consumption of dirty commodities causes a classical negative externality in a manner that is delineated below.

Optimal choices of labor and consumption are governed by the following first-order conditions:[10]

$$\frac{-u_l}{u_c} = (1-t)n, \quad \forall n, \tag{3}$$

$$\frac{u_q}{u_c} = 1+\tau, \quad \forall n. \tag{4}$$

The marginal rate of substitution between labor and consumption in equation (3) equals the net wage rate. A larger tax rate on labor earnings induces substitution towards leisure. This is distortionary, since the tax drives a wedge between the social marginal benefit and the private marginal benefit of an hour of work effort. According to equation (4), the individual optimally decides upon the allocation of resources between dirty and clean consumption goods. A higher tax on dirty goods harms non-environmental welfare by distorting the optimal composition of consumption as a result of substitution towards the consumption of clean commodities.

The indirect utility function is designated by $v_n(T, t, \tau, E) \equiv u\left(\hat{c}_n, \hat{q}_n, \hat{l}_n, E\right)$, $\forall n$, where hats denote optimized values of each commodity and labor supply. Application of Roy's lemma produces the following derivatives of the indirect utility function: $\frac{\partial v_n}{\partial T} = \lambda_n$, $\frac{\partial v_n}{\partial t} = -\lambda_n n l_n$, $\frac{\partial v_n}{\partial \tau} = -\lambda_n q_n$, and $\frac{\partial v_n}{\partial E} = \lambda_n \frac{u_E}{u_c}$, $\forall n$, where λ_n stands for the private marginal utility of income.

3.2 Environmental quality

Environmental quality (E) is modeled as a pure public good. In particular, environmental quality is specified as a linear function of aggregate consumption of dirty goods:

$$E \equiv E_0 - \alpha N \int_{\mathcal{N}} q_n \mathrm{d}F(n), \quad \alpha > 0, \tag{5}$$

where E_0 denotes the exogenously given initial stock of environmental quality. The linearity of the specification for environmental quality is without loss of generality, since the utility function features diminishing marginal utility of environmental quality. The latter assumption ensures increasing social marginal damage from pollution. Alternatively, one can also interpret equation (5) as the production technology of environmental quality.

3.3 Government

The government maximizes a Samuelson-Bergson social welfare function, which is a concave sum of individual utilities:

$$N \int_{\mathcal{N}} \Psi(u_n) \mathrm{d}F(n), \quad \Psi'(u_n) > 0, \quad \Psi''(u_n) \leq 0. \tag{6}$$

If $\Psi'(u_n) = 1$, the social welfare function is utilitarian. If $\Psi'(u_n) = 0$, except for the lowest skill level, the social welfare function is Rawlsian.

[10] Strict quasi-concavity of the utility function ensures that second-order conditions for a maximum are fulfilled.

Total tax revenues from the labor income tax and the dirt tax should be equal to total outlays on non-individualized transfers NT, and an exogenous revenue requirement to finance public goods R:

$$N\int_{\mathcal{N}}(tnl_n+\tau q_n)\,\mathrm{d}F(n)=NT+R. \tag{7}$$

4 Optimal linear taxation

The Lagrangian for maximizing social welfare is given by (where the whole expression has been divided by the population size N to save on notation):

$$\begin{aligned}\max_{\{T,t,\tau,E\}}\mathcal{L} \;\equiv\; & \int_{\mathcal{N}}\Psi\left(v_n\left(T,t,\tau,E\right)\right)\mathrm{d}F(n) \\ & +\eta\left(\int_{\mathcal{N}}(tnl_n+\tau q_n)\,\mathrm{d}F(n)-T-\frac{R}{N}\right)-\mu\left(\frac{E-E_0}{N}+\alpha\int_{\mathcal{N}}q_n\mathrm{d}F(n)\right),\end{aligned} \tag{8}$$

where the Lagrange multiplier η denotes the marginal social value of public resources and the Lagrange multiplier μ denotes the marginal social cost per capita (measured in social welfare units) of providing a better environmental quality E.

The optimization program (8) is solved for the optimal linear taxes on labor income and the optimal tax on polluting goods. Moreover, environmental quality E is employed as a separate control variable for the government. The first-order conditions for an optimal allocation are given by:

$$\frac{\partial\mathcal{L}}{\partial T}=\int_{\mathcal{N}}\left[\Psi'\lambda_n-\eta+\eta tn\frac{\partial l_n}{\partial T}+(\eta\tau-\alpha\mu)\frac{\partial q_n}{\partial T}\right]\mathrm{d}F(n)=0, \tag{9}$$

$$\frac{\partial\mathcal{L}}{\partial t}=\int_{\mathcal{N}}\left[-nl_n\Psi'\lambda_n+\eta nl_n+\eta tn\frac{\partial l_n}{\partial t}+(\eta\tau-\alpha\mu)\frac{\partial q_n}{\partial t}\right]\mathrm{d}F(n)=0, \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial \tau}=\int_{\mathcal{N}}\left[-q_n\Psi'\lambda_n+\eta q_n+\eta tn\frac{\partial l_n}{\partial \tau}+(\eta\tau-\alpha\mu)\frac{\partial q_n}{\partial \tau}\right]\mathrm{d}F(n)=0, \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial E}=\int_{\mathcal{N}}\left[\frac{u_E}{u_c}\Psi'\lambda_n-\mu+\eta tn\frac{\partial l_n}{\partial E}+(\eta\tau-\alpha\mu)\frac{\partial q_n}{\partial E}\right]\mathrm{d}F(n)=0, \tag{12}$$

where the derivatives of the indirect utility function are used in each expression.[11]

In the rest of this section, the optimal income tax, the optimal environmental tax, and the optimal provision of environmental quality are derived by employing the Slutsky equations for labor supply, the demand for the dirty commodity, and the demand for environmental quality: $\frac{\partial l_n}{\partial t}=\frac{\partial l_n^*}{\partial t}-nl_n\frac{\partial l_n}{\partial T}$, $\frac{\partial q_n}{\partial t}=\frac{\partial q_n^*}{\partial t}-nl_n\frac{\partial q_n}{\partial T}$, $\frac{\partial l_n}{\partial \tau}=\frac{\partial l_n^*}{\partial \tau}-q_n\frac{\partial l_n}{\partial T}$, $\frac{\partial q_n}{\partial \tau}=\frac{\partial q_n^*}{\partial \tau}-q_n\frac{\partial q_n}{\partial T}$, $\frac{\partial l_n}{\partial E}=\frac{\partial l_n^*}{\partial E}+\frac{u_E}{u_c}\frac{\partial l_n}{\partial T}$, and $\frac{\partial q_n}{\partial E}=\frac{\partial q_n^*}{\partial E}+\frac{u_E}{u_c}\frac{\partial q_n}{\partial E}$. To compute the income effect of the change in environmental quality, the property has been used that $\frac{u_E}{u_c}$ measures the marginal change in (virtual) income when environmental quality improves by one unit, see Wildasin (1984).

[11] We always assume that the solution to the optimal tax problem is interior and that second-order conditions are met.

4.1 Marginal cost of public funds

The marginal cost of public funds (MCF) is defined as the ratio between social marginal value of one unit of public income (η) and the average of the social marginal value of one unit of private income. The traditional literature on the marginal cost of public funds generally takes $\Psi'\lambda_n$ as a measure for the social marginal value of private income of individual n (see, for example, Pirttilä (2000)). Consequently, the traditional measure for the marginal cost of public funds is defined as $MCF \equiv \eta/\bar{\lambda}$, where $\bar{\lambda} \equiv \int_{\mathcal{N}} \Psi'\lambda_n \mathrm{d}F(n)$. Jacobs (2009) and Jacobs (2010) demonstrate that this traditional definition of the marginal cost of public funds suffers from three major defects.

First, the traditional marginal cost of public funds for lump-sum taxes is generally not equal to one. In particular, a higher lump-sum tax stimulates labor supply if leisure is a normal good. Hence, if labor income is taxed at a positive rate, a larger lump-sum tax raises revenues from the income tax. Consequently, under the standard definition, the marginal cost of public funds for the lump-sum tax is smaller than one (cf. Wilson (1991) and Sandmo (1998)).

Second, the marginal cost of public funds for the distortionary labor income tax is not directly related to the excess burden of the tax. The marginal cost of public funds measure increases in the *un*compensated wage elasticity of labor supply, which does not fully reflect the distortionary impact of the tax, due to the presence of income effects. Indeed, the marginal cost of public funds of the distortionary tax could even be smaller than one if there is a backward-bending labor-supply curve, cf. Atkinson and Stern (1974) and Ballard and Fullerton (1992).

Third, the marginal cost of public funds is highly sensitive to the normalization of the tax system. Jacobs (2010) shows that, for identical allocations, the marginal cost of public funds for the lump-sum tax switches from a number smaller than one to a number larger than one if consumption is taxed instead of income. The reason is that if consumption is a normal good, a higher lump-sum tax reduces tax revenues from consumption taxes. Moreover, the marginal cost of public funds for the consumption tax is unambiguously larger than one, since the uncompensated elasticity of consumption demand is always negative. Intuitively, income and substitution effects of consumption taxes reinforce each other. Thus, at identical allocations, the marginal cost of public funds of a consumption tax is always larger than the marginal cost of public funds of an income tax.

These findings point to a problematic concept of the marginal cost of public funds, which renders its usefulness for both theory and applied analysis doubtful. Nevertheless, these problems disappear if the income effects on taxed bases and environmental welfare are included in the definition for the marginal social value of private income. Intuitively, the *social* value of a marginal unit of private income should capture *all* effects of transferring a marginal unit of resources from the public to the private sector. Thus, as Diamond (1975) proposes, the indirect (income) effects of lump-sum taxes on taxed bases should be included in the *social* value of private income, and not only the direct effect on private utility ($\Psi'\lambda_n$).

Definition 1 *The* social *marginal value of transferring a marginal unit of income to individual n is defined as*

$$\lambda_n^* \equiv \Psi'\lambda_n + \eta t n \frac{\partial l_n}{\partial T} + (\eta\tau - \alpha\mu)\frac{\partial q_n}{\partial T}. \tag{13}$$

λ_n^* gives the net increase in social welfare (measured in social utils) of transferring a marginal unit of resources to person n. Note that λ_n^* consists of four elements. First, when the individual receives a marginal unit of income his private welfare rises by λ_n, and social welfare thus increases by $\Psi'\lambda_n$. This is the traditional measure of the social value of private income. Second, an individual receiving a marginal unit of income reduces labor supply as long as leisure is a normal good. If labor income is taxed ($t > 0$), lower labor supply reduces tax revenues. This results in a reduction of social welfare by $\eta t n \frac{\partial l_n}{\partial T}$. Third, by receiving a marginal unit of income, the individual will consume more dirty commodities if the dirty commodity is a normal good. If dirty commodities are taxed ($\tau > 0$), the government receives more revenues, and social welfare expands by $\eta\tau\frac{\partial q_n}{\partial T}$. Finally, the social value of a unit of private income should subtract the value of the larger environmental damage arising from a larger demand for dirty commodities, as represented by $\alpha\mu\frac{\partial q_n}{\partial T}$.

