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Preface

These notes had their origin in a course in invariant theory, given at the University of Utrecht in the autumn of 1975. The purpose of the course was to give an introduction to invariant theory on an elementary level, illustrated by some examples from 19^{th} century invariant theory. No completeness was striven at. From the brief review of the contents, given below, the informed reader will gather that these notes give a very incomplete picture of invariant theory. We have tried to compensate a little for this incompleteness, by mentioning additional results in the notes at the end of the chapters.

Chapter 1 introduces the basic notions and discusses some examples. Some elementary facts from algebraic geometry are introduced. In chapter 2 the finiteness theorem is proved, following Nagata, for reductive linear algebraic groups. The last notion is the same as what was formerly called geometrically reductive groups. This chapter also contains some more or less familiar results about Poincaré series of graded algebras, which essentially go back to Hilbert.

In chapter 3 we prove that the group $SL_2(k)$ (k algebraically closed) is reductive. This is done via the argument recently given by W.Haboush, to prove reductivity of semi-simple groups. Using full reducibility of rational representations of $SL_2(\mathbb{C})$ we then derive a number of classical results from the invariant theory of "binary forms", such as the formula of Cayley-Sylvester for dimensions of spaces of invariants, and Hilbert's asymptotic formula for such dimensions.

Chapter 4 is devoted to finite groups. We view Chevalley's theorem, which states that invariant algebras of finite reflection groups over $\mathbb{C}$ are free graded polynomial algebras, as one of the basic results in the invariant theory of finite groups. We show that it is useful also for obtaining descriptions of invariant algebras of non-reflection groups. The binary polyhedral groups are discussed, as well as some classical examples of 3-dimensional linear groups.

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References to the literature are given for each chapter separately. Formulas are numbered consecutively in each chapter.

CONTENTS