The field strength correlator from QCD sum rules

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The gauge invariant two-point correlator for the gluon field strength tensor is analysed by means of the QCD sum rule method. To this end, we make use of a relation of this correlator to a two-point function for a quark–gluon hybrid in the limit of the quark mass going to infinity. From the sum rules a relation between the gluon correlation length and the gluon condensate is obtained.

1. Introduction

An important role in non-perturbative approaches to QCD is played by the gauge invariant non-local gluon field strength correlator

$$\begin{split} \mathcal{D}_{\mu\nu\rho\sigma}(z) &\equiv \langle 0| T\{g_s^2 F_{\mu\nu}^a(y) \mathcal{P} e^{gf^{abc}z^{\tau}} \int_0^1 dt A_{\tau}^c(x+tz) \\ &\times F_{\rho\sigma}^b(x)\} |0\rangle \,, \end{split} \tag{1}$$

where the field strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$, z = y - x and $\mathcal P$ denotes path ordering of the exponential. In general, the gauge invariant field strength correlator could be defined with an arbitrary gauge string connecting the end points x and y, but in this work we shall restrict ourselves to a straight line.

It is the basic ingredient in the stochastic model of the QCD vacuum [1,2] and in the description of high energy hadron-hadron scattering [3–6]. In the spectrum of heavy quark bound states it governs the effect of the gluon condensate on the level splittings [7–10] and it is useful for the determination of the spin dependent parts in the heavy quark potential [11,12].

The correlator can be related to a correlator of a colour singlet current composed of a (fictitious) infinitely heavy octet quark and the gluon field strength tensor. This fact has already been employed in ref. [13] in order to calculate the perturbative corrections by means of Heavy Quark Effective Theory (HQET). In this work we again use this relation to estimate the correlation length from QCD sum rules [14] using as ingredients the value of the gluon condensate and the results for the perturbative calculation.

2. The field strength correlator

Instead of dealing with the non-local string operator which makes a calculation rather tedious one can replace the string by local heavy quark fields and use the methods developed in HQET. To this aim, we introduce an infinitely heavy quark field in the octett representation, $h^a(x)$, which is constructed from the field $Q^a(x)$ analogous to HQET by

$$h^a(x) = \lim_{m_Q \to \infty} \frac{1}{2} \left(1 + \not v \right) e^{i m_Q v x} Q^a(x) \,, \tag{2} \label{eq:definition}$$

v is the four–velocity of the heavy quark. The propagator of the free heavy quark field in coordinate space is given by

$$S(z) = \langle 0|T\{h^{a}(y)\bar{h}^{b}(x)\}|0\rangle$$
$$= \delta^{ab}\frac{1}{v^{0}}\theta(z^{0})\delta\left(\mathbf{z} - \frac{z^{0}}{v^{0}}\mathbf{v}\right), \tag{3}$$

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where v^0 is the zero–component of the velocity. The δ –function constrains the heavy quark on a straight line. With the effective HQET action $S_{eff} = \int dx \ \bar{h} i v^{\mu} D_{\mu} h$, the following equation can be shown analytically [15]:

$$\langle 0|T\{h^{a}(y)\bar{h}^{b}(x)e^{iS_{eff}}\}|0\rangle$$

$$= S(z)\langle 0|\mathcal{P}e^{gf^{abc}z^{\tau}}\int_{0}^{1}dtA_{\tau}^{c}(x+tz)|0\rangle. \tag{4}$$

The physical picture of this result is a heavy quark moving from point x to y with a fourvelocity v, acquiring a phase proportional to the path-ordered exponential. The limit of $m_Q \to \infty$ is necessary in order to constrain the heavy quark on a straight line and in order to decouple the spin interactions which are suppressed by a power of $1/m_Q$ and can therefore be neglected. By introducing a new correlator $\tilde{\mathcal{D}}(z)$, we can now express our correlator (1) in terms of heavy quark fields

$$\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(z)
\equiv \langle 0|T\{g_s^2 F_{\mu\nu}^a(y) h^a(y) F_{\rho\sigma}^b(x) \bar{h}^b(x) e^{iS_{eff}}\}|0\rangle
= S(z) D_{\mu\nu\rho\sigma}(z),$$
(5)

which establishes the relation between the field strength correlator and HQET.

3. The sum rules

Our next aim is to evaluate this correlator in the framework of QCD sum rules [14] and in that way obtain information on the correlation length of the field strength correlator. We may view the composite operator $(g_s h^a F^a_{\mu\nu})(x)$ as an interpolating field of colourless quark gluon hybrids and evaluate $\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(z)$ by introducing these as intermediate states in the absorption part of $\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(z)$. The lowest lying state will govern the long–range behaviour and hence the inverse of its energy is the correlation length.

For the sum rule analysis it is preferable to work with the correlator in momentum space. Thus we define

$$\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(w) = i \int dz \, e^{iqz} \langle 0|T\{g_s^2 F_{\mu\nu}^a(y)h^a(y) \times F_{\rho\sigma}^b(x)\bar{h}^b(x)\}|0\rangle \,, \tag{6}$$

where w = vq is the residual heavy quark momentum.