The definition of the marginal cost of public funds is thus given by the following:

Definition 2 $MCF \equiv \eta/\bar{\lambda}^*$, *where* $\bar{\lambda}^* \equiv \int_{\mathcal{N}} \lambda_n^* \mathrm{d}F(n)$.

Armed with the Diamond (1975) definition of the social marginal value of income, the first main result of the paper can be established:

Proposition 1 *The marginal cost of public funds in the optimal linear tax system is equal to one:*

$$MCF = 1. \tag{14}$$

Proof. Substituting definition 2 for MCF in the first-order condition of the lump-sum transfer in equation (9) yields the result. ■

The government thus sets the average resource cost of the lump-sum transfer (unity) equal to the average social welfare gain expressed in monetary units of the transfer: $\bar{\lambda}^*/\eta$. This result demonstrates that representative-agent models, which yield a marginal cost of public funds larger than one, need reconsideration. Allowing for redistributional concerns has two important consequences. First, at the margin, the government always has access to a non-distortionary source of finance. This source of finance is arbitrarily ruled out in representative-agent models. However, by allowing for a non-distortionary marginal source of finance, the marginal cost of public funds for this instrument is one in the optimal tax system. Second, all other tax instruments must then have a marginal cost of public funds equal to one as well, since an optimal tax system equates the marginal cost of public funds across all instruments. As a result, the marginal excess burden of distortionary taxes should be exactly compensated by the social marginal benefits of redistribution or a higher environmental quality, see below.

4.2 Optimal income tax

To derive the optimal income tax from equation (10), the Feldstein (1972) distributional characteristic of the labor income tax base is introduced.

Definition 3 *The distributional characteristic of labor income is*

$$\xi_l \equiv 1 - \frac{\int_{\mathcal{N}} \lambda_n^* n l_n \mathrm{d}F(n)}{\int_{\mathcal{N}} \lambda_n^* \mathrm{d}F(n) \int_{\mathcal{N}} n l_n \mathrm{d}F(n)} > 0. \tag{15}$$

ξ_l corresponds to the normalized covariance of earnings of individual n (nl_n), and the net social welfare weight λ_n^* of individual n. ξ_l measures the marginal gain in social welfare (in monetary equivalents), expressed as a percentage of taxed labor income, of raising revenue via the tax on labor income. The distributional characteristic is positive because the covariance between labor earnings and welfare weights is negative.[12] Individuals with higher incomes feature lower welfare weights because of diminishing social marginal utility of income. This is caused by diminishing private marginal utility of income and the concavity of the social welfare function. A positive distributional characteristic ξ_l therefore implies that taxing labor income yields distributional benefits. A stronger social preference for redistribution increases the distributional characteristic. Similarly, more pre-tax inequality in labor earnings raises the demand for redistribution. A zero distributional characteristic is obtained either if the government is not interested in redistribution and attaches the same welfare weight λ_n^* to all n, or if taxable income nl_n is the same for all n so that there is no inequality. The distributional characteristic reaches a maximum of one with the strongest possible distributional concerns, i.e., if the government has Rawlsian social preferences.

The compensated elasticities of labor supply and dirty consumption with respect to the labor tax rate are defined as $\varepsilon_{lt} \equiv \frac{\partial l_n^*}{\partial t}\frac{1-t}{l_n} < 0$ and $\varepsilon_{qt} \equiv \frac{\partial q_n^*}{\partial t}\frac{1-t}{q_n}$. The direct tax elasticity of labor supply ε_{lt} is unambiguously negative, i.e., higher taxes reduce labor supply due to substituting leisure for consumption. The cross-tax elasticity of dirty consumption cannot be signed unambiguously[13]. If dirty consumption is complementary to leisure, higher income taxes will stimulate dirty consumption, i.e., $\varepsilon_{qt} > 0$. If dirty consumption is a substitute for leisure, income taxes will reduce dirty consumption, i.e., $\varepsilon_{qt} < 0$.[14] Further, $\gamma_n \equiv \frac{(1+\tau)q_n}{(1-t)nl_n}$ is defined as the net expenditure share of dirty commodities in net labor income. Armed with these definitions, the following proposition derives the optimal income tax.

Proposition 2 *The optimal linear income tax satisfies*

$$\xi_l = \frac{t}{1-t}\left(-\overline{\varepsilon_{lt}}\right) + \frac{\left(\tau - \alpha\frac{\mu}{\eta}\right)}{1+\tau}\left(-\overline{\gamma\varepsilon_{qt}}\right), \tag{16}$$

where $\overline{\varepsilon_{xt}} \equiv \left[\int_{\mathcal{N}} \varepsilon_{xt} nl_n \mathrm{d}F(n)\right]\left[\int_{\mathcal{N}} nl_n \mathrm{d}F(n)\right]^{-1}$ *is the income-weighted average of the elasticity* ε_{xt}*,* $x = l, q$*.*

Proof. Substituting the Slutsky equations and the distributional characteristic (15) into the first-order condition for the income tax in equation (10), using the definitions for the elasticities and rearranging the resulting equation yields the expression. ∎

$\frac{\mu}{\eta}$ is the *price* of one unit of environmental quality. μ is the social cost of reducing aggregate dirty consumption by one unit. By dividing through η, the marginal value of public resources, this utility cost is converted into monetary equivalents. The ratio $\frac{\mu}{\eta}$ can also be interpreted as

[12]Note that the covariance is negative only if the indirect income effect does not result in increasing welfare weights λ_n^*. This possibility is assumed not to occur. At low tax rates, one can be sure that this holds true, see also Kaplow (2008).

[13]The ambiguity in sign of this and all subsequently discussed cross-elasticities is formally derived in the Appendix

[14]The text refers to Hicksian complements and substitutes, i.e., the signs of the cross-derivatives of the compensated labor supply and consumption demand functions.

the marginal rate of transformation of producing one unit of environmental quality per capita (E/N). A reduction of aggregate dirty consumption by one unit improves the quality of the environment by α. Hence, $\alpha\frac{\mu}{\eta}$ corresponds to the marginal external cost of one unit of aggregate consumption of dirty commodities. The first-best Pigouvian tax thus equals $\alpha\frac{\mu}{\eta}$.

The expression for the optimal linear income tax (16) demonstrates that the marginal distributional benefits of the income tax (ξ_l) are equated with the sum of the marginal deadweight loss in labor supply $-\frac{t}{1-t}\overline{\varepsilon_{lt}} > 0$ and the marginal deadweight loss in the demand for dirty goods $-\frac{\left(\tau-\alpha\frac{\mu}{\eta}\right)}{1+\tau}\overline{\gamma\varepsilon_{qt}}$. The labor tax rate increases with its redistributional benefits (larger ξ_l). It falls with the compensated labor supply elasticity (larger $-\overline{\varepsilon_{lt}}$). If the demand for dirty goods falls with a larger labor tax ($\overline{\gamma\varepsilon_{qt}} < 0$), the income tax helps to internalize the environmental externality, and should be set higher as long as the dirt tax is below the Pigouvian tax ($\tau < \alpha\frac{\mu}{\eta}$). However, if the dirt tax is above the Pigouvian tax ($\tau > \alpha\frac{\mu}{\eta}$), a higher income tax exacerbates the non-environmental distortion in consumption and should be set lower as a result. If $\overline{\gamma\varepsilon_{qt}} > 0$, the opposite reasoning holds.

4.3 Modified Pigouvian tax

To find the optimal tax on the externality-generating good, the first-order condition for the dirt tax in equation (11) can be rewritten by introducing the Feldstein (1972)-distributional characteristic of the dirty commodity:

Definition 4 *The distributional characteristic of dirty goods consumption is*

$$\xi_q \equiv 1 - \frac{\int_{\mathcal{N}} \lambda_n^* q_n \mathrm{d}F(n)}{\int_{\mathcal{N}} \lambda_n^* \mathrm{d}F \int_{\mathcal{N}} q_n \mathrm{d}F(n)}. \tag{17}$$

Generally, this normalized covariance cannot be signed, since it depends on how the demand for dirty commodities covaries with the social welfare weights. If individuals with a high ability (low ability) consume relatively more from the dirty good, the distributional characteristic is positive (negative), i.e., $\xi_q > 0$ ($\xi_q < 0$).

The compensated elasticities of labor supply and dirty commodities with respect to the environmental tax are defined as $\varepsilon_{l\tau} \equiv \frac{\partial l_n^*}{\partial \tau}\frac{1+\tau}{l_n}$ and $\varepsilon_{q\tau} \equiv \frac{\partial q_n^*}{\partial \tau}\frac{1+\tau}{q_n} < 0$, respectively. Hence, the direct tax elasticity of dirty consumption, $\varepsilon_{q\tau}$, is unambiguously negative. However, the cross-elasticity of labor supply cannot be signed unambiguously. If dirty consumption is complementary to leisure, higher environmental taxes will encourage labor supply ($\varepsilon_{l\tau} > 0$), while if dirty consumption is a substitute for leisure, environmental taxes will discourage labor supply ($\varepsilon_{l\tau} < 0$).

Proposition 3 *The optimal commodity tax satisfies*

$$\xi_q = \frac{t}{1-t}\left(-\frac{\overline{\varepsilon_{l\tau}}}{\bar{\gamma}}\right) + \frac{\left(\tau - \alpha\frac{\mu}{\eta}\right)}{1+\tau}\left(-\frac{\overline{\gamma\varepsilon_{q\tau}}}{\bar{\gamma}}\right), \tag{18}$$

where $\bar{\gamma} \equiv \left[\int_{\mathcal{N}} \gamma_n n l_n \mathrm{d}F(n)\right]\left[\int_{\mathcal{N}} n l_n \mathrm{d}F(n)\right]^{-1}$, *denotes the income-weighted average of* γ_n, *and* $\overline{\varepsilon_{x\tau}} \equiv \left[\int_{\mathcal{N}} \varepsilon_{x\tau} n l_n \mathrm{d}F(n)\right]\left[\int_{\mathcal{N}} n l_n \mathrm{d}F(n)\right]^{-1}$ *is the income-weighted average of the elasticity* $\varepsilon_{x\tau}$,

$x = l, q$.

