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PREFACE

Rudolf Halin: 1934–2014

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When I began to read mathematics at Hamburg as an undergraduate, I was able to study happily for three years without noticing the presence of Rudolf Halin. These were turbulent times, and Halin was keeping his head down. It troubled him deeply that students at that time seemed, occasionally, to be looking for a fight: while their plight always moved him, he found no way to communicate this in ways that the noisier ones amongst us, too, would hear. So few of us got to know him; and I was not one of these lucky ones.

After a few years at Hamburg, I went to Cambridge, initially for a year. My aim was to study graph theory, at last. When I sat in what must have been Andrew Thomason's first graduate course, I was not a little surprised to be literally bombarded with two names: Halin and Mader. Andrew was teaching from Béla Bollobás's *Extremal Graph Theory*, and giving us an animated account of these two young researchers' ping-pong of results from the 1960s, each improving on the previous, which started with Halin's seminal theorem that minimally k-connected graphs have a vertex of degree k [25].

Although Halin had early made a name for himself in structural finite graph theory, his heart was with the infinite. One of Wagner's top students, he started his mathematical life by expanding the tool of 'simplicial decompositions' of graphs, which Wagner had developed to prove his now famous Equivalence Theorem (that Hadwiger's conjecture for 5 is equivalent to the then 4-colour-conjecture), into a powerful theory for infinite graphs; see, e.g., [6,15,17,42,46,49]. Halin then applied this to obtain the first excluded minor theorems in infinite graph theory, to study Hadwiger's conjecture for infinite cardinals, and so on: results several of which have remained unsurpassed to this day.

Another of Halin's early papers that is still influential today is his 1973 study of the *Automorphisms and endomorphisms of locally finite graphs* [32]. When such a graph is connected, then every automorphism fixes either a finite subgraph or an *end*: a point at infinity in its Freudenthal compactification. This is a fundamental fact, for example, in the study of finitely generated groups via their Cayley graphs, on which the group elements themselves act as automorphisms.

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Halin's approach to ends differed radically from how they were previously conceived. I do not know whether Halin was aware of Freudenthal's earlier work on the ends of groups, and later of more general locally compact spaces; he was certainly aware of Carathéodory's *Primenden*. But while, for Carathéodory and Freudenthal, ends were something quite abstract, like boundary points in an inverse limit of compact spaces obtained by contracting the components of what is left after deleting a compact subspace, Halin exploited the discrete nature of graphs to define ends constructively: as equivalence classes of infinite paths that cannot be finitely separated [5].

Representing ends by concrete paths in this way has made them much more accessible to combinatorial analysis, a fact he used to great avail in his work, for example, on Menger-type connectivity between ends [33].

Halin worked in graph theory right from the start, an area which was just emerging at the time and barely recognised as mathematics. Yet he was already, and remained throughout his life, an archetypical mathematician. What spurred him on was the search for the essential. For example, he suggested [54] that the 'essence' of an infinite graph might be captured by declaring two graphs as equivalent if one could be obtained from the other in finitely many steps in such a way that of any two graphs adjacent in this finite sequence one was a contraction minor of the other, with finite branch sets. Are there 'typical' representatives of these equivalence classes that are easy to describe? Can we always make do with a sequence of length 2? Neither of these questions, to the best of my knowledge, has been answered.

Another example has been taken up more readily. Wagner's simplicial decompositions of a graph, obtained recursively by splitting it along complete separators, will always lead to parts that fit together in a tree-like way that describes a coarse overall structure of the original graph. The drawback is that few graphs have enough complete separators to make this worthwhile. So why insist? Separations along complete separators are nested, and *this* is what produces the tree-structure. In modern terminology, every collection of nested separations of a graph G defines a tree-decomposition of G, and these are eminently useful even when the separators are not complete. Halin recognised this, and proved the first known theorems about tree-width and graph minors [38] nearly ten years before Robertson and Seymour reinvented these notions, gave them their modern name, and made them famous.

As a final example for how his search for the essential inspired Halin's work let me mention his famous 'end-faithful spanning tree' problem. The simplest graphs that can have ends are trees, and so Halin asked whether every connected graph G had a spanning tree T whose end structure captured that of G in the sense that the natural map from the ends of T to those of G (inclusion, if we think of ends as sets of infinite paths) was bijective [5]. He proved this for countable G, and later for graphs without a topological $K_{\aleph_0}$ minor [42]. The general problem remained open for over 25 years until Thomassen, and independently Seymour and Thomas, constructed counterexamples in 1989.

However, even this story has a happy ending. Another 25 years later it was shown that arbitrary connected graphs have end-faithful spanning trees after all—as long as we seek to represent not all their 'Halin ends' but just those that are ends also in the sense of Freudenthal. Fittingly, this deep result was obtained in a Hamburg Ph.D. thesis (2015), by one of Halin's many mathematical grandchildren: Johannes Carmesin.

Rudolf Halin himself had twelve Ph.D. students, and he inspired many more. His search for the essential lives on. This volume, with contributions by many who admired Halin both for his work and as a person, bears witness to his legacy.

Reinhard Diestel, September 2016

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