For a sum rule analysis the states have to be classified according to different quantum numbers. The projections onto the two independent subspaces is done with

$$\widetilde{\mathcal{D}}^{-}(w) \equiv g^{\mu\rho}v^{\nu}v^{\sigma}\,\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(w)
\widetilde{\mathcal{D}}^{+}(w) \equiv (g^{\mu\rho}g^{\nu\sigma} - 2\,g^{\mu\rho}v^{\nu}v^{\sigma})\,\widetilde{\mathcal{D}}_{\mu\nu\rho\sigma}(w)\,,\,(7)$$

where $\widetilde{\mathcal{D}}^-$ contains a vector and $\widetilde{\mathcal{D}}^+$ an axialvector intermediate state.

We model the correlators by a contribution from the lowest lying resonance plus the perturbative continuum above a threshold s_0 . Inserting the matrix elements and performing the heavy quark phase space integrals one obtains

$$\widetilde{\mathcal{D}}^{\mp}(w) = \frac{\kappa^{\mp} |f^{\mp}|^2}{w - E^{\mp} + i\epsilon} + \int_{s_0^{\mp}}^{\infty} d\lambda \, \frac{\rho^{\mp}(\lambda)}{\lambda - w - i\epsilon} \,, \quad (8)$$

where E represents the energy of the glue around the heavy quark, f^{\mp} are the hadronic matrix elements and κ^{\mp} are constants. The spectral densities are defined by $\rho^{\mp}(\lambda) \equiv 1/\pi \operatorname{Im} \widetilde{\mathcal{D}}^{\mp}(\lambda + i\epsilon)$ and are known at the next-to-leading order [13].

After Fourier transformation to coordinate space the above representation reads:

$$\widetilde{\mathcal{D}}(z) = S(z) \left\{ -\kappa |f|^2 e^{-iE|z|} + \int_{s_0}^{\infty} d\lambda \, \rho(\lambda) \, e^{-i\lambda|z|} \right\}. \tag{9}$$

Since the heavy quark propagator factorises, we identify the expression inside the brackets with our original correlator $\mathcal{D}(z)$. The long-range behaviour will be dominated by the term containing

the exponential with the energy E. Therefore the correlator decays exponentially and the correlation length is given by 1/E.

Now we turn to the theoretical side of the sum rules which is obtained by calculating the correlator of eq. (6) in the framework of the operator product expansion [14,16].

The perturbative contributions in momentum space have the form

$$\widetilde{\mathcal{D}}_{PT}^{\mp}(w) = (-w)^3 a \left[p_{10}^{\mp} + p_{11}^{\mp} L + a \left(p_{20}^{\mp} + p_{21}^{\mp} L + p_{22}^{\mp} L^2 \right) \right], \quad (10)$$

where $a \equiv \alpha_s/\pi$, $L = \ln(-2w/\mu)$ and the coefficients p_{ij}^{\mp} can be found in ref. [17].

Essential for the sum rule analysis are the contributions coming from the condensates. In our case the dimension three condensate $\langle \bar{h}h \rangle$ vanishes since the quark mass is infinite. The lowest nonvanishing term is the gluon condensate of dimension four:

$$\widetilde{\mathcal{D}}_{FF}^{-}(w) = \frac{1}{2} \, \widetilde{\mathcal{D}}_{FF}^{+}(w) = -\frac{\pi^2}{w} \, \langle aFF \rangle \ . \tag{11}$$

The next condensate contribution would be of dimension six, but we shall neglect all higher condensate contributions in this work and restrict ourselves to the gluon condensate.

The correlators satisfy homogeneous renormalisation group equations. Thus we can improve the perturbative expressions by resumming the logarithmic contributions. Calculation of the first coefficients γ_1^{\mp} for the anomalous dimensions from eq. (10) leads to

$$\gamma_1^- = 0, \qquad \gamma_1^+ = 3.$$
 (12)

The correlator $\widetilde{\mathcal{D}}^-(w)$ which corresponds to the vector intermediate state does not depend on the renormalisation scale μ at this order.

In order to suppress contributions in the dispersion integral coming from higher exited states and from higher dimensional condensates, it is convenient to apply a Borel transformation \hat{B}_T with T being the Borel variable. After renormalisation group improvement and Borel transformation all the ingredients needed for a sum rule analysis are known. Explicit formulas for the expressions can be found in ref. [17]. Now we turn to the numerical analysis.

4. Numerical analysis

Let us denote by $\chi(T,s_0)$ the Borel transformed expression for the continuum part $\chi(T,s_0) \equiv \widehat{B}_T \int_{s_0}^{\infty} d\lambda \ \rho(\lambda)/(\lambda-\omega-i\epsilon)$. After equating the phenomenological and the theoretical part we end up with the sum rule

$$-\kappa^{\mp} |f^{\mp}|^{2} e^{-E^{\mp}/T}$$

$$= \widehat{\mathcal{D}}_{FF}^{\mp} + \widehat{\mathcal{D}}_{PT}^{\mp}(T) - \chi^{\mp}(T, s_{0}).$$
(13)

By taking the logarithmic derivative we get an equation for the energy E.