Proof. Substituting the Slutsky equations and the distributional characteristic (17) into the first-order condition for the commodity tax in equation (11), using the definitions for the elasticities and rearranging the resulting equation yields the expression. ■

The expression for the optimal dirt tax in equation (18) equates the distributional benefits of the dirt tax (ξ_q) on the left-hand side with the marginal deadweight losses of the dirt tax on the right-hand side. The latter consist of the deadweight loss in labor supply $-\frac{t}{1-t}\frac{\overline{\varepsilon_{l\tau}}}{\bar{\gamma}}$, and the deadweight loss in the demand of dirty goods $-\frac{\left(\tau-\alpha\frac{\mu}{\eta}\right)}{1+\tau}\frac{\overline{\gamma\varepsilon_{q\tau}}}{\bar{\gamma}} > 0$. Hence, dirt taxes increase if they serve well as a redistributive device, i.e., if dirty goods are mainly consumed by the high-ability types (i.e., when ξ_q is positive and large). Dirt taxes rise as well if the external cost associated with environmental damage is large (i.e., $\alpha\frac{\mu}{\eta}$ is large), and if the demand for dirty consumption is not sensitive to the dirt tax (i.e., $\overline{\gamma\varepsilon_{q\tau}}$ is small). The dirt tax may indirectly affect labor supply, depending on the complementarity of dirty goods with leisure. Dirt taxes should be set lower when dirty goods are complementary to work ($-\overline{\varepsilon_{l\tau}} > 0$), but higher when dirty goods are complementary to leisure ($-\overline{\varepsilon_{l\tau}} < 0$). In analogy of the modified Samuelson rule for the optimal provision of public goods Atkinson and Stern (1974), the optimal second-best corrective tax from (18) is labeled as the *modified Pigouvian tax*.

4.4 Modified Samuelson rule for optimal environmental quality

Finally, the optimal provision of environmental quality is derived. The distributional characteristic of the environmental quality E is defined as the normalized covariance between the social marginal value of private income λ_n^* and the marginal willingness to pay for the environment $\frac{u_E}{u_c}$:

Definition 5 *The distributional characteristic of environmental quality is*

$$\xi_E \equiv 1 - \frac{\int_{\mathcal{N}} \frac{u_E}{u_c}\lambda_n^* \mathrm{d}F(n)}{\int_{\mathcal{N}} \frac{u_E}{u_c}\mathrm{d}F(n)\int_{\mathcal{N}} \lambda_n^* \mathrm{d}F(n)}. \tag{19}$$

If mainly high-ability (low-ability) types benefit from a better environmental quality, then $\xi_E > 0$ ($\xi_E < 0$). $\xi_E = 0$ if environmental quality is distributionally neutral. In that case, social welfare weights λ_n^* are equal across individuals, or all individuals share the same willingness to pay for a better environment $\frac{u_E}{u_c}$.

The compensated cross-elasticities of labor supply and dirty consumption with respect to environmental quality are defined as $\varepsilon_{lE} \equiv \frac{\partial l_n^*}{\partial E}\frac{E}{l_n}$ and $\varepsilon_{qE} \equiv \frac{\partial q_n^*}{\partial E}\frac{E}{q_n}$. If improvements in environmental quality raise (reduce) compensated labor supply, $\varepsilon_{lE} > 0$ ($\varepsilon_{lE} > 0$) is obtained. Similarly, if compensated demand for dirty commodities increases with a higher (lower) environmental quality, $\varepsilon_{qE} > 0$ ($\varepsilon_{qE} > 0$) is found.

Proposition 4 *The modified Samuelson rule for the optimal provision of environmental quality is given by*

$$(1-\xi_E)\, N \int_{\mathcal{N}} \frac{u_E}{u_c}\mathrm{d}F(n) = \frac{\mu}{\eta} - \delta\left[\frac{t}{1-t}\overline{\varepsilon_{lE}} + \left(\frac{\tau-\alpha\frac{\mu}{\eta}}{1+\tau}\right)\overline{\gamma\varepsilon_{qE}}\right], \tag{20}$$

where $\delta \equiv (1-t)\int_{\mathcal{N}} nl_n \mathrm{d}F(n)/E$ *measures the ratio of net labor income to environmental quality, and* $\overline{\varepsilon_{xE}} \equiv \left[\int_{\mathcal{N}} \varepsilon_{x\tau} nl_n \mathrm{d}F(n)\right]\left[\int_{\mathcal{N}} nl_n \mathrm{d}F(n)\right]^{-1}$ *denotes the income-weighted average of the elasticity* ε_{xE}, $x = l, q$.

Proof. Substituting the Slutsky equations and the distributional characteristic (19) into the first-order condition for environmental quality in equation (12), using the definitions for the elasticities and rearranging the resulting equation yields the expression. ■

Equation (20) gives a modified Samuelson rule for the provision of the environmental public good. The left-hand side of equation (20) indicates the benefits of a cleaner environment. It is equal to the sum of the marginal rates of substitution between environmental quality and consumption, i.e., the sum of the marginal willingness to pay for a cleaner environment. The distributional characteristic of environmental quality deflates (inflates) the marginal benefits if mainly the rich (poor) benefit from a better environmental quality, i.e., when $\xi_E > 0$ ($\xi_E < 0$).

The right-hand side of equation (20) gives the marginal cost of a cleaner environment. The first term, $\frac{\mu}{\eta}$, can be interpreted as the marginal rate of transformation of producing one unit of environmental quality. If environmental quality is more costly to produce in terms of reducing the consumption of dirty goods, $\frac{\mu}{\eta}$ increases. The second, and third terms on the right-hand side of (20) reflect the direct impact of changes in the public good of environmental quality on, respectively, labor supply and dirty consumption (cf. Atkinson and Stern (1974) and Jacobs (2010) for ordinary public goods). If a better environmental quality reduces (increases) compensated labor supply, i.e., $\overline{\varepsilon_{lE}} < 0$ ($\overline{\varepsilon_{lE}} > 0$), the marginal cost of providing a better environment increases (decreases) as long as labor income is taxed. In that case, the government loses (gains) tax revenues from the labor tax if the quality of the environment improves. Similarly, if a better environment would reduce (increase) the compensated demand for dirty commodities, i.e., $\overline{\gamma\varepsilon_{qE}} < 0$ ($\overline{\gamma\varepsilon_{qE}} > 0$), the marginal cost of providing a better environment increases (decreases) if environmental externalities are more (less) than fully internalized by the dirt tax, i.e., $\tau - \alpha\frac{\mu}{\eta} > 0$ ($\tau - \alpha\frac{\mu}{\eta} < 0$).

4.5 First-best

Note that in the absence of distributional concerns, the first-best policy rules for the environment can be obtained.

Proposition 5 *In the absence of distributional concerns, the marginal income tax rate is set to zero* ($t = 0$)*. The optimal environmental tax* τ *satisfies the first-best Pigouvian tax rate:*

$$\tau = \alpha\frac{\mu}{\eta}. \tag{21}$$

Moreover, the Pigouvian tax sustains a first-best level of environmental quality:

$$N\int_{\mathcal{N}} \frac{u_E}{u_c}\mathrm{d}F(n) = \frac{\mu}{\eta}. \tag{22}$$

Proof. In the absence of distributional concerns, all distributional characteristics are zero ($\xi_l = \xi_q = \xi_E = 0$). Substitution in (16), (18) and (20) yields the results. ■

Equation (22) is the uncorrected Samuelson rule for environmental quality stating that the sum of the marginal rates of substitution should be equal to the marginal rate of transformation of providing a better environment. This proposition demonstrates most clearly that positive marginal tax rates on labor income are introduced only to redistribute income.

4.6 Optimal tax structure

The optimal tax structure of income and environmental taxes can be solved as functions of the elasticities and distributional terms.

Corollary 1 *The optimal income tax and the optimal pollution tax are given by*

$$\frac{t}{1-t} = \frac{\xi_l + \overline{\gamma\varepsilon_{qt}}\left(\frac{\bar{\gamma}\xi_q}{-\overline{\gamma\varepsilon_{q\tau}}}\right)}{-\overline{\varepsilon_{lt}} - \overline{\gamma\varepsilon_{qt}}\left(\frac{\overline{\varepsilon_{l\tau}}}{-\overline{\gamma\varepsilon_{q\tau}}}\right)}, \tag{23}$$

$$\frac{\tau - \alpha\frac{\mu}{\eta}}{1+\tau} = \frac{\bar{\gamma}\xi_q + \overline{\varepsilon_{l\tau}}\left(\frac{\xi_l}{-\overline{\varepsilon_{lt}}}\right)}{-\overline{\gamma\varepsilon_{q\tau}} - \overline{\varepsilon_{l\tau}}\left(\frac{\overline{\gamma\varepsilon_{qt}}}{-\overline{\varepsilon_{lt}}}\right)}. \tag{24}$$

Proof. Solve equations (16) and (18) for $\frac{t}{1-t}$ and $\frac{\tau-\alpha\frac{\mu}{\eta}}{1+\tau}$. ■

Equations (23) and (24) give the optimal tax rates on labor and dirty consumption. The left-hand side in equation (24) shows the environmental tax in deviation from its first-best Pigouvian rate. Hence, the right-hand side of (24) reflects the non-environmental role of the environmental tax.

To gain some intuition for these expressions, suppose that dirty consumption and labor are neither relative substitutes nor relative complements, i.e., when compensated consumption demand and labor supply are independent ($\overline{\gamma\varepsilon_{qt}} = 0$ and $\overline{\varepsilon_{l\tau}} = 0$). This requires a separable utility function without income effects. In that case, the optimal tax rates are determined by the ratios of a distributional term (respectively ξ_l and $\bar{\gamma}\xi_q$) and the compensated tax elasticity (respectively, $-\overline{\varepsilon_{lt}}$ and $-\overline{\gamma\varepsilon_{q\tau}}$). Thus, tax rates increase with distributional concerns and decrease with the compensated elasticities. This is the standard trade-off between equity and efficiency, which applies to both policy instruments.

The compensated cross-elasticities are generally not zero. This implies that two additional terms determine optimal taxes. First, the second term in the denominators of both tax expressions indicates whether the elasticity of the tax base of one tax instrument increases due to employing the other tax instrument. $-\overline{\gamma\varepsilon_{qt}}\left(\frac{\overline{\varepsilon_{l\tau}}}{-\overline{\gamma\varepsilon_{q\tau}}}\right)$ in equation (23) captures the interaction between the demand for dirty commodities and labor supply. For example, $\overline{\gamma\varepsilon_{qt}}\frac{\overline{\varepsilon_{l\tau}}}{-\overline{\gamma\varepsilon_{q\tau}}} > 0$ if taxing dirty commodities reduces labor supply ($\overline{\varepsilon_{l\tau}} < 0$), and taxing labor income reduces demand for dirty commodities ($\overline{\gamma\varepsilon_{qt}} < 0$). Therefore, the less distortionary dirt taxes for labor supply are, the more the elasticity of the labor tax base decreases, and the higher optimal labor taxes should be set. Similarly, $-\overline{\varepsilon_{l\tau}}\left(\frac{\overline{\gamma\varepsilon_{qt}}}{-\overline{\varepsilon_{lt}}}\right)$ in equation (24) indicates whether the elasticity of the base of the dirt tax increases with labor-tax distortions. For example, $\overline{\varepsilon_{l\tau}}\left(\frac{\overline{\gamma\varepsilon_{qt}}}{-\overline{\varepsilon_{lt}}}\right) > 0$, if the income tax lowers the demand for dirty commodities ($\overline{\gamma\varepsilon_{qt}} < 0$), and the dirt tax reduces labor supply ($\overline{\varepsilon_{l\tau}} < 0$), a higher dirt tax mitigates the distortion of the labor tax on dirty goods consumption, and dirt taxes should be set higher, accordingly. The signs of the cross-elasticities are generally ambiguous. Consequently, cross-substitution effects could theoretically either increase or decrease optimal tax rates on income or dirty consumption.

Second, the second term in the numerator of both tax expressions indicates whether one tax instrument improves the equity-efficiency trade-off of employing the other tax instrument. The term $\overline{\gamma\varepsilon_{qt}}\left(\frac{\bar{\gamma}\xi_q}{-\overline{\gamma\varepsilon_{q\tau}}}\right)$ in equation (23) is negative if dirty commodities are consumed more by the affluent ($\xi_q > 0$), and income taxes reduce the demand of dirty commodities ($\overline{\gamma\varepsilon_{qt}} < 0$). Consequently, a higher labor tax worsens the trade-off between equity and efficiency of the dirt tax. As a result, the dirt tax loses its redistributive power, and the income tax should be lowered. The reverse reasoning holds for the case in which dirty commodities are consumed by the poor ($\xi_q < 0$) or if income taxes increase the demand for dirty commodities ($\overline{\gamma\varepsilon_{qt}} > 0$). From the numerator in equation (24) follows that dirt taxes improve the equity-efficiency trade-off for the income tax if $\overline{\varepsilon_{l\tau}}\frac{\xi_l}{-\bar{\varepsilon}_{lt}} > 0$. If dirt taxes boost (reduce) labor supply, i.e., $\overline{\varepsilon_{l\tau}} > 0$ ($\overline{\varepsilon_{l\tau}} < 0$), then it becomes more (less) attractive to fight inequality with the income tax.

Important to note is that in equation (24) there is *no* correction for the marginal cost of public funds. The optimal second-best environmental tax requires a correction for distributional concerns and interactions with labor supply, but not for pre-existing tax distortions. Intuitively, in the optimal tax system, the deadweight loss of a distortionary labor tax is exactly offset by the distributional gain of a more equal welfare distribution (or a better environmental quality). Since the marginal cost of public funds equals one, the marginal unit of revenue is valued equally by the government and the private sector. Hence, a better environmental quality no longer competes with other public goods. This finding contrasts with models that assume a representative agent (e.g. Sandmo (1975) and Bovenberg and van der Ploeg (1994b)). These models have emphasized that the interactions of environmental taxes with preexisting tax distortions push the optimal second-best pollution tax below its first-best level. This result also sheds a different light on the findings by Pirttilä (2000), who adopts the traditional concept for the social value of private income. Using the Diamond measure for the social value of private income, the marginal cost of public funds plays no role in the optimal rule for corrective taxes.

4.7 Conditions for first-best rules in second-best

A special case can be derived where the optimal environmental tax in the second-best equals the first-best expression for the Pigouvian tax.

Proposition 6 *If preferences are given by*

$$u_n \equiv \upsilon\left(c_n, q_n\right) - h\left(l_n\right) + \Gamma(E), \quad \upsilon_c, \upsilon_q, h', \Gamma' > 0, \quad \upsilon_{cc}, \upsilon_{qq}, -h'', \Gamma'' < 0, \quad \forall n, \tag{25}$$

where $\upsilon\left(.\right)$ denotes total real consumption from clean and dirty commodities, and $\upsilon\left(.\right)$ is a linear homogeneous sub-utility function over clean and dirty commodities, then the optimal income tax is given by $\frac{t}{1-t} = \frac{\xi_l}{-\bar{\varepsilon}_{lt}}$, the modified Pigouvian tax equals the first-best Pigouvian tax, $\tau = \alpha\frac{\mu}{\eta}$, and environmental quality follows the first-best Samuelson rule, $N\int_{\mathcal{N}}\frac{u_E}{u_c}\mathrm{d}F(n) = \frac{\mu}{\eta}$.

Proof. Marginal utility of income is constant, due to the homotheticity of $\upsilon\left(.\right)$, which follows from substitution of the first-order condition $\frac{u_q}{u_c} = 1+\tau$ in the definition for $\lambda_n \equiv u_c$. There-fore,

$\frac{u_E}{u_c}$ is constant, so that $\xi_E = 0$, cf. (19). Furthermore, $\varepsilon_{lt} = \gamma_n \varepsilon_{l\tau}$, which follows from totally differentiating the first-order conditions (3), (4) and the utility function (1), and setting the change in utility to zero in order to find the compensated elasticities. Finally, it can be derived that $\int_{\mathcal{N}} (1-\lambda_n^*) \left(\frac{1+\tau}{1-t}\right) q_n \mathrm{d}F(n) = \int_{\mathcal{N}} (1-\lambda_n^*) \gamma_n n l_n \mathrm{d}F(n)$, since $\frac{(1+\tau)q_n}{(1-t)nl_n+T}$ is constant with homothetic preferences and using the first-order condition for T from (9). Substitution of these results in the first-order conditions – equations (16), (18) and (12) – proves the proposition. ■

The preferences given in equation (25) are quasi-linear in total real consumption $\upsilon(.)$, and income effects in labor supply are absent. Due to the homotheticity of sub-utility $\upsilon(.)$, taxing dirty commodities has no distributional advantage over taxing labor income. Intuitively, consumption of dirty commodities is proportional to labor income. Hence, taxing dirty goods at a higher rate than the Pigouvian tax yields no distributional gains, but results in larger labor market distortions by distorting the composition of consumption, thereby harming non-environmental welfare. Hence, dirt taxes are not used for distributional reasons. Similarly, dirt taxes cannot be used to alleviate the tax burden on labor, since both clean and dirty goods are equally complementary to leisure. Thus, the consumption tax imposes the same distortions on labor supply as the income tax, while it introduces additional distortions in the optimal composition of consumption. These distortions can be avoided by not taxing dirty commodities at a different rate than the Pigouvian rate.[15] Naturally, optimal allocations differ between first-best and second-best settings.

This example most clearly illustrates why it is misleading to ignore distributional concerns in second-best tax analysis. The assumptions on preferences in equation (25) are similar to those in Bovenberg and van der Ploeg (1994b). They find that the optimal second-best Pigouvian tax should be corrected for the marginal cost of funds. However, this paper shows that this correction disappears in the presence of redistributional concerns.

5 Optimal non-linear environmental taxation

This section derives the optimal non-linear taxes on income and polluting consumption goods. The government can observe only the total labor income of an individual, $z_n \equiv nl_n$, but cannot observe either labor supply l_n, or ability n. Therefore, individualized lump-sum taxes are ruled out. Moreover, the government is assumed to be able to verify consumption of polluting goods at the individual level. Hence, a non-linear commodity tax can be levied. The non-linear income tax schedule is designated by $T(z_n)$. The marginal tax rate is $T'(z_n) \equiv \mathrm{d}T(z_n)/\mathrm{d}z_n$. The non-linear tax on the dirty commodity is given by $\tau(q_n)$, where $\tau'(q_n) \equiv \mathrm{d}\tau(q_n)/\mathrm{d}q_n$ stands for the marginal commodity tax. The individual's optimization problem is not affected, except that non-linear marginal tax rates replace the linear ones in the first-order conditions for utility maximization. The social welfare function remains identical as well.

To determine the non-linear policy schedules $T(.)$ and $\tau(.)$, a standard mechanism-design approach is employed. First, making use of the revelation principle, the optimal direct mechanism is derived, which induces individuals to reveal their ability truthfully through self-selection. This direct mechanism yields the optimal second-best allocation. Then, this allocation is decentralized as the outcome of a competitive equilibrium by employing the non-linear policy

[15] A version of this utility function is employed by Jacobs and van der Ploeg (2010) in a dynamic framework with stochastic environmental damages and the optimal setting of environmental and income taxes.

instruments.

Any second-best allocation must satisfy the resource- and incentive-compatibility constraints. The economy's resource constraint is

$$N \int_{\mathcal{N}} (z_n - c_n - q_n)\,\mathrm{d}F(n) = R. \tag{26}$$

If the government maximizes social welfare subject to the resource constraint, and all individuals respect their budget constraints, the government budget constraint is automatically satisfied by Walras' law.