The analysis shows that the different sign of the perturbative and non–perturbative term in the 1⁻ state leads to a stabilisation for the energy sum rule, whereas the equal sign in the 1⁺ state destabilises.

As our input parameters for the case of three light quark flavours we use $\langle aFF \rangle = 0.024 \pm 0.012 \text{ GeV}^4$, $\Lambda_{3fl} = 0.323 \text{ GeV}$ and $\mu = 2 \text{ GeV}$. To estimate the errors we have varied the scale μ , the continuum threshold s_0 and the gluon condensate. In Fig. 1 we have displayed the energy E^- as a function of the Borel parameter T for different values of μ and s_0 . A good balance between stability for the energy and sensitivity for the resonance is found around $E^- = 1.5 \text{ GeV}$.

Including the errors we get for the energy and correlation length

$$E_{3fl}^- = 1.5 \pm 0.4 \ {\rm GeV}, \ a_{3fl}^- = 0.13^{+0.05}_{-0.02} \ {\rm fm}. \ \ (14)$$

In a world without light quarks, i.e. $n_f = 0$,

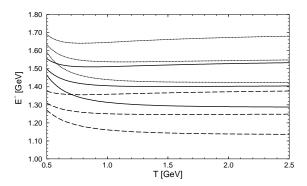


Figure 1. The energy as a function of the Borel-parameter T. Dashed curves $\mu=1\,\mathrm{GeV}$: lowest $s_0=1.3\,\mathrm{GeV}$, middle $s_0=1.5\,\mathrm{GeV}$, upper $s_0=1.7\,\mathrm{GeV}$. Solid curves $\mu=2\,\mathrm{GeV}$: lowest $s_0=1.5\,\mathrm{GeV}$, middle $s_0=1.7\,\mathrm{GeV}$, upper $s_0=1.9\,\mathrm{GeV}$. Dotted curves $\mu=4\,\mathrm{GeV}$: lowest $s_0=1.7\,\mathrm{GeV}$, middle $s_0=1.9\,\mathrm{GeV}$, upper $s_0=2.1\,\mathrm{GeV}$.

the main influence is the expected change of the gluon condensate which might increase by a factor two to three [23]. If we perform an analysis as above, we get for $\Lambda_{0fl}=0.250~{\rm GeV}, < aFF>=0.048\pm0.024~{\rm GeV}^4$ and $s_0=2.3~{\rm GeV}$ an energy and correlation length of

$$E^-_{0fl} = 1.9 \pm 0.5 \ {\rm GeV} \ , \ a^-_{0fl} = 0.11^{+0.04}_{-0.02} \ {\rm fm}. \ (15)$$

For E^+ , the energy of the axial vector, we obtain no stable sum rule. The expressions for E^- and E^+ are equal in lowest order perturbation theory, higher order perturbative contributions and the gluon condensate lead to a splitting in such a way that for the same values of s_0 and T the expression for E^- is higher than that for E^+ .

5. Summary and conclusions

The analysis of the gauge invariant gluon field strength correlator by QCD sum rules allows to establish a relation between the gluon condensate and the correlation length. In order to apply the sum rule technique which consists in the comparison of a phenomenological ansatz with a theoretical expression obtained from operator product expansion we interpret the gluon correlator as the correlator of two colour neutral hybrid states composed of a (fictitious) heavy quark transforming under the adjoint representation and a gluon field. The former serves as the source for the gauge string in the correlator.

The value of the lowest intermediate 1^- state (the inverse correlation length of the correlator) with three flavours can be determined to $E_{3fl}^- = 1/a_{3fl}^- \approx 1.5 \pm 0.4$ GeV and with zero flavours to $E_{0fl}^- = 1/a_{0fl}^- \approx 1.9 \pm 0.5$ GeV. The main sources of uncertainty are the choice of the continuum threshold s_0 and the value of the gluon condensate.

Though we find no stable sum rule for the axial vector state we have from the difference of the expressions for the 1⁻ and 1⁺ state strong evidence for the counterintuitive result that the 1⁺ state is lighter than the vector state.

The field strength correlator has been calculated on the lattice using the cooling technique [18,19] and field insertions into a Wilson loop [20]. For a discussion see ref. [17]. Recently in an analysis of heavy quarkonium in the framework of NRQCD [21] a splitting between the vector and the axialvector part has been observed in the same direction as proposed by the sum rules.

The sum rule analysis shows that the state investigated here namely a gluon confined by an octet source has a much higher energy than a corresponding state in HQET. A similar analysis of a light quark bound by a source in the fundamental representation [22] yielded an energy which is by a factor 2 to 4 smaller. This is to be expected from general grounds [23] since the case treated here is nearer to a glueball than to a heavy meson.

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