Since n is not observable by the government, every bundle $\{c_n, q_n, z_n\}$ for individual n must be such that individual n does not want to have another bundle $\{c_m, q_m, z_m\}$ intended for individual $m \neq n$. If utility is written as $u(c_n, q_n, l_n, E) = u(c_n, q_n, \frac{z_n}{n}, E) \equiv U(c_n, q_n, z_n, E, n)$, then incentive compatibility requires

$$U(c_n, q_n, z_n, E, n) \geq U(c_m, q_m, z_m, E, n), \quad \forall m \in \mathcal{N}, \forall n \in \mathcal{N}. \tag{27}$$

By adopting the first-order approach, these incentive constraints can be replaced by a differential equation on utility:[16]

$$\frac{\mathrm{d}u(c_n, q_n, l_n, E)}{\mathrm{d}n} = -\frac{l_n u_l(c_n, q_n, l_n, E)}{n}. \tag{28}$$

After integrating the incentive-compatibility constraint by parts, the Lagrangian for maximizing social welfare can be formulated as

$$\begin{aligned} \max_{\{l_n, q_n, u_n, E\}} \mathcal{L} \;\; \equiv & \;\; \int_{\mathcal{N}} \left(\Psi\left(u_n\right) + \eta \left(n l_n - c_n - q_n - \frac{R}{N} \right) \right) f(n)\mathrm{d}n \qquad (29) \\ - & \;\; \mu \left(\frac{E - E_0}{N} + \alpha \int_{\mathcal{N}} q_n \mathrm{d}F(n) \right) + \int_{\mathcal{N}} \left(\theta_n \frac{l_n u_l}{n} - u_n \frac{\mathrm{d}\theta_n}{\mathrm{d}n} \right) \mathrm{d}n + \theta_{\overline{n}} u_{\overline{n}} - \theta_{\underline{n}} u_{\underline{n}}, \end{aligned}$$

where θ_n is the Lagrangian multiplier associated with the differential equation for utility (28). Intuitively, θ_n equals the marginal increase in social welfare of redistributing one unit of income from individuals above n to individuals below n. η is the Lagrangian multiplier on the economy's resource constraint. As before, η measures the marginal social value of public resources. The transversality conditions for this control problem are given by

$$\lim_{n \to \overline{n}} \theta_n = 0, \quad \lim_{n \to \underline{n}} \theta_n = 0. \tag{30}$$

[16] This is a valid procedure only if second-order conditions for utility maximisation are fulfilled at the optimum allocation. This requires single-crossing of the utility function ($\frac{d(-U_z/U_c)}{dn} < 0$, $\frac{d(U_q/U_c)}{dn} > 0$) and strictly non-decreasing gross incomes and consumption of dirty goods with skill n at the optimal allocation ($\frac{dz_n}{dn} \geq 0$, $\frac{dq_n}{dn} \geq 0$), see Appendix for a formal proof. In the remainder it is assumed that these conditions are always met.

5.1 Optimal redistribution

Proposition 7 *The optimal non-linear income tax is given by*

$$\begin{aligned} \frac{T'(z_n)}{1-T'(z_n)} &= \frac{\left(1+\frac{1}{\varepsilon_n}\right)}{nf(n)}\frac{\theta_n\lambda_n}{\eta} \\ &= \frac{\left(1+\frac{1}{\varepsilon_n}\right)}{nf(n)}u_c(.)\int_n^{\overline{n}}\left(\frac{1}{u_c(.)}-\frac{\Psi'(u_n)}{\eta}\right)\exp\left[\int_n^m -\frac{l_s u_{lc}(.)}{s u_c(.)}\mathrm{d}s\right]f(m)\mathrm{d}m, \quad \forall n, \end{aligned} \tag{31}$$

where $\varepsilon_n \equiv \left(\frac{l_n u_{ll}}{u_l}-\frac{l_n u_{lc}}{u_c}\right)^{-1} = \left(\frac{\partial \ln(-u_l/u_c)}{\partial \ln l_n}\right)^{-1} > 0$ *is the compensated labor-supply elasticity.*

Proof. The first-order condition of (29) with respect to l_n is

$$\frac{\partial \mathcal{L}}{\partial l_n} = \eta\left(n - \left.\frac{\mathrm{d}c_n}{\mathrm{d}l_n}\right|_{\bar{q},\bar{u},\bar{E}}\right)f(n) + \frac{\theta_n u_l}{n}\left(1+\frac{l_n u_{ll}}{u_l}+\frac{l_n u_{lc}}{u_l}\left.\frac{\mathrm{d}c_n}{\mathrm{d}l_n}\right|_{\bar{q},\bar{u},\bar{E}}\right) = 0, \quad \forall n. \tag{32}$$

Totally differentiating the utility function at constant dirty goods consumption, utility and environmental quality, and substituting the result in the first-order condition for labor supply yields $\left.\frac{\mathrm{d}c_n}{\mathrm{d}l_n}\right|_{\bar{q},\bar{u},\bar{E}} = \frac{-u_l}{u_c} = (1-T')n$. Substitution of this result and some simplification yield

$$\frac{T'(z_n)}{1-T'(z_n)} = \frac{\left(1+\frac{1}{\varepsilon_n}\right)}{nf(n)}\frac{\theta_n\lambda_n}{\eta}, \quad \forall n. \tag{33}$$

The first-order condition for u_n is given by

$$\frac{\partial \mathcal{L}}{\partial u_n} = \left(\Psi'(.) - \eta\left.\frac{\mathrm{d}c_n}{\mathrm{d}u_n}\right|_{\bar{q},\bar{l},\bar{E}}\right)f(n) + \frac{\theta_n l_n}{n}u_{lc}\left.\frac{\mathrm{d}c_n}{\mathrm{d}u_n}\right|_{\bar{q},\bar{l},\bar{E}} - \frac{\mathrm{d}\theta_n}{\mathrm{d}n} = 0, \quad \forall n. \tag{34}$$

Note that $\left.\frac{\mathrm{d}c_n}{\mathrm{d}u_n}\right|_{\bar{q},\bar{l},\bar{E}}$ is equal to $\frac{1}{u_c(.)}$. This equation is a first-order differential equation in θ_n. Using a transversality condition from (30), the equation can be integrated to find:

$$\frac{\theta_n}{\eta} = \int_n^{\overline{n}}\left(\frac{1}{u_c(.)}-\frac{\Psi'(u_n)}{\eta}\right)\exp\left[\int_n^m -\frac{l_s u_{lc}(.)}{s u_c(.)}\mathrm{d}s\right]f(m)\mathrm{d}m, \quad \forall n. \tag{35}$$

Substitution of (35) in (33) yields (31). ■

This expression is identical to Mirrlees (1971). At each point in the income distribution, the government trades off imposing the distortions of a higher marginal tax rate against its distributional gains. If the compensated labor supply elasticity ε_n is larger, the optimal tax rate should be lower. This conforms to standard Ramsey intuition. If the taxed labor base is larger, $nf(n)$ is higher, and the welfare losses increase, because more individuals or individuals with higher wages are affected by higher marginal tax rates. The marginal tax rates should be higher if the marginal value of redistribution (expressed in money units) $\frac{\theta_n\lambda_n}{\eta}$ increases. The marginal value of redistribution θ_n is zero at the end points of the skill distribution. In the absence of bunching at zero labor effort, the marginal tax rate is zero at the bottom of the skill distribution. The intuition is that it is not optimal to distort the labor supply for the lowest-skilled person if there is no one below that person who benefits from redistribution. If there is bunching at zero labor supply, the marginal tax rate at the bottom is positive, see Seade (1977) and Ebert (1992). Positive marginal tax rates then help to redistribute income from individuals with positive labor supply to individuals with zero labor earnings. Similarly, if the skill distribution is bounded, the marginal tax rate at the top should be zero. Intuitively, it makes no sense to distort labor supply of the highest-skilled individual, since there will be no one contributing to redistribution above him, cf. Sadka (1976) and Seade (1977). However, if the skill distribution is Pareto, then the top rate does not converge to zero, see also Diamond (1998), and Saez (2001). The reader is referred to Mirrlees (1971), Seade (1977), Tuomala (1984), Diamond (1998), and Saez (2001) for further discussion of the optimal non-linear income tax. Note that the expression for the optimal non-linear income tax does not depend on the presence of the externality.

The non-linear income tax does not provide a direct measure for the marginal cost of public funds. To determine the marginal cost of public funds, the welfare effect of a unit-increase in the intercept $-T(0)$ of the tax function is analyzed. In particular, each tax payer is provided with one unit of extra income. This is equivalent to an increase in the lump-sum transfer of one unit of income under the linear tax schedule. This method is also employed by Saez (2001) and Jacquet et al. (2010).

Proposition 8 *The marginal cost of public funds in the optimal non-linear tax system is equal to one:*

$$MCF \equiv \frac{\eta}{\int_{\mathcal{N}}\left(\Psi'\lambda_n + \eta n T'(z_n)\frac{\partial l_n}{\partial(-T(0))} + (\eta\tau'(q_n) - \alpha\mu)\frac{\partial q_n}{\partial(-T(0))}\right)\mathrm{d}F(n)} = 1. \tag{36}$$

Proof. Raising $-T(0)$ by one unit has the following three effects. First, the government loses N units of resources. Second, this policy mechanically raises social welfare – measured in monetary equivalents – for each individual by $\frac{\Psi'\lambda_n}{\eta}$. Third, a larger $-T(0)$ generates behavioral changes in labor supply and demand for polluting commodities. Since marginal tax rates are unaffected by this policy reform, substitution effects are zero and only the income effects matter. The income effect in labor supply changes tax revenues by $T'(z_n)\frac{\partial z_n}{\partial(-T(0))} = T'(z_n)\,n\frac{\partial l_n}{\partial(-T(0))}$. Moreover, a larger lump-sum transfer generates an income effect on the demand for polluting goods, which both raises tax revenues by $\tau'(q_n)$ and lowers environmental welfare by $\alpha\frac{\mu}{\eta}$ per unit of polluting consumption. The combined effect is $\left(\tau'(q_n) - \alpha\frac{\mu}{\eta}\right)\frac{\partial q_n}{\partial(-T(0))}$. The total change in welfare should be zero when the intercept of the tax function is optimized, i.e., when

$$N\int_{\mathcal{N}}\left(\frac{\Psi'\lambda_n}{\eta} + nT'(z_n)\frac{\partial l_n}{\partial(-T(0))} + \left(\tau'(q_n) - \alpha\frac{\mu}{\eta}\right)\frac{\partial q_n}{\partial(-T(0))}\right)\mathrm{d}F(n) = N.$$

Rewriting yields the desired result. ■

At the optimum, the marginal cost of public funds for all tax instruments must be equalized. Since $-T(0)$ is equivalent to a non-distortionary non-individualized lump-sum tax, at the optimum the marginal cost of public funds for all other tax rates must be equal to the marginal cost of public funds of $-T(0)$. Thus, at each point in the income distribution, tax distortions should be equal to distributional and/or environmental gains for all marginal tax rates.

5.2 Modified Pigouvian tax

Proposition 9 *The optimal non-linear tax on polluting commodities is given by*

$$\frac{\tau'(q_n) - \alpha\frac{\mu}{\eta}}{1+\tau'(q_n)} = -\frac{u_c\theta_n/\eta}{nf(n)}\left(\frac{\partial \ln(u_q/u_c)}{\partial \ln l_n}\right), \quad \forall n. \tag{37}$$

Proof. The first-order condition for q_n is

$$\frac{\partial \mathcal{L}}{\partial q_n} = \eta\left(-1 - \left.\frac{\mathrm{d}c_n}{\mathrm{d}q_n}\right|_{\bar{l},\bar{u},\bar{E}}\right) f(n) - \mu\alpha f(n) + \frac{\theta_n l_n}{n}\left(u_{lc}\left.\frac{\mathrm{d}c_n}{\mathrm{d}q_n}\right|_{\bar{l},\bar{u},\bar{E}} + u_{lq}\right) = 0, \quad \forall n. \tag{38}$$

Totally differentiate the utility function at constant l_n, u_n, and E, to obtain $\left.\frac{\mathrm{d}c_n}{\mathrm{d}q_n}\right|_{\bar{l},\bar{u},\bar{E}} = -\frac{u_q}{u_c}$. Use this result and the individual first-order condition ($\frac{u_q}{u_c} = 1+\tau'(q_n)$) to find the desired result. ■

Again, the optimal corrective tax does not depend on measures for the marginal cost of public funds. Environmental taxes could be useful for non-environmental reasons, i.e., to reduce labor market distortions. The environmental tax optimally deviates from the Pigouvian level if this helps to relax the incentive-compatibility constraint associated with income redistribution. If $\frac{\partial \ln(u_q/u_c)}{\partial \ln l_n} > 0$ (< 0), then the marginal willingness to pay for the dirty commodity increases (decreases) with labor supply. Consequently, individuals with a higher ability are less tempted to mimic those with lower ability, since the commodity tax boosts labor supply. Hence, the optimal environmental tax in second-best is lower (higher) than the Pigouvian tax, $\tau'(q_n) < \alpha\frac{\mu}{\eta}$ ($\tau'(q_n) > \alpha\frac{\mu}{\eta}$). Thus, the environmental tax alleviates some of the tax-induced distortions of the income tax on labor supply.

In contrast to linear taxes, environmental taxes are not directly used for redistributional reasons. If there are no other sources of heterogeneity than in skills, the non-linear income tax is the informationally most efficient instrument to redistribute income, which renders indirect instruments for redistribution redundant. Intuitively, indirect instruments cannot redistribute more income than the income tax, but introduce additional distortions by distorting the composition of consumption demands.[17]

The following proposition states the condition under which Pigouvian, flat rate pollution taxes are optimal.

Proposition 10 *If the utility function is weakly separable between labor and private commodities, i.e.,* $u(h(c_n, q_n), l_n, E)$*, then the optimal marginal environmental tax in second-best equals the first-best Pigouvian tax:*

$$\tau' = \alpha\frac{\mu}{\eta}. \tag{39}$$

Proof. If $u(h(c_n, q_n), l_n, E)$, it is immediately established that $\frac{\partial \ln(u_q/u_c)}{\partial \ln l_n} = 0$. Hence, the right-hand side of (37) is zero. ■

[17] However, this result disappears if there is also preference heterogeneity, see also Saez (2002). If the willingness to pay for dirty commodities (i.e., the marginal rate of substitution of dirty and clean commodities) correlates with skills, dirt taxes (or subsidies) should also be employed for redistributional reasons.

Thus, with the relatively weak assumption that consumption of both commodities is weakly separable from leisure, the optimal second-best policy rule for environmental taxation is identical to the optimal first-best policy rule. The intuition is that environmental taxes are used neither for redistribution nor to improve efficiency only to internalize the environmental externality.[18] Moreover, the optimal marginal environmental tax is constant, i.e., flat, in the case in which labor supply is weakly separable from clean and dirty commodities – even though the environmental tax is allowed to be non-linear. The intuition is twofold. First, the marginal social damage of the pollution from consuming one unit of the dirty commodity, α, is constant across skill types. Second, differentiated commodity taxes do not help to raise social welfare by indirectly reducing the distortions of the labor income tax.

The term $\frac{\partial \ln(u_q/u_c)}{\partial \ln l_n}$ can be rewritten to relate the modified Pigouvian tax to empirically observable elasticities of consumer behavior:

$$-\frac{\partial \ln\left(u_q/u_c\right)}{\partial \ln l_n} = \omega_l \left(\rho_{ql} - \rho_{cl}\right), \tag{40}$$

where $\omega_l \equiv \frac{-u_l l_n}{u} > 0$ is the utility share of labor supply and $\rho_{ql} \equiv \frac{u_{ql} u}{u_q u_l}$ and $\rho_{cl} \equiv \frac{u_{cl} u}{u_c u_l}$ are Hicks' partial elasticities of complementarity between labor and dirty or clean goods in the utility function. Alternatively, one can write

$$-\frac{\partial \ln\left(u_q/u_c\right)}{\partial \ln l_n} = -\left(\theta_{ql} - \theta_{cl}\right), \tag{41}$$

where $\theta_{ql} \equiv \frac{\partial u_q}{\partial l_n}\frac{l_n}{u_q}$ and $\theta_{cl} \equiv \frac{\partial u_c}{\partial l_n}\frac{l_n}{u_c}$ denote the elasticities of the marginal utility of dirty and clean goods with respect to labor supply. Empirical estimation of the parameters of the utility function therefore makes it possible to test whether environmental taxes should be set above or below the Pigouvian level.

5.3 Modified Samuelson rule environmental quality

Proposition 11 *The modified Samuelson rule for environmental quality is given by*

$$\int_{\mathcal{N}} \left(\frac{N\frac{u_E}{u_c} - \frac{\mu}{\eta}}{N\frac{u_E}{u_c}}\right) \mathrm{d}F(n) = -\int_{\mathcal{N}} \frac{u_c \theta_n/\eta}{n f(n)} \left(\frac{\partial \ln\left(u_E/u_c\right)}{\partial \ln l_n}\right) \mathrm{d}F(n). \tag{42}$$

Proof. The first-order condition for optimal environmental quality E is:

$$\frac{\partial \mathcal{L}}{\partial E} = \int_{\mathcal{N}} \left[\left(-\frac{\mu}{N} - \eta \left.\frac{\mathrm{d}c_n}{\mathrm{d}E}\right|_{\bar{l},\bar{q},\bar{u}}\right) f(n) + \frac{\theta_n l_n}{n}\left(u_{lE} + u_{lc} \left.\frac{\mathrm{d}c_n}{\mathrm{d}E}\right|_{\bar{l},\bar{q},\bar{u}}\right)\right] \mathrm{d}n = 0. \tag{43}$$

Totally differentiating utility at constant labor, dirty goods consumption and utility, yields $\left.\frac{\mathrm{d}c_n}{\mathrm{d}E}\right|_{\bar{l},\bar{q},\bar{u}} = -\frac{u_E}{u_c}$. Substitution in the first-order condition yields the desired result. ■

$\int_{\mathcal{N}} \left(\frac{N\frac{u_E}{u_c} - \frac{\mu}{\eta}}{N\frac{u_E}{u_c}}\right) \mathrm{d}F(n)$ can be interpreted as the 'virtual tax' on the provision of environmen-

[18] Browning and Meghir (1991) and Crawford et al. (2010) empirically reject weak separability for the UK. Some externality-generating goods are found to be leisure complements (domestic fuels), while others are leisure substitutes (motor fuels). Pirttilä and Suoniemi (2010) find that a joint expenditure category of housing and energy consumption is a leisure complement in Finland.

tal quality relative to the first-best policy rule. That is, environmental quality is overprovided compared to the first-best rule, $N\int_{\mathcal{N}}\frac{u_E}{u_c}\mathrm{d}F(n) < \frac{\mu}{\eta}$, if the willingness to pay for a cleaner environment rises with labor supply: $\frac{\partial \ln(u_E/u_c)}{\partial \ln l_n} > 0$. Similarly, environmental quality is underprovided compared to the first-best rule, $N\int_{\mathcal{N}}\frac{u_E}{u_c}\mathrm{d}F(n) > \frac{\mu}{\eta}$, if the individual's marginal willingness to pay for a cleaner environment falls with his/her labor effort, i.e., $\frac{\partial \ln(u_E/u_c)}{\partial \ln l_n} < 0$. Intuitively, in the second-best with labor market distortions, the government employs environmental quality to indirectly alleviate these distortions by underproviding leisure complements or overproviding leisure substitutes – relative to the first-best policy rule. Again, a higher (lower) environmental quality can help to relax the incentive constraints, if a higher (lower) environmental quality boosts labor supply. In that case, individuals with a higher ability are less tempted to mimic those with lower ability by boosting their labor supply.

Note that the ordinary Samuelson rule for the optimal provision of environmental quality results if the marginal willingness to pay for the environment does not depend on labor supply, i.e., $\frac{\partial \ln(u_E/u_c)}{\partial \ln l_n} = 0$:

$$N\int_{\mathcal{N}}\frac{u_E}{u_c}\mathrm{d}F(n) = \frac{\mu}{\eta}. \tag{44}$$

That is, the sum of the marginal rates of substitution of better environmental quality should be equal to the marginal rate of transformation of better environmental quality. Hence, with the relatively weak assumption that consumption and environmental quality are weakly separable from leisure, the optimal second-best policy rule for environmental quality is identical to the optimal first-best policy rule. The intuition is again that the provision of environmental quality neither is useful to redistribute more income nor to alleviate income tax distortions on labor supply.

5.4 First-best Pigouvian tax and environmental quality in second-best

The findings of this paper reveal that if environmental taxes are set at the Pigouvian level, see (39), this does not automatically imply that the optimal level of environmental quality follows the standard first-best decision rule in (42). Similarly, if environmental quality follows the first-best Samuelson rule, as in (44), this does not automatically imply that the optimal environmental tax coincides with the first-best Pigouvian tax, see (37). Whether they coincide or not depends on the preference structure of individuals. In the second-best, indirect policies are employed if they help to reduce tax distortions on labor supply. However, it can be derived that the first-best rules for environmental taxes and environmental quality are applicable in second-best settings if the utility function is weakly separable between labor and *all* other commodities (including environmental quality).

Proposition 12 *If utility can be written as $u\left(h\left(c_n, q_n, E\right), l_n\right)$, the optimal modified Pigouvian tax equals the first-best Pigouvian tax, and environmental quality follows the first-best Samuelson rule:*

$$\tau' = \alpha\frac{\mu}{\eta} = \alpha N\int_{\mathcal{N}}\frac{u_E}{u_c}\mathrm{d}F(n). \tag{45}$$

Proof. If $u\left(h\left(c_n, q_n, E\right), l_n\right)$ it is immediately derived that $\frac{\partial \ln(u_E/u_c)}{\partial \ln l_n} = \frac{\partial \ln(u_q/u_c)}{\partial \ln l_n} = 0$. Substitution in (37) and (42) yields the result. ■

6 Policy implications

The results of this paper have important policy implications. In particular, if the marginal cost of public funds is equal to one, public resources are equally as scarce as private resources are. Accordingly, the modified Pigouvian tax and the optimal provision of environmental quality should not depend on the marginal cost of public funds in the second-best. Therefore, environmental taxes should mainly be levied for their impact on environmental welfare.[19]

Another implication is that, in the optimal tax system, the revenue-raising capacity of environmental policy instruments does *not* make them superior compared to revenue-neutral instruments (such as regulation) or revenue-reducing instruments (such as subsidies on non-polluting commodities). Intuitively, the optimal second-best allocation is independent of the way in which this allocation is implemented. Price instruments, such as environmental taxes, can sustain the second-best allocation equally as well as quantity controls, regulation or auctions. The reason is that the fundamental informational constraint – ability is private information – cannot be relaxed by simply adopting a different policy instrument. Only if one policy instrument has additional costs or benefits over another instrument, can one discriminate between the desirability of various instruments. Thus, one needs to include additional constraints in the analysis in order to assess the desirability of various environmental policy instruments.[20]

As a corollary, the findings of this paper also cast a new light on the 'double-dividend' hypothesis pioneered by Oates (1991) and Pearce (1991). These authors argued that environmental taxes could both improve the environment and allow for a reduction in distortionary labor taxes. Subsequently, a large literature emerged exploring the conditions under which such a double dividend would indeed be feasible.[21] However, if the marginal cost of public funds equals one in the optimal tax system, then it is irrelevant whether environmental taxes raise revenue to cut labor taxes. Hence, the issue of the double dividend of environmental taxes has become moot.

If markets or governments fail, it is no longer guaranteed that the marginal cost of funds is one. The extensive environmental tax literature has demonstrated that if governments fail to set optimal taxes (e.g. by letting fixed factors go untaxed or by setting sub-optimal non-environmental taxes/subsidies), it is no longer clear in which direction optimal second-best environmental taxes should be modified compared to the first-best Pigouvian tax, see, for example, Bovenberg and van der Ploeg (1994a), de Mooij and Bovenberg (1998), and Ligthart and van der Ploeg (1999). The literature also provides examples in which markets fail (e.g. due to involuntary unemployment), see also Bovenberg and van der Ploeg (1996), Koskela and Schöb (1999) and Holmlund and Kolm (2000). Also in this case, the marginal cost of public funds differs from unity, and the implications for how the modified Pigouvian tax should be set are ambiguous. See also Goulder (1995), Fullerton and Metcalf (1998), Bovenberg (1999), Sandmo (2000), Goulder

[19] Naturally, under non-separable preferences, modified Pigouvian taxes and the modified Samuelson rule for environmental quality should take into account the relative complementarity of dirty consumption goods and environmental quality with leisure.

[20] Fullerton and Metcalf (2001) demonstrate that environmental policies can create scarcity rents. The government can skim off these rents without distortions by using either revenue-raising instruments (e.g. a tax or auctioning permits) or a profit tax. As long as a pure profit tax is available, instrument equivalence is obtained. In our model, environmental policy does not give rise to scarcity rents.

[21] See for surveys of this literature, e.g., Goulder (1995), Fullerton and Metcalf (1998), Bovenberg (1999), Sandmo (2000), Goulder and Bovenberg (2002), and Fullerton et al. (2010)

and Bovenberg (2002) and Fullerton et al. (2010) for extensive discussions of these arguments.

In order to analyze the impact of government or market failures on optimal environmental taxes, one should explicitly model these government and market failures from first principles, and not impose them in an ad hoc fashion. This paper reveals that restricting the initial tax code, for reasons that do not originate endogenously from technological or informational constraints, can have profound effects on optimal policy rules in the second-best. This should be true in more general settings as well. Once government and market failures are explicitly modeled, the marginal cost of public funds should probably not differ from one at an optimal tax system, since otherwise the tax system would not have been optimal.

7 Conclusions

This paper shows that it is erroneous to adjust the optimal second-best policy rule for internalizing environmental externalities for the marginal cost of public funds. The optimal second-best environmental tax does generally depend on the complementarity of labor with dirty commodities and with environmental quality. When linear taxes are employed, second-best pollution taxes also depend on the distributional impact of the dirt tax. Under non-linear commodity and income taxation, however, the optimal second-best commodity tax is independent of distributional concerns. If utility is weakly separable between consumption and labor, and the government can levy non-linear income and environmental taxes, then the second-best environmental tax is equal to the Pigouvian tax. When linear instruments are considered, sub-utility over dirty consumption goods and leisure needs to be homothetic, and income effects need to be absent in individual choices in order to obtain the optimal Pigouvian tax in the second-best.

Appendix

Second-order incentive compatibility

This appendix derives the conditions under which the first-order approach yields an allocation that respects second-order conditions for an optimal allocation, see also Fudenberg and Tirole (1991). Define the net utility $\Omega(m,n)$ of an n-type mimicking an m-type as $\Omega(m,n) \equiv U(c_m, q_m, z_m, E, n) - U(c_n, q_n, z_n, E, n)$. Under the direct mechanism, each individual reports an ability type to the government. Each report m is associated with a commodity bundle $\{c_m, q_m, z_m\}$. The optimal utility-maximizing report m by agent n follows from the first-order condition $\frac{\mathrm{d}\Omega(m,n)}{\mathrm{d}m} = U_c \frac{\mathrm{d}c_m}{\mathrm{d}m} + U_q \frac{\mathrm{d}q_m}{\mathrm{d}m} + U_z \frac{\mathrm{d}z_m}{\mathrm{d}m} = 0$. The second-order condition for this utility maximization problem is $\frac{\partial^2\Omega(m,n)}{\partial m^2} \leq 0$. This second-order condition can be simplified in a number of steps. Totally differentiating the first-order condition yields $\frac{\partial^2\Omega(m,n)}{\partial m^2} + \frac{\partial^2\Omega(m,n)}{\partial m \partial n} = 0$. Differentiating the first-order condition with respect to n gives $\frac{\partial^2\Omega(m,n)}{\partial m \partial n} = \frac{\partial U_c}{\partial n}\frac{\mathrm{d}c_m}{\mathrm{d}m} + \frac{\partial U_q}{\partial n}\frac{\mathrm{d}q_m}{\mathrm{d}m} + \frac{\partial U_z}{\partial n}\frac{\mathrm{d}z_m}{\mathrm{d}m}$. The second-order condition for incentive compatibility – evaluated at $n = m$ – thus requires $\frac{\partial U_c}{\partial n}\frac{\mathrm{d}c_n}{\mathrm{d}n} + \frac{\partial U_q}{\partial n}\frac{\mathrm{d}q_n}{\mathrm{d}n} + \frac{\partial U_z}{\partial n}\frac{\mathrm{d}z_n}{\mathrm{d}n} \geq 0$. Substitute the first-order condition $\frac{\mathrm{d}c_n}{\mathrm{d}n} = -\frac{U_q}{U_c}\frac{\mathrm{d}q_n}{\mathrm{d}n} - \frac{U_z}{U_c}\frac{\mathrm{d}z_n}{\mathrm{d}n}$ to find $\left(\frac{\partial U_q}{\partial n} - \frac{U_q}{U_c}\frac{\partial U_c}{\partial n}\right)\frac{\mathrm{d}q_n}{\mathrm{d}n} + \left(\frac{\partial U_z}{\partial n} - \frac{U_z}{U_c}\frac{\partial U_c}{\partial n}\right)\frac{\mathrm{d}z_n}{\mathrm{d}n} \geq 0$. Differentiate $\frac{U_q}{U_c}$ and $\frac{U_z}{U_c}$ to obtain $\frac{\mathrm{d}\left(\frac{U_q}{U_c}\right)}{\mathrm{d}n} = \frac{1}{U_c}\left(\frac{\partial U_q}{\partial n} - \frac{U_q}{U_c}\frac{\partial U_c}{\partial n}\right)$ and $\frac{\mathrm{d}\left(\frac{U_z}{U_c}\right)}{\mathrm{d}n} = \frac{1}{U_c}\left(\frac{\partial U_z}{\partial n} - \frac{U_z}{U_c}\frac{\partial U_c}{\partial n}\right)$. Substitute these results to find the final condition: $\frac{\mathrm{d}\left(\frac{U_q}{U_c}\right)}{\mathrm{d}n}\frac{\mathrm{d}q_n}{\mathrm{d}n} - \frac{\mathrm{d}\left(\frac{-U_z}{U_c}\right)}{\mathrm{d}n}\frac{\mathrm{d}z_n}{\mathrm{d}n} \geq 0$. Sufficient conditions for second-order incentive compatibility are: $\frac{\mathrm{d}(U_q/U_c)}{\mathrm{d}n} > 0$, $\frac{\mathrm{d}(-U_z/U_c)}{\mathrm{d}n} < 0$, $\frac{\mathrm{d}q_n}{\mathrm{d}n} \geq 0$, and $\frac{\mathrm{d}z_n}{\mathrm{d}n} \geq 0$. Thus, in the optimal mechanism both z_n and q_n should be non-decreasing in n, and two Spence-Mirrlees conditions on the marginal rates of substitution for goods and for labor supply are required.

Compensated elasticities

This appendix formally derives the compensated elasticities under alternative specifications for the utility function. It demonstrates that the compensated cross-elasticities of the utility function are generally ambiguous.

This appendix starts from a general specification of utility. For notational simplicity, the ability-indices n are omitted and environmental quality is suppressed in the utility function as it remains constant for the individual:

$$u(c,q,v), \quad u_c, u_q, u_v > 0, \quad u_{cc}, u_{qq}, u_{vv} < 0. \tag{46}$$

The time endowment equals one so that $l \equiv 1 - v$ denotes labor supply. For later reference, $\mu \equiv l/v$ is defined as the labor-leisure ratio. The individual budget constraint is given by:

$$c + (1+\tau)q = (1-t)n(1-v). \tag{47}$$

Maximization of utility gives the following first-order conditions (FOC's): $\frac{u_v}{u_c} = (1-t)n$ and $\frac{u_q}{u_c} = 1+\tau$.

The model is log-linearized around an initial equilibrium. A tilde ($\tilde{}$) designates a relative change, for example, $\tilde{c} \equiv \mathrm{d}c/c$, except for the relative changes in taxes that are defined as: $\tilde{\tau} \equiv \frac{\mathrm{d}\tau}{1+\tau}$ and $\tilde{t} \equiv \frac{\mathrm{d}t}{1-t}$. To find the compensated elasticities, the utility function is log-linearized at a fixed level of utility ($\tilde{u} = 0$). For the general form of the utility function, the linearized first-order conditions can be written as: $(\rho_{cv} + \rho_{cc})\,\omega_c\tilde{c} - (\rho_{vv} + \rho_{cv})\,\omega_v\tilde{v} + (\rho_{vq} - \rho_{cq})\,\omega_q\tilde{q} = -\tilde{t}$, and $(\rho_{cq} + \rho_{cc})\,\omega_c\tilde{c} + (\rho_{vq} - \rho_{cv})\,\omega_v\tilde{v} - (\rho_{qq} + \rho_{cq})\,\omega_q\tilde{q} = \tilde{\tau}$, where $\omega_c \equiv \frac{u_c c}{u}$, $\omega_q \equiv \frac{u_q q}{u}$ and $\omega_v \equiv \frac{u_v v}{u}$ are the utility shares of clean consumption, dirty consumption, and leisure, respectively. $\rho_{cc} \equiv -\frac{u_{cc}u}{u_c^2}$, $\rho_{qq} \equiv -\frac{u_{qq}u}{u_q^2}$, $\rho_{vv} \equiv -\frac{u_{vv}u}{u_v^2}$, $\rho_{cv} \equiv \frac{u_{cv}u}{u_c u_v}$, $\rho_{cq} \equiv \frac{u_{cq}u}{u_c u_q}$, and $\rho_{vq} \equiv \frac{u_{vq}u}{u_v u_q}$ designate Hick's partial elasticities of complementarity. Together with the linearized utility function $\omega_c\tilde{c} + \omega_q\tilde{q} + \omega_v\tilde{v} = 0$, these three equations can be solved in terms of the changes in the policy variables only (not shown). It is then established that the compensated 'own' price elasticities are always negative and the compensated cross-elasticities are ambiguous in sign. For three different CES nestings the remainder of this appendix explicitly derives the signs of the elasticities.

Nested CES I: $u\left(x(c,q),v\right)$

Assume that utility is a nested CES function according to $u\left(x(c,q),v\right)$, where $u\,(.)$ and $x\,(.)$ are linear homogeneous (and therefore homothetic) functions exhibiting a constant elasticity of substitution equal to ρ and θ, respectively. Log-linearize the utility function to keep utility fixed yields: $\omega_x\tilde{x} + \omega_v\tilde{v} = 0$, where $\omega_x \equiv \frac{u_x x}{u}$, and $\omega_v \equiv \frac{u_v v}{u}$. Log-linearize sub-utility to find: $\tilde{x} = \omega_{xc}\tilde{c} + \omega_{xq}\tilde{q}$, where $\omega_{xc} = \frac{x_c c}{x}$, and $\omega_{xq} = \frac{x_q q}{x}$. Linear homogeneity implies that

$\omega_{xc}+\omega_{xq}=1$ and $\omega_x+\omega_v=1$. Log-linearize the FOC's to find $\tilde{c}-\tilde{q}=\theta\tilde{\tau}$, $\frac{1}{\rho}\left(\tilde{x}-\tilde{v}\right)=-\tilde{t}+\tilde{x}_c$, and $\tilde{x}_c=\frac{x_{cc}c}{x_c}\tilde{c}+\frac{x_{cq}q}{x_c}\tilde{q}=\frac{\omega_{xq}}{\theta}\left(\tilde{q}-\tilde{c}\right)$, where $x_{cc}c=-x_{cq}q$ and $\theta^{-1}=\frac{x_{cq}x}{x_c x_q}$ are used. Hence, $\tilde{x}-\tilde{v}=-\rho\tilde{t}+\omega_{xq}\frac{\rho}{\theta}\left(\tilde{q}-\tilde{c}\right)$. Combine the expressions to get the compensated demand equations $\tilde{v}=\omega_x\omega_{xq}\rho\tilde{\tau}+\omega_x\rho\tilde{t}$, and $\tilde{q}=-\left(\omega_v\omega_{xq}\rho+\omega_{xc}\theta\right)\tilde{\tau}-\omega_v\rho\tilde{t}$. The compensated elasticities are therefore given by $\varepsilon_{q\tau}\equiv-\left(\rho\omega_v\omega_{xq}+\theta\omega_{xc}\right)<0$, $\varepsilon_{qt}\equiv-\rho\omega_v<0$, $\varepsilon_{l\tau}\equiv-\frac{1}{\mu}\omega_x\omega_{xq}\rho<0$, and $\varepsilon_{lt}\equiv-\frac{1}{\mu}\omega_x\rho<0$.

Nested CES II: $u\left(x(c,v),q\right)$

Assume that utility is a nested CES function according to $u\left(x(c,v),q\right)$, where $u\left(.\right)$ and $x\left(.\right)$ are linear homogeneous (and therefore homothetic) functions exhibiting a constant elasticity of substitution equal to ρ and θ, respectively. Log-linearize the utility function to keep utility fixed yields: $\omega_x\tilde{x}+\omega_q\tilde{q}=0$, where $\omega_x\equiv\frac{u_x x}{u}$, and $\omega_q\equiv\frac{u_q q}{u}$. Log-linearize sub-utility to find: $\tilde{x}=\omega_{xc}\tilde{c}+\omega_{xv}\tilde{v}$, where $\omega_{xc}=\frac{x_c c}{x}$, and $\omega_{xv}=\frac{x_v v}{x}$. Linear homogeneity implies that $\omega_{xc}+\omega_{xv}=1$ and $\omega_x+\omega_q=1$. Log-linearize the FOC's: $\tilde{x}-\tilde{q}=\rho\tilde{\tau}+\rho\tilde{x}_c$, $\tilde{c}-\tilde{v}=-\theta\tilde{t}$, $\tilde{x}_c=\frac{x_{cc}c}{x_c}\tilde{c}+\frac{x_{cv}v}{x_c}\tilde{v}=\frac{\omega_{xv}}{\theta}\left(\tilde{v}-\tilde{c}\right)$, where $x_{cc}c=-x_{cv}v$ and $\theta^{-1}=\frac{x_{cv}x}{x_c x_v}$ are used. Hence, $\tilde{x}-\tilde{q}=\rho\tilde{\tau}+\omega_{xv}\frac{\rho}{\theta}\left(\tilde{v}-\tilde{c}\right)$. Combine the expressions to get the demand equations: $\tilde{v}=\omega_q\rho\tilde{\tau}+\left(\omega_q\omega_{xv}\rho+\omega_{xc}\theta\right)\tilde{t}$, and $\tilde{q}=-\omega_x\rho\tilde{\tau}-\omega_x\omega_{xv}\rho\tilde{t}$. The compensated elasticities are therefore given by $\varepsilon_{q\tau}\equiv-\omega_x\rho<0$, $\varepsilon_{qt}\equiv-\omega_x\omega_{xv}\rho<0$, $\varepsilon_{l\tau}\equiv-\frac{1}{\mu}\omega_q\rho<0$, and $\varepsilon_{lt}\equiv-\frac{1}{\mu}\left(\omega_{xc}\theta+\omega_q\omega_{xv}\rho\right)<0$.

Nested CES III: $u\left(x(q,v),c\right)$

Assume that utility is a nested CES function according to $u\left(x(q,v),c\right)$, where $u\left(.\right)$ and $x\left(.\right)$ are linear homogeneous (and therefore homothetic) functions exhibiting a constant elasticity of substitution equal to ρ and θ, respectively. Log-linearize the utility function to keep utility fixed yields: $\omega_x\tilde{x}+\omega_c\tilde{c}=0$, where $\omega_x\equiv\frac{u_x x}{u}$, and $\omega_c\equiv\frac{u_c c}{u}$. Log-linearize sub-utility to find: $\tilde{x}=\omega_{xq}\tilde{q}+\omega_{xv}\tilde{v}$, where $\omega_{xq}=\frac{x_q q}{x}$, and $\omega_{xv}=\frac{x_v v}{x}$. Linear homogeneity implies that $\omega_{xq}+\omega_{xv}=1$ and $\omega_x+\omega_c=1$. Log-linearize the FOC's: $\tilde{c}-\tilde{x}=-\rho\tilde{x}_q+\rho\tilde{\tau}$, $\tilde{q}-\tilde{v}=-\theta\left(\tilde{t}+\tilde{\tau}\right)$, $\tilde{x}_q=\frac{x_{qq}q}{x_q}\tilde{q}+\frac{x_{qv}v}{x_q}\tilde{v}=\frac{\omega_{xv}}{\theta}\left(\tilde{v}-\tilde{q}\right)$, where $x_{qq}q=-x_{qv}v$ and $\theta^{-1}=\frac{x_{qv}x}{x_q x_v}$ are used. Hence, $\tilde{c}-\tilde{x}=\rho\tilde{\tau}+\omega_{xv}\frac{\rho}{\theta}\left(\tilde{q}-\tilde{v}\right)$. Combine the expressions to get the demand equations: $\tilde{v}=\omega_{xq}\left(\theta-\omega_c\rho\right)\tilde{\tau}+\left(\omega_c\omega_{xv}\rho+\omega_{xq}\theta\right)\tilde{t}$, and $\tilde{q}=-\left(\omega_{xv}\theta+\omega_c\omega_{xq}\rho\right)\tilde{\tau}+\omega_{xv}\left(\omega_c\rho-\theta\right)\tilde{t}$. The compensated elasticities are therefore given by $\varepsilon_{q\tau}\equiv-\left(\omega_{xv}\theta+\omega_c\omega_{xq}\rho\right)<0$, $\varepsilon_{qt}\equiv\omega_{xv}\left(\omega_c\rho-\theta\right)$, $\varepsilon_{l\tau}\equiv-\frac{1}{\mu}\omega_{xq}\left(\theta-\omega_c\rho\right)$, and $\varepsilon_{lt}\equiv-\frac{1}{\mu}\left(\omega_c\omega_{xv}\rho+\omega_{xq}\theta\right)<0$.

Cobb-Douglas

With a Cobb-Douglas utility function, all the substitution elasticities in the nested CES-structures equal one. Hence, they are all equivalent and $\alpha\tilde{c}+\beta\tilde{v}+\gamma\tilde{q}=0$, where $\alpha\equiv\frac{u_c c}{u}$, $\beta\equiv\frac{u_v v}{u}$, and $\gamma=\frac{u_q q}{u}$ denote the utility shares. The compensated elasticities are thus given by: $\varepsilon_{q\tau}\equiv-(1-\gamma)<0$, $\varepsilon_{qt}\equiv-\beta<0$, $\varepsilon_{l\tau}\equiv-\frac{1}{\mu}(1-\beta)<0$, and $\varepsilon_{lt}\equiv-\frac{1}{\mu}\gamma<0$.